## Research Article

# Experimental and numerical modeling of sidewall orifices 

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#### Abstract

The discharge coefficient of an orifice is important for the outflow through the orifice. While it cannot be quantified theoretically as the outflow through an orifice depends on a number of parameters such as the pipe pressure, the liquid velocity and the shape and the area of the orifice. In this study, experiments and computational fluid dynamics (CFD) simulations were performed to find out the factors influencing discharge coefficient and the corresponding mechanism. The CFD simulation is based on Navier-Stokes equations combined with RNG $k-\varepsilon$ turbulence model. The results show that a negative exponential function could fit the relationship between orifice discharge coefficient, pipe pressure, and orifice area more accurately. The relationship between the discharge coefficient of the orifice and the velocity was linear. In general, the simulation results fit well with the experimental results, which indicates that CFD simulation could be used to study pipeline leakage.


Keywords Outflow • Orifice • Velocity • Pressure • Numerical model • Computational fluid dynamics (CFD)

## Abbreviations

$Q_{\text {total }} \quad$ The inlet flow rate, $\mathrm{m}^{3} / \mathrm{s}$
$Q_{\text {out }} \quad$ The outflow rate, $\mathrm{m}^{3} / \mathrm{s}$
$Q_{\text {end }} \quad$ The outlet flow rate, $\mathrm{m}^{3} / \mathrm{s}$
$A_{\text {cross }} \quad$ The area of the cross-section, $\mathrm{m}^{2}$
The ration of the orifice area which represents the relative size of the orifice area
$H_{1} \quad$ The upstream water head, $m$
$P_{\mathrm{a}} \quad$ Atmospheric pressure, $\mathrm{P}_{\mathrm{a}}$
$\rho \quad$ The water density, $\mathrm{kg} / \mathrm{m}^{3}$
$g \quad$ The gravity constant, $9.8 \mathrm{~m} / \mathrm{s}^{2}$
$a_{0} \quad$ The kinetic energy correction factor of the inlet flow
$v_{\text {come }} \quad$ The upstream velocity, $\mathrm{m} / \mathrm{s}$
$P_{\text {out }} \quad$ The pressure of the shrinkage section out of the orifice, $\mathrm{P}_{\mathrm{a}}$
$a_{\text {out }} \quad$ The kinetic energy correction factor of the orifice outflow
$h_{\mathrm{w}} \quad$ Local head loss, m
$h_{\mathrm{f}} \quad$ Frictional head loss, m
$\mu \quad$ Discharge coefficient of orifices
$H_{0} \quad$ The outflow pressure in the pipeline, $m$
$F_{1} \quad$ The surface tension
$\sigma \quad$ The water surface tension coefficient
I The perimeter of the orifice
$\alpha \quad$ The contact angle of the pipe wall
$h_{1} \quad$ The change of the water head
A Pipe section area, $\mathrm{m}^{2}$
$k_{\mathrm{h}} \quad k_{h}=\frac{H}{d}$
$d$ The pipe diameter, m
$k_{v} \quad k_{v}=\frac{v}{\sqrt{g d}}$
$v \quad$ The velocity, $\mathrm{m} / \mathrm{s}$
$k_{\mathrm{a}} \quad k_{\mathrm{a}}=\frac{A_{\text {out }}}{A}$
$A_{\text {out }} \quad$ The orifice area, $\mathrm{m}^{2}$
$\kappa_{\mathrm{a}}, \kappa_{\mathrm{h}}, \kappa_{\mathrm{V}}, \xi_{\mathrm{a}}$,
$\xi_{h}, \theta_{h}, \theta_{v}, \kappa_{h}^{\prime}$. Coefficients that are all positive numbers
$\kappa_{v^{\prime}}^{\prime} \kappa_{v h}, \kappa_{\mathrm{h} 1}, \xi_{\mathrm{h} 1}$
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| $\mu^{\prime}$ | The discharge coefficient of the orifice of Model II |
| :---: | :---: |
| $\chi_{\mathrm{h}}, \lambda_{\mathrm{h}}, \chi_{\mathrm{a}}, \lambda_{\mathrm{a}}$ | Positive coefficients |
| $\underline{\eta}, \underline{\phi}, \omega$ |  |
| $\overline{u_{i}}, \overline{u_{j}}$ | The average velocities in different coordinate axis, $\mathrm{m} / \mathrm{s}$ |
| $S_{t}$ | The source item |
| $G_{k}$ | The turbulence production caused by the mean velocity gradient |
| $G_{\mathrm{b}}$ | The turbulence production caused by the buoyancy |
| $Y_{M}$ | The influence of the compressible turbulent flow pulsation expansion on the total dissipation rate |
| $\alpha_{k} \alpha_{\varepsilon}$ | Respectively the reciprocal of the effective turbulent Prandtl number of turbulence energy $k$ and dissipation rating $\varepsilon$ |
| H | $\left(H_{1}+H_{2}\right) / 2$ |

## 1 Introduction

Orifice outflow is a common hydrodynamic phenomenon that occurs in various industries such as the chemical industry, energy engineering, agricultural irrigation, and hydraulic engineering. Particularly, the safety of the current urban water supply network, suffering from frequent incidents of pipe leakage and explosion, is a very serious problem. Thus, in order to effectively reduce water leakage in the pipe network, it is necessary to establish models diagnosing water leakage and controlling water pressure. To this end, the discharge coefficient in the orifice outflow model must be determined.

Orifice outflow in a pipe is influenced by the pressure, the velocity, the area, and the shape of the orifice. Goodwin, Hiki, and May found that the outflow of water increased with pressure [1-3]. However, the discharge coefficient was not considered in their model. Germanopoulos et al. made up for this oversight by introducing the discharge coefficient related to the pressure to their model [4, 5].

In the field of urban water supply, empirical formulas have often been utilized to describe pipeline leakage. For example, the discharge coefficient of the point model in China is 0.421 , while the coefficient of the model proposed by Shinozuka equals 0.64 . Thus, the discharge coefficients used in leakage models differ significantly and cannot accurately describe the leakage situation [6, 7].

Lateral velocity prevents liquid from turning at the edge of an orifice, resulting in flow separation at the orifice. Heggeman et al. found that orifice flow in a liquid distributor was influenced by lateral velocity [8]. Through a series of pipe section simulation experiments, Jia studied
leakage under different pipe diameters, pressures and leakage areas. This result showed that the influence of water pressure and leakage area on leakage was interrelated, and their relationship was described by an 'S' model [9]. The study of Prohaska et al. indicated that as the ratio of the orifice diameter to pipe diameter increased, the discharge coefficient decreased and eventually reached an asymptotic value with respect to the riser pipe [10]. And the discharge coefficient is lower for the lager pipe with the other same variables. Yu found that the discharge coefficient decreased with an increase in lateral flow velocity in the trough of a filled tower [11]. The lateral flow might lead to large-scale maldistribution in trough-type liquid distributors with larger throughput. Investigating the flow characteristics of a liquid distributor could help to avoid large-scale maldistribution. Astaraki's showed that when the length-width ratio of a rectangular opening was larger, the turbulence generated at a corner had less effect on flow reduction [12].

In this paper, to study the influence of pressure, velocity, area, and orifice shape on the orifice outflow an orifice outflow experiment with replaceable pipe sections was conducted. and the orifice outflow in a pipe was also assessed through a computational fluid dynamics (CFD) simulation. The fluid flow in the leakage pipeline includes the free flow in the pipeline and the flow through the leakage orifice. Therefore, the numerical model applied to the CFD simulation is based on Navier-Stokes equations combined with RNG k- $\varepsilon$ turbulence model. The results show that the experimental data perfectly match the CFD simulation data. While experiments are expensive due to the costs of experimental setup, CFD simulations can be performed relatively easily. Moreover, CFD simulations have been employed to solve hydraulic engineering problems successfully. Hence, developing a simulation which is capable of reflecting the results of actual experiments is of great practical value [13-18]. The models presented in this paper could be potentially applied in engineering applications, such as leak control in the water supply industry, filledtower optimization in the chemical industry, and drip irrigation design in agricultural irrigation.

### 1.1 Experimental equipment and method

### 1.1.1 Experimental equipment

The purpose of the experiment is to investigate the influences of the pressure and velocity of water, the orifice area, and the orifice shape on the orifice discharge coefficient. To this end, an experiment setup which is able to simulate the physical flow state and process of an orifice outflow was designed as schematically displayed in Fig. 1.

In the experimental setup, the upper water tank provided the inflow condition, and a collecting water tank was under the orifice. The water in the collecting water tank would flow into flume 1, while the downstream water would flow into flume two at the end of the pipe. The water in the flumes would flow back to the lower water tank. The water in the lower tank could be pumped back to the upper tank. In order to collect and measure the outlet flow rate and the terminal flow rate of the pipe, measuring cylinders were placed in flume 1 and 2 . Piezometer tubes were used to measure the upstream and downstream pressure of the orifice in the experimental equipment.

Ghazali found that the relationship between flux and pressure was related to the shape of an orifice [19]. A replaceable pipe section was thus adopted in this
experiment to explore the orifice shape effects. Valves were provided on the upstream and downstream sides of the orifice to control the working conditions. To obtain high accuracy, the mass method was used to measure the flow data.

