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MEASUREMENT THEORETIC SEMANTICS AND THE SEMANTICS OF NECESSITY

ABSTRACT. In the first two sections I present and motivate a formal semantics program that is modeled after the application of numbers in measurement (e.g., of length). Then, in the main part of the paper, I use the suggested framework to give an account of the semantics of necessity and possibility: (i) I show that the measurement theoretic framework is consistent with a robust (non-Quinean) view of modal logic, (ii) I give an account of the semantics of the modal notions within this framework, and (iii) I defend the suggested account against various objections.

1. MEASUREMENT THEORETIC SEMANTICS

Several views of linguistic meaning involve two kinds of holism: language (or theory) holism, and sentence holism. In order to avoid confusion I shall reserve here the term 'holism' for the first feature (language holism) alone. Roughly put, holism will be understood here as the view that what a natural language sentence means depends in an essential way on its place in a whole structure that many sentences of the language form together. Thus holism is not exhausted by the claim that the sentences of a natural language are semantically interconnected; this claim is trivially true. Rather, according to meaning holism many of the semantic interconnections that sentences have among them (typically inferential connections) are constitutive for their meaning what they do.[1]

What is often called sentence holism will be labeled here *sentence priority*; it is the view that the level of direct semantic connection between language and the non-linguistic world is that of the complete sentence, rather than its parts. Put a little differently, sentence priority is the view that the semantically primitive facts in virtue of which our utterances mean what they do have to do only with *complete* sentences, and not with sentence parts. (Semantically primitive facts are facts that are not accounted for in further semantic terms.) Quine's view of language, for example, satisfies sentence priority (as does Davidson's), while in Kripke's causal theory of reference, the reference relation between a name and its bearer

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is semantically primitive (although it is explained causally), and therefore this theory is inconsistent with sentence priority.[2]

Let us label the conjunction of holism and sentence priority HSP. The basic observation underlying Measurement Theoretic Semantics is this: according to a conception of meaning incorporating HSP a natural language resembles a *system of physical objects under measurement*. In order to see why, consider what happens in the measurement of length, for example. We start with a basic procedure of length comparison among the objects in a certain class: for each two objects we can check which is longer by putting them side by side. Due to this basic procedure the objects in the given class (together with their possible concatenations) form a structured relational system, called in measurement theory an *empirical relational structure*, satisfying various axioms. (For example, the relation 'longer than' is transitive and anti-symmetric, the order of concatenation of pairs of objects does not affect the overall length of the result, etc.) Now in virtue of the special properties of this empirical relational structure we can homomorphically embed it within the *real numbers*, i.e., map the structure into the numbers in a structure-preserving way; such a mapping results in our assigning numbers to objects as their length. The homomorphism between the empirical structure and the abstract mathematical one ensures that the representation of length through the assignment of numbers to objects does indeed fully capture all there is to capture in the physical, length-theoretic reality.[3]

Note the holistic aspect of measurement: according to the above analysis the measurement of some property of the objects in a given a class consists in finding a homomorphism between one whole – the relational structure that the objects make up together – and another, mathematical whole, i.e., the numbers.

Going back now to the linguistic case, we see that according to the above considered holistic views of meaning a given natural language can be construed semantically as an empirical relational structure, in the measurement theoretic sense. The constituents of the structure will be the sentences of the language, and the basic structure-imputing relation among them will typically be that of inference (construed in truth-functional terms, or otherwise). Inner sentence composition comes into play through its enabling us to keep track of inferential connections among sentences, and through its allowing us to systematically construct new sentences with inferential properties that suit our needs and purposes.

As in numeric measurement, we can attempt to represent such a semantic empirical structure by homomorphically embedding it into an adequate abstract mathematical structure. Obviously, structures that are adequate for the purpose at hand will not be numeric, but rather algebraic structures of other kinds. Each structure-preserving mapping from a given language to an abstract mathematical structure will result in an assignment of an element of the abstract structure to each sentence in the language (and possibly to sentence parts as well), and this in a way that represents the semantic properties of the sentence (or sentence part) in question within the system, exactly on a par with what happens in numeric length measurement.

In this way we get a new approach to formal semantics, one that is modeled after a standard application of mathematics to the non-mathematical world, viz. measurement. I call this approach *Measurement Theoretic Semantics* (MTS). In what follows I show how Measurement Theoretic Semantics can be implemented and formally developed (Section 2), and then I turn to apply it to the main subject matter of this paper – the semantics of necessity.

2. BOOLEAN ALGEBRAS AND CYLINDRIC ALGEBRAS IN MEASUREMENT THEORETIC SEMANTICS

2.1. *Boolean Algebras*[4]

According to the philosophical theses outlined in Section 1 the meaning of utterances of natural language expressions arises from the place of the expressions uttered in a system that is structured due to semantically primitive (e.g., inferential) data pertaining to complete sentences. Let us consider inferential data, and suppose they are given in terms of *truth* or *falsity* of sentence-utterances.[5] Furthermore, suppose that the class of utterances is of English sentences, and that we are interested only in structure applicable to this class of utterances due to appearances (in sentences) of the words 'and', 'or', and 'not' as sentential connectives. The 'raw' data (or the basic facts) in this case will consist in connections between appearances of the above words in sentences and truth/falsity of utterances of sentences. For example, we could find that whenever (an utterance of) the conjunction of two sentences is true, then each of the conjuncts is true (i.e., a concomitant utterance of each of the conjuncts would be true), that an utterance of 'p or q' (where p and q are any two sentences and the quotes are Quine's square quotes) is true if and only if a concomitant utterance of 'q or p' would also be true (i.e., that disjunction is commutative), and so forth.[6] We could go on and collect a large body of data of this kind.

Now this body of data about the given class of sentences (call it K) can be represented systematically through the use of mathematical means. The truth-functional structure that K has (only due to appearances of 'and', 'or' and 'not' in sentences[7]) can be captured by an application of an adequate algebraic structure to K.[8] An algebraic structure adequate for this purpose would be a Boolean algebra – an abstract algebraic structure (i.e., a set with operations defined on it) that has two binary operations (* and $+$), one unary operation ($\sim$), and two designated elements (0 and 1) such that for any x, y and z from the structure the following axioms hold:

(i)	$x + (y + z) = (x + y) + z.$	(ii)	$x^*(y^*z) = (x^*y)^*z.$
(iii)	$x + y = y + x.$	(iv)	$x^*y = y^*x.$
(v)	$x^*(y + z) = (x^*y) + (x^*z).$	(vi)	$x + (y^*z) = (x + y)^*(x + z).$
(vii)	$x + (x^*y) = x.$	(viii)	$x^*(x + y) = x.$
(ix)	$x + (\sim x) = 1.$	(x)	$x^*(\sim x) = 0.$

An application of such an algebra to K consists in *an assignment of an element of the Boolean algebra to each sentence in* K, an assignment that represents the data connecting truth/falsity of utterances with appearances of the logical connectives in sentences as sentential connectives.[9] For example, if an element x is assigned to sentence S_1 and an element y is assigned to sentence S_2, then the element x^*y should be assigned to the conjunction of S_1 and S_2 (i.e., to the sentence created from S_1, 'and' and S_2 in the standard way); any instance of a propositional logical truth would be assigned the Boolean 1 and any absurdity 0, and so forth.[10] (1 and 0 are the distinguished elements of a Boolean algebra, not integers.) In short, an adequate assignment of a Boolean algebra to K captures exactly all the data delineated in the previous paragraph (i.e., truth-functional data arising only from appearances of the three logical connectives in sentences[11] as sentential connectives).[12]

2.2. *Cylindric Algebras*

After we have considered algebraic means with which to capture what amounts to propositional truth-functional structure, the next step to take is to look for an algebraic representation of structure expressible in first-order logic. Such representation does indeed exist, in the form of cylindric algebras. Before these structures are defined, though, let me introduce a technical term that will be of use in presenting them. Let A be an algebra, and let X be a subset of the underlying set of A (i.e., X is a set of elements of A). The *closure* of X under the operations in A ($\text{Cl}(X)$ in symbols) is the set of all the elements in A you can get by repeated application of the operations of A to elements in X. X is then called *a set of generators* of $\text{Cl}(X)$; in particular, if $\text{Cl}(X) = A$ (i.e., if you can get every element in A

by repeated application of the operations of A to elements of X), then X is called a set of generators of A.

