# Anamorphic Visualization of the Space-Time-Paradigm by Geodetic Network Adjustment

Tamara Feicht[1] · Andreas Schmitt[1]

## Abstract

The steadily increasing urbanization results in a continuous areal expansion of urban agglomerations and generates the need for effective intra- and inter-urban mobility. In fact, travel time plays the key role for daily commuters, not necessarily the distance between residence and work. Thus, the space-time-ratio is a quality measure for the respective traffic network. This study explores the spatio-temporal distance ratio between 97 German cities along the current road and railroad infrastructure by the help of geodetic network adjustment methods. The method bases on a highly dynamic network of 9312 connections and estimates new locations for the cities with respect to their geographic location and the required travel time. Cartographic design methods are applied to generate a non-central spatio-temporal cartogram. The resulting anamorphic map clearly denotes urban agglomerations with outstanding transport infrastructure, but also cities that are not adequately integrated in the national mobility network. This approach provides insight information with a high impact on future transport infrastructure planning.

## Anamorphe Visualisierung eines Raum-Zeit-Paradigmas durch geodätische Netzausgleichung

### Zusammenfassung

Die stetig ansteigende Urbanisierung trägt eine kontinuierliche räumliche Ausdehnung der städtischen Ballungsräume zur Folge und schafft den Bedarf an effektiver inner- und zwischenstädtischer Mobilität. Für den täglichen Pendler spielt dabei die effektive Reisezeit eine erheblichere Rolle als die reine Entfernung zwischen dem Wohnort und dem Arbeitsplatz. Dadurch wird das Raum-Zeit-Verhältnis ein Qualitätsmaß für das jeweilige Verkehrsnetz. In dieser Studie wird das raum-zeitliche Entfernungsverhältnis zwischen 97 deutschen Städten entlang der aktuellen Straßen- und Schieneninfrastruktur mit Hilfe von geodätischen Netzausgleichsverfahren untersucht. Die Methode basiert auf einem hochdynamischen Netz von 9312 Verbindungen und schätzt neue Standorte für die untersuchten Städte in Bezug auf ihre geographische Lage und die benötigte Reisezeit. Für die Erstellung eines dezentralen raum-zeitliches Kartogramm kommen kartographische Gestaltungsmethoden zum Einsatz. Die resultierende anamorphe Karte zeigt deutlich die strukturstarken und schwachen Gebiete von Deutschland sowie wichtige Verbindungen zwischen den Städten. Dieser Ansatz liefert Informationen, welche für die künftige Verkehrsinfrastrukturplanung von großer Bedeutung sind.

✉ Tamara Feicht
Tamarafeicht@icloud.com

Andreas Schmitt
Andreas.Schmitt@hm.edu

1    Department of Geoinformatics, Munich University
of Applied Sciences, Karlstraße 6, 80333 Munich, Germany

## 1 Introduction

Travel time cartograms (TTC) are nothing new. But, today's standard approaches still show some deficiencies regarding the non-central evaluation of travel time and thus, leave room for further enhancements.

## 1.1 Travel Time Cartograms

The deviations from an assumed proportional relationship between distance and time can be visualized in various TTCs. Hereby, two different types can be distinguished: The central and the non-central TTC. In the central variant, a fixed location in the map is defined. From this root node, the travel time is calculated to all other points and the temporal distances can be represented by isochrones. It thus resembles a tree graph. Inter-connections between leaf nodes are omitted. The visualization of this tree graph knows two types: first, nodes fixed at their geographic location with flexible isochrones (Büttner et al. 2010) and second, equidistant isochrones with shifted location s of the nodes (Wang et al. 2018). The central TTC is most commonly used due to its simple generation and interpretation.

The non-central TTC typically illustrates the travel time between neighboring nodes in the mobility network, i.e., city-to-city connections which results in a graph where only edges are considered regardless of paths. As every node is equivalent, the visualization of the travel times becomes much more complex (Bies and van Kreveld 2012; Buchin et al. 2014). This problem is usually solved by so-called anamorphic maps. Similar to central TTCs, two different visualization strategies are followed in literature: first, fixed geographic locations of the nodes with graphic representation of travel time by the edges Buchin et al. (2014) and second, constant travel time scale and therewith, shifted locations of the nodes in a distance map Carosio et al. (2005).

In summary, it is indispensable so far to take the decision whether to illustrate the exact geographic location of the cities or the exact travel time in-between. If we choose the location, the travel time can only be indicated by isochrones or by the style of the edges. If we choose exact travel time, the locations are shifted by the travel time and the resulting map may be beyond all recognition. Aiming at the simultaneous visualization of time and space in one cartogram, we briefly review the approaches for non-central TTCs with shifted locations published so far to develop an enhanced combined approach afterwards.

## 1.2 Travel Time Dependent Locations

In previous studies, three methods have been applied to compute new travel time dependent locations in a non-central TTC: Multidimensional scalation (MDS), stepwise multidimensional scalation (SMDS), and moving or minimum least squares estimation (MLS).

MDS is based on the similarity or dissimilarity between objects. These have fixed locations within a feature space. The closer two objects are to each other in the feature space, the more similar they are and vice versa (Backhaus et al.

2018). Including the travel time in the feature space leads to new locations of the cities (Spiekermann and Wegener 1994). Two major problems occur: The method locates calibration nodes based on travel time only and does not consider the topological characteristics of the map, i.e., edges between adjacent nodes might be broken. So it is possible, that some nodes switch their relative location. The second problem is caused by the chosen interpolation method that applies a weighted average of offsets from local calibration nodes. The author states certain, but inevitable discontinuities in the resulting transformation (Spiekermann and Wegener 1994). So it is possible, that continuous polylines like coastlines, roads, or river courses show several jumps in the transformed map. Both issues impede the interpretation of TTCs generated by MDS. To avoid the above-mentioned problems, the method was extended to SMDS, which performs the method step by step. Thus, the extent of the spatio-temporal network considers the geographic locations of the cities. The stepwise scalation prevents topological errors in the transformed result. Doing so, polylines keep their continuous course, because the interpolation parameters are restricted to the respective neighborhood (Spiekermann and Wegenerhood 1994).