Pipes with a variety of orifice shapes, including oval, semi-oval and a small circle, were used in this experiment (Fig. 2). The arc directions of semi-oval 1 and semi-oval 2 were different (assuming the ratio of the long axis to the short axis of the oval is $2: 1$ ). In the experiment, the shape and the area of the orifice was determined by a replaceable pipe section, as shown in Fig. 3. The length of the replaceable section was more than 20 times the pipe diameter, and thus the normal outflow would be unaffected by the fluctuation flow at the interface of the sections.

Fig. 1 Experiment setup to investigate pipe section leakage


Fig. 2 The shape of the orifice

(a) Illustration

(b) Test photo

(a) Illustration
(b) Test photo

Fig. 3 The active oval section

### 1.1.2 Experimental method

The experiments with different hydraulic conditions of the orifice were performed according to the following procedures.

Step 1: the pressure, the area, and the shape of orifice were set to be constant, while the velocities were altered by adjusting the valves upstream and downstream of the orifice to study the change of the outflow rate. In this step, pressure, outflow rate and flow rate at the end of the pipe were recorded.

Step 2: under constant orifice area and orifice shape, experiments using different pressures were conducted. The pressures were $2.35 \mathrm{~cm}, 2.85 \mathrm{~cm}$, and 3.35 cm when the shape of the orifice was oval (except under the condition when the orifice area ratio $k_{\mathrm{a}}$ is $10 \%$ ). The pressures were $2.35 \mathrm{~cm}, 2.85 \mathrm{~cm}, 3.35 \mathrm{~cm}$ and 6.85 cm when the shape of the orifice was oval, and the orifice area ratio $k_{a}$ is $10 \%$. When the shape of the orifice is semi-oval 1 and semi-oval 2 , the pressures were $2.35 \mathrm{~cm}, 2.85 \mathrm{~cm}$ and 3.85 cm . When the shape of the orifice was a small circle and the orifice area ratio $k_{\mathrm{a}}$ was $2.8 \times 10^{-3}$, the pressures were $7.55 \mathrm{~cm}, 8.35 \mathrm{~cm}$, and 9.35 cm . When the shape of the orifice was a small circle, and the orifice area ratio $k_{\mathrm{a}}$ was 0.11 , the pressures were $6.85 \mathrm{~cm}, 7.55 \mathrm{~cm}$, and 8.35 cm . Step 1 should be repeated for every pressure.

Step 3: with a constant orifice shape, experiments using different orifice areas were conducted. When the orifice shape was oval, the orifice area ratios $k_{a}$ were $10 \%, 20 \%$, $40 \%, 60 \%$, and $80 \%$. When the orifice shape was semioval 1 , the orifice area ratios $k_{\mathrm{a}}$ were $20 \%, 30 \%$, and $40 \%$. When the orifice shape was semi-oval 2 , the orifice area ratio $k_{\mathrm{a}}$ was $40 \%$. When the shape was a small circle, the orifice area ratios $k_{\mathrm{a}}$ were $2.8 \times 10^{-3}$ and 0.011 . Steps 1 and 2 should be repeated for every orifice area.

Step 4: experiments using different orifice shapes were conducted. Steps 1, 2 and 3 were repeated for different orifice shapes (oval, semi-oval 1, semi-oval 2 and circle).

In this experiment, the main measurements are the pressure (upstream and downstream), the orifice outflow rate and the terminal flow rate. The total pipeline flow is calculated by the orifice outflow rate and the flow rate at the end of the pipe. The formula is as follows:
$Q_{\text {total }}=Q_{\text {out }}+Q_{\text {end }}$
$v_{\text {come }}=Q_{\text {total }} / A_{\text {cross }}$
The detailed experimental conditions are present in Table 1.

### 1.2 Establishment of models and data analysis

### 1.2.1 Theoretical analysis of orifice outflow characteristics

To analyze and discuss experimental results, a theoretical explanation of orifice outflow characteristics is given. As Fig. 4 shows, when the upstream water flows through the orifice, a certain amount of flow would leak through the orifice, and the rest continues downstream. The flow lines in the section through the orifice are unparallel, and thus the water flow continues to shrink until the flow lines become parallel. This section is called the shrinkage section outside the orifice. The shrinkage factor $\varepsilon$ is the ratio of the shrinkage section area to the outflow rate section area.

The energy Eq. (3) between the orifice cross-section and upstream cross-section is established according to the Bernoulli equation. The equation can be expressed as:
$H_{1}+\frac{p_{\mathrm{a}}}{\rho g}+\frac{\alpha_{0} v_{\text {come }}^{2}}{2 g}=\frac{p_{\text {out }}}{\rho g}+\frac{\alpha_{\text {out }} v_{\text {come }}^{2}}{2 g}+h_{\mathrm{w}}+h_{\mathrm{f}}$
where $H_{1}$ is the upstream water head ( m ), $P_{\mathrm{a}}$ is atmospheric pressure $\left(P_{\mathrm{a}}\right), v_{\text {come }}$ is upstream velocity $(\mathrm{m} / \mathrm{s}), P_{\text {out }}$ is the pressure of the shrinkage section out of the orifice $\left(P_{\mathrm{a}}\right), v_{\text {out }}$ is the average velocity of the shrinkage section out of the orifice ( $\mathrm{m} / \mathrm{s}$ ), $h_{\mathrm{w}}$ is local head loss ( m ), and $h_{\mathrm{f}}$ is frictional head loss ( m ). Assuming that the water head of the orifice outflow is $H_{0}=H_{1}+\frac{\alpha_{0} v_{c o m e}^{2}}{2 g}-h_{f}$, an equation for the outflow rate can be expressed as follows:
$Q_{\text {out }}=\mu A_{\text {out }} \sqrt{2 g H_{0}}$
where $A_{\text {out }}$ is orifice area $\left(\mathrm{m}^{2}\right), Q_{\text {out }}$ is outflow rate $\left(\mathrm{m}^{3} / \mathrm{s}\right)$, $H_{0}$ is the outflow pressure in the pipeline ( m ); $\mu$ is the discharge coefficient of the orifice, and its value is usually $0.60-0.62$. However, the outflow in a pipe or a channel with a transverse flow is significantly different from a general orifice discharge. When the orifice is small, the effect of the surface tension on discharge is great. This effect of the surface tension on the outflow rate is described as follows.
$F_{1}=\sigma \cdot l \cdot \cos \alpha$
where $F_{1}$ is the surface tension, $\sigma$ is the water surface tension coefficient, $l$ is the perimeter of the orifice; $a$ is the contact angle of the pipe wall.

The variation in the water head $\left(h_{1}\right)$ can be described as follows:
$h_{1}=\frac{F_{1}}{\rho g A}=\frac{\sigma \cdot l \cdot \cos \beta}{\rho g A}$
The impact of the surface tension on the outflow pressure is also expressed by $\frac{h_{1}}{H_{0}}$.

Table 1 The experimental conditions


Fig. 4 The flow conditions at the orifice point

## $\mathrm{H}_{1}$ <br> Measuring point

$\stackrel{\mathrm{H}_{2}}{\text { Measuring point }}$


To satisfy the needs of research, three dimensionless numbers, namely $k_{\mathrm{h}}, k_{v^{\prime}}$ and $k_{\mathrm{a}}$, were introduced.
$k_{\mathrm{h}}=\frac{H}{d}$
where $k_{\mathrm{h}}$ is the ratio of the pressure to the pipe diameter and represents the average pressure; $H(\mathrm{~m})$ is defined as the average of $H_{1}$ and $H_{2}$, i.e., $\left(H_{1}+H_{2}\right) / 2$, and $d(\mathrm{~m})$ stands for the pipe diameter.
$k_{v}=\frac{v}{\sqrt{g d}}$
where $k_{v}$ is the flow velocity, and $v(\mathrm{~m} / \mathrm{s})$ represents the velocity of water; $d(m)$ denotes the section diameter.
$k_{a}=\frac{A_{\text {leak }}}{A}$
where $k_{\mathrm{a}}$ is the ratio of the orifice area and represents the relative size of the orifice; $A_{\text {out }}$ and $A\left(\mathrm{~m}^{2}\right)$ indicate the orifice area and pipe section area, respectively.

Computational results indicate that the impact of surface tension on the total head is less than $5 \%$ with a $k_{\mathrm{a}}$ greater than 0.2 , while the impact on the outflow rate is relatively large, up to $22 \%$, with a $k_{\mathrm{a}}$ less than 0.2 .


The equation for the outflow rate is $Q_{\text {leak }}=\mu A_{\text {out }} \sqrt{2 g\left(H_{0}-h_{1}\right)}$. The effect of surface tension on the value of $Q_{\text {out }}$ is less than $5 \%$, which can be ignored in most engineering situations. Therefore, ignoring the effect of surface tension would likely not influence the results of this experiment.

### 1.3 Data analysis and fitting

According to the experimental data, there is a negative linear relationship between the discharge coefficient of the orifice ( $\mu$ ) and the velocity, as shown in Fig. 5. A linear function could be used to describe the relationship between the discharge coefficient of the orifice $(\mu)$ and the velocity.

In Fig. 5a-c, different fitting curves correspond to the oval, semi-oval 1 and semi-oval 2 orifices, the orifice area of which are the same. This illustrates that the orifice shape influences the discharge coefficient of an orifice.