Let K be a class of sentences, and suppose that we want to represent truth-functional data about K. This time, though, we want to represent data that arise from sentence-structure that can be captured by first-order logic. For example, we want to capture such facts as these: if an utterance of "Everybody talks" is true, then a concomitant utterance of "All the men talk" would be also true; if an utterance of "There are big black swans" is true then "There are black swans" is also true, and so forth. In order to facilitate the presentation, let us behave as if the sentences in K are rewritten in first-order form (e.g., "Everybody talks" turns into "$(\forall x_1)$talks(x_1)", etc.). Another simplification is this: it will be assumed that the first-order rendition of sentences in K is purely relational, i.e., that it does not make any use of function symbols or constant symbols.[13]

As before, we want to capture the truth-functional connections among sentences through an assignment of the elements of an algebraic structure to them. What could a set of *generators* of such an algebraic structure be? Well, it seems that in order to capture the data in question this time, these generators will have to be assigned not to complete sentences but to sentence parts; i.e., to the predicate and relation symbols. (Recall that we do not have any function symbols or individual constants.) And what should the operations in this algebra be? Clearly the three Boolean operations will have to appear again, but on top of them we shall need operations corresponding to existential quantification (or universal quantification – each of the quantifiers can be used to express the other). In fact, we shall need an operation C_i for each variable x_i, because the contribution of '$(\exists x_i)$' to a sentence is different from that of '$(\exists x_j)$' for $i \neq j$.

In order to see better where we are heading, consider the analogy between what has been done in the previous subsection and what is suggested here. In Subsection 2.1 we considered sentences such as S_1: "Everybody talks and no one listens". The propositionally atomic sentences "Everybody talks" and "No one listens" were assigned elements of a Boolean algebra, say a_1 and a_2, and S_1 was assigned their Boolean conjunction, i.e., $a_1{}^*a_2$. The Boolean axioms ensured that this assignment will represent correctly the truth-functional relations among S, its two sub-sentences, and other sentences that might be constructed from these sub-sentences. Now in this subsection we do the same thing with sentences such as $S2$: "$(\exists x_1)$(talks$(x_1) \wedge$ listens(x_1))". "talks(x_1)" and "listens(x_1)" will be assigned elements of a new algebra B, say b_1 and b_2, "talks$(x_1) \wedge$ listens(x_1)" will be assigned their Boolean conjunction, i.e., $b_1{}^*b_2$, and the whole sentence S_2 will be assigned $C_1(b_1{}^*b_2)$, another element of the algebra, where C_1 is the operation in the algebra standing for existential quantification over x_1. As before, the axioms of the new algebra are supposed to ensure us that this assignment will represent correctly the truth-functional relations among S_2 and other sentences that might be constructed from the same (and other) building blocks. So the question, of course, is this: can such axioms be found?

Before this question is answered (positively) we need to overcome one more obstacle. The contribution of a predicate such as "talks( )" to the truth conditions of (first-order) sentences in which it appears depends on the variable you put inside the brackets: the sentences "$(\forall x_1)(\exists x_2)$ (talks $(x_2) \wedge$ listens (x_1))" and "$(\forall x_1)(\exists x_2)$(talks$(x_1) \wedge$ listens (x_2))" are different in meaning because the variables appear in different places. Hence we cannot just assign the same element of the algebra both to "talks(x_1)" and to "talks(x_2)". So must we use more than one element to stand for the predicate "talks( )", i.e., a distinct element b_i for each expression "talks(x_i)"? No. Luckily we have an alternative to this (intuitively unsatisfactory) solution. If we add to the algebra elements that stand for identity statements such as "$x_1 = x_2$", then we shall be able to restrict ourselves to sentences in which unary predicates always appear with the same variable, say 'x_1', binary predicates always appear with 'x_1' and 'x_2' (in this order), etc. In other words, it can be verified that every first-order formula is equivalent to a formula of first-order logic with identity in which "talks( )" and "listens( )" appear only as "talks(x_1)" and "listens(x_1)", "loves(,)" appears only as "loves(x_1, x_2)", etc.[14] If we use only formulas of this form (called *restricted* form), we shall retain the full expressive power of the language while assigning only one element of the algebra to each predicate or relation. (For example, we shall assign an element only to "talks(x_1)", because this is the only way in which "talks( )" will appear in sentences.) The trade off is this: for any natural numbers i and j there must be an element D_{ij} assigned to the formula '$x_i = x_j$'.[15]

We can now define Cylindric Algebras (CAs).[16] A CA is an algebraic structure such that:

(1) It has the following operations: two binary operations, * and +, a unary operation $\sim$, and a unary operation C_i for every natural number i.[17]

(2) It has the following designated elements (i.e., elements for which there are constants in the language of cylindric algebras): 0, 1, and D_{ij} for any natural numbers $i, j > 0$.

(3) It satisfies the following axioms:

(i)	$x + (y + z) = (x + y) + z.$	(ii)	$x^*(y^*z) = (x^*y)^*z.$
(iii)	$x + y = y + x.$	(iv)	$x^*y = y^*x.$
(v)	$x^*(y + z) = (x^*y) + (x^*z).$	(vi)	$x + (y^*z) = (x + y)^*(x + z).$
(vii)	$x + (x^*y) = x.$	(viii)	$x^*(x + y) = x.$
(ix)	$x + (\sim x) = 1.$	(x)	$x^*(\sim x) = 0.$

(xi) $C_i 0 = 0.$

(xii) $x + C_i x = C_i x.$

(xiii) $C_i(x^*C_i y) = C_i x^* C_i y.$

(xiv) $C_i C_j x = C_j C_i x.$

(xv) $D_{ii} = 1.$

(xvi) $D_{ij} = C_k(D_{ik}{}^* D_{kj})$ if $k \neq i, j.$

(xvii) $C_i(D_{ij}{}^*x)^*C_i(D_{ij}{}^* \sim x)$
$= 0$ if $i \neq j.$[18]

(Note that (xi) to (xvii) are *axiom schemes*, i.e., they apply for all C_i's and D_{ij}'s.)

Cylindric algebras are the solution to the problem posed two paragraphs above. That is, let K be a class of sentence-utterances in a natural language such as English, and suppose that the sentences in K are paraphrased into a relational first-order language L. In this case there is a Cylindric algebra B such that an appropriate assignment of the elements of B to sentences and sentence-parts of L fully represents the truth-functional relations that the sentences in K have among them due to their structure. The proof of this fact will not be presented here;[19] it should be noted, however, that the proof relies on the equivalence that obtains between the suggested semantics and standard Tarskian semantics. This equivalence yields the soundness and completeness of the standard proof systems for first order logic with respect to the assignment of elements of B to L-expressions.

