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Fuzzy inventory model for NVOCC's returnable containers under empty container repositioning with leasing option

S. Rajeswari¹ · C. Sugapriya¹ · D. Nagarajan²

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Abstract

At present, the entire globe gets engaged in importing and exporting the products for promoting their business in which supply chain management is playing a vital role. The main aspect of any effective supply chain management is the transportation of cargoes. To avoid the damages of cargoes during transportation and for minimizing the cost, the returnable containers are used. The present research deals with an inventory model of Non-Vessel Operating Common Carrier (NVOCC) for returnable containers with price dependent demand under fuzzy environment. In this study, it is presumed that the import of cargoes is less than the export. The Empty Container Repositioning (ECR) and the leasing options are utilized to replace the deficit containers which prevent shortages. The proportion of the used containers returned, the proportion of the repositioned containers and the fraction of repairable from the returned containers are considered as Triangular Fuzzy Numbers [TFNs]. Fuzzy inventory model is framed for the purpose of attaining optimal length of the screening, the repositioning cycle and the leasing cycle which are used to minimize the expected total cost and the proposed model is illustrated with the numerical example. The sensitivity analysis is performed to show the effect of fuzziness of return rate, repositioning rate and repairable percentage along with the changes in parameters.

Keywords Inventory model \cdot Returnable containers \cdot Empty container repositioning \cdot Container leasing \cdot Fuzzy \cdot Price dependent demand

Introduction

In general, the supply chain management is an integrated process which includes several links such as transportation of goods, import, export etc. This research paper studies a closed-loop supply chain management which focuses on returnable containers used for storing and safe transportation of cargoes as stated by Kim and Glock [12]. According to Cobb [3], for the safe and the cost minimizing transportation process, the returnable containers are used numerous times.

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² Department of Mathematics, Hindustan Institute of Technology and Science, Chennai, Tamil Nadu, India A Non-Vessel Operating Common Carrier (NVOCC) is an ocean carrier which transports cargoes without operating vessels. They own their own fleet of containers and in some situations they also operate containers as freight forwarders.

This paper examines the issues encountered by the NVOCC for its returnable containers in a closed-loop supply chain. The NVOCC allows the shipper to use their returnable containers at some competitive rate and the empty containers are collected after the usage. The imbalance of cargo supply and the demand lead to imbalanced flow of containers over the various territories. In this study, it is presumed that the import of the cargoes is less than the exports which will lead to the scarcity of containers. So that, the Empty Container Repositioning (ECR) option and the leasing option are scrutinized in this model instead of purchasing new containers for deficit units that include scarcity as well as salvaged containers. The deficit containers are repositioned from dry port as well as from the shortest transshipment port and the number of ECR depends upon the slot allocation and the container leasing option is considered for some fraction of units that are unable to be repositioned. The costs incurred



for ECR are the handling charges per container at terminal point and the transportation charges of the reposition per container from the surplus area to the slack area. For uncertain situation, the fraction of return rate, the proportion of repairable units and the proportion of repositioned containers are considered as Triangular Fuzzy Numbers (TFNs). The inventory model is developed for screen, repair, reposition and lease of the containers with price dependent demand. The optimal screening length, the optimal repositioning cycle length and the optimal leasing cycle length are formulated which minimize the total cost under fuzzy environment. The present research paper helps the container

Table 1 Notations

- **d** Demand rate as a function of the renting price, say $\alpha \beta R_p$
- R_p Customers' rent price per container
- α Constant demand rate coefficient, $\alpha > 0$
- β Price dependent demand rate coefficient, $\beta > 0$
- λ Proportion of the containers returned, $0 \le \lambda \le 1$
- $\tilde{\lambda}$ Fuzzy proportion of the containers returned, where $\lambda = (\lambda_1, \lambda_2, \lambda_3)$
- ρ Proportion of restorable containers from returned units, $0 \le \rho \le 1$
- $\tilde{\rho}$ Fuzzy proportion of restorable containers from returned units, where $\tilde{\rho} = (\rho_1, \rho_2, \rho_3)$
- λ' Proportion of the containers repositioned, $0 \le \lambda' \le 1$
- λ' Fuzzy proportion of the containers repositioned, $\lambda' = (\lambda'_1, \lambda'_2, \lambda'_3)$
- K_i Fixed screening charge per screening lot
- K_r Fixed repairing charge for one mending lot
- K_{cr} Fixed charge for ECR per lot
- K_l Fixed ordering charge per order for leasing containers
- k_i Variable screening charge for one screened container
- k_r Variable repairing charge per mended container
- *k_h* Terminal Handling charge per repositioned container
- k_t Transportation charge per repositioned container
- k_l Rent for a leasing container
- s' Scrap price of a non-restorable container
- H_{u} Carrying charge for a returned container
- H_r Carrying charge for a restorable container
- $H_{\rm s}$ Carrying charge for a serviceable container
- H_{cr} Carrying charge for a repositioned container
- H_l Carrying charge for a leased container
- H_s Carrying charge for a serviceable container
- η Screening rate
- r Repair rate
- n Number of working days per year
- D Annual demand, where D = n.d
- T_i Screening period
- T_r Mending period
- T_{cr} Time between successive container repositioning processes
- T_l Time between successive leasing processes