Figure 6 shows that $\mu$ increases with $k_{\mathrm{a}}$ and $k_{\mathrm{h}}$, and the relationship tends to slow. This indicates that there is a positive correlation in how $\mu$ is affected by the orifice area and the pressure.

Figure 5 indicates that $\mu$ is influenced by the shape of the orifices, which means the equation of the model changes with the shape of the orifice. This study focused

(b) semi-oval $1, k_{a}=0.4, k_{h}=1.76$

(d) hole, $k_{a}=2.77 \times 10^{3}, k_{h}=1.76$

Fig. 5 The change of $\mu$ with $k_{v}$ in different orifices

Fig. 6 The change of $\mu$ with $k_{\mathrm{a}}$ and $k_{h}$

on the equation of the oval orifice, which is called the oval orifice Model I.

### 1.4 The establishment of oval orifice Model I

As pipeline pressure increases, it gradually becomes the main factor affecting orifice outflow. If the outflow rate loss caused by a transverse flow can be neglected, then the value of $\mu$ should approach 1 . With a gradual increase in the orifice area, it can be seen from the Fig. 6a that the value of $\mu$ is gradually approaching 1 . Therefore, a negative exponential function could describe the relationship between orifice discharge coefficient, pipe pressure, and orifice area more accurately. Combined with the linear relationship between the discharge coefficient of the orifice and the velocity, the relationship between $\mu, k_{h}, k_{v}$ and $k_{\mathrm{a}}$ is as follows:
$\mu=\left(1-\kappa_{\mathrm{a}} e^{-\xi_{\mathrm{a}} k_{\mathrm{a}}}\right) \cdot\left[\theta_{\mathrm{h}} \cdot\left(1-\kappa_{\mathrm{h}} e^{-\xi_{\mathrm{h}} k_{\mathrm{h}}}\right)\right] \cdot\left[\theta_{v} \cdot\left(1-\kappa_{v} k_{v}\right)\right]$
where $K_{\mathrm{a}^{\prime}} \kappa_{v^{\prime}} \kappa_{\mathrm{h}^{\prime}} \xi_{\mathrm{a}^{\prime}} \xi_{v^{\prime}} \xi_{h}$ are coefficients that are all positive numbers. Because the upper limit of $\mu$ is 1, Eq. (10) can be developed into:
$\mu=\left(1-\kappa_{\mathrm{a}} e^{-\xi_{\mathrm{a}} k_{\mathrm{a}}}\right) \cdot\left(1-\kappa_{\mathrm{h}}^{\prime} e^{-\xi_{\mathrm{h}} k_{\mathrm{h}}}-\kappa_{v}^{\prime} k_{v}+\kappa_{v \mathrm{~h}} k_{v} \cdot e^{-\xi_{\mathrm{h}} k_{\mathrm{h}}}\right)$
where $\kappa_{h}^{\prime}, \kappa_{v}^{\prime}$ and $\kappa_{v h}$ are coefficients that are positive numbers.

Equation (4) shows that $\mu$ is mainly influenced by the upstream pressure $H_{1}$; Thus $\kappa_{h}$ could be replaced by $\kappa_{h 1}$ ( $k_{\mathrm{h} 1}=H_{1} / d$ ), which represents the upstream pressure. The equation is developed as follows:
$\mu=\left(1-\kappa_{\mathrm{a}} e^{-\xi_{\mathrm{a}} k_{\mathrm{a}}}\right) \cdot\left(1-\kappa_{\mathrm{h} 1} e^{-\xi_{h 1} k_{\mathrm{h} 1}}-\kappa_{v}^{\prime} k_{v}+\kappa_{v h 1} k_{v} \cdot e^{-\xi_{h 1} k_{\mathrm{h} 1}}\right)$
where $\kappa_{h 1}, \varepsilon_{h 1}$ and $\kappa_{v h 1}$ are positive coefficients.
After using the experimental data in the case of the oval orifice to fit Model I, a unified equation can be obtained as follows:

$$
\begin{equation*}
\mu=\left(1-0.5 e^{-5.6 k_{\mathrm{a}}}\right) \cdot\left(1-0.37 e^{-k_{\mathrm{h} 1}}-0.2 k_{v}-0.32 k_{v} \cdot e^{-k_{\mathrm{h} 1}}\right) \tag{13}
\end{equation*}
$$

The multiple correlation coefficient $R^{2}$ of Model I is 0.997 . Figure 7 shows the degree of curve fitting.

Combined with Eq. (13), Eq. (2) can be developed into orifice outflow Model I.

$$
\begin{align*}
Q_{\text {out }}= & \left(1-0.5 e^{-5.6 k_{\mathrm{a}}}\right) \cdot\left(1-0.37 e^{-k_{\mathrm{h} 1}}-0.2 k_{\mathrm{v}}-0.32 k_{v} \cdot e^{-k_{\mathrm{hl}}}\right) \\
& A_{\text {out }} \sqrt{2 g H_{0}} \tag{14}
\end{align*}
$$

Note that Model I has some limitations. Under conditions of high velocity and low pressure, the real outflow rate stops. But in Model I, the outflow rate is always greater than zero. Also, the computational process is relatively complicated because it needs to measure the upstream velocity in the pipe.

### 1.5 The establishment of Model II

The linear correlation of $\frac{Q_{\text {out }}}{Q_{\text {total }}}$ and $\frac{A_{\text {out }} \sqrt{2 g H}}{Q_{\text {total }}}$ can be seen in Fig. 8. The relationship between the outflow rate, the


Fig. 7 The distribution map of computing data and experimental data

(a) oval

(c) semi-oval 2

Fig. $8 k_{\mathrm{a}}=0.4, k_{\mathrm{h}}=1.76$, the change of $\frac{Q_{\text {out }}}{Q_{\text {total }}}$ with $\frac{A_{\text {out }} \sqrt{2 g \mathrm{H}}}{Q_{\text {total }}}$
orifice area, the pressure, and the total flow rate is as follows:
$Q_{\text {out }}=\mu^{\prime} A_{\text {out }} \sqrt{2 g H}-\alpha Q_{\text {total }}$
The above equation is Model II. $\mu^{\prime}$ is the discharge coefficient of the orifice of Model II. $a Q_{\text {total }}$ measures the influence of the horizontal flow on the outflow of lateral orifices, and $\alpha$ is the effective coefficient of the lateral orifices.

Orifice outflow Model II is also the equation of the oval orifice outflow. Figure 9a and b show the relationship curve of $\mu^{\prime}, k_{\mathrm{a}}$ and $k_{\mathrm{h}}$ in Model II. Figure 9 c shows the relationship curve of $a$ and $k_{a}$.

Figure 9 a and b indicate that $\mu^{\prime}$ has a positive correlation with $k_{\mathrm{a}}$ and $k_{\mathrm{h}}$, and the rising trend gradually slows. Figure 9c shows that $\alpha$ increases with $k_{\mathrm{a}}$, and the rising trend gradually accelerates, which reflects the relationship between $a$ and the orifice area.

Orifice Model II could be described as follows:
$Q_{\text {out }}=\left(1-\chi_{\mathrm{h}} e^{-\lambda_{\mathrm{h}} k_{\mathrm{h}}}\right) *\left(1-\chi_{\mathrm{a}} e^{-\lambda_{\mathrm{a}} k_{\mathrm{a}}}\right) \cdot A_{\text {leak }} \sqrt{2 g H}-\left(\eta e^{\phi k_{\mathrm{a}}}-\omega\right) Q$
where $\chi_{\mathrm{h}}, \lambda_{\mathrm{h}}, \chi_{\mathrm{a}}, \lambda_{\mathrm{a}}, \eta, \Phi$, and $\omega$ are positive coefficients, $A$ is the orifice area $\left(\mathrm{m}^{2}\right), H$ is the orifice pressure ( m ), and $Q$ is the total flow rate ( $\mathrm{m}^{3} / \mathrm{s}$ ). After using the experimental

(b) semi-oval 1

(d) hole
data of the oval orifice to fit Model II, an equation could be obtained as follows:

$$
\begin{align*}
Q_{\text {leak }}= & \left(1-0.215 e^{-6.3 k_{\mathrm{h}}}\right) \cdot\left(1-0.44 e^{-4.1 k_{\mathrm{a}}}\right) \\
& \cdot A_{\text {leak }} \sqrt{2 g H}-\left[0.12\left(e^{1.67 k_{\mathrm{a}}}\right)-0.117\right] Q \tag{17}
\end{align*}
$$

The multiple correlation coefficient $R^{2}$ of Model II is 0.999, which shows the simulation results fit the experimental data well. This model could be used to describe the relationship between $\mu^{\prime}$ and other factors accurately and conveniently.

### 1.6 CFD simulation

### 1.6.1 The principle of the CFD model

Orifice outflow of a pipe includes free flow in the pipe and flow at the orifice. Assuming that the flow is incompressible, the governing equations of the flow are obtained by using continuity equations and Navier-Stokes equations:
$\frac{\partial \rho \overline{u_{i}}}{\partial x_{i}}=0$


Fig. 9 The change of $\mu^{\prime}$ and $a$ with $k_{\mathrm{a}}$ and $k_{\mathrm{h}}$
$\frac{\partial}{\partial t}\left(\rho \overline{u_{i}}\right)+\frac{\partial \rho \overline{u_{i} u_{j}}}{\partial x_{i}}=-\frac{\partial p}{\partial x_{i}}+\mu \frac{\partial^{2} \overline{u_{i}}}{\partial x_{i} x_{j}}-\frac{\partial\left(\rho \overline{u_{i} u_{j}}\right)}{\partial x_{i}}+S_{t}$
where $\rho$ is the density of the fluid, $\overline{u_{i}}$ and $\overline{u_{j}}$ are the average velocities in different coordinate axes, $p$ is the pressure and $\rho \overline{u_{i} u_{j}}$ is the Reynolds stress. $S_{t}$ is the source term.