As is made clear by the above remarks, the suggested semantics for propositional and first order logical structure is logically equivalent to the standard, classical ones. The measurement theoretic framework is of interest not because it yields a deviant notion of logical consequence. Rather, due to the more abstract conceptual and formal resources it offers this framework can be used to address major problems in the study of language, and this in ways that are beyond the scope of other semantic programs. I argue to this effect below, and elsewhere.[20]

3. MEASUREMENT THEORETIC SEMANTICS AND THE BOUNDS OF LOGIC

The two algebraic assignments considered in Section 2 represent the truth-functional structure that a class of English utterances K has due to (i) appearances of the logical connectives in sentences (as sentential connectives), and (ii) syntactic structure that can be paraphrased into relational 1st-order logic. These assignments capture (part of) what is often called the *logical* structure of sentences, or part of our knowledge of *logic*.

Consider now the following addition of structure to a cylindric algebra. We add three designated elements: B, U and M; furthermore, we add a new axiom: $B = U \wedge M$. The way a structure of this type applies to a class of sentence-utterances in English is this: B is assigned to the primitive predicate 'bachelor(x_1)', U is assigned to 'unmarried(x_1)', and M is assigned to 'man(x_1)'.[21] The new axiom captures the familiar piece of truth-functional data: bachelors are unmarried men.[22] This new piece of data is clearly 'legitimate', i.e., it is of exactly the same nature as the other data we have considered so far. Similarly, the way this piece of data is captured by the new structure is on a par with what has been done above. However, this new piece of data is usually viewed as having to do not with *logic*, but with the *meaning* of the English words 'bachelor', 'man', and 'unmarried'. Thus the new structure proposed above would be usually said to capture a piece of our knowledge of *meaning*, not of logic.

Next, consider a different expansion of cylindric algebras. This time the new designated elements are E and N, the new axiom is $E \wedge N = E$, and the truth-functional data that this axiom is designed to capture are that electrons are negatively charged. Again, this is a perfectly legitimate piece of truth-functional data, and it is captured in (what we have come to regard as) the standard way. But would capturing this information count as representing the *logical* structure of sentences, or the *meaning* of words that appear in them? No. An algebraic structure that captures this information would usually count as representing some knowledge that we have of the *world*.

We see that the measurement theoretic framework does not differentiate among the types of structure considered in the last three paragraphs: these structures capture data of the same kind in the same way. This is a formal expression of the following thesis:

(C) There are no meaning-theoretic grounds for distinguishing within our semantic knowledge between knowledge of logic, meaning, and world.[23]

As just shown, (C) arises in a natural way from HSP (the conjunction of holism and sentence priority), and it is of course closely related to Quine's rejection of the analytic-synthetic distinction:[24] what Quine rejects is the (empiricists') claim that there are meaning-theoretic grounds for distinguishing between our knowledge of linguistic convention and our knowledge of the world.

Thesis (C) is completely negative, in the following sense: it says only that certain distinctions cannot be grounded where it was hoped that they would be, not that these distinctions are groundless. Therefore (C) can be strengthened in several different ways. That is, when one shows that certain grounds do not yield a specific conclusion, one can still believe in this conclusion and try to argue for it differently, or one could reject the conclusion and argue against it. In the case at hand, Quine made several such choices. As regards the distinction between analytic and synthetic sentences, Quine leans the second way: he believes that the notion of analyticity cannot be given any precise content. (Quine allows that a graded and vague distinction might reflect our 'security' in sentences.) On the other hand, I claim that as regards the boundaries of logic Quine takes a different path. According to (C) there are no meaning-theoretic grounds for saying that one specific type of truth-functional structure should be called logical, and not another. This allows for an inclusive (or graded) view of what should count as logic and what should not. However, Quine does not espouse such an inclusive view. Contrary to his position on the analytic-synthetic issue, Quine sticks here to a clear-cut distinction: he often talks as if he is convinced that logic is *first-order predicate logic*, nothing more and nothing less.[25] Clearly HSP and (C) do not offer any grounds for this belief.

We see, then, that (C) is equivalent to a 'core view' of Quine's, and that Quine's two choices as regards how to strengthen this core view are independent of it. Notice now that both these choices motivate Quine's dismissal of *modal logic*.

First, (C) indeed rules out grounding the notion of *necessity* in linguistic convention: such grounding requires a robust distinction between knowledge of conventional meaning and knowledge of world, a distinction which (C) repudiates. Does it follow that the notion of necessity cannot be embedded in a coherent semantic theory for a natural language such as English? Of course not. However, Quine thinks that the notion of *analyticity* cannot be made clear and explicit, and he seems to hold (as least in earlier stages of his career) that the only way to make sense of necessity is to tie to analyticity.[26] Therefore his conclusion is that clear and precise semantic sense cannot be made of necessity. Thus Quine's first choice on how to strengthen (C) (i.e., his ruling out analyticity) supports his dismissive view of modal logic.

Quine's second way of strengthening (C), i.e., his decision to stick with a conservative view of logic, has the same effect. If first-order logic is viewed as the only formal tool to be applied to natural language, then it follows that there should not be admitted into formal semantics any new connectives in general, and a necessity connective in particular.

Thus Quine's two choices as regards how to strengthen (C) motivate his dismissal of modal logic. (This dismissal is subsequently supported by several arguments, the most significant of which is the (notorious) so called Slingshot Argument.) But since both of these two choices of Quine's are independent of HSP and (C) it is clear that his rejection of modal logic too is independent of these theses as well.

4. ALGEBRAIC SEMANTICS FOR NECESSITY

Our conclusion, then, is that HSP and (C) were not shown to preclude the integration of modal logic into a semantic theory of natural language. Indeed, we turn now to see that (i) the measurement theoretic framework all but forces upon us the means with which to achieve such an integration, and that (ii) the resulting formalization of modal semantics has significant merits that recommend it over other formal presentations.

Recall that in both Boolean algebras and cylindric algebras the Boolean 1 is a designated element. Furthermore, structures of the above two types were applied to classes of sentences in the following way: an assignment of 1 to a sentence *S* indicates that according to the data captured by the structure[27] every possible[28] utterance of *S* is true.[29] In other words: if a sentence *S* is assigned 1, then *S* is necessary.

Suppose now that in some of the sentences to which we assign the elements of an algebra *A* (either a Boolean algebra or a cylindric algebra) there appears a necessity connective. To make things easier, let us suppose (as we did in Section 2) that the utterances we are dealing with are paraphrased into a semi-formal language in which the symbol for necessity is a unary connective, say 'nec'. In order to capture the truth-functional relations that sentences have among them due to appearances of 'nec' we *expand A*, i.e., we add to it a new operator $\Box$ which is governed by this new axiom: for any element *p* of *A*, $\Box p = 1$ if $p = 1$; otherwise $\Box p = 0$.

Now the only thing that remains to be done is to say how the expanded structure (call it A') is applied to the class of sentences in question. This is easy to do. Elements of A' are applied to sentences (or sentence parts, in the case of cylindric algebras) exactly like the elements of *A*, except when these sentences have appearances of the necessity connective 'nec' in them. In cases of this second kind we add the following rule: if an expression *e* (a sentence or a sentence part) is assigned an element *p*, then the expression 'nec *e*'[30] will be assigned $\Box p$.

Remarks: 1. For Boolean algebras and a variety of their expansions (including cylindric algebras) the suggested formalization of necessity satisfies the axioms of S5. In the case of Boolean algebras this is an easy result of Stone's representation theorem,[31] and in the general case the claim follows from representation theorems by B. Jonsson and A. Tarski.[32] In a nut shell, the reason for which the suggested notion of necessity satisfies S5 is this: (i) The representation theorems referred to show that the algebraic structures in question can be represented set theoretically, (ii) the sets representing these algebras can be thought of as sets of *possible worlds*, on which there is defined a full (and thus trivial) *accessibility* relation, and therefore (iii) by Kripke's semantics for modality the necessity notion that is underlied by these sets of possible worlds satisfies S5. There are other operators (satisfying different axioms) definable in Boolean algebras and their expansions that correspond in a similar fashion to other modal inference schemes (e.g., T and S4).[33]

It could be justifiably asked now whether the suggested semantics for necessity is just the standard Kripke semantics in (algebraic) disguise. The answer to this question is negative, for reasons that are implicit in several of the following remarks, and that are stated explicitly in clause (c) of Remark 7.