The following notations will be used throughout the paper



management organizations to rectify their problems faced by the imbalance of container flow. Instead of purchasing new containers, the idea of utilizing ECR and leasing option in this paper will be very useful for NVOCC organizations. The idea of implementing fuzziness of some parameters will be resulted in more accuracy. Some of the research works related to this topic are discussed below.

Literature review

Schardy [20] introduced an inventory model for the repairable products and he computed the optimal number of purchasing products and repairing items. Kelle and Silver [11] established four types of predicting strategies for returning reusable containers. Mabini et al. [18] framed a model for regulating repairable products under two cases in which the mending space for the first case is infinite and the space for the second case is finite. Buchanan and Abad [1] developed an optimal inventory policy for a routine analysis of returnable containers under single period and N-period analysis. Clott [2] presented an inquiry of NVOCC and maritime improvement.

Yun et al. [22] designed the (s, S) inventory policy to ECR problem including the leasing option. Further, Kim and Glock [12] examined the use of RFID tracking system to manage the returnable containers by considering stochastic return rate. Glock and Kim [6] designed a supply chain inventory model by presuming to supply the finished lot to the customers along with RTIs. Subsequently, they extended the model with some preventive measures [7]. Hariga et al. [9] formulated a single vendor single retailer inventory model for the combined products and the returnable containers. They also incorporate the renting option of returnable containers for delay returns. The study of Cobb [3] initially analyzed the optimal time duration of screening, mending and purchasing process of returnable containers in which the screening and the mending operations proceed concurrently. Further, the research discussed the buffer stock provided by early returns. Recently, Fan et al. [4] have designed an inventory model of RTI which associates with the investment of the retailer for minimizing the loss percentage. Three kinds of issues of RTI under the closed-loop supply chain process are analyzed by Lakshmi et al. [13]. An optimization inventory decision model for the uncertain situations was framed by Hosseini and Sahlin [10], by utilizing the case-study of European Logistics Service Provider. Luo and Chang [18] designed an ECR inventory model for the intermodal transportation strategy by considering the model with collaboration and without collaboration between dry port and seaport. Göçen et al. [8] framed the cost optimization model to reposition the serviceable containers from surplus area to slack area. Currently, Liu et al. [15] have established a decision model for RTI managing organization which reinforces the regular planning process along with the sharing strategy.

For uncertain situations, many researchers of inventory and production management areas used to adopt the perception of fuzzy sets which was established by Zadeh [23]. An approach of expected value operator of fuzzy variable was conferred and a fuzzy simulation was framed to calculate the expected value [14]. Wang and Tian [21] proposed a paper to formulate the expected values and the variance of various fuzzy variables. Furthermore, an EOQ model with the substandard items, shortages, and screening errors under fuzzy situation was framed by Liu and Zheng [16]. An analysis of pentagonal fuzzy number along with its matrix was established by Panda and Pal [19]. Garai et al. [5] designed a fully fuzzy inventory model by presuming the demand as price sensitive and the carrying charge as time dependent.

Proposed methodology

Preliminaries

Let us first epitomize a few fundamental definitions which are adopted from [19].

Definition 1. Fuzzy set A fuzzy set Δ in the universal set *Y* is defined as a set of ordered pairs and it is expressed as

$$\Delta = \left(\mathbf{y}', \mu_{\Delta}(\mathbf{y}')\right) : \mathbf{y}' \in Y,$$

in which $\mu_{\Delta}(y')$ is a membership function of y' which assumes values in the range from 0 to 1 (ie.,) $\mu_{\Lambda}(y') \in [0,1]$.

Definition 2. Fuzzy number A fuzzy number Δ is a subset of real line *R*, which has the membership function μ_{Δ} fullfilling the given features:

- 1. $\mu_{\Delta}(y')$ is piecewise continuous in its domain.
- 2. Δ is normal, i.e., there is a $y'_0 \in \Delta$ such that $\mu_{\Delta}(y'_0) = 1$.