The governing equation of the renormalization $k-\varepsilon$ model is as follows:
$\rho \frac{d k}{d t}=\frac{\partial}{\partial x_{i}}\left[\left(\alpha_{k} \mu_{\mathrm{eff}}\right) \frac{\partial k}{\partial x_{i}}\right]+G_{k}+G_{b}-\rho \varepsilon-Y_{M}$
$\rho \frac{d \varepsilon}{d t}=\frac{\partial}{\partial x_{i}}\left[\left(\alpha_{g} \mu_{\mathrm{eff}}\right) \frac{\partial \varepsilon}{\partial x_{i}}\right]+C_{1 \varepsilon} \frac{\varepsilon}{k}\left(G_{k}+G_{3 g} G_{b}\right)-C_{2 s} \rho \frac{\varepsilon^{2}}{k}-R$
where $G_{k}$ is the turbulent kinetic energy due to the average velocity gradient, $G_{b}$ is the turbulent kinetic energy due to buoyancy and $Y_{M}$ is the effect of the fluctuating expansion of the compressible turbulent flow on the total dissipation rate. These parameters are the same as those in the standard $k-\varepsilon$ model. $\alpha_{k}$ and $\alpha_{\varepsilon}$ are reciprocals of the turbulent kinetic energy $k$ and the effective Prandtl number of the dissipation rate $\varepsilon$, respectively.

(b) $k_{a}=0.1$, the change of $\mu^{\prime}$ with $k_{h}$


Fig. 10 The distribution map of simulation data of the model and experimental data


Fig. 11 The computing domain of the CFD model

The computing domain was a circular tube ( 570 mm long, 19 mm diameter and 3 mm thickness). The center of the orifice was 38 mm away from the upstream flow. The medium in the computing domain was the water of $25^{\circ}$ Centigrade, and the boundary conditions at the import and the export of the model were both velocity inlet. The boundary condition at the orifice was pressure outlet. As the water at the outflow hole was exposed to the atmosphere, the pressure value was set to 0 . The boundary conditions at the piezometric surfaces of both sides were interior. The boundary conditions at other surfaces were set
as well, which was no slip walls with velocities of 0 . In the computing field, Hex grid was adopted at the piezometric surfaces of both sides and the two ends, while the rest of the area used Tet grid. The grid at the orifice was finer to make the description of flow conditions more precise.

### 1.6.3 Comparison of the simulation and experimental results

The experimental data and the simulation results under the same conditions of pressures and orifice areas are compared in the curve of $\mu-k_{v}$, as shown in Figs. $12,13,14$, 15 and 16. The curves of experimental data and simulation results fit well with a deviation of less than 10\% (except for Fig. 15). The experimental data curve trend was generally consistent with the simulation curve, which indicated that CFD could precisely describe the influence of velocities on the orifice discharge coefficient. Figure 15 presents the experimental data and results of the simulation have the biggest deviation. This is mainly because the surface tension and viscous force had a great effect on the outflow at the orifice when the orifice area ratio was $10 \%$.

## 2 Discussion and conclusion

To investigate the effect of velocities on the outflow rate, a piece of self-designed equipment was used to simulate an orifice outflow under different pressures, orifice areas, and shapes. After analysis, the relationships between outflow rate and pressure, velocities, orifice areas, and shapes were determined. The experimental results indicate that the discharge coefficient of an orifice ( $\mu$ ) could be affected by velocities, pressures, orifice areas, and orifice shape. Specifically, $\mu$ had a negative linear correlation with velocity, increased with orifice area and pressure, and the relationships tended to slow as $\mu$ increases. Oval orifice outflow Model I was obtained based on the experiment results.

Combined with the analysis of experiment results and data fitting, oval orifice outflow Model II was established, which could accurately describe the relationships between $\mu$ and pressure, velocity, and orifice area.

The effects of velocities on the outflow rate under different pressures and orifice areas could be obtained through CFD simulation to analyze the flow condition at

(a) $k_{h}=1.24$

(c) $k_{h}=1.76$

(b) $k_{h}=1.5$

(d) $k_{h}=3.6$

Fig. 12 The $\mu-k_{v}$ curve in the orifice area ratio of $10 \%$


Fig. 13 The $\mu-k_{v}$ curve in the orifice area ratio of $20 \%$


(a) $k_{h}=1.24$

(c) $k_{h}=1.76$

(b) $k_{h}=1.5$
都
Appendix: Experimental data and analysis results

| The shape of the crack | $Q_{\text {out }}$ | $Q_{\text {total }}$ | $v$ | $\mathrm{H}_{1}$ | $\mathrm{H}_{2}$ | H | $\mathrm{H}_{0}$ | $k_{\text {a }}$ | $\mu$ | $\frac{Q_{\text {out }}}{Q_{\text {total }}}$ | $\frac{A_{\text {out }} \sqrt{g H}}{Q_{\text {total }}}$ | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Oval | 27.07 | 27.07 | 0.096 | 2.36 | 2.34 | 2.35 | 2.39 | 0.2 | 0.697 | 1 | 1.42415 | 1802 |
|  | 26.16 | 41.82 | 0.148 | 2.36 | 2.34 | 2.35 | 2.43 | 0.2 | 0.669 | 0.625 | 0.92163 | 2784 |
|  | 25.69 | 53.29 | 0.188 | 2.35 | 2.35 | 2.35 | 2.47 | 0.2 | 0.651 | 0.482 | 0.72179 | 3548 |
|  | 24.42 | 75.86 | 0.268 | 2.36 | 2.34 | 2.35 | 2.62 | 0.2 | 0.602 | 0.322 | 0.50815 | 5050 |
|  | 23.96 | 86.79 | 0.306 | 2.37 | 2.33 | 2.35 | 2.71 | 0.2 | 0.58 | 0.276 | 0.44507 | 5778 |
|  | 22.96 | 105.1 | 0.371 | 2.44 | 2.26 | 2.35 | 2.95 | 0.2 | 0.533 | 0.219 | 0.37309 | 6994 |
|  | 30.2 | 30.2 | 0.107 | 2.85 | 2.85 | 2.85 | 2.89 | 0.2 | 0.708 | 1 | 1.40265 | 2010 |
|  | 29.27 | 47.83 | 0.169 | 2.85 | 2.85 | 2.85 | 2.95 | 0.2 | 0.68 | 0.612 | 0.88556 | 3184 |
|  | 29.07 | 53.6 | 0.189 | 2.85 | 2.85 | 2.85 | 2.97 | 0.2 | 0.672 | 0.542 | 0.79031 | 3568 |
|  | 28.11 | 68.41 | 0.241 | 2.85 | 2.85 | 2.85 | 3.06 | 0.2 | 0.641 | 0.411 | 0.6192 | 4554 |
|  | 27.22 | 86.62 | 0.306 | 2.86 | 2.84 | 2.85 | 3.2 | 0.2 | 0.607 | 0.314 | 0.48988 | 5766 |
|  | 26.09 | 106.2 | 0.375 | 2.9 | 2.8 | 2.85 | 3.42 | 0.2 | 0.562 | 0.246 | 0.4024 | 7069 |
|  | 25.44 | 118.6 | 0.419 | 2.95 | 2.75 | 2.85 | 3.61 | 0.2 | 0.534 | 0.214 | 0.36326 | 7898 |
|  | 33.2 | 33.2 | 0.117 | 3.35 | 3.35 | 3.35 | 3.4 | 0.2 | 0.717 | 1 | 1.38331 | 2210 |
|  | 32.49 | 51.25 | 0.181 | 3.35 | 3.35 | 3.35 | 3.46 | 0.2 | 0.696 | 0.634 | 0.89608 | 3412 |
|  | 31.4 | 65.5 | 0.231 | 3.35 | 3.35 | 3.35 | 3.54 | 0.2 | 0.665 | 0.479 | 0.70116 | 4360 |
|  | 30.98 | 77.21 | 0.272 | 3.35 | 3.35 | 3.35 | 3.62 | 0.2 | 0.649 | 0.401 | 0.59478 | 5140 |
|  | 30.24 | 90.11 | 0.318 | 3.35 | 3.35 | 3.35 | 3.72 | 0.2 | 0.625 | 0.336 | 0.50967 | 5999 |
|  | 29.22 | 108.6 | 0.383 | 3.37 | 3.33 | 3.35 | 3.92 | 0.2 | 0.589 | 0.269 | 0.42419 | 7229 |
|  | 55.28 | 55.28 | 0.195 | 2.33 | 2.37 | 2.35 | 2.46 | 0.4 | 0.702 | 1 | 1.38579 | 3680 |
|  | 54.46 | 63.79 | 0.225 | 2.33 | 2.37 | 2.35 | 2.51 | 0.4 | 0.685 | 0.854 | 1.20083 | 4247 |
|  | 53.25 | 69.02 | 0.244 | 2.33 | 2.37 | 2.35 | 2.54 | 0.4 | 0.666 | 0.772 | 1.10989 | 4595 |
|  | 52.13 | 81.7 | 0.288 | 2.28 | 2.42 | 2.35 | 2.58 | 0.4 | 0.647 | 0.638 | 0.92746 | 5439 |
|  | 50.36 | 89.07 | 0.314 | 2.28 | 2.42 | 2.35 | 2.64 | 0.4 | 0.618 | 0.565 | 0.85078 | 5929 |
|  | 48.57 | 104.6 | 0.369 | 2.26 | 2.44 | 2.35 | 2.76 | 0.4 | 0.582 | 0.464 | 0.72121 | 6964 |
|  | 46.54 | 129.1 | 0.455 | 2.27 | 2.43 | 2.35 | 3.05 | 0.4 | 0.531 | 0.361 | 0.58583 | 8592 |
|  | 45.24 | 145.2 | 0.512 | 2.3 | 2.4 | 2.35 | 3.3 | 0.4 | 0.496 | 0.312 | 0.52411 | 9667 |