2. The algebraic representation of the meaning of the connective 'nec' does not rely in any way on the notion of proof. In other words, the necessity of a sentence is not equated with its provability in a certain deductive system. This is easy to see from the way the suggested assignment was introduced above, and it is also indicated by the reasoning in the previous paragraph: in both cases the considerations involve a semantic notion (i.e., truth), and there is no appeal to any syntactic notion of derivability.

3. In the two types of algebraic structures we have considered so far (Boolean algebras and cylindric algebras) the sentences that were assigned 1 were logical truths: propositional logical truths were assigned 1 by adequate Boolean assignments, and logical truths of 1st-order logic were assigned 1 by cylindric assignments. Thus in these two cases a sentence 'nec *s*' could have true utterances only if '*s*' is a truth of logic. However, we employ in our speech notions of necessity other than the logical, and therefore it might be objected that the suggested formal representation of the semantics of necessity is too restricted, and hence inadequate.

However, this objection does not take into account the discussion in Section 3 above. We saw there that (i) according to HSP there are no clear-cut distinctions to be made among our knowledge of logic, meaning and world, and that (ii) we can construct increasingly richer algebraic structures that capture not only logical, but also (what are usually called) conceptual and factual truth-functional data. Now if we expand these hypothetical richer structures so that they represent also necessity (this can be done in exactly the same way as described above), then the resulting notions of necessity will not be called logical, but rather conceptual, metaphysical, or physical. Therefore the suggested formalism is not restricted to representing logical necessity, and hence the objection is unjustified.

Note that the formal account of the semantics of necessity suggested here does not give grounds for making distinctions (clear-cut or otherwise) among the different notions of necessity just mentioned, e.g., logical necessity, metaphysical necessity and physical necessity. What is presented here is an account of the semantics of necessity, not a conceptual analysis of our various notions of necessity. And indeed, this is as it should be: as Davidson says,[34] it is not the task of the semanticist to provide an analysis or explication of all the concepts in the language he deals with; otherwise semantics will have to be the science of everything.

It should not be held against the measurement theoretic account, then, that it does not offer much help in answering questions such as whether something is necessary, and if it is in what way (i.e., logically, or metaphysically, etc.). Similarly, it is indeed true that our intuitions about counterfactuals and necessity/possibility are unclear and messy, but this fact is not inconsistent with the simplicity and clarity of the account suggested here. Presumably we have a very flexible notion of necessity; that is, at different times and contexts we take into account different parts and reducts of the nebulous and ever-changing structure of our language and belief, and hence we end up with the shifty and vague intuitions about necessity that we have.[35]

4. It should be stressed again that the robust view of necessity suggested here is consistent with a rejection of the positivist view of analyticity. Here, again, is why.

Analytic sentences are supposed to be true or false due to linguistic convention; the truth (falsity) of other sentences depends also on how the world is. Now as we saw in the previous section, HSP does not offer any grounds for such a distinction among sentences. According to HSP an assignment of meaning to the sentences in a certain class K consists in an assignment of structure to K, an assignment that represents (truth-functional) data about complete sentences. Thus the structure assigned to (or manifested by) K represents the data relating sentences to the world, and therefore it is wrong to say that this structure (or some of it) is merely a matter of convention (i.e., that it is *independent* of the way language relates to the world). In other words, according to HSP a natural language *applies to the world* as a *structured whole*, and hence the structure of this whole cannot be dissociated from the way this whole is applied. To see this more clearly, consider the use of the real numbers in the measurement of length. The axioms that govern the real numbers are exactly what makes this algebraic structure suitable for measuring length. Therefore the axioms that govern the reals do not play merely a conventional role in length measurement. For example, the open equation '$x + y = y + x$' is an axiom of the real numbers, but it is not *a convention* of the way we measure length; rather, the axiom represents a structural feature of our conception of length. Similarly, the (so called) logical axioms governing language are not conventions; rather, they represent structural features of meaning.

However, all this has nothing to do with necessity as it is construed here, which has nothing to do with convention (or with any other meta-linguistic questions, for that matter). 'Nec' is just another connective in the language, and its meaning is represented like that of other connectives, i.e., through an assignment of the elements of an algebraic structure to expressions in which this connective appears.

The dispute over the role of convention in language is meta-structural; it has to do with the way we should construe (logical or other) structure 'from without', so to speak. The account of necessity given here, on the other hand, is 'from within': the axiom for necessity is *a part of* the structure of an algebra that is used to represent the contribution of 'nec' to sentences in which it appears; this axiom *does not* capture meta-theoretic observations *about* this structure (e.g., that it arises from convention).

The dissociation between necessity (as construed here) and analyticity might be challenged as follows. Suppose an utterance of 'nec s' is true (for some sentence s). This means that s is necessary, i.e., that any possible utterance of s is true. Therefore the truth of s does not depend on the way the world is: s is going to be true whatever the circumstances may be. Hence the source of s's truth has to be independent of the world, and the only plausible way this can be (say the positivists) is that the source of s's truth is linguistic convention.

However, this objection is misguided; it fails to take into account the holistic aspect of HSP. The fact that s is necessary arises from its place in a structure. (Put formally, 'nec s' is true because (in a loose sense of 'because') s is assigned the Boolean 1 by an adequate application of an algebraic structure to a class of sentences K.) This structure, in turn, applies *as a whole* to a class of sentences – not merely due to convention, but because of data (or facts) relating utterances of these sentences to each other and to the world. It is wrong, then, to consider a single sentence and the algebraic element assigned to it in isolation. Rather, a whole algebraic structure is assigned to a whole class of sentences, this global assignment is clearly not a matter of mere convention, and therefore each particular association of a sentence with an algebraic element cannot be only a matter of convention either.

5. There is close affinity between the treatment of necessity suggested here and Quine's notion of stimulus analyticity.[36] Quine calls a sentence 'stimulus analytic' if the sentence will be assented to by a speaker under any kind of stimulus. Similarly, we agreed above that an utterance of 'nec *s*' is true iff *s* is assigned the Boolean 1, i.e., iff all possible utterances of *s* are true. Now there are two apparent differences between Quine's notion and the one suggested here: (i) Quine talks of (stimulus) *analyticity*, while here we talk of *necessity*, and (ii) Quine makes no appeal to *truth* in his account of stimulus analyticity,[37] while here truth is appealed to. Let me address these two differences; my aim will be to show that neither is substantial.