Obviously, the complexity and therewith, the number of inconsistencies increases with the number of nodes and edges taken into account. At this point, the MLS approach that is explicitly designed to find the optimal solution of an overdetermined equation system comes into play. The new locations of the cities are treated as parameters to be estimated from the measured travel time. This method aims at minimizing the total square error by adjusting the locations (Navratil and Staudinger 2006). MLS has already been used in several research papers dealing with TTC (e.g., (Miura and Tajima 2019; Ullah and Kraak 2015; Wang et al. 2018)). Though MLS is able to find the optimal solution for a given linear or linearized functional interrelationship, it requires further extensions to take into account the topology as well. Wang et al. (2018) extended the Bentley-Ottman algorithm to avoid topological cross problems in the results. Briefly, they verify whether a polygon shows intersections after shifting its nodes according to the MLS model. If yes, the nodes of conflict are moved again in order to restore the original topology. Miura and Tajima (2019) introduce a weighting function within MLS so that the configuration of the nodes in their geographic location is taken into account in the TTC as well. Although we focus on non-central TTCs, we should mention that MLS has also been applied in the generation of centrals TTCs of railroad networks by Ullah and Kraak (2015). For this, a vector calculation was performed in the first step. Starting from a fixed location, the train station was displaced according to the travel time. MLS then deforms the railroads and the map boundaries on the basis of the new locations of the considered train stations.

The homeomorphism condition guarantees, that the transformation is bijective and thus, possesses a continuous inverse. With regard to the map, this means inter alia that regions do not overlap, and the train stations are not shifted outside the borders.

## 1.3 Motivation

From the literature review above, we can identify the preservation of the topology as the major challenge with an increasing number of connections under study. Therefore, we propose a two-step approach consisting of the estimation of new locations with one consistent adjustment model for the whole country under study in the first and the estimation of an appropriate spatial transformation to an anamorphic map in the second step.

We rely on MLS integrated in the classical geodetic network adjustment (GNA) and create one mobility network including all nodes and all possible edges, not only direct connections. The $n \times n$ connectivity would produce $n^2$ connections from $n$ cities under study, but also contains several defects: The connection from one city to itself does not provide any useful information on the mobility network. Furthermore, the model only treats distances regardless of their orientation, i.e., outward journey and return are fused together. The number of connections between $n$ cities is thus restricted to a maximum of $\frac{n^2-n}{2}$ in the functional model. Possible intermediate stops and compulsory parallel connections could be imaged in the stochastic model that weights single connections in comparison to each other. As none of the locations in the network is fixed, a conditional model has to be added that retains the location, rotation, and size of the complete network. The new locations are substantiated with accuracy measures. In the same way, the importance of the individual connections for the network can be estimated. The resulting map directly reflects what we call the perceived distance, i.e., the distance between two points introduced by travel time. The perceived distance might deviate extremely from the geographic distance. For instance, the time from Munich airport to Berlin airport is around 1 h by airplane for 500 km. In contrast to that, it takes almost 3 h to reach the Alps from Munich airport by train, although they are only 100 km away.

The anamorphosis then is derived from the original and shifted locations of the nodes on a coarse grid to reduce topological deficiencies by spatial smoothing. The resulting TTC contains the original and the distorted political boundary of Germany for better recognition as well as a distorted coordinate grid for easier interpretation of the local performance of the mobility network: a contraction of the grid indicates outstanding connectivity whereas an inflation marks weak connectivity. As first illustrative example, we choose 97 cities within Germany according to the central-location principle. This results in 9312 connections for which the travel time is derived from routing apps. We distinguish between traveling individually by car and using public transport, especially the train. The subsequent section introduces the methodology for estimating the new locations and generating the TTC. Afterwards, the results for Germany are presented and discussed in detail. The article concludes with a short summary and outlook to future studies.

## 2 Methodology

Our approach comprises two separate steps: the geodetic network adjustment (GNA) to derive the shifted positions based on travel times as well as the anamorphosis including the cartographic visualization of the perceived distance.

### 2.1 Geodetic Network Adjustment

GNA is considered a standard procedure in Engineering Geodesy to minimize variations within a set of measurements or to determine unknown parameters of a terrestrial network. Nowadays, it is commonly used for quality assessment, i.e., to check the measurements for plausibility, reliability, and accuracy. The GNA typically composes of node points and measurements in-between. In the usual application, the underlying MLS method adjusts redundant measurements like distances (slant, horizontal, or vertical) or angles (horizontal and vertical) in a geodetic network. In this work, we apply the methodology to compensate the deviations between travel time and spatial distance. The GNA joins one stochastic, one functional, and one conditional model in the normal equation.

$$A^T P A \cdot \Delta \hat{x} = A^T P l \qquad (1)$$

The core is the design matrix $A$ of the functional model, that reflects the relation between shifts of the node's locations and deviations in the travel time. The matrix $P$ of the stochastic model weights the individual travel times. The vector $l$ denotes the reduced measurements, i.e., the differences between the travel time expected from the geographic distance and the real travel time derived by routing apps along the mobility network. The linear residuals to the parameters, i.e., the estimated shifts, are given in $\Delta \hat{x}$. The adjusted parameters, i.e., the new locations, are derived by

$$\hat{x} = x_0 + \Delta \hat{x}. \qquad (2)$$

Certain conditions are required to unambiguously solve the above equation. Due to the necessary linearization, the estimation has to be calculated iteratively.

### 2.1.1 Stochastic Model

The stochastic model typically investigates the accuracy of the observations, in our case, the travel times between two cities. Thus, the more accurate the travel time is known, the higher is the weight of the respective connection and vice versa. We start with a simple uncorrelated model that comprises an absolute (minimum) and a proportional contribution to the individual accuracy. The minimum accuracy is set to 6 min and the travel-time-dependent coefficient is set to 6min h$^{-1}$ where $T_{real}$ refers to travel time reported by routing apps:

$$\sigma = 0.1h + 0.1 \cdot T_{real}[h] \tag{3}$$

From these standard deviations the variances of each of the 9,312 connections which form the main diagonal of the initial covariance matrix $C$ are calculated. Correlations between the individual connections, e.g., because of using the same track, are not taken into account in this first attempt. To form the cofactor matrix $Q$ as inverse of the weighting matrix, a reference variance must be set. Here, the median of the standard deviations in $\sigma$ was used. By building the inverse, the weight matrix $P$ is created, which defines the weighting for each connection (Navratil and Staudinger 2006):

$$P = Q^{-1} = \left( \frac{C}{median(\sigma^2)} \right)^{-1} = diag\left( \frac{median(\sigma^2)}{\sigma^2} \right) \tag{4}$$

This procedure ensures numerically stable results. As all standard deviations range between six minutes and some more than one hour, the division by their median finally results in weights between zero and roughly one hundred.

The unknown parameters on $\hat{x}$ in Eq. 2 represent the desired new location of the cities. Since the geographic location of the cities is known, this can be used as the starting value in $x_0$. In addition, the average speed along all connections of the respective means of transport was initialized as supplementary parameter.