| The shape of the crack | $Q_{\text {out }}$ | $Q_{\text {total }}$ | $v$ | $H_{1}$ | $\mathrm{H}_{2}$ | H | $\mathrm{H}_{0}$ | $k_{\text {a }}$ | $\mu$ | $\frac{Q_{\text {out }}}{Q_{\text {total }}}$ | $\frac{A_{\text {out }} \sqrt{g H}}{Q_{\text {total }}}$ | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 37.87 | 213.5 | 0.753 | 2.45 | 2.25 | 2.35 | 4.68 | 0.4 | 0.349 | 0.177 | 0.36799 | 14,210 |
|  | 64.12 | 64.12 | 0.226 | 2.8 | 2.9 | 2.85 | 2.98 | 0.4 | 0.74 | 1 | 1.30957 | 4269 |
|  | 62.95 | 71.12 | 0.251 | 2.8 | 2.9 | 2.85 | 3.02 | 0.4 | 0.721 | 0.885 | 1.18074 | 4734 |
|  | 61.18 | 79.75 | 0.281 | 2.78 | 2.92 | 2.85 | 3.06 | 0.4 | 0.696 | 0.767 | 1.04924 | 5309 |
|  | 60.6 | 91.67 | 0.323 | 2.77 | 2.93 | 2.85 | 3.15 | 0.4 | 0.68 | 0.661 | 0.91113 | 6102 |
|  | 59.41 | 102.4 | 0.361 | 2.76 | 2.94 | 2.85 | 3.24 | 0.4 | 0.657 | 0.58 | 0.81412 | 6817 |
|  | 57.81 | 111.1 | 0.392 | 2.73 | 2.97 | 2.85 | 3.3 | 0.4 | 0.634 | 0.52 | 0.74648 | 7395 |
|  | 55.47 | 134 | 0.473 | 2.71 | 2.99 | 2.85 | 3.56 | 0.4 | 0.586 | 0.414 | 0.61659 | 8919 |
|  | 53.06 | 155.2 | 0.548 | 2.7 | 3 | 2.85 | 3.85 | 0.4 | 0.539 | 0.342 | 0.53126 | 10,333 |
|  | 44.77 | 224.5 | 0.792 | 2.85 | 2.85 | 2.85 | 5.32 | 0.4 | 0.387 | 0.199 | 0.37732 | 14,947 |
|  | 70.14 | 70.14 | 0.248 | 3.3 | 3.4 | 3.35 | 3.52 | 0.4 | 0.745 | 1 | 1.29973 | 4669 |
| Oval | 68.91 | 77.97 | 0.275 | 3.27 | 3.43 | 3.35 | 3.54 | 0.4 | 0.73 | 0.884 | 1.16384 | 5191 |
|  | 67.67 | 86.5 | 0.305 | 3.25 | 3.45 | 3.35 | 3.59 | 0.4 | 0.712 | 0.782 | 1.04585 | 5759 |
|  | 65.47 | 97.23 | 0.343 | 3.24 | 3.46 | 3.35 | 3.67 | 0.4 | 0.681 | 0.673 | 0.92903 | 6473 |
|  | 62.23 | 129.5 | 0.457 | 3.17 | 3.53 | 3.35 | 3.96 | 0.4 | 0.623 | 0.481 | 0.68989 | 8622 |
|  | 59.23 | 152.6 | 0.538 | 3.15 | 3.55 | 3.35 | 4.26 | 0.4 | 0.572 | 0.388 | 0.58379 | 10,156 |
|  | 56.13 | 176.8 | 0.624 | 3.17 | 3.53 | 3.35 | 4.68 | 0.4 | 0.517 | 0.317 | 0.50526 | 11,772 |
|  | 48.97 | 243.8 | 0.86 | 3.35 | 3.35 | 3.35 | 6.29 | 0.4 | 0.389 | 0.201 | 0.37668 | 16,233 |
|  | 84.79 | 84.79 | 0.299 | 2.26 | 2.44 | 2.35 | 2.58 | 0.6 | 0.701 | 1 | 1.33468 | 5644 |
|  | 83.71 | 91.21 | 0.322 | 2.25 | 2.45 | 2.35 | 2.63 | 0.6 | 0.686 | 0.918 | 1.23797 | 6072 |
|  | 82.05 | 96.59 | 0.341 | 2.24 | 2.46 | 2.35 | 2.67 | 0.6 | 0.667 | 0.849 | 1.16645 | 6430 |
|  | 79.87 | 107.1 | 0.378 | 2.23 | 2.47 | 2.35 | 2.76 | 0.6 | 0.639 | 0.746 | 1.04959 | 7130 |
|  | 76.03 | 127 | 0.448 | 2.16 | 2.54 | 2.35 | 2.92 | 0.6 | 0.591 | 0.599 | 0.87111 | 8455 |
|  | 71.97 | 147.7 | 0.521 | 2.1 | 2.6 | 2.35 | 3.14 | 0.6 | 0.54 | 0.487 | 0.73868 | 9831 |
|  | 63.56 | 185.9 | 0.656 | 2.13 | 2.57 | 2.35 | 3.8 | 0.6 | 0.433 | 0.342 | 0.59096 | 12,376 |
|  | 53.83 | 242.3 | 0.855 | 2.26 | 2.44 | 2.35 | 5.16 | 0.6 | 0.315 | 0.222 | 0.46705 | 16,130 |
|  | 92.87 | 92.87 | 0.328 | 2.74 | 2.96 | 2.85 | 3.13 | 0.6 | 0.697 | 1 | 1.34175 | 6182 |
|  | 91 | 101.1 | 0.357 | 2.7 | 3 | 2.85 | 3.17 | 0.6 | 0.679 | 0.9 | 1.22349 | 6730 |
|  | 89.64 | 110.6 | 0.39 | 2.65 | 3.05 | 2.85 | 3.22 | 0.6 | 0.664 | 0.81 | 1.10795 | 7363 |
|  | 86.17 | 125.2 | 0.442 | 2.62 | 3.08 | 2.85 | 3.35 | 0.6 | 0.625 | 0.688 | 0.97313 | 8335 |
|  | 81.74 | 146 | 0.515 | 2.57 | 3.13 | 2.85 | 3.58 | 0.6 | 0.574 | 0.56 | 0.82635 | 9722 |
|  | 74.97 | 174.1 | 0.614 | 2.5 | 3.2 | 2.85 | 3.96 | 0.6 | 0.5 | 0.431 | 0.68379 | 11,587 |
|  | 68.57 | 204.9 | 0.723 | 2.55 | 3.15 | 2.85 | 4.6 | 0.6 | 0.425 | 0.335 | 0.58667 | 13640 |
|  | 57.24 | 287.2 | 1.014 | 2.84 | 2.86 | 2.85 | 6.95 | 0.6 | 0.288 | 0.199 | 0.44167 | 19,121 |