As already said in Section 3, Quine views the notions of analyticity and necessity as closely related (maybe inter-definable) and similarly ill-founded.[38] Therefore Quine's choice of the term 'stimulus analyticity' (and not 'stimulus necessity') for the (allegedly new) notion that he introduces might seem arbitrary and innocent: having stripped the words of their old meaning it does not matter which of them you use. However, I claim that the situation here is not as simple and straightforward as that. Here is why. Quine himself admits (Quine 1960, 66) that his notion of 'stimulus analyticity' does bear close resemblance to (the ill-founded, by his lights) notion of necessity: a sentence is supposed to be necessary if it will be true 'come what may', while a sentence is stimulus analytic if it will be assented to 'come what stimulation may'(p. 66). Analyticity, on the other hand, seems to have much less to do with Quine's new notion: to say that a sentence will be assented to under any stimulation is a far cry from saying that its truth is due to convention, or due to conceptual analysis (two of the traditional conceptions of analyticity). I think it is clear, then, that from the two options that he had *Quine picked the one that makes less sense*; the term he chose for stimulus analyticity is such that the continuity with previous usage is ill-preserved, and he could have done better. (That is, Quine could have used 'stimulus necessity' for his new notion.) It is now natural to ask: why did Quine make this seemingly unreasonable choice? I think a probable answer to this question might be as follows. Quine wants to present himself as breaking away from a philosophical tradition that preceded him (and of course, he *does* break away from that tradition). Therefore he tries to make the distinction between his ‘strictly vegetarian’ (p. 67) notion of stimulus analyticity and older notions as clear as possible. For this reason Quine makes the (misleading) choice that he does: stimulus analyticity is indeed radically different from analyticity (as traditionally construed), and thus the new term serves Quine’s purpose well.

We see, then, that nothing should be read into the difference between Quine’s term (i.e., ‘stimulus *analyticity*’) and our talk here of necessity. ‘Stimulus necessity’ is probably the right name (or a better name, anyway) for Quine’s notion.[39]

Let us move on to the second difference mentioned above: Quine defines stimulus analyticity in proximal terms (i.e., in terms of assent to sentences in response to patterns of stimulation), while necessity is construed here in distal terms (i.e., in terms of objective truth-values of sentence-utterances). This does look like a real and significant difference between the two notions; indeed, Quine seems to rely on this difference when he calls stimulus analyticity a ‘strictly vegetarian’ notion (as opposed to the older and more full-blooded notions that he rejects). However, it turns out that recently Quine gave up on this point. In Quine (1996), Quine finds a way to reconcile his stimulus-based theory of meaning (and epistemology) with the obvious fact that translation is made “in terms of the external objects of reference, bypassing consideration of neural receptors” (p. 160). Quine says that he has long “recognized the adequacy of this object-oriented line in describing the procedures of translation, having described them thus [himself]” (p. 160), and he goes on to say that this externalist talk is consistent with his views because of the “preestablished harmony of standards of perceptual similarity” (p. 160) that is induced among the members of our species by natural selection. Now putting aside the intricacies of Quine’s argument, the upshot of his conclusion is clear: Quine is willing to give up talking about stimulation in favor of talking about *objectively holding circumstances*. Instead of associating with a sentence a pattern of stimulation (which is the sentence’s stimulus meaning) Quine allows us now to associate with the sentence a type of circumstances[40] where it is assented to, i.e., a set of circumstances where it is held true.[41] And if we apply this transformation to what interests us here, i.e., to stimulus analyticity, we get the following result: a sentence that is assented to under all patterns of stimulation is described now as a sentence that is assented to in all circumstances, i.e., a sentence that is held true ‘come what may’ (and not ‘come what stimulus may’). Thus due to Quine’s change (or restatement) of his position the second gap between stimulus analyticity (as it is called in *Word and Object*) and necessity (as introduced here) is closed. To put it in a nutshell: in view of the discussion in the last several paragraphs Quine's stimulus analyticity seems to deserve better the term 'objective necessity', and this is exactly the notion we have been considering here.[42]

6. In the previous remark it was shown that the view of necessity suggested here is in fact *close* to Quine's view of *stimulus analyticity*. In this remark, on the other hand, I want to stress the *difference* between the measurement theoretic view and Quine's long held position as regards the notion of *necessity* itself. As is well known, this position is two-pronged: Quine thinks that (i) it is not coherent to quantify into modal contexts, and that (ii) necessity should be viewed as a *predicate* applying (truly or falsely) only to complete sentences; it should not be viewed as a connective, in the mold of negation. Moreover, Quine claims that (iii) these two aspects (or parts) of his view are logically related. That is, he says that the only reason to construe necessity as a connective is to enable quantification into modal contexts, and that therefore if one accepts the claim that such quantification is incoherent then one has to agree with Quine that necessity is better construed as a predicate (applicable only to complete sentences) and not as a connective.[43]

I disagree with Quine on all of the claims (i) to (iii); however, I shall argue here only against the last two of them. For this purpose all that needs to be done is to justify the view of necessity as a connective, and this on grounds that are independent of the question whether 'quantifying in' is coherent or not. (Clearly this would refute both (ii) and (iii) above.)[44]

The required justification of the view of necessity as a connective is straightforward. Although the question whether 'nec' is a connective or a predicate is stated in syntactic terms, clearly it must be answered on semantic grounds. Now the suggested measurement theoretic framework was shown above to yield a formal account of the semantics of necessity, so this account should help us decide the question at hand. Looking at this account we see that the natural approach to necessity within the measurement theoretic framework is to treat it like a connective – i.e., to capture its semantic value through the use of an operator in the algebra – and not like a predicate. Therefore the semantics gives us grounds to hold that the logic of necessity is indeed that of a connective.

To see this, suppose an algebraic element p is assigned to an expression e. In order to get the algebraic element assigned to 'neg e' (negation of e) we apply the Boolean operation $\sim$ to p, and we get $\sim p$. Similarly, in order to get the algebraic element assigned to 'nec e' (necessity of e) we apply the operation $\Box$ to p, and we get $\Box p$. Things are exactly the same here. If we compare necessity with a predicate, on the other hand, we see a striking difference: a predicate is assigned an element of a cylindric algebra, while necessity ('nec') is associated with an operator (i.e., $\Box$). The roles played by an element of an algebra and by an operator in it are clearly different, and therefore the semantic behavior of necessity (as represented by the algebraic structure) is different from that of any predicate.

Of course, there are differences between necessity and negation. One of them is this: when applied to complete sentences negation is truth-functional at the utterance-level while necessity is not. That is, given only the truth value of an utterance of a sentence s we can predict the truth value of a hypothetical concomitant utterance of '$\sim s$' (according to the truth table for negation); however, we cannot do the same for a hypothetical utterance of 'nec s'. (The truth value of an utterance of 'nec s' depends on the truth/falsity of all possible utterances of s, not only on one of them.) However, this difference is of no significance to our discussion here. The truth-conditional behavior of 'nec s' (exactly like that of '$\sim s$') *can be derived from the truth-conditional behavior of s*, and this is all that is important: if we are able to assign truth values to utterances of s in a law-like, counterfactual-supportive fashion, then we know how to assign truth values to utterances of 'nec s' as well.[45] In this respect the semantic behavior of 'nec' is similar to that of '$\sim$' and the other (so called) logical connectives, and therefore the above mentioned difference between 'nec' and negation does not present any problem to the suggested semantics of 'nec' in general, and to the construal of 'nec' as a connective in particular.

In order to support the foregoing remarks let us consider again Quine's discussion of stimulus analyticity in *Word and Object*. (As shown in Remark 5, Quine's treatment of stimulus analyticity is analogous to the view of necessity suggested here.) Here is how Quine recapitulates what his imagined linguist has been able to come up with by the end of an early stage of radical translation (p. 68):

> We have had our linguist observing native utterances and their circumstances passively, to begin with, and then selectively querying native sentences for assent and dissent under varying circumstances. Let us sum up the possible yield of such methods. (1) Observation sentences can be translated. There is uncertainty, but the situation is the normal inductive one. (2) Truth-functions can be translated. (3) *Stimulus analytic sentences can be recognized. So can the sentences of the opposite type, the "stimulus-contradictory" sentences, which command irreversible dissent* [my italics].