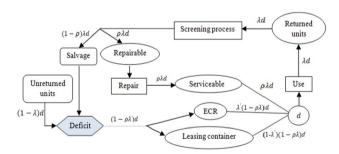


Fig. 1 Container flow per cycle

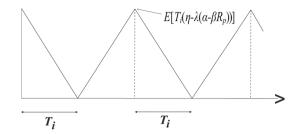


Fig. 2 Inventory of returned containers

3. Δ is convex, i.e., $\mu_{\Delta}(\epsilon y'_1 + (1 - \epsilon)y'_2) \ge min(\mu_{\Delta}(y'_1), \mu_{\Delta}(y'_2)) \forall y'_1, y'_2$ in *Y*.

Definition 3. Triangular Fuzzy number: A fuzzy number $\Delta = (\sigma_1, \sigma_2, \sigma_3)$ is considered as a triangular fuzzy number suppose it possesses the ensuing membership function (Table 1)

$$\mu_{\Delta}(\mathbf{y}') = \begin{cases} 0, & \mathbf{y}' < \sigma_1 \\ \frac{\mathbf{y}' - \sigma_1}{\sigma_2 - \sigma_1}, & \sigma_1 \le \mathbf{y}' \le \sigma_2 \\ 1, & \mathbf{y}' = \sigma_2 \\ \frac{\sigma_3 - \mathbf{y}'}{\sigma_3 - \sigma_2}, & \sigma_2 \le \mathbf{y}' \le \sigma_3 \\ 0, & \mathbf{y} > \sigma_3 \end{cases}.$$

Assumptions The problem formulation is developed under the following assumptions and some of the assumptions have been retained from [3]. It is considered that the demand rate of the container depends upon the customer rent price instead of constant demand rate. As the customer rent price increases, the demand rate decreases and vice-versa.

- 1. Rate of demand is price dependent.
- 2. Deficit of containers is considered.
- 3. The received containers are subject to inspection as well as repair process simultaneously and the rate of screening is more than the rate of mending process, $\eta > r$.
- 4. ECR and container leasing option for deficit units are considered.

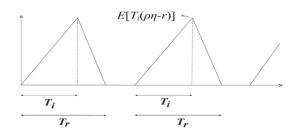


Fig. 3 Inventory of repairable containers

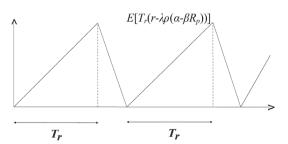
مدينة الملك عبدالعزيز KACST 🖄 للعلوم والتفنية KACST 5. Fuzzy model is proposed by scrutinizing the fraction of returned units, the proportion of repairable units and the proportion of repositioning units as TFNs.

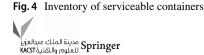
During the particular time period, NVOCC supplies the containers to its customer with the demand of d, which is considered as price dependent demand say, $d = \alpha - \beta R_n$. It is hypothesized that the import of the cargoes is less than the export, the proportion λd of containers is returned after the use and the remaining containers $(1 - \lambda)d$ being unreturned in preferred cycle. Once the used containers are received, the screening process and the mending process arise at constant rate and they proceed simultaneously. The time taken for screening process is T_i and it is found that the proportion $\rho \lambda d$ of the used containers are reusable while the left over $(1 - \rho)\lambda d$ containers are unable to be used again and so, such containers are sold at scrap price. The time taken to mend $\rho \lambda d$ is given as T_r . Once the mending process gets over, the $\rho \lambda d$ containers are stocked as serviceable ones. To replace the unreturned units and the salvaged containers, the ECR is considered and because of slot allocation issues, the NVOCC is unable to reposition all the deficit containers. So that, the fraction of deficit containers are presumed to be repositioned and the remaining units are leased from the local dealer. Thus, $\lambda'(1 - \lambda \rho)d$ containers are repositioned and the remaining $(1 - \lambda')(1 - \lambda \rho)d$ units are leased from the local dealer at some cost and stock them along with the serviceable containers to satisfy the customers' demand without shortages.

The number of containers screened during the time T_i is given as $I_i = \eta T_i$ and the repairing time duration is given as $T_r = \frac{\rho I_i}{r} = \frac{\rho \eta T_i}{r}$. It is clear that, the mending period T_r is a function of the screening period T_i .

Maximum inventory level

The maximum inventory level of all the categories such as returned containers on time, repairable containers, serviceable containers and leased containers are discussed below (Fig. 1).