| The shape of the crack | $Q_{\text {out }}$ | $Q_{\text {total }}$ | $v$ | $H_{1}$ | $\mathrm{H}_{2}$ | H | $\mathrm{H}_{0}$ | $k_{\text {a }}$ | $\mu$ | $\frac{Q_{\text {out }}}{Q_{\text {total }}}$ | $\frac{A_{\text {out }} \sqrt{g H}}{Q_{\text {total }}}$ | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 102.4 | 102.4 | 0.361 | 3.23 | 3.47 | 3.35 | 3.71 | 0.6 | 0.706 | 1 | 1.32146 | 6815 |
|  | 98.47 | 117.5 | 0.414 | 3.14 | 3.56 | 3.35 | 3.78 | 0.6 | 0.673 | 0.838 | 1.13561 | 7819 |
|  | 95.73 | 129.5 | 0.457 | 3.07 | 3.63 | 3.35 | 3.86 | 0.6 | 0.647 | 0.739 | 1.01822 | 8623 |
|  | 93.06 | 142.1 | 0.502 | 3.05 | 3.65 | 3.35 | 4.01 | 0.6 | 0.618 | 0.655 | 0.92493 | 9462 |
|  | 87.96 | 166.1 | 0.586 | 2.99 | 3.71 | 3.35 | 4.31 | 0.6 | 0.563 | 0.53 | 0.78364 | 11,058 |
|  | 80 | 199.6 | 0.704 | 2.96 | 3.74 | 3.35 | 4.9 | 0.6 | 0.48 | 0.401 | 0.64885 | 13,287 |
|  | 72.1 | 241.3 | 0.852 | 3.05 | 3.65 | 3.35 | 5.92 | 0.6 | 0.394 | 0.299 | 0.54473 | 16,066 |
|  | 61.93 | 310.2 | 1.095 | 3.28 | 3.42 | 3.35 | 8.1 | 0.6 | 0.289 | 0.2 |  | 20,650 |
|  | 101.3 | 101.3 | 0.357 | 2.2 | 2.5 | 2.35 | 2.67 | 0.8 | 0.617 | 1 | 1.47007 | 6741 |
|  | 97.87 | 113.2 | 0.4 | 2.15 | 2.55 | 2.35 | 2.75 | 0.8 | 0.589 | 0.864 | 1.29961 | 7538 |
|  | 93.18 | 126.9 | 0.448 | 2.06 | 2.64 | 2.35 | 2.82 | 0.8 | 0.553 | 0.734 | 1.13534 | 8447 |
|  | 88.61 | 143.3 | 0.506 | 2.04 | 2.66 | 2.35 | 3.01 | 0.8 | 0.508 | 0.618 | 1.00005 | 9543 |
|  | 82.9 | 162.2 | 0.573 | 1.97 | 2.73 | 2.35 | 3.23 | 0.8 | 0.459 | 0.511 | 0.86825 | 10,801 |
|  | 73.63 | 190.9 | 0.674 | 1.97 | 2.73 | 2.35 | 3.74 | 0.8 | 0.379 | 0.386 | 0.73791 | 12,709 |
|  | 60.83 | 242.2 | 0.855 | 2.14 | 2.56 | 2.35 | 5.03 | 0.8 | 0.27 | 0.251 | 0.60628 | 16,122 |
|  | 113.9 | 113.9 | 0.402 | 2.65 | 3.05 | 2.85 | 3.25 | 0.8 | 0.629 | 1 | 1.4349 | 7580 |
|  | 110 | 126.1 | 0.445 | 2.56 | 3.14 | 2.85 | 3.31 | 0.8 | 0.603 | 0.872 | 1.27316 | 8397 |
|  | 104.5 | 141.3 | 0.499 | 2.5 | 3.2 | 2.85 | 3.45 | 0.8 | 0.561 | 0.739 | 1.12272 | 9410 |
|  | 100.7 | 152.1 | 0.537 | 2.45 | 3.25 | 2.85 | 3.55 | 0.8 | 0.532 | 0.662 | 1.0329 | 10,125 |
|  | 96.57 | 163.9 | 0.578 | 2.43 | 3.27 | 2.85 | 3.72 | 0.8 | 0.499 | 0.589 | 0.95443 | 10,913 |
| Oval | 89.03 | 185.6 | 0.655 | 2.4 | 3.3 | 2.85 | 4.07 | 0.8 | 0.44 | 0.48 | 0.83779 | 12,355 |
|  | 84.1 | 200.8 | 0.709 | 2.38 | 3.32 | 2.85 | 4.34 | 0.8 | 0.402 | 0.419 | 0.77112 | 13,367 |
|  | 70.66 | 258.9 | 0.914 | 2.55 | 3.15 | 2.85 | 5.87 | 0.8 | 0.291 | 0.273 | 0.61894 | 17,238 |
|  | 125 | 125 | 0.441 | 3.1 | 3.6 | 3.35 | 3.83 | 0.8 | 0.636 | 1 | 1.41367 | 8322 |
|  | 120.2 | 138 | 0.487 | 3.03 | 3.67 | 3.35 | 3.93 | 0.8 | 0.604 | 0.871 | 1.26578 | 9188 |
|  | 115.9 | 150.2 | 0.53 | 2.95 | 3.75 | 3.35 | 4.02 | 0.8 | 0.576 | 0.772 | 1.14795 | 9997 |
|  | 109.8 | 167 | 0.589 | 2.85 | 3.85 | 3.35 | 4.19 | 0.8 | 0.534 | 0.657 | 1.01479 | 11,115 |
|  | 100.3 | 195.2 | 0.689 | 2.8 | 3.9 | 3.35 | 4.65 | 0.8 | 0.463 | 0.514 | 0.8602 | 12,997 |
|  | 90.86 | 225.9 | 0.797 | 2.76 | 3.94 | 3.35 | 5.27 | 0.8 | 0.394 | 0.402 | 0.73799 | 15,041 |
|  | 76.13 | 291.4 | 1.028 | 2.95 | 3.75 | 3.35 | 7.19 | 0.8 | 0.283 | 0.261 | 0.59159 | 19,399 |
| Semi-oval 1 | 29.9 | 29.9 | 0.106 | 2.85 | 2.85 | 2.85 | 2.89 | 0.2 | 0.701 | 1 | 1.41658 | 1991 |
|  | 29.4 | 35.34 | 0.125 | 2.85 | 2.85 | 2.85 | 2.91 | 0.2 | 0.687 | 0.832 | 1.1988 | 2352 |
|  | 29.07 | 41.37 | 0.146 | 2.85 | 2.85 | 2.85 | 2.92 | 0.2 | 0.678 | 0.703 | 1.02396 | 2754 |
|  | 28.37 | 51.84 | 0.183 | 2.84 | 2.86 | 2.85 | 2.95 | 0.2 | 0.658 | 0.547 | 0.81573 | 3451 |
|  | 27.57 | 63.53 | 0.224 | 2.84 | 2.86 | 2.85 | 3.02 | 0.2 | 0.633 | 0.434 | 0.66558 | 4229 |
|  | 26.07 | 84.01 | 0.296 | 2.85 | 2.85 | 2.85 | 3.17 | 0.2 | 0.584 | 0.31 | 0.50424 | 5592 |
|  | 24.17 | 109.6 | 0.387 | 2.89 | 2.81 | 2.85 | 3.45 | 0.2 | 0.519 | 0.221 | 0.38932 | 7294 |