For the car an average speed of 87 km h$^{-1}$ and for the train an average speed of 78 km h$^{-1}$ was initialized. It was calculated as mean speed over all routes under consideration. The respective standard deviation amounts 8 km h$^{-1}$ for the car and 20 km h$^{-1}$ for the train due to different acceleration and deceleration behaviors as well as different duration of stay at traffic lights or train stations. For easier computation and better visualization in Eq. 7, the reciprocal value $m$ in s km$^{-1}$ is used that transforms a given distance into an expected travel time by simple multiplication

$$T_{expected} = m \cdot \sqrt{(\Delta N)^2 + (\Delta E)^2} \tag{6}$$

with $\Delta N$ the difference in Northing and $\Delta E$ the difference in Easting in the respective UTM coordinate system. After the calculation of $l$, the design matrix $A$ is formed and filled with the partial derivatives of the expected travel time at the starting points $x_0$. The design matrix thus describes the influence of a minimal deviation in the parameters (UTM coordinates in Northing und Easting) on the single observations (travel times). For instance, it contains one line for the connection between Hamburg and Munich that states that a north shift of Munich by 1 km shortens, but a north shift of Hamburg of equal length increases the travel time by 45 s. A shift in Eastern direction is less important as it only produces a lengthening or shortening of 9 s.

$$A = \begin{array}{cccccccc|c}
\cdots & \Delta N_H & \Delta E_H & \cdots & \Delta N_M & \Delta E_M & \cdots & \Delta m & \leftarrow Parameters \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\cdots & +45\frac{s}{km} & -9\frac{s}{km} & \cdots & -45\frac{s}{km} & +9\frac{s}{km} & \cdots & 612km & \Delta T_{M \to H}[s] \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\hline
& [km] & [km] & & [km] & [km] & & [\frac{s}{km}] & \uparrow Observations
\end{array} \tag{7}$$

### 2.1.2 Functional Model

The functional model describes the relation between the unknown new locations $\hat{x}$ and the observed travel times $T_{real}$. It is formed by the design matrix $A$ and the reduced observation vector $l$. The reduction is necessary to apply this linear model. The reduced observation vector is obtained as the difference between the real travel time from routing apps in $T_{real}$ and the estimated travel times along the beeline $T_{expected}$ which is calculated from the original geographic locations of the 97 cities.

$$l = T_{real} - T_{expected} \tag{5}$$

A change in the speed parameter $m$ always affects the whole route. Therefore, the distance along the beeline is given in the last column of the design matrix. This matrix consequently mediates between parameters (in columns) and observations (in rows). Only small coordinate differences symbolized by $\Delta N$ and $\Delta E$ are treated in $\Delta\hat{x}$ to assure the linearity of the model. Being aware that the linear residuals do not directly lead to a adequate solution due to the extreme variations in the spatio-temporal relation, an iterative adjustment is performed.

### 2.1.3 Conditional Model

The estimation of $\Delta\hat{x}$ in each iteration step requires the inversion of $A^TPA$, which is called the network configuration. As we are pursuing a free network adjustment, there are no fixed points in contrast to traditional TTCs. In principle, we have an infinite number of possible solutions for the posed problem. We necessarily have to shrink the solution space to only one optimum by introducing certain conditions: (1) The gravity center of all points in the network should not shift. This is realized by setting the mean residuals in Northing and Easting to zero. (2) The network should not rotate. This is realized by setting the correlation between the residuals in Northing and Easting to zero. (3) The network in total should neither swell nor shrink. This is realized by setting the auto-correlation of the residuals to zero. With these conditions applied, we can assure that the network stays at the same place with the same orientation and in the same size. Furthermore, the matrix inversion becomes stable.

## 2.2 Anamorphic Cartogram Generation

Based on the temporal positions, an anamorphic transformation is calculated, visualized and supplemented with additional information.
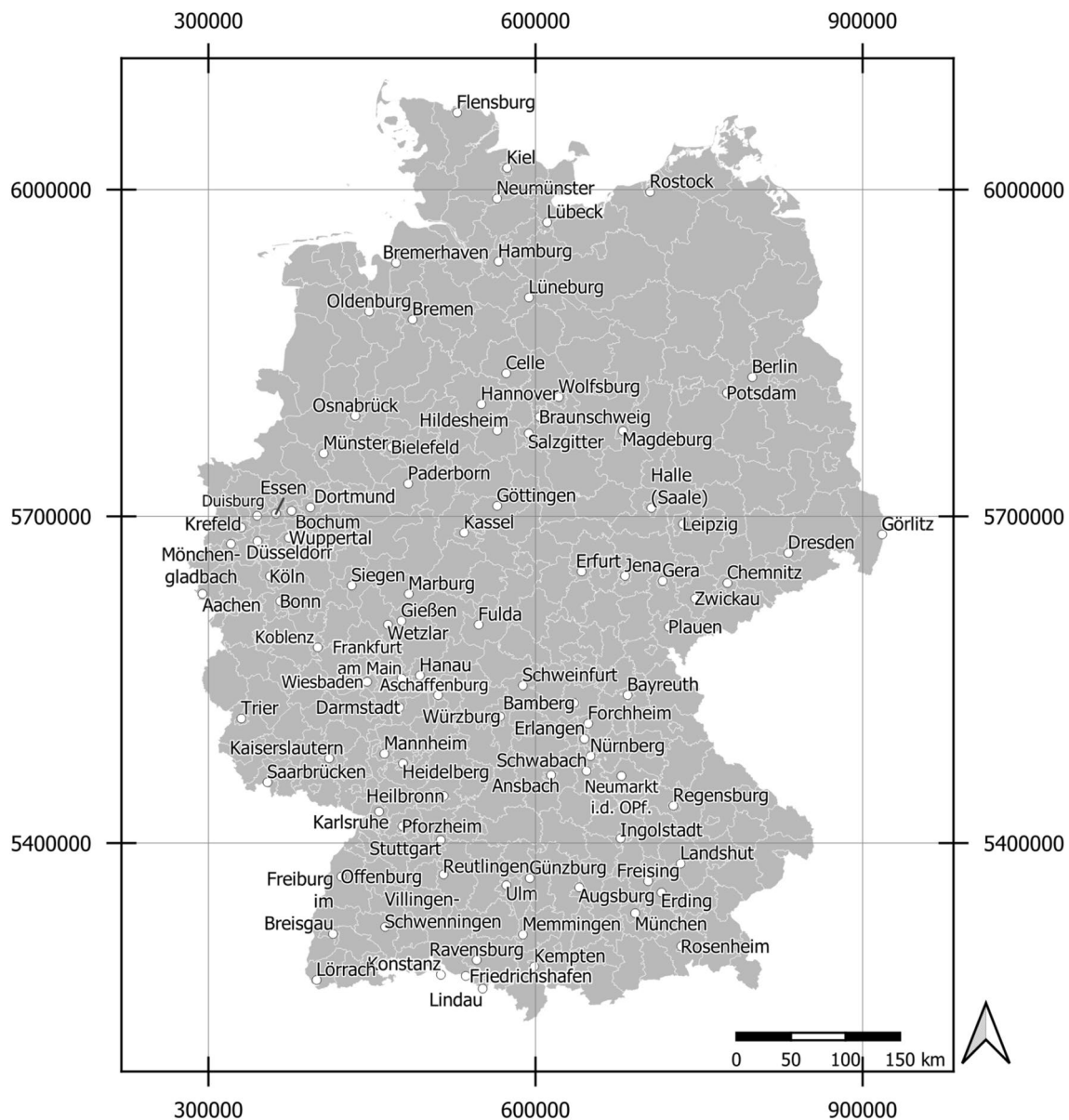