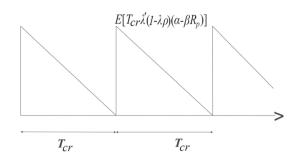


Fig. 5 Inventory of repositioned containers

Expected maximum inventory level of returned containers

The customer should return the empty containers to NVOCC's depot after the usage. According to the hypothesis of this study, only the proportion λd of containers is returned and the remaining containers $(1 - \lambda)d$ are being unreturned in preferred cycle. Once the used containers are received, the screening process and the mending process arise at constant rate and they proceed simultaneously. Figure 2 clearly indicates that the duration of screening process is T_i and the number of containers screened per length is ηT_i . Thus, the proportion of a year elapsed in each screening process is $\frac{\eta T_i}{\lambda D}$. The idle time between the screening process per unit time and the containers compiled over that time is $n \frac{\eta T_i}{\lambda D} - T_i$.

Expected maximum inventory of returned containers

$$= E\left[\left(n\frac{\eta T_i}{\lambda D} - T_i\right)\lambda d\right] = E\left[T_i\left(\eta - \lambda\left(\alpha - \beta R_p\right)\right)\right].$$
 (1)

At the time of screening process, the stock level depletes at a rate of $\eta - \lambda(\alpha - \beta R_p)$ per day and it compiles at a rate of $\lambda(\alpha - \beta R_p)$ per day when the length of the screening is idle.

Expected maximum inventory level of repairable containers

While observing Figs. 2 and 3, it is clear that, the screening process and the repairing process commence together. Once the screening of each container gets over, then it has been

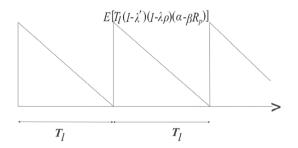


Fig. 6 Inventory of leased containers

sent to repairing process. So that, the inventory of repairable units increases as the inventory level of returned containers reduces. The time taken for mending procedure is considered as T_r . From the equation $T_r = \frac{\rho \eta T_i}{r}$, the number of containers sent for the repairing process is $\rho \eta T_i = rT_r$. During the screening period T_i , the rate of $\rho \eta - r$ containers compile in the inventory of repairable containers. Thus, the expected maximum inventory of the repairable containers, the number of days repaired per annum and the percentage of active repair per annum are given below.

Expected maximum inventory of repairable containers

$$= \mathbf{E} \left[T_i(\rho \eta - r) \right]. \tag{2}$$

A number of days for the repair process per annum = $\frac{\rho \eta D}{r}$. (3)

Percentage of active repair per annum = $\frac{\rho \eta D}{nr}$. (4)

During the time period $E[T_i(\rho\eta - r)]/r$, it is observed that the inventory of repairable container depletes at a rate of *r* per unit time.

Expected maximum inventory level of serviceable containers

When the repairing process gets over every container is stored as a serviceable one. Hence, the inventory level of repairable containers reduces as the inventory level serviceable containers increases. On analyzing Figs. 3 and 4, it is clear that the repair runs during the time T_r simultaneously the inventory of serviceable containers increased by $E[r - \rho\eta(\alpha - \beta R_p)]$ per unit time which depletes by the rate of $E[\rho\eta(\alpha - \beta R_p)]$ containers if the repairing time is idle. Thus,

The expected maximum inventory of repairable containers

$$= E \left[T_r \left(r - \rho \eta \left(\alpha - \beta R_p \right) \right) \right].$$
(5)

Expected maximum inventory level of repositioned containers

To satisfy the customers' demand *d* for each cycle, the unreturned units and the salvaged containers are restored. Initially, the ECR option is presumed to replace the deficit containers and because of slot allocation problem, all the deficit containers are unable to be repositioned. Thus, the fraction of deficit containers $\lambda'(1 - \lambda \rho)d$ is repositioned. From Fig. 5, it is observed that the time between the successive ECR cycles is T_{cr} and the maximum number of containers repositioned for each cycle is given below.

Expected maximum inventory of leased containers

$$= E \left[T_{cr} \lambda' (1 - \lambda \rho) \left(\alpha - \beta R_p \right) \right].$$
⁽⁶⁾

And hence, the container depletes at the rate of $E[\lambda'(1-\lambda\rho)(\alpha-\beta R_p)]$ containers per period.

Expected maximum inventory level of leased containers

The fractions of deficit container that are unable to reposition are leased from the local dealer at some cost. From the above figure, it is noted that the time between the successive leasing cycles is T_l and the maximum number of containers leased for each cycle is given below.