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| The shape of the crack | $Q_{\text {out }}$ | $Q_{\text {total }}$ | $v$ | $H_{1}$ | $\mathrm{H}_{2}$ | H | $\mathrm{H}_{0}$ | $k_{\text {a }}$ | $\mu$ | $\frac{Q_{\text {out }}}{Q_{\text {total }}}$ | $\frac{A_{\text {out }} \sqrt{g H}}{Q_{\text {total }}}$ | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 22.63 | 136.1 | 0.48 | 2.95 | 2.75 | 2.85 | 3.82 | 0.2 | 0.461 | 0.166 | 0.31668 | 9060 |
|  | 32.97 | 32.97 | 0.116 | 3.35 | 3.35 | 3.35 | 3.4 | 0.2 | 0.712 | 1 | 1.39303 | 2195 |
|  | 32.2 | 39.17 | 0.138 | 3.35 | 3.35 | 3.35 | 3.41 | 0.2 | 0.695 | 0.822 | 1.17257 | 2607 |
|  | 31.7 | 47.73 | 0.168 | 3.35 | 3.35 | 3.35 | 3.44 | 0.2 | 0.681 | 0.664 | 0.96218 | 3177 |
|  | 31.4 | 52.97 | 0.187 | 3.34 | 3.36 | 3.35 | 3.46 | 0.2 | 0.673 | 0.593 | 0.86575 | 3526 |
|  | 30.54 | 66.97 | 0.236 | 3.33 | 3.37 | 3.35 | 3.53 | 0.2 | 0.648 | 0.456 | 0.68368 | 4458 |
|  | 29.7 | 78.07 | 0.275 | 3.33 | 3.37 | 3.35 | 3.6 | 0.2 | 0.624 | 0.38 | 0.58653 | 5197 |
|  | 28.23 | 95.18 | 0.336 | 3.35 | 3.35 | 3.35 | 3.76 | 0.2 | 0.58 | 0.297 | 0.48251 | 6336 |
|  | 27.3 | 110.1 | 0.388 | 3.36 | 3.34 | 3.35 | 3.92 | 0.2 | 0.549 | 0.248 | 0.41785 | 7328 |
|  | 24.37 | 157.6 | 0.556 | 3.55 | 3.15 | 3.35 | 4.74 | 0.2 | 0.446 | 0.155 | 0.30004 | 10,489 |
|  | 35.4 | 35.4 | 0.125 | 3.85 | 3.85 | 3.85 | 3.91 | 0.2 | 0.713 | 1 | 1.3907 | 2357 |
|  | 34.93 | 43.43 | 0.153 | 3.85 | 3.85 | 3.85 | 3.93 | 0.2 | 0.702 | 0.804 | 1.13358 | 2891 |
|  | 34.4 | 54.4 | 0.192 | 3.84 | 3.86 | 3.85 | 3.97 | 0.2 | 0.688 | 0.632 | 0.90382 | 3622 |
|  | 32.58 | 73.24 | 0.258 | 3.82 | 3.88 | 3.85 | 4.06 | 0.2 | 0.645 | 0.445 | 0.6696 | 4876 |
|  | 31.07 | 95.87 | 0.338 | 3.84 | 3.86 | 3.85 | 4.26 | 0.2 | 0.6 | 0.324 | 0.51287 | 6382 |
|  | 28.6 | 128.9 | 0.455 | 3.87 | 3.83 | 3.85 | 4.65 | 0.2 | 0.528 | 0.222 | 0.38286 | 8583 |
|  | 27.33 | 146.4 | 0.517 | 3.95 | 3.75 | 3.85 | 4.97 | 0.2 | 0.489 | 0.187 | 0.34064 | 9746 |
|  | 43.16 | 43.16 | 0.152 | 2.85 | 2.85 | 2.85 | 2.93 | 0.3 | 0.67 | 1 | 1.47235 | 2873 |
|  | 42.42 | 51.79 | 0.183 | 2.84 | 2.86 | 2.85 | 2.95 | 0.3 | 0.656 | 0.819 | 1.22474 | 3448 |
|  | 41.47 | 62.8 | 0.222 | 2.83 | 2.87 | 2.85 | 3 | 0.3 | 0.636 | 0.66 | 1.00825 | 4181 |
|  | 39 | 84.6 | 0.299 | 2.8 | 2.9 | 2.85 | 3.12 | 0.3 | 0.586 | 0.461 | 0.74444 | 5632 |
|  | 36.04 | 110.8 | 0.391 | 2.84 | 2.86 | 2.85 | 3.41 | 0.3 | 0.519 | 0.325 | 0.57239 | 7377 |
|  | 32.71 | 143.1 | 0.505 | 2.86 | 2.84 | 2.85 | 3.83 | 0.3 | 0.444 | 0.229 | 0.4449 | 9524 |
|  | 47.09 | 47.09 | 0.166 | 3.35 | 3.35 | 3.35 | 3.44 | 0.3 | 0.674 | 1 | 1.46295 | 3135 |
|  | 46.22 | 54.32 | 0.192 | 3.33 | 3.37 | 3.35 | 3.46 | 0.3 | 0.661 | 0.851 | 1.26436 | 3616 |
|  | 45.44 | 59.44 | 0.21 | 3.3 | 3.4 | 3.35 | 3.45 | 0.3 | 0.65 | 0.765 | 1.15026 | 3957 |
| Semi-oval 1 | 43.87 | 75.74 | 0.267 | 3.3 | 3.4 | 3.35 | 3.55 | 0.3 | 0.618 | 0.579 | 0.90277 | 5042 |
|  | 41.09 | 99.03 | 0.349 | 3.3 | 3.4 | 3.35 | 3.75 | 0.3 | 0.564 | 0.415 | 0.69042 | 6593 |
|  | 38.47 | 122.3 | 0.432 | 3.3 | 3.4 | 3.35 | 4 | 0.3 | 0.511 | 0.315 | 0.55908 | 8141 |
|  | 35.13 | 158.8 | 0.56 | 3.36 | 3.34 | 3.35 | 4.57 | 0.3 | 0.437 | 0.221 | 0.43437 | 10574 |
|  | 50.09 | 50.09 | 0.177 | 3.85 | 3.85 | 3.85 | 3.96 | 0.3 | 0.669 | 1 | 1.4744 | 3334 |
|  | 49.62 | 59.76 | 0.211 | 3.8 | 3.9 | 3.85 | 3.95 | 0.3 | 0.663 | 0.83 | 1.22783 | 3978 |
|  | 49.2 | 66.2 | 0.234 | 3.77 | 3.93 | 3.85 | 3.96 | 0.3 | 0.657 | 0.743 | 1.10395 | 4407 |
|  | 47.27 | 82.97 | 0.293 | 3.76 | 3.94 | 3.85 | 4.07 | 0.3 | 0.623 | 0.57 | 0.87967 | 5523 |
|  | 45.16 | 100.5 | 0.355 | 3.75 | 3.95 | 3.85 | 4.21 | 0.3 | 0.584 | 0.449 | 0.72528 | 6690 |
|  | 42.04 | 128.8 | 0.455 | 3.77 | 3.93 | 3.85 | 4.55 | 0.3 | 0.524 | 0.326 | 0.56731 | 8575 |
|  | 38.36 | 163.9 | 0.578 | 3.86 | 3.84 | 3.85 | 5.15 | 0.3 | 0.449 | 0.234 | 0.45122 | 10,910 |


| The shape of the crack | $Q_{\text {out }}$ | $Q_{\text {total }}$ | $v$ | $H_{1}$ | $\mathrm{H}_{2}$ | H | $\mathrm{H}_{0}$ | $k_{\text {a }}$ | $\mu$ | $\frac{Q_{\text {out }}}{Q_{\text {total }}}$ | $\frac{A_{\text {out }} \sqrt{g H}}{Q_{\text {total }}}$ | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 60.93 | 60.93 | 0.215 | 2.89 | 2.81 | 2.85 | 3.05 | 0.4 | 0.695 | 1 | 1.4001 | 4056 |
|  | 58.13 | 78.16 | 0.276 | 2.84 | 2.86 | 2.85 | 3.11 | 0.4 | 0.657 | 0.744 | 1.08198 | 5203 |
|  | 55.69 | 91.49 | 0.323 | 2.8 | 2.9 | 2.85 | 3.18 | 0.4 | 0.622 | 0.609 | 0.91783 | 6091 |
|  | 50.67 | 123.6 | 0.436 | 2.77 | 2.93 | 2.85 | 3.49 | 0.4 | 0.541 | 0.41 | 0.67573 | 8228 |
|  | 49.2 | 134.6 | 0.475 | 2.76 | 2.94 | 2.85 | 3.62 | 0.4 | 0.516 | 0.365 | 0.61923 | 8963 |
|  | 45.64 | 160.2 | 0.565 | 2.8 | 2.9 | 2.85 | 4.03 | 0.4 | 0.453 | 0.285 | 0.52403 | 10668 |
|  | 65.53 | 65.53 | 0.231 | 3.33 | 3.37 | 3.35 | 3.52 | 0.4 | 0.696 | 1 | 1.39741 | 4363 |
|  | 62.8 | 78.07 | 0.275 | 3.27 | 3.43 | 3.35 | 3.54 | 0.4 | 0.665 | 0.804 | 1.16239 | 5197 |
|  | 61.69 | 88.46 | 0.312 | 3.25 | 3.45 | 3.35 | 3.6 | 0.4 | 0.648 | 0.697 | 1.02276 | 5889 |
|  | 58.29 | 105.6 | 0.373 | 3.25 | 3.45 | 3.35 | 3.76 | 0.4 | 0.599 | 0.552 | 0.85646 | 7032 |
|  | 54.56 | 130.7 | 0.461 | 3.2 | 3.5 | 3.35 | 4 | 0.4 | 0.543 | 0.418 | 0.68708 | 8698 |
|  | 51.56 | 153.2 | 0.541 | 3.23 | 3.47 | 3.35 | 4.35 | 0.4 | 0.493 | 0.336 | 0.58862 | 10200 |
|  | 48.96 | 175.7 | 0.62 | 3.25 | 3.45 | 3.35 | 4.74 | 0.4 | 0.448 | 0.279 | 0.51492 | 11,696 |
|  | 69.67 | 69.67 | 0.246 | 3.83 | 3.87 | 3.85 | 4.04 | 0.4 | 0.69 | 1 | 1.40974 | 4638 |
|  | 66.07 | 89.93 | 0.317 | 3.75 | 3.95 | 3.85 | 4.12 | 0.4 | 0.649 | 0.735 | 1.08059 | 5987 |
|  | 63.42 | 105.9 | 0.374 | 3.73 | 3.97 | 3.85 | 4.25 | 0.4 | 0.613 | 0.599 | 0.91562 | 7047 |
|  | 60.47 | 124.7 | 0.44 | 3.7 | 4 | 3.85 | 4.43 | 0.4 | 0.573 | 0.485 | 0.7741 | 8301 |
|  | 57.87 | 142.5 | 0.503 | 3.7 | 4 | 3.85 | 4.66 | 0.4 | 0.534 | 0.406 | 0.67739 | 9487 |
|  | 53.2 | 179.1 | 0.632 | 3.73 | 3.97 | 3.85 | 5.28 | 0.4 | 0.461 | 0.297 | 0.54117 | 11923 |
| Semi-oval 2 | 60.76 | 60.76 | 0.214 | 2.83 | 2.87 | 2.85 | 2.99 | 0.4 | 0.7 | 1 | 1.38954 | 4045 |
|  | 59.44 | 70.22 | 0.248 | 2.8 | 2.9 | 2.85 | 3.02 | 0.4 | 0.682 | 0.847 | 1.19591 | 4674 |
|  | 57.53 | 86.04 | 0.304 | 2.76 | 2.94 | 2.85 | 3.09 | 0.4 | 0.652 | 0.669 | 0.96904 | 5727 |
|  | 55.04 | 105 | 0.37 | 2.75 | 2.95 | 2.85 | 3.26 | 0.4 | 0.608 | 0.524 | 0.79268 | 6989 |
|  | 49.33 | 145.7 | 0.514 | 2.75 | 2.95 | 2.85 | 3.76 | 0.4 | 0.507 | 0.339 | 0.57129 | 9697 |
|  | 45.91 | 171.2 | 0.604 | 2.78 | 2.92 | 2.85 | 4.19 | 0.4 | 0.447 | 0.268 | 0.48881 | 11395 |
|  | 67.13 | 67.13 | 0.237 | 3.33 | 3.37 | 3.35 | 3.53 | 0.4 | 0.712 | 1 | 1.36411 | 4469 |
|  | 63.96 | 83.29 | 0.294 | 3.26 | 3.44 | 3.35 | 3.57 | 0.4 | 0.674 | 0.768 | 1.08789 | 5545 |
|  | 62.42 | 95.99 | 0.339 | 3.25 | 3.45 | 3.35 | 3.67 | 0.4 | 0.649 | 0.65 | 0.94247 | 6390 |
|  | 58.04 | 127.5 | 0.45 | 3.2 | 3.5 | 3.35 | 3.96 | 0.4 | 0.581 | 0.455 | 0.70399 | 8489 |
|  | 55.33 | 148.8 | 0.525 | 3.2 | 3.5 | 3.35 | 4.25 | 0.4 | 0.535 | 0.372 | 0.60311 | 9909 |
|  | 50.02 | 191.5 | 0.676 | 3.25 | 3.45 | 3.35 | 5.03 | 0.4 | 0.444 | 0.261 | 0.4725 | 12747 |
|  | 71.07 | 71.07 | 0.251 | 3.8 | 3.9 | 3.85 | 4.02 | 0.4 | 0.706 | 1 | 1.37655 | 4731 |
| Semi-oval 2 | 69.02 | 83.19 | 0.294 | 3.77 | 3.93 | 3.85 | 4.08 | 0.4 | 0.681 | 0.83 | 1.17125 | 5538 |
|  | 66.87 | 100.9 | 0.356 | 3.73 | 3.97 | 3.85 | 4.2 | 0.4 | 0.65 | 0.663 | 0.9609 | 6715 |
|  | 63.84 | 123.5 | 0.436 | 3.7 | 4 | 3.85 | 4.41 | 0.4 | 0.606 | 0.517 | 0.78176 | 8220 |
|  | 60.13 | 150.3 | 0.53 | 3.65 | 4.05 | 3.85 | 4.73 | 0.4 | 0.551 | 0.4 | 0.63776 | 10008 |
|  | 55.58 | 188.9 | 0.667 | 3.7 | 4 | 3.85 | 5.43 | 0.4 | 0.475 | 0.294 | 0.5109 | 12578 |