**Fig. 1** This base map serves as overview of all studied cities in their actual geographic location

**Fig. 2** Resulting spatio-temporal cartogram for Germany based on the road network. It is obvious that east-west connections are significantly faster than north-south connections by car. Motorway axes contract the adjacent cities
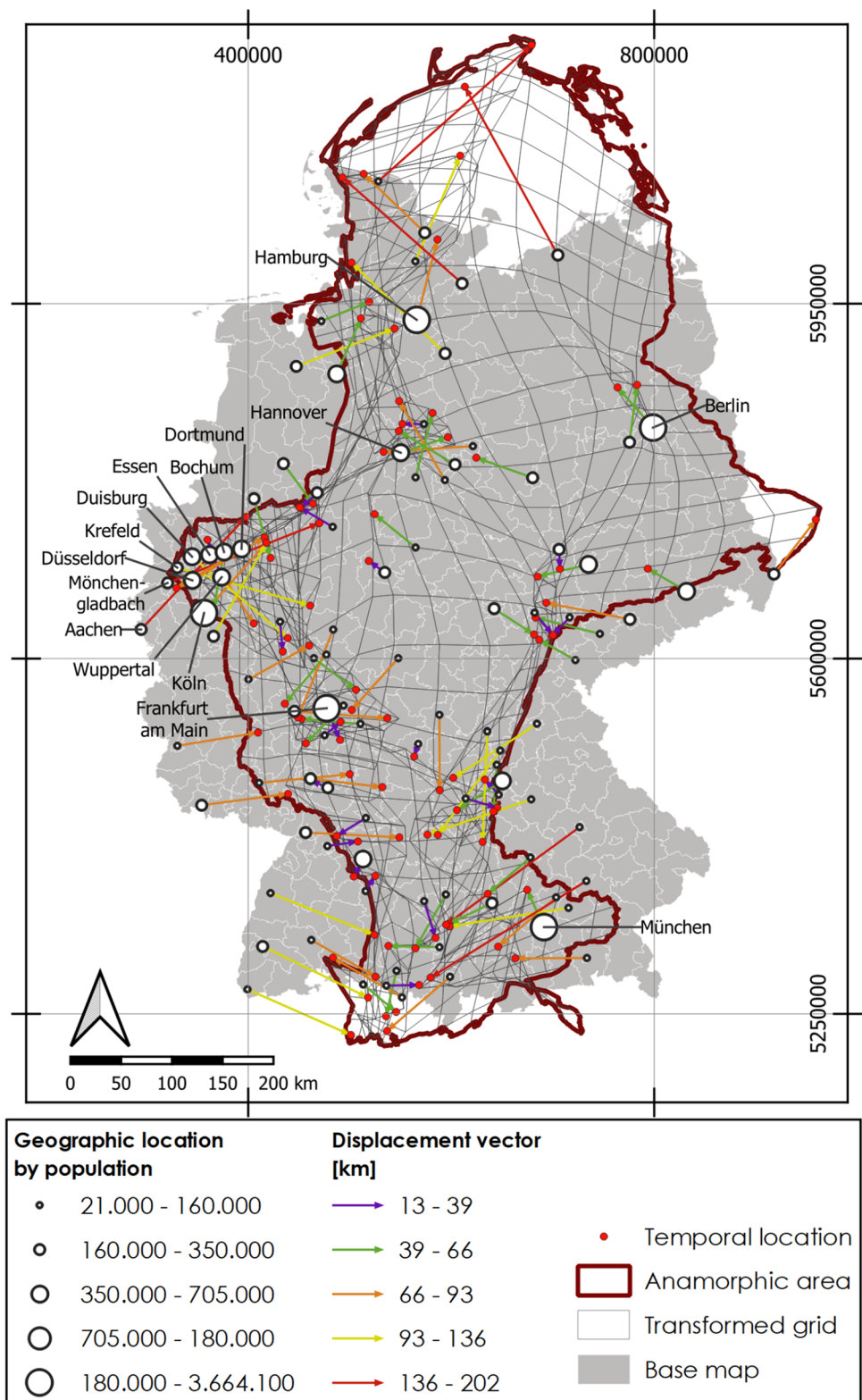


### 2.2.1 Anamorphic Map

The anamorphosis converts the observed shifts from geographic to temporal positions to an area-wide distortion of

the geographic map. This is realized by the plugin 'create distance cartogram' in QGIS (Viry 2018). It interpolates a deformation of a base map by the help of point pairs before and after GNA. From these shift vectors, a reference