The expected maximum inventory of leased containers

$$= E[T_l(1 - \lambda')(1 - \lambda\rho)(\alpha - \beta R_p)].$$
⁽⁷⁾

It is clear that, the container depletes at the rate of $E[(1 - \lambda')(1 - \lambda\rho)(\alpha - \beta R_p)]$ containers per period (Fig. 6).

Cost model

The total cost for each cycle is the sum of the fixed cost, variable cost and the holding cost and it is derived as follows.

The fixed cost is obtained once from the inventory of returned containers, repairable containers repositioned containers and leased containers which is given as,

$$FC = \frac{\lambda D(K_i + K_r)}{\eta T_i} + \frac{nK_{cr}}{T_{cr}} + \frac{nK_l}{T_l}$$
$$= \frac{n\lambda(\alpha - \beta R_p)(K_i + K_r)}{\eta T_i} + \frac{nK_{cr}}{T_{cr}} + \frac{nK_l}{T_l}.$$
(8)

The variable costs are incurred from all the inventories which are constructed depend upon the quantity of containers processed. The charges incurred for repositioning per container are the terminal handling charge and the transportation charge. Thus, the variable cost is obtained as,

$$VC = \lambda D(k_i - s' + \rho(s' + k_r - \lambda' k_{cr} - (1 - \lambda')k_l))$$

+ $\lambda' D(k_t + 2k_h) + (1 - \lambda')Dk_l$
= $n\lambda(\alpha - \beta R_p)(k_i - s' + \rho(s' + k_r - \lambda'(k_t + 2k_h) - (1 - \lambda')k_l))$
+ $n\lambda'(\alpha - \beta R_p)(k_t + 2k_h) + n(1 - \lambda')(\alpha - \beta R_p)k_l.$ (9)

The carrying charge is the cost incurred by the empty depot for the inventory of returned containers, repairable containers, serviceable containers, repositioned containers and leased containers, which is derived as,



$$HC = \frac{T_i}{2} \left[\left(\eta - \lambda \left(\alpha - \beta R_p \right) \right) H_u + \frac{\lambda \left(\alpha - \beta R_p \right) \left(\eta \rho^2 - r \rho \right)}{r} \right] \\ H_r + \frac{\eta \left(r \rho - \rho^2 \lambda \left(\alpha - \beta R_p \right) \right)}{r} H_s \right] \\ + \frac{T_{cr}}{2} \left[\lambda' (1 - \lambda \rho) \left(\alpha - \beta R_p \right) H_{cr} \right] \\ + \frac{T_l}{2} \left[\left(1 - \lambda' \right) (1 - \lambda \rho) \left(\alpha - \beta R_p \right) H_l \right].$$

$$(10)$$

The total cost for each cycle is represented as,

$$TC(T_i, T_{cr}, T_l) = FC + VC + HC.$$
(11)

Therefore,

$$TC(T_{i}, T_{cr}, T_{l}) = \frac{n\lambda(\alpha - \beta R_{p})(K_{i} + K_{r})}{\eta T_{i}} + \frac{nK_{cr}}{T_{cr}} + \frac{nK_{l}}{T_{l}} + n\lambda(\alpha - \beta R_{p}) (k_{i}-s' + \rho(s' + k_{r} - \lambda'(k_{r} + 2k_{h}) - (1 - \lambda')k_{l})) + n\lambda'(\alpha - \beta R_{p})(k_{t} + 2k_{h}) + n(1 - \lambda')(\alpha - \beta R_{p})k_{l} + \frac{T_{i}}{2} \left[(\eta - \lambda(\alpha - \beta R_{p}))H_{u} + \frac{\lambda(\alpha - \beta R_{p})(\eta \rho^{2} - r\rho)}{r}H_{r} + \frac{\eta(r\rho - \rho^{2}\lambda(\alpha - \beta R_{p}))}{r}H_{s} \right] + \frac{T_{cr}}{2} \left[\lambda'(1 - \lambda\rho)(\alpha - \beta R_{p})H_{cr} \right] + \frac{T_{l}}{2} \left[(1 - \lambda')(1 - \lambda\rho)(\alpha - \beta R_{p})H_{l} \right].$$
(12)