We see that Quine's linguist can assign stimulus analyticity to sentences in the native's language. It follows that if (1) the native's language has a connective that expresses stimulus analyticity, call it 's.a.', and if (2) the linguist's language has such a connective too, call it 'S.A.', then (3) the linguist should have no trouble in translating the native's 's.a.' into his own

'S.A'. The linguist's reasoning will simply be as follows: utterances of 's.a. *s*' are true just in case s is stimulus analytic, i.e., just in case utterances of 'S.A. *S*' are true (where *S* is the translation of the native's *s*). If we now factor in the points made in Remark 5, i.e., that *stimulus analyticity* is *objective necessity* in disguise, we get the desired result; that is, we see that there is no problem in treating 'nec' as an intensional connective.

Moreover, the passage from *Word and Object* quoted above does not only show that treating 'nec' as an intensional connective is coherent (and consistent with HSP); rather, the passage also gives strong reasons for rejecting Quine's later suggestion that we view necessity as a *predicate*, applicable (truly or falsely) to sentences. Quine ascribes to his linguist the ability to translate the logical connectives and to identify the stimulus analytic sentences (and thus translate a necessity connective if there is one in the native's language) *at a very early stage of the translation process*. In particular, the linguist can handle the logical connectives and stimulus analyticity before he tries to break into the quantificational structure of the native's language, i.e., before he starts translating predicates (e.g., verbs and adjectives), names, and quantificational constructions (such 'every man', 'some students', etc.). It follows that according to the above quotation from Quine, *stimulus analyticity cannot be viewed as expressed within the language by a predicate applying to sentences*: a stimulus-analyticity expression within the native's language (if there is one) can be translated before any subject-predicate structure has been assigned to the language, and therefore the logic of this notion is not of the subject-predicate form. And if we appeal again to the direct analogy between stimulus analyticity and objective necessity (as explained in Remark 5), we get the same result for the latter notion: the logic of necessity is not of the subject-predicate form, because if it were then a necessity connective would not have been amenable to radical translation at the stage that it actually is (according to Quine), i.e., together with the truth-functional sentential connectives. Notice the difference between the modal notion discussed here and, e.g., temporal locutions: a sentence of the form 'always *s*' *does* involve quantification over times (or events), and thus it is of a different logical structure than 'nec *s*' (which has just been shown not to involve quantification).

This very point is captured quite clearly by the algebraic formalism that is being used here. It is easy to see that the axiom for $\Box$ (i.e., the algebraic operation that captures the significance of 'nec') makes use only of the Boolean 0 and 1; the axiom does not appeal to any further structure that appears only in cylindric algebras and not in Boolean algebras. This observation amounts to what was said in the previous paragraph: 'nec' can be assigned meaning independently of quantificational structure (i.e., without such structure being introduced), and therefore it is misguided to ascribe to necessity the logic of quantification (i.e., to view 'nec' as a predicate symbol).[46]

Let me sum up this remark. The algebraic formalization of the semantics of necessity provides (or uncovers) good reasons (i) in favor of viewing 'nec' as a connective, and (ii) against viewing it as a predicate. These same reasons arise also from Quine's own account of stimulus analyticity, a notion that was shown above to be equivalent to objective necessity (modulo some changes undergone by Quine's position). Thus it turns out that Quine's remarks on stimulus analyticity can be used to undermine his position as regards *necessity*, i.e., that necessity should be viewed as a predicate applicable to complete sentences.[47]

7. Having considered in detail Quine's Scylla we turn now to David Lewis. The well known alternative to Quine's view of necessity is Lewis's so called modal realism, so after having contrasted Quine's position with the measurement theoretic approach to necessity let us see how this approach compares with possible worlds semantics.

(a) In Remark 6 above it was claimed that the logic of necessity does not involve quantification/predication, and this claim was used against Quine's view that necessity should be understood as a predicate (of sentences). The same claim can be used now against Lewis's reduction of necessity to quantificational structure, i.e., his view that sentences of the form 'nec *s*' have the logical form of universal generalizations (every possible world is such that ..., etc.). The argument will be the same: according to the semantics suggested above, 'nec' can be assigned significance already in a Boolean algebra, before quantificational structure has been brought into the picture, and therefore it is wrong to attribute to (at least some notions of) necessity the logic of full-fledged objectification/predication. Some of the standard objections against the analysis of necessity/possibility in terms of quantification over possible worlds can be easily seen to be concrete derivatives of this general point.[48]

Thus the same feature of the measurement theoretic account can be used against both Quine's and Lewis's views. Of course, this is no coincidence. Both Quine and Lewis subscribe to the same reductionist dogma, i.e., to the view that an acceptable logical analysis of natural language has to be given in terms of 1st-order logic (or at least predicate logic).[49] HSP frees us from this dogma (because it allows for the application of a wide verity of types of structures to natural language), and so it is no surprise that the formal semantics program arising from it conflicts with the views of Quine and Lewis on the very issue that it does.

(b) In Lewis (1979) Lewis writes as follows (p. 183):

If our modal idioms are not quantifiers over possible worlds, then what else are they? (1) We might take them as unanalyzed primitives; this is not an alternative theory at all, but an abstinence from theorizing. (2) We might take them as mentalistic predicates analyzable in terms of consistency: '*Possibly* Φ' means that Φ is a consistent sentence. But what is consistency? If a consistent sentence is one that could be true, or one that is not necessarily false, then the theory is circular; of course, one can be more artful than I have been in hiding the circularity. If a consistent sentence is one whose denial is not a theorem of some specified deduction system, then the theory is incorrect rather than circular: no falsehood of arithmetic is possibly true, but for any deductive system you care to specify either there are falsehoods among its theorems or there is some falsehood of arithmetic whose denial is not among its theorems.

Now in clause (a) above it was shown that the algebraic formalization of some modal idioms rules out treating these idioms as quantifiers (over possible worlds or any other entities). Hence we should be able to place the approach suggested by the algebraic treatment in the array of views that Lewis considers as alternatives to his own. Working our way backwards by elimination we first rule out the second option in Lewis's 2nd branch (the branch that deals with explaining possibility in terms of consistency): as shown in Remark 2 above, the measurement theoretic approach to necessity does not appeal to any notion of proof whatsoever. We can rule out the first option in the 2nd branch as well; the reason is that the measurement theoretic treatment does not "take them [i.e., the modal notions] as mentalistic predicates analyzable in terms of consistency", nor does the algebraic approach appeal to any circular argumentation involving necessity and consistency.

The measurement theoretic approach to necessity must fall under the first heading, then; this is the only one that is left. That is, according to Lewis this approach "take[s] them [the modal notions] as unanalyzed primitives", and therefore it "is not an alternative theory at all, but an abstinence from theorizing". Is this characterization fair? Only partly so. The semantics for necessity suggested here does indeed treat necessity as an irreducible notion, but this feature does not render it a non-theory.

Necessity is indeed irreducible in the measurement theoretic account. The axiom for $\Box$ (the operator that is used to capture the semantics of 'nec') is specified in terms of a pre-existing structure (a Boolean algebra, or a cylindric algebra, or a richer structure), *but this does not mean that* $\Box$ *is definable in terms of pre-existing designated elements and operations*. $\Box$ is similar to other operations and designated elements that are introduced when an algebra is expanded. For example, clearly the structure of cylindric algebras goes beyond what is definable in Boolean terms; otherwise there would be no point in introducing the new cylindric primitives. So the first part of the claim made in the previous paragraph is correct: the algebraic formalization of the semantics of necessity treats this notion as primitive.