**Fig. 3** Resulting spatio-temporal cartogram for Germany based on the rail network. Even by train, the east-west connections are significantly faster than north-south connections. Important rail axes become clearly visible
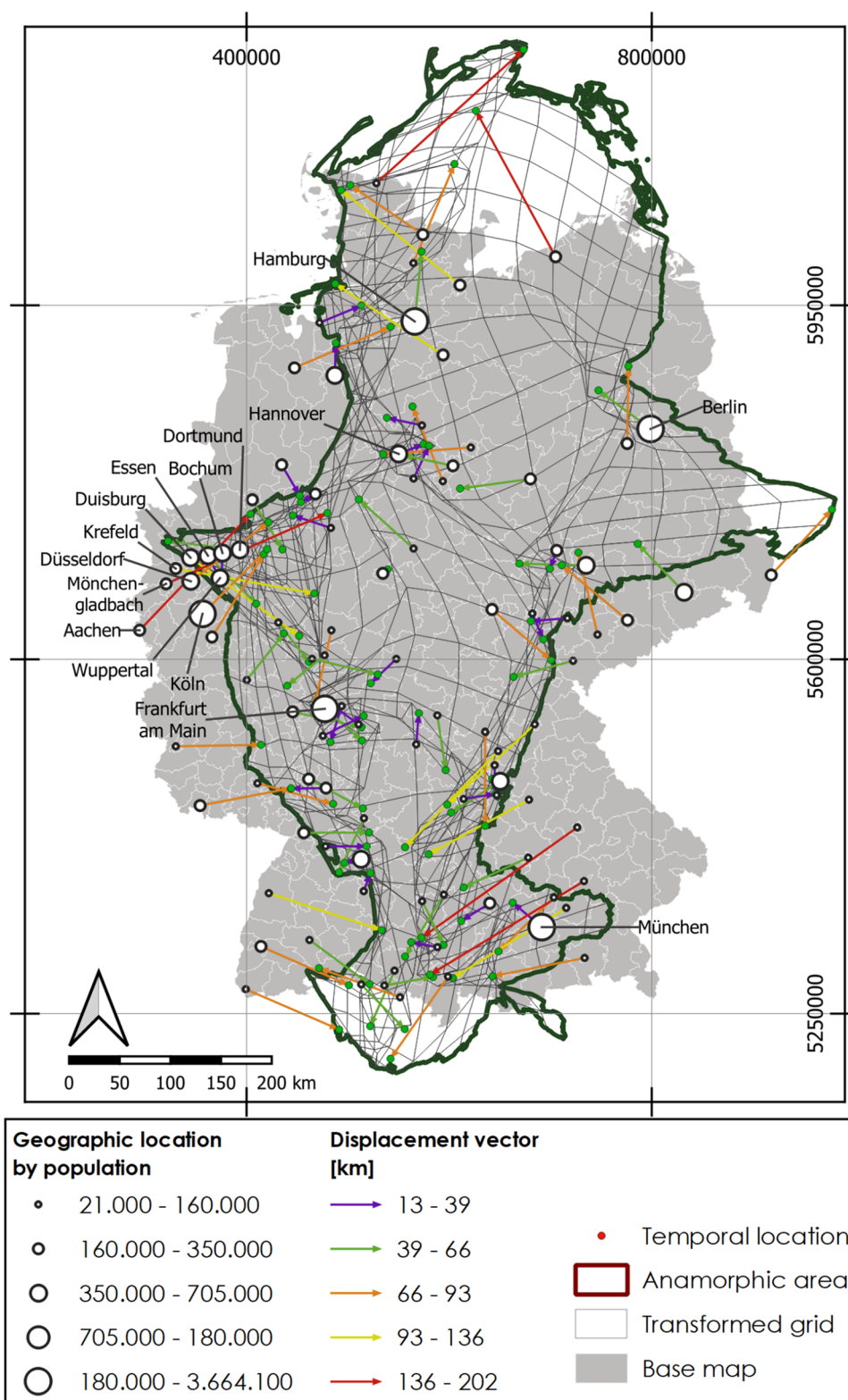


grid is generated and projected. Here, different grid sizes ('0.5' until '5') can be chosen. It should be chosen in a way that the point pairs in the map are best reflected without oversampling and the topology is preserved to the greatest

possible extent. We have chosen the size '3' for both maps as a compromise between the densely and less densely populated areas. Densely populated areas would allow for a finer grid, while widely rural landscapes do not provide
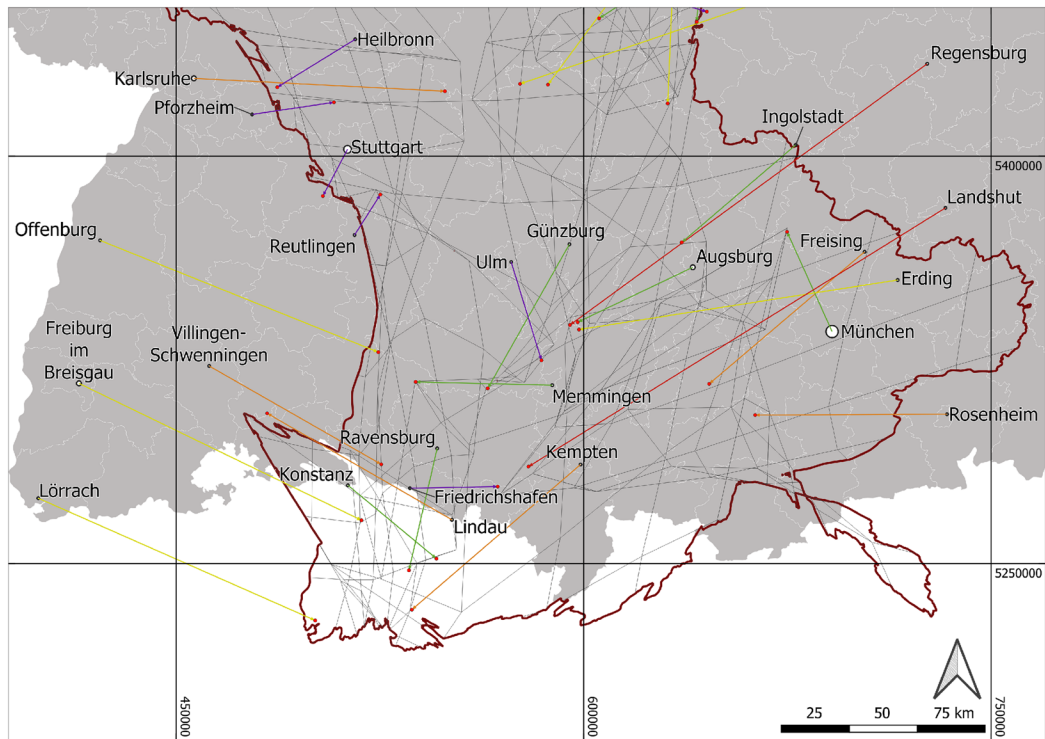
**(a)** View of detail for the road network in southern Germany. Converging vectors and the contraction of the grid clearly indicate a road axis from Lindau via Memmingen to Ulm which is the motorway A7.
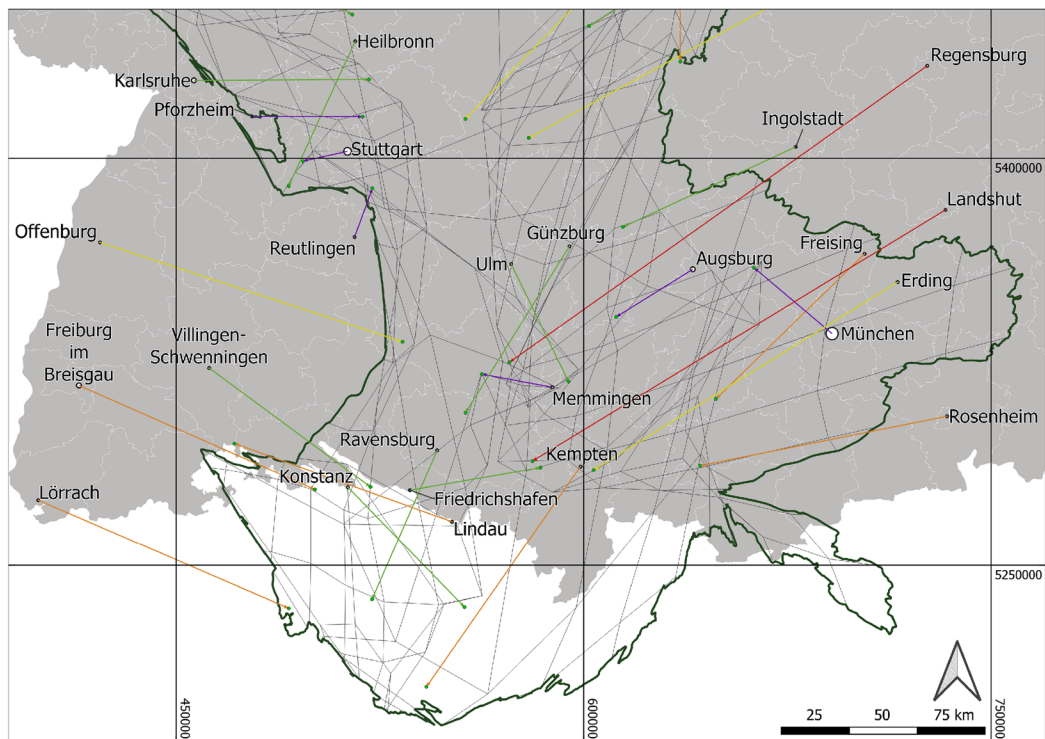