The expected total cost is obtained as

$$E\left[\operatorname{TC}(T_{i}, T_{cr}, T_{l})\right] = \frac{nE[\lambda](\alpha - \beta R_{p})(K_{i} + K_{r})}{\eta T_{i}} + \frac{nK_{cr}}{T_{cr}} + \frac{nK_{l}}{T_{l}} + nE[\lambda](\alpha - \beta R_{p}) \left(k_{i} - s' + E[\rho](s' + k_{r} - E[\lambda'](k_{t} + 2k_{h}) - \left(1 - E[\lambda']\right)k_{l}\right)\right) + nE[\lambda'](\alpha - \beta R_{p})(k_{t} + 2k_{h}) + n\left(1 - E[\lambda']\right)(\alpha - \beta R_{p})k_{l} + \frac{T_{i}}{2}\left[(\eta - E[\lambda](\alpha - \beta R_{p}))H_{u} + \frac{E[\lambda](\alpha - \beta R_{p})(\eta E[\rho^{2}] - rE[\rho])}{r}H_{r} + \frac{\eta(rE[\rho] - E[\rho^{2}]E[\lambda](\alpha - \beta R_{p}))}{r}H_{s}\right] + \frac{T_{cr}}{2}\left[E[\lambda'](1 - E[\lambda]E[\rho])(\alpha - \beta R_{p})H_{cr}\right] + \frac{T_{i}}{2}\left[\left(1 - E[\lambda']\right)(1 - E[\lambda]E[\rho])(\alpha - \beta R_{p})H_{l}\right].$$
(13)

Fuzzy inventory model

The expected total cost in fuzzy sense is derived by considering the proportion of used containers returned, the proportion of repairable units from returned containers and the proportion of repositioned containers as fuzzy numbers. In this paper, they are considered as the Triangular Fuzzy Numbers, that is,

$$\widetilde{\boldsymbol{\lambda}} = \left(\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \boldsymbol{\lambda}_3\right), \widetilde{\boldsymbol{\rho}} = \left(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \boldsymbol{\rho}_3\right) \text{ and } \widetilde{\boldsymbol{\lambda}'} = (\boldsymbol{\lambda}_1', \boldsymbol{\lambda}_2', \boldsymbol{\lambda}_3').$$

Therefore, the expected total cost in fuzzy view, $E\left[TC(T_i, T_{cr}, T_l)\right]$ is written as,

$$E\left[TC(T_{i}, T_{cr}, T_{l})\right] = \frac{nE\left[\tilde{\lambda}\right](\alpha - \beta R_{p})(K_{i} + K_{r})}{\eta T_{i}} + \frac{nK_{cr}}{T_{cr}} + \frac{nK_{l}}{T_{l}} + nE\left[\tilde{\lambda}\right](\alpha - \beta R_{p}) \left(k_{i} - s' + E\left[\tilde{\rho}\right]\left(s' + k_{r} - E\left[\tilde{\lambda}'\right](k_{i} + 2k_{h})\right) - \left(1 - E\left[\tilde{\lambda}'\right]\right)k_{l}\right)\right) + nE\left[\tilde{\lambda}'\right](\alpha - \beta R_{p})(k_{i} + 2k_{h}) + n\left(1 - E\left[\tilde{\lambda}'\right]\right)(\alpha - \beta R_{p})k_{l} + \frac{T_{i}}{2}\left[\left(\eta - E\left[\tilde{\lambda}\right](\alpha - \beta R_{p})\right) H_{u} + \frac{E\left[\tilde{\lambda}\right](\alpha - \beta R_{p})\left(\eta E\left[\tilde{\rho}^{2}\right] - rE\left[\tilde{\rho}\right]\right)}{r} H_{r} + \frac{\eta\left(rE\left[\tilde{\rho}\right] - E\left[\tilde{\rho}^{2}\right]E\left[\tilde{\lambda}\right](\alpha - \beta R_{p})\right)}{r} H_{s} \right] + \frac{T_{cr}}{2}\left[E\left[\tilde{\lambda}'\right]\left(1 - E\left[\tilde{\lambda}\right]E\left[\tilde{\rho}\right]\right)(\alpha - \beta R_{p})H_{cr}\right] + \frac{T_{i}}{2}\left[\left(1 - E\left[\tilde{\lambda}'\right]\right)\left(1 - E\left[\tilde{\lambda}\right]E\left[\tilde{\rho}\right]\right)(\alpha - \beta R_{p})H_{l}\right].$$
(14)