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| The shape of the crack | $Q_{\text {out }}$ | $Q_{\text {total }}$ | $v$ | $\mathrm{H}_{1}$ | $\mathrm{H}_{2}$ | H | $\mathrm{H}_{0}$ | $k_{\text {a }}$ | $\mu$ | $\frac{Q_{\text {out }}}{Q_{\text {totat }}}$ | $\frac{A_{\text {out }} \sqrt{g H}}{Q_{\text {total }}}$ | Re |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hole | 0.8 | 20.67 | 0.073 | 7.55 | 7.55 | 7.55 | 7.57 | 0.00277 | 0.837 | 0.039 | 0.0462 | 1376 |
|  | 0.797 | 40.63 | 0.143 | 7.6 | 7.5 | 7.55 | 7.67 | 0.00277 | 0.828 | 0.02 | 0.02358 | 2705 |
|  | 0.79 | 74.26 | 0.262 | 7.65 | 7.45 | 7.55 | 7.89 | 0.00277 | 0.81 | 0.011 | 0.01294 | 4944 |
|  | 0.787 | 102.8 | 0.363 | 7.74 | 7.36 | 7.55 | 8.23 | 0.00277 | 0.789 | 0.008 | 0.00941 | 6843 |
|  | 0.778 | 145.2 | 0.512 | 7.86 | 7.24 | 7.55 | 8.86 | 0.00277 | 0.752 | 0.005 | 0.00671 | 9664 |
|  | 0.769 | 173.9 | 0.614 | 7.95 | 7.15 | 7.55 | 9.41 | 0.00277 | 0.721 | 0.004 | 0.00563 | 11577 |
|  | 0.854 | 28.98 | 0.102 | 8.36 | 8.34 | 8.35 | 8.4 | 0.00277 | 0.848 | 0.029 | 0.03467 | 1930 |
|  | 0.848 | 61.08 | 0.216 | 8.45 | 8.25 | 8.35 | 8.61 | 0.00277 | 0.831 | 0.014 | 0.01654 | 4066 |
|  | 0.842 | 92.22 | 0.325 | 8.5 | 8.2 | 8.35 | 8.89 | 0.00277 | 0.813 | 0.009 | 0.01099 | 6139 |
|  | 0.839 | 111.1 | 0.392 | 8.55 | 8.15 | 8.35 | 9.12 | 0.00277 | 0.799 | 0.008 | 0.00915 | 7397 |
|  | 0.832 | 145.1 | 0.512 | 8.67 | 8.03 | 8.35 | 9.67 | 0.00277 | 0.77 | 0.006 | 0.00705 | 9662 |
|  | 0.826 | 173.2 | 0.611 | 8.75 | 7.95 | 8.35 | 10.2 | 0.00277 | 0.744 | 0.005 | 0.00594 | 11527 |
|  | 0.91 | 28.71 | 0.101 | 9.37 | 9.33 | 9.35 | 9.41 | 0.00277 | 0.853 | 0.032 | 0.03705 | 1911 |
|  | 0.905 | 64.14 | 0.226 | 9.45 | 9.25 | 9.35 | 9.63 | 0.00277 | 0.839 | 0.014 | 0.01666 | 4270 |
|  | 0.9 | 99.5 | 0.351 | 9.54 | 9.16 | 9.35 | 9.99 | 0.00277 | 0.819 | 0.009 | 0.01079 | 6624 |
|  | 0.894 | 126.8 | 0.447 | 9.6 | 9.1 | 9.35 | 10.4 | 0.00277 | 0.8 | 0.007 | 0.00849 | 8439 |
|  | 0.887 | 167.2 | 0.59 | 9.75 | 8.95 | 9.35 | 11.1 | 0.00277 | 0.766 | 0.005 | 0.00649 | 11,128 |
|  | 2.546 | 21.41 | 0.076 | 6.86 | 6.84 | 6.85 | 6.88 | 0.011 | 0.698 | 0.119 | 0.17003 | 1426 |
|  | 2.52 | 44.32 | 0.156 | 6.87 | 6.83 | 6.85 | 6.95 | 0.011 | 0.688 | 0.057 | 0.0822 | 2951 |
|  | 2.487 | 67.78 | 0.239 | 6.95 | 6.75 | 6.85 | 7.15 | 0.011 | 0.669 | 0.037 | 0.05407 | 4512 |
|  | 2.429 | 101.4 | 0.358 | 7 | 6.7 | 6.85 | 7.47 | 0.011 | 0.639 | 0.024 | 0.03627 | 6751 |
|  | 2.414 | 132.4 | 0.467 | 7.1 | 6.6 | 6.85 | 7.93 | 0.011 | 0.617 | 0.018 | 0.02798 | 8812 |
|  | 2.369 | 177.9 | 0.628 | 7.24 | 6.46 | 6.85 | 8.77 | 0.011 | 0.575 | 0.013 | 0.02103 | 11843 |
|  | 2.641 | 32.84 | 0.116 | 7.57 | 7.53 | 7.55 | 7.62 | 0.011 | 0.688 | 0.08 | 0.11646 | 2186 |
|  | 2.612 | 45.51 | 0.161 | 7.59 | 7.51 | 7.55 | 7.68 | 0.011 | 0.678 | 0.057 | 0.08415 | 3030 |
|  | 2.58 | 60.28 | 0.213 | 7.64 | 7.46 | 7.55 | 7.8 | 0.011 | 0.665 | 0.043 | 0.06375 | 4013 |
|  | 2.571 | 70.8 | 0.25 | 7.65 | 7.45 | 7.55 | 7.87 | 0.011 | 0.659 | 0.036 | 0.0543 | 4714 |
|  | 2.562 | 84.83 | 0.299 | 7.66 | 7.44 | 7.55 | 7.98 | 0.011 | 0.652 | 0.03 | 0.04536 | 5647 |
|  | 2.545 | 95.08 | 0.336 | 7.67 | 7.43 | 7.55 | 8.08 | 0.011 | 0.644 | 0.027 | 0.04049 | 6330 |
|  | 2.509 | 136 | 0.48 | 7.8 | 7.3 | 7.55 | 8.67 | 0.011 | 0.613 | 0.018 | 0.02854 | 9054 |
|  | 2.825 | 28.93 | 0.102 | 8.37 | 8.33 | 8.35 | 8.41 | 0.011 | 0.701 | 0.098 | 0.13904 | 1926 |
|  | 2.812 | 43.95 | 0.155 | 8.4 | 8.3 | 8.35 | 8.48 | 0.011 | 0.695 | 0.064 | 0.09168 | 2926 |
|  | 2.772 | 77.54 | 0.274 | 8.45 | 8.25 | 8.35 | 8.72 | 0.011 | 0.675 | 0.036 | 0.05212 | 5162 |
|  | 2.718 | 117.4 | 0.414 | 8.55 | 8.15 | 8.35 | 9.19 | 0.011 | 0.645 | 0.023 | 0.03463 | 7814 |
|  | 2.63 | 187.8 | 0.663 | 8.75 | 7.95 | 8.35 | 10.5 | 0.011 | 0.585 | 0.014 | 0.0219 | 12,501 |

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