**(b)** View of detail for the railroad network in southern Germany. The southern area outside the geographic borderline even inflates showing that the connection by train are worse in comparison to the car in the figure above.

◄**Fig. 4** Considering the details between the means of transportation in the southern part of Germany. Symbols are used according to the legend in Figs. 2 and 3. Though the coarse shape is similar, distinct discrepancies become obvious in detail

sufficient vectors to interpolate the shift for each node in the grid. After the creation of the grid, the plugin also visualizes displacement vectors showing the shift between the point pairs. The vectors show the spatial distance as well as the direction of the displacement. In the final step, the base map is interpolated from the study area by an holistic observation. The previous steps are used as an interpolation tool to distort the base map according to the shift vectors (Cauvin et al. 2010). For better recognition we restrict the base map to the political boundary of Germany. We explicitly forego vector information with finer spatial resolution like state or district boundaries because of possibly misleading results by topological errors due to the unequal distribution of the cities all over Germany, see Fig. 1.

### 2.2.2 Cartographic Visualization

We are well aware that our approach is very challenging with regard to the cartographic visualization as we try to fuse locally adaptive, but topologically erroneous shifts from GNA with the quite coarse, but topologically widely reliable anamorphic map. Therefore, we have endeavored to exploit all the whole strengths of cartographic visualization methods like color, shape, size, and labels to generate an impressive TTC. In this way, the results of the GNA as well as of the anamorphosis are directly visualized in one single TTC without further correction or adaption. The color can be used in many different ways. First, it has an illustrative purpose. For instance, a blue surface generally is associated with water. Besides the illustrative purpose, it is also strong in symbolic variation. Here, different colors were used for geographic and temporal location to provide a clear distinction. Another application of color in this work is the color table to underline the vector length. A qualitative color scale was used to color the displacement vectors by their Euclidean distance. The scale shows a division of equivalent colors with the same brightness and contrast. Due to the different background colors of the temporal positions, the color of the displacement vector must be clearly visible. We need to take into account that different background colors can have different effects on the visual perception (Hake et al. 2002). Therefore, we decided in favor of a simply gray visualization of the original shape, that does not affect the interpretation of the distorted map.

### 2.2.3 Supplementary Information

As color and shape are already needed to visualize the results of GNA and anamorphosis as described above, we still have the size and labels left to include additional information such as the name of the cities under study and their respective number of inhabitants. The unequally distributed metropolitan areas pose another big challenge because each city should be clearly visible on the one hand, but also clearly distinguishable from the surrounding cities on the other hand. So we decided to distinguish five population categories (Figs. 2 and 3) symbolized by circles with increasing diameter. Only the biggest cities are labeled by name. The labels are mostly placed outside the boundary to prevent the overlay with other results and connected by a simple gray line with their respective marker. In this way, it is possible to add supplementary information which might be of high value for the visual interpretation and recognition of interrelationships (Hake et al. 2002).

## 3 Results

The GNA anamorphosis (GNAA) of Germany comprises Fig. 1 as base map with 97 cities of interest. The travel times are retrieved from the road and the railroad network and evaluated separately. The respective TTC can be found in Fig. 2 for the car and in Fig. 3 for the train as means of transportation.

Considering both TTCs, the shape of Germany obviously pulls together in the east-west and stretches in north-south direction. This fact indicates that east-west connections are rather faster than north-south connections both in the road and the railroad network. Likewise for both means of transportation a bloated area in the middle of the transformed grid which runs from northern Germany to southern regions is apparent. This area separates the two main traffic axes: a western axis from Freiburg in the south to the Ruhr metropolitan region in the north and one eastern axis from Munich in the south to Leipzig in the north. It has to be noted, that the results based on road and train connections are quite similar regarding the shape of Germany which means that places that are well integrated in the road network usually also share good train connections. In other words, road and railroad network in Germany show similar strengths and weaknesses. Deviations between car and train can only be observed locally.