However, this feature of the suggested approach does not render it useless. The algebraic formalization captures formally our intuitions about necessity and the contribution of 'nec' to the truth-conditions of sentences. The formalism shows how to assign algebraic elements to expressions in which the connective 'nec' appears, and this assignment represents data (given in terms of truth) about utterances of complete sentences.[50] According to HSP (which we grant for the argument) this is all that should be required from a semantic theory, so why is the algebraic treatment inadequate? What is the difference between the algebraic treatment of necessity and the algebraic treatment of negation that makes the first defective and the second acceptable? There is no such difference; the accounts are analogous. Now Lewis does not suggest that negation should somehow be reduced to 'negationless' quantification logic, so it is not clear why he insists on such a reduction in the case of necessity. As said in remark (a), the reason might be that Lewis gives 1st-order logic a special status: he thinks that a reduction of our modal notions to 1st-order logic consists in a substantial theory, while an account that goes beyond 1st-order logic and makes use of a formalism that is tailor-made for modality is viewed by him as "an abstinence from theorizing". Lewis's implicit assignment of special status to 1st-order logic seems to me deserving to be called dogmatic, and I think it is unjustified.

The modal notions belong to the list of distinguished concepts that are the most fundamental in our conceptual system. We should not expect to be able to reduce these concepts to more primitive ones, because *they are* the most primitive.[51] Rather, what we should try to achieve is an explication of the interrelations among these basic concepts, and the algebraic formalization of the semantics of necessity given above consists in a step towards this goal.

(c) It cannot be denied that talk about possible worlds (reborn formally in Kripke's work) is very useful and successful in capturing and conveying our intuitions about modality. Therefore it is required from any account that rejects Lewis's possible worlds realism to explain why this realism is so successful and useful. This will be done here only in outline.

There are (at least) two kinds of representations in which abstract algebraic structures can be involved. One is the representation of some structure in the non-mathematical world through the assignment the elements of an abstract algebraic structure to objects, as is done in measurement. The other kind is the representation *of the abstract structure* by a more concrete and familiar mathematical entity. Now if these two kinds of representation obtain for the same algebraic structure, then they can be composed and yield a third one; that is, if the concrete mathematical entity E represents the abstract structure A, and if A represents the structure of a class of non-mathematical objects K,[52] then E can be used directly to represent the structure in K *without any appeal to* A. Such a direct use of E arises from what may be called *transitivity of representation*.

In the case at hand (i.e., in the meaning-theoretic case) we have the following instantiation of this general phenomenon. (i) According to the foregoing discussion Boolean algebras and (some of) their expansions can be used to represent the structure that systems of sentences have due to primitive semantic data (or facts). Suppose that an algebra A represents in this way the structure of a system of sentences K. (ii) According to the theorems cited in footnotes 31–32 Boolean Algebras and (some of) their expansions can be represented set theoretically. That is, for the algebra A from (i) there can be found a set U and a mapping from the elements of A to subsets of U *that preserves all of A's structure*. In such a mapping the Boolean 1 of A will be mapped to the whole of U, the Boolean 0 to the empty set, the Boolean operations of conjunction, disjunction and negation will be mapped to the set-theoretic intersection, union and complementation to U; the other operations on A (if there are any) will be mapped to well-defined operations on the subsets of U. (iii) As said in the previous paragraph, we can now skip the mediation of A. That is, we can represent the structure of K by an assignment of the subsets of U to sentences: U represents A (in the purely mathematical fashion), so it can serve to represent directly the structure in K as well, without any appeal to A whatsoever.

If we assign the subsets of a set U directly to sentences, *then we can think of the elements of U as possible worlds*. For each sentence s in K the subset of U assigned to it will be the set of possible worlds in which it true, the logical truths (captured by the original structure A) are assigned the whole of U (i.e., all possible worlds, as it should be), and so forth. Now suppose A includes the operator $\Box$ (representing necessity); by hypothesis U represents correctly A, so ipso facto *U construed as a set of possible worlds* captures correctly the semantics of necessity. Thus we get the answer to the question we started with: we explain, on the basis of HSP and measurement theoretic semantics, why Kripke's possible worlds semantics works.

Moreover, we have here also the means through which to explain why possible worlds semantics for at least some of the modalities is inadequate in the way it was shown above to be (i.e., in that it imposes quantificational structure on modal logic without justification). Transitivity of representation has the following problem. The concrete mathematical object E through which we represent the abstract structure A could be 'too rich'; that is, it could have properties that go beyond what is required to represent A. If we use now E directly to represent the structure of (the non-mathematical) K, then it would seem to be called for that we apply to K also the extra structure in E, that goes beyond what there is in A. However, this could be a mistake: the extra structure in E need not be applicable to K at all, and 'forcing' this structure on K might lead to a distorted view of the data on the basis of which structure was assigned to K in the first place.

The application of this generalized discussion to the case at hand is clear. The set-theoretic representation of Boolean algebras (and their expansions) is richer than the original abstract algebraic structures. For example, cardinality considerations definitely do apply to sets, but it is meaningless to apply them to the elements of a mere Boolean algebra: these elements are *not* sets, and therefore you cannot ask how large (cardinality-wise) these elements are. Now when we apply the set-theoretic representations of the algebraic structures directly to sentences, we are tempted to look for the extra structure of these representations where it is not to be found. For example, we take seriously the objectification that is implicit in the set-theoretic representation of Boolean algebras, and we apply this objectification to our talk about modality; that is, we postulate possible worlds as objects. However, such postulation is without justification: as is shown by the pure algebraic representation of the semantics of necessity, talk of objects in the precise articulation of such semantics is neither required nor called for.

I claim to have supplied in Remarks 5–7 sufficient grounds for the account presented in this paper to be preferred over its to quantificational alternatives: Quine's on the one hand and Lewis's on the other. Previous remarks recommend other aspects of the suggested account and the measurement theoretic framework that gives rise to it. I therefore contend that this framework and its products should be given their place within the array of formal and conceptual tools that are to be appealed to in the research of meaning.

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NOTES

[1] In Dresner (1998, Ch. 1), I present this definition of meaning holism in detail, and discuss the way it relates to other definitions, such as Dummett's and Fodor and Lepore's.

[2] Note that sentence priority is perfectly consistent with another well known thesis of Kripke's concerning reference, namely the thesis that names are rigid designators. Also, sentence priority is consistent with (at least some versions of) compositionality.

[3] For more details, see, e.g., Krantz et al. (1971, 1–35).

[4] For a more detailed account of the measurement theoretic approach of Boolean algebras and natural language, see Dresner (1999).

[5] If the semantic data are to be empirically observable, then they should comprise of facts about language speakers *holding* utterances to be true or false, not facts about utterances *being* true or false; indeed, this is how Davidson views the data available to his radical interpreter. Alternatively, one can go along with D. Lewis and view the data that the formal model must account for in metaphysical terms – as a body of facts about the language in question – and think of the way we can come to know these facts as irrelevant to semantics; in this case the data *could* be made up of truth values of utterances. In order to make the following examples simple I shall ignore this issue, which is not essential to the presentation at hand, and talk of utterances simply as being true or false. This is not to say, though, that I endorse the 'metaphysical' point of view.

[6] In order to simplify things I assume here (counterfactually) that the English connectives behave like pure logical connectives.

[7] As already said above, we consider here appearances of these words only as sentential logical connectives. This remark applies throughout the remainder of this subsection.

[8] In fact, the class of sentences which is assigned structure is larger than (i.e., contains as a proper subset) K, the class of sentences for which there are data; in other words, the structure is projected from those sentences about which there is data to infinitely many others. This projection is made on the basis of inner sentence structure, as follows: (i) the structured system of sentences is such that the inner composition of a sentence determines its place (i.e., its properties) within the system, (ii) the inner composition of sentences is projectable (i.e., new sentences can be created in a systematic way from parts that appear in old sentences, and through the use of rules according to which these old sentences are built up from their parts), and therefore (iii) data (or facts) about a limited number of sentences can give rise to a structured-system that includes many (possibly infinitely many) other sentences. This remark applies throughout this section.