Lemma 1 If $\tilde{\lambda}$ and $\tilde{\rho}$ is a triangular fuzzy number, where $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3), \quad \tilde{\rho} = (\rho_1, \rho_2, \rho_3) \text{ and } \lambda' = (\lambda'_1, \lambda'_2, \lambda'_3) \text{ then}$

$$E\left[\tilde{\lambda}\right] = \frac{\lambda_1 + 2\lambda_2 + \lambda_3}{4},\tag{15}$$

$$E\left[\stackrel{\sim}{\rho}\right] = \frac{\rho_1 + 2\rho_2 + \rho_3}{4},\tag{16}$$

$$E\begin{bmatrix}\tilde{\lambda}'\\\lambda'\end{bmatrix} = \frac{\lambda_1' + 2\lambda_2' + \lambda_3'}{4},\tag{17}$$

and

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$$E\left[\tilde{\rho}^{2}\right] = \frac{\rho_{1}^{2} + 2\rho_{2}^{2} + \rho_{3}^{2} + \rho_{1}\rho_{2} + \rho_{2}\rho_{3}}{6}.$$
 (18)

Proof Let λ_{1} , ρ_{1} and λ_{1}' be TFNs given as $\lambda = (\lambda_{1}, \lambda_{2}, \lambda_{3}), \rho = (\rho_{1}, \rho_{2}, \rho_{3})$ and $\lambda' = (\lambda_{1}', \lambda_{2}', \lambda_{3}')$ with the membership function and the credibility distribution

$$\mu_{\lambda}^{-}(x) = \begin{cases} 0, & x < \lambda_{1} \\ L_{1}(x) = \frac{x - \lambda_{1}}{\lambda_{2} - \lambda_{1}}, & \lambda_{1} \le x \le \lambda_{2} \\ 1, & x = \lambda_{2} \\ R_{1}(x) = \frac{\lambda_{3} - x}{\lambda_{3} - \lambda_{2}}, & \lambda_{2} \le x \le \lambda_{3} \\ 0, & x > \lambda_{3} \end{cases}$$

and $\Omega_{\lambda}^{-}(x) = \begin{cases} 0x < \lambda_{1} \\ \frac{x - \lambda_{1}}{2(\lambda_{2} - \lambda_{1})}, & \lambda_{1} \le x \le \lambda_{2} \\ 1 - \frac{\lambda_{3} - x}{2(\lambda_{3} - \lambda_{2})}, & \lambda_{2} \le x \le \lambda_{3} \\ 1, x \ge \lambda_{3} \end{cases}$;

$$\mu_{\rho}(x) = \begin{cases} 0, & x < \rho_{1} \\ L_{2}(x) = \frac{x - \rho_{1}}{\rho_{2} - \rho_{1}}, & \rho_{1} \le x \le \rho_{2} \\ 1, & x = \rho_{2} \\ R_{2}(x) = \frac{\rho_{3} - x}{\rho_{3} - \rho_{2}}, & \rho_{2} \le x \le \rho_{3} \\ 0, & x > \rho_{3} \end{cases}$$

and
$$\Omega_{\tilde{\rho}}(x) = \begin{cases} 0x < \rho_1 \\ \frac{x - \rho_1}{2(\rho_2 - \rho_1)}, \rho_1 \le x \le \rho_2 \\ 1 - \frac{\rho_3 - x}{2(\rho_3 - \rho_2)}, \rho_2 \le x \le \rho_3 \end{cases};$$

 $1, x \ge \rho_3$

$$\mu_{\tilde{\lambda}'}(x) = \begin{cases} 0, & x < \lambda'_1 \\ L_2(x) = \frac{x - \lambda'_1}{\lambda'_2 - \rho_1}, & \lambda'_1 \le x \le \lambda'_2 \\ 1, & x = \lambda'_2 \\ R_2(x) = \frac{\lambda'_3 - x}{\lambda'_3 - \lambda'_2}, & \lambda'_2 \le x \le \lambda'_3 \\ 0, & x > \lambda'_3 \end{cases}$$

and $\Omega_{\tilde{\lambda}'}(x) = \begin{cases} 0x < \lambda'_1 \\ \frac{x - \lambda'_1}{2(\lambda'_2 - \lambda'_1)}, & \lambda'_1 \le x \le \lambda'_2 \\ 1 - \frac{\lambda'_3 - x}{2(\lambda'_3 - \lambda'_2)}, & \lambda'_2 \le x \le \lambda'_3 \\ 1, x \ge \lambda'_3 \end{cases}$

where $\lambda_1 \leq \lambda_2 \leq \lambda_3$, $\rho_1 \leq \rho_2 \leq \rho_3$ and $\lambda_1^{'} \leq \lambda_2^{'} \leq \lambda_3^{'}$, then it has an expected value which is defined as,

$$E[\tilde{\lambda}] = \int_{0}^{\lambda_{1}} (1-0)dx + \int_{\lambda_{1}}^{\lambda_{2}} \left(1 - \frac{L_{1}(x)}{2}\right)dx + \int_{\lambda_{2}}^{\lambda_{3}} \left(\frac{R_{1}(x)}{2}\right)dx + \int_{\lambda_{3}}^{\infty} (1-1)dx = \frac{\lambda_{1} + 2\lambda_{2} + \lambda_{3}}{4}.$$