Already in this rough view of the anamorphic structures, the distribution of German settlements is excellently shown. So, the weak and strong regions of the mobility network come to light. Two examples for strong infrastructures are the 'Ruhr-' region in West Germany and the local agglomeration of fast connection around the city of Hannover. Here you can see, that the new positions of the cities seem to contract. In comparison, the north-eastern regions are especially weak in infrastructure, because the new positions were pulled apart. The reason for this is not only the infrastructure
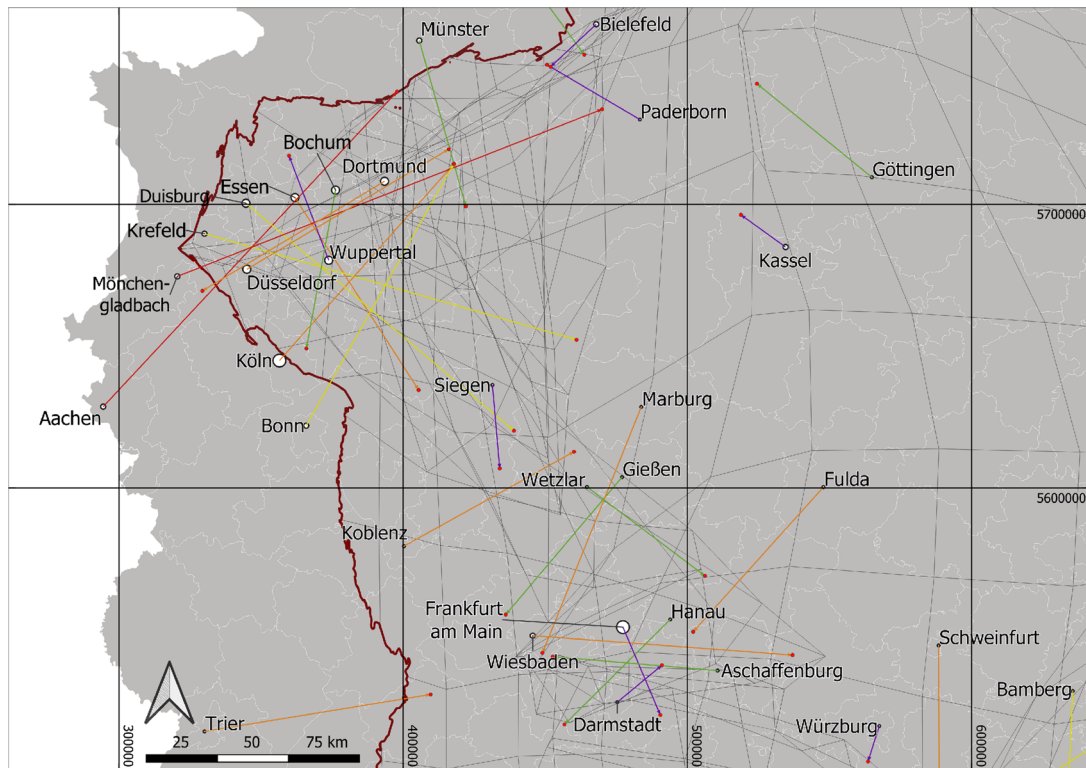
**(a)** View of detail for the road network in western Germany. The high urban density creates a strong network, especially in the most western area.
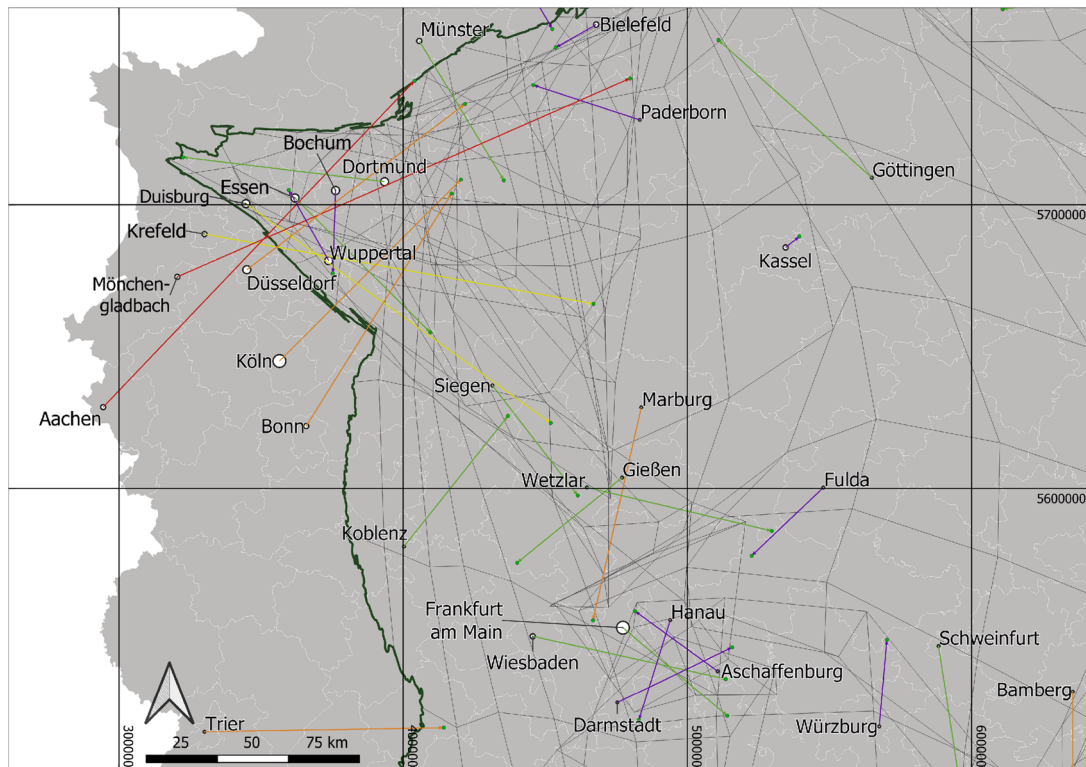


**(b)** View of detail for the railroad network in western Germany. In the railroad network fine differences are shown, f.e. the displacement of Dortmund. Here the connections run via western railroad stations.

◄**Fig. 5** Considering the details between the means of transportation in the western part of Germany. Symbols are used according to the legend in Figs. 2 and 3. Though the coarse shape is similar, distinct discrepancies become obvious in detail

of road and railroad , but also that the sparse settlements do not allow for establishing a stable mobility network in the sense of GNA. In the absence of sufficient more or less equally spaced nodes, the mobility network reduces to only few connections which lowers the redundancy and therewith, destabilizes the network.

## 4 Discussion

The GNAA resembles a design proposal that visualizes the optimal fusion of spatio-temporal information in a most accurate way. It reveals unseen characteristics of the mobility infrastructure of Germany with respect to the road and railroad network.

Although it is clear that spatial distance and travel time are often contradictory, the map represents an optimal trade-off according to the three models of the GNA. Thanks to the combined representation of the distorted boundary and the transformed grid via anamorphosis, the spatio-temporal shape of the whole of Germany becomes visible. A further advantage is that the base map can be placed in the background. Hence, the viewer does not have to reorient, because the displacement vectors show the direct connection between geographic (well-known) and travel time dependent (new) position. The general findings above are now substantiated and refined by detailed views of selected subsets. The detailed view in Fig. 4 reveals exciting findings, e.g., the converging and diverging colored displacement vectors. Thanks to the color scale, special features stand out more clearly. Some cities show a nearly equidistant parallel displacement (e.g., Landshut and Regensburg, see Fig. 4a and b). Thus, the local topology is preserved in this case. But, also discrepancies in the local topology are recognized more easily by the special color scheme. In the case of the road network, the cities Villingen-Schwenningen and Lindau are impressive examples for this. With a comparably equidistant, but opposing displacement, these cities almost swap places with each other (Fig. 4a). A nearly identical example can be seen for the railroad network. Here, the phenomenon can be perceived regarding the cities Freiburg im Breisgau (geographically not far from Villingen-Schwenningen) and Lindau. The TTC shows that the location of these cities contract to the western axis. It does not automatically imply that these cities have a strong interconnection, but, that they belong to the strong western north-south axis in southern Germany realized by the motorways A5, A7, and A81 as well as the corresponding railroads running more or less parallel to the motorways.