[9] In order to simplify this example, I assume that K is made up of sentences without any element of indexicality.

[10] A complete specification of the conditions that an adequate assignment must satisfy is easy to produce and I omit it here. The only interesting point about it is that the elements assigned to the propositionally atomic sentences have to be independent from each other. (For a detailed explanation of this last remark see Bell and Demopoulos (1996) – what is said there about propositions applies also to the algebraic elements that are assigned here to sentences.)

[11] Material implication can be defined in terms of these connectives.

[12] Notice that sentences are *not* assigned truth values here. Rather, sentences are assigned mathematical entities that represent information about truth-functional relations among their utterances.

[13] I indicate how this simplified example can be generalized in Dresner (1998, 106–107).

14 For example: "$(\exists x_1)(\exists x_2)(\text{loves}(x_1, x_2)\&\text{loves}(x_2, x_1))$" can be rewritten as

"$(\exists x_3)(\exists x_4)((\exists x_1)(\exists x_2)(\text{loves}(x_1, x_2)\&x_1 = x_3\&x_2 = x_4)\&(\exists x_1)(\exists x_2)(\text{loves}(x_1, x_2)\&x_2 = x_3\&x_1 = x_4))$".

That every first-order formula is equivalent to a formula in this form follows from an observation made by Tarski (1986, 253–271).

15 For a discussion of the applicability of the restricted form to natural language semantics, see Dresner (forthcoming2).

16 Cylindric algebras were first defined by Tarski. The *locus classicus* is Leon Henkin, Donald Monk and Alfred Tarski (1971) and (1984).

17 In order to avoid technical complication the class of algebraic structures defined here is that of a particular kind of Cas, called CA_ω's. In the standard, more general definition the set of indices of the C_i's (and also the D_{ij}'s) need not be the natural numbers, but could rather be any ordinal number.

18 The cylindric axioms schemas were designed to be minimal in number, and are therefore somewhat removed from intuitive axiomatization of first order logic.

19 Such a proof can be found, e.g., in Henkin (1973, 115, proposition 1).

20 Dresner (1998).

21 As in subsection 2.2, it will be supposed here that the English utterances are paraphrased into relational 1st-order logic.

22 As in Section 2, algebra-elements are assigned to sentence parts (unary predicates in this case) so as to capture truth-functional data concerning sentences in which these sentence-parts appear.

23 Notice that (C) is consistent with our having also non-semantic (e.g., pragmatic) knowledge of language, as well as non-linguistic knowledge of the world. The thesis is concerned only with what lies within the domain of semantics, i.e., literal meaning.

24 As presented, e.g., in Quine (1953a).

25 A good example can be found in Quine (1970, 72) (where Quine says that the boundary between logic and set theory is '*not* [my italics] a vague one'), and p. 79 (where he explains why logic just is 1st-order logic).

26 See, for example, Quine (1943, 120–121).

27 Structures of different type capture different ranges of data; see Section 2.

28 As is the case with all systems of measurement, the regularities that they capture are counterfactual supportive (or law-like), and can therefore be used to talk about what could happen.

29 Similarly, sentences that are assigned the Boolean 0 are necessarily false (i.e., all their utterances are expected to be false).

30 The quotation marks should be understood as Quine's square-quotes, i.e., the expression designated by 'nec *s*' is the result of concatenating 'nec' to the expression *s*. There are similar cases below.

31 See, for example, Monk and Bonnet (1989, 31–34).

32 Jonsson and Tarski (1951).

33 See, for example, Hughes and Cresswell (1968, 311–330).

34 E.g., in Davidson (1984b, 32).

35 Of course, this observation is old news; see, for example, Kaplan (1979).

36 In fact, this concept is actually due to Davidson. (See Quine (1960, footnote 4 on p. 66).)

[37] That is, for Quine, truth is not a basic notion that should be used in the theory of meaning, but rather a notion that should be explained *away* in such a theory. See, for example, Quine (1970, 10–14).

[38] Again, this might apply only to Quine's earlier writings.

[39] Remarks to this effect were made by Martin Davies in Rudolf Fara (1994).

[40] My use here of 'circumstances' is loose; I am not committed to the existence of any kind of entities (e.g., facts, or situations) that are circumstances.

[41] Notice, though, that in Quine's theory stimulus meaning is assigned only to *some* sentences, i.e., to so called observation sentences.

[42] It might be objected that one further difference between Quine's stimulus analyticity and objective necessity has not been acknowledged: sentences that are necessary but very complex might not elicit affirmation from language speakers on every occasion, and thus they would not count as stimulus analytic.

My answer to this objection is as follows. Quine must concede (and at least in his later writings *has* conceded) that a sentence can be observational even if it is very complex and is actually usually assented to on the basis of knowledge of theory, not straightforward conditioning. For example, in Quine (1990) he says (p. 5): 'Observation sentences as I have defined them far exceed the primitive ones that are the child's entering wedged. Many of them are learned not by simple conditioning or imitation, but by *subsequent construction from sophisticated vocabulary* [my italics]'. Now what Quine says here applies also to stimulus analyticity: a sentence can be stimulus analytic even if the knowledge that it should be assented to 'come what stimulus may' is based on acquaintance with sophisticated vocabulary. Thus stimulus analytic sentences *are* similar to necessary sentences after all: some of them can be very complex, and therefore might not elicit affirmation from language speakers on every occasion, without reflection.

[43] In Quine (1976a).

[44] It is of interest to note here that many philosophers who disagree with Quine on (i) and (ii) are willing to grant him (iii). That is, these philosophers agree with Quine that the whole dispute over modal semantics starts and ends with 'quantification in'. My view, on the other hand, is that there are other considerations that should be brought into the discussion.

[45] Utterances of 'nec *s*' are true only if all possible utterances of *s* are true; otherwise utterances of 'nec *s*' are false.

[46] This claim clearly goes also against Lewis's reduction of modal logic to quantification over possible worlds. See Remark 7 below.

[47] Let me acknowledge again that I have not tackled the main issue under dispute between Quine and his opponents, viz. whether quantification into modal contexts is coherent. As already said above, this is partially because I reject the view (shared by both Quine and his adversaries) that the 'quantifying in' question is the only one that should be considered when one decides whether to view 'nec' as a connective or as a predicate. However, it should be noticed that if a cylindric algebra is expanded with the operator $\Box$, then this operator can be applied to all the elements of the algebra, including those that are assigned to incomplete expressions (e.g., the elements assigned to the primitive predicate symbols in the language). This seems to indicate that the algebraic formalism is consistent with quantification into modal contexts, but I do not want to commit myself to this view here.

[48] An example is the objection that possible world semantics has to count as meaningful such seemingly meaningless question as whether a certain statement is true in *exactly two* possible worlds.

[49] Quine's commitment to 1st-order logic was already discussed in Section 3; Lewis's implicit in remarks such as the one quoted in (b) below.
[50] The data is projected by the structure from uttered sentences to sentences utterances of which have not been made. Also, the structure is used for counterfactual-supportive predictions.
[51] See Davidson (1996) for a recent statement of this basic 'meta-philosophical' point. Davidson applies this point to the discussion of truth, but it seems to me that the discussion of the modal notions could benefit too from taking the same point into account.
[52] *K* has its structure due to the application of some primitive predicate or relation to the objects in it, as elaborated in Section 1.

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