Looking at the northern part of the western axis, some peculiarities stand out here as well. At first glance, it is obvious that the local topology is confusing. For both road and railroad, this can be recognized, e.g., with view to the cities of Duisburg and Krefeld or Hanau and Aschaffenburg. But also a de facto 'exchange of the positions' is evident, e.g., in Fig. 5a the cities of Dortmund and Düsseldorf. For the reliable interpretation of the results, it is important to consider not only the direct interconnections, but the complete mobility network in the GNA. For instance, in the west ern part of Germany it is clear that cities in the south, such as Bonn and Cologne have good connections in a northerly direction because they belong to the metropolitan megaregion 'Rhine-Ruhr'. The remaining cities in this area move together. The city of Aachen is even shifted from a location outside to a more central position in the north of the Rhine-Ruhr region. This is also an effect of missing connections towards the west beyond the German border that would fix its location in east-west direction. With respect to the complete mobility network of Germany, it reveals that the embedding of the cities in the European mobility context is most important. The next step thus should be a Europe-wide analysis of the mobility network.

## 5 Conclusion

The calculation and visualization of TTCs has been a challenge in the cartographic context for quite some time. Numerous approaches refer to the central TTC. Practicable solutions for the non-central TTCs are rare and suffer from the same problem: how to calculate new locations while preserving the topology. However, it is the change of perspective that highlights strengths and weaknesses of the mobility infrastructure that connects the cities to each other. In this article, the complex structures of the urban network of Germany can be managed easily by our GNAA method. Thereby, many advantages of this traditional approach become obvious. Next to a fast calculation time, the user can modify the estimation by adapting the three models of GNA (e.g., by introducing group or even individual weights for different connection types or by fixing certain locations etc.) to meet new challenges.

Our non-central TTC reflects and validates the structure of the mobility network. Geographic location and travel time are fused according to special means of transportation: car and train. Thereby, it becomes evident, which spatial areas or which corridors are especially strong in the network. Likewise, the integration of cities in the mobility network can be assessed. Here, it is important to consider the information holistically. Because of the large number of connections considered in the model, the influence of single connections on the network is restricted on the one hand. On the other

hand, this allows for the evaluation of every single connection in the context of the complete mobility network. The geographic borders of the model still pose problems because of the connections in only one direction. Hence, future studies should be extended to the whole European continent.

We further propose to adapt the models to the dynamic of the travel data, which would lead to an interactive map. Here, the time of day at which the travel takes place (morning, noon, evening, night) as well as the means of transportation can be chosen by the user, so that the map changes according to the particular situation. An extension by further means of transportation is also feasible, e.g., the network of intercity coaches will probably produce most interesting results in comparison with the railroad network. For intra-urban studies (e.g.) we can also create TTCs with pedestrian, cyclists or public transportation. With distinct variations of the GNA models , it is also imaginable to regroup a city around fixed locations like critical infrastructure (e.g., fire stations) or recreation areas (e.g., green spaces). At global scale, the visualization of the international trade relations expressed in strong transport connections for passengers or goods and their variation according to the current political situation also becomes feasible.

In summary, we exemplified that the spatio-temporal extension of the traditional GNA method from geodesy by anamorphosis to GNAA is possible and leads to a non-central TTC which allows for an in-depth analysis of the underlying mobility network. Besides new applications, future studies will focus on methodological refinements of the GNAA models including more spatio-temporal characteristics of the considered connections.

## References

Backhaus K, Erichson B, Plinke W, Weiber R (2018) Multivariante Analysemethoden-Eine Anwendungsorientierte Einführungsmethode. Springer, Gabler. https://doi.org/10.1007/978-3-662-56655-8

Bies S, van Kreveld M (2012) Time-space maps from triangulations. In: Didimo W, Patrignani M (eds). Graph Drawing, Springer, Berlin Heidelberg. pp. 511– 516. https://doi.org/10.1007/978-3-642-36763-2_45

Buchin K, van Goethem A, Hoffmann M, van Kreveld MJ, Speckmann B (2014) Travel-time maps: Linear cartograms with fixed vertex locations. In: Duckham M, Pebesma E, Stewart K, Frank AU (eds). GIScience, vol. 8728, Springer, Cham, pp. 18– 33. https://doi.org/10.1007/978-3-319-11593-1_2

Büttner B, Keller J, Wulfhorst G (2010) Ein Erreichbarkeitsatlas für die Europäische Metropolregion München (EMM)–Ein Pilotprojekt der AG Mobilität des EMM e.V

Carosio A, Dolci C, Scherer M (2005) Erreichbarkeitsveränderungen in der schweiz-eine kartographische darstellung. In: Axhausen KW, Hurni L (eds). Zeitkarten der Schweiz 1950 - 2000, pp. 41– 61. Eidgenössische Technische Hochschule, Zürich. Chap. 3. https://doi.org/10.3929/ethz-a-005231298

Cauvin C, Escobar F, Serradj A (2010) Cartography and the Impact of the Quantitative Revolution, vol. 2. John Wiley & Sons, New Jersey. https://doi.org/10.1002/9781118558126.index

GeoBasis-DE/BKG (2021) Open data. https://gdz.bkg.bund.de/index.php/default/open-data.html. Data retrieved on: 30.03.2021, Data status: 31.12.2019

Google: Getting directions through the Directions API. (2021) https://developers.google.com/maps/documentation/directions/get-directions#required-parameters. Data retrieved on: 24.09.2021 (n.d.)

Hake G, Grünreich D, Meng L (2002) Kartographie: Visualisierung Raum-zeitlicher Informationen vol. 8., vollständig neu bearbeitete und erweiterte Auflage. De Gruyter, Berlin. https://doi.org/10.1515/9783110870572

Miura T, Tajima K (2019) Improvement of moving least squares transformation in distance cartogram construction. IEE J Trans Electrical Electronic Eng 14:1879–1880. https://doi.org/10.1002/tee.23015

Navratil G, Staudinger M (2006) Ausgleichsrechnung I Oder Die Anwendung Statistischer Methoden in Vermessungswesen und GIS. Technische Universität Wien, Institut für Geoinformation (2006). Lecture notes

Spiekermann K, Wegener M (1994) The shrinking continent: new time-space maps of Europe. Envi Plan B: Plan Design 21:653–673. https://doi.org/10.1068/b210653

Ullah R, Kraak M-J (2015) An alternative method to constructing time cartograms for the visual representation of scheduled movement data. J Maps 11(4):674–687. https://doi.org/10.1080/17445647.2014.935502

Viry M (2018) DistanceCartogram. https://plugins.qgis.org/plugins/dist_cartogram/. Retrieved on: 28.01.2022

Wang L, Ding L, Krisp JM, Li X (2018) Design and implementation of travel-time cartograms. KN-J Cartogr Geogr Inf. https://doi.org/10.1007/BF03545340