Similarly, $E[\tilde{\rho}] = \frac{\rho_{1} + 2\rho_{2} + \rho_{3}}{4}$; and $E[\tilde{\lambda}'] = \frac{\lambda'_{1} + 2\lambda'_{2} + \lambda'_{3}}{4}.$
Now, $E[\tilde{\rho}^{2}] = \int_{0}^{+\infty} C_{r}(\rho^{2} \ge r)dr - \int_{-\infty}^{0} C_{r}(\rho^{2} \le r)dr = \int_{0}^{+\infty} C_{r}(\rho \ge \sqrt{r})dr.$
By letting $x = \sqrt{r}$, which implies
 $E[\tilde{\rho}^{2}] = \int_{0}^{+\infty} (2x(1 - C_{r}(\rho \le x))dx = \frac{\rho_{1}^{2} + 2\rho_{2}^{2} + \rho_{3}^{2} + \rho_{1}\rho_{2} + \rho_{2}\rho_{3}}{4}.$

$$E[\tilde{\rho}] = \int_{0}^{\infty} (2x(1 - C_r(\rho \le x))dx) = \frac{\rho_1 + 2\rho_2 + \rho_3 + \rho_1\rho_2 + \rho_2\rho_3}{6}$$

This completes the proof.

Lemma 2

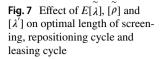
(a) $E[TC(T_i, T_{cr}, T_l)]$ is strictly convex. (b) The optimum length of the screening period and the optimum time duration for leasing the containers are

 Table 2
 Effect of fuzziness of return rate, repairable percentage and repositioned persentage on the optimal solutions

Parameter	Value of the parameter	$T_i^*(\text{days})$	$T_{cr}^{*}(\text{days})$	T_l^* (days)
	(0.55, 0.60, 0.70)	1.8600	3.0071	3.0832
$\tilde{\lambda}$	(0.65, 0.70, 0.80)	2.1321	3.4287	3.5155
	(0.75, 0.80, 0.90)	2.4403	4.0983	4.2021
	(0.85, 0.90, 1.00)	2.8030	5.4223	5.5596
	(0.725, 0.75, 0.80)	2.8795	3.4768	3.5648
	(0.775, 0.80, 0.85)	2.8533	3.7649	3.8603
$\tilde{\rho}$	(0.825, 0.85, 0.90)	2.8320	4.1391	4.2439
	(0.875, 0.90, 0.95)	2.8153	4.6529	4.7707
	(0.925, 0.95, 1.00)	2.803	5.4223	5.5596
	(0.25, 0.30, 0.40)	2.803	6.9439	4.6816
	(0.35, 0.40, 0.50)	2.803	6.0439	5.0644
λ'	(0.45, 0.50, 0.60)	2.803	5.4223	5.5596
	(0.55, 0.60, 0.70)	2.803	4.9599	6.2358
	(0.65, 0.70, 0.80)	2.803	4.5987	7.2395



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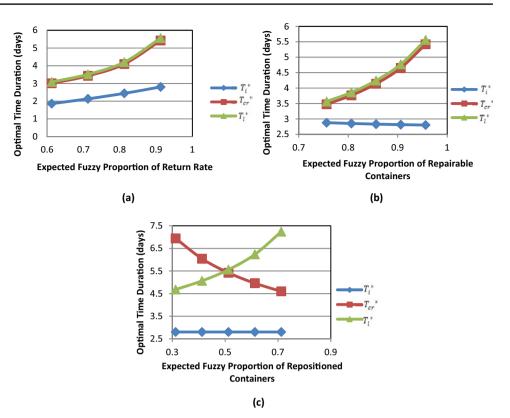


Table 3 Effect of customer rent price per container on the optimal screening length, ECR cycle, leasing cycle period and the expected annual cost

(USD)	T_i^* (days)	T_{cr}^{*} (days)	T_l^* (days)	$E\left[TC(T_i, T_{cr}, T_l)\right]$ (USD)
40	2.9543	5.3170	5.4516	3,969,100
45	2.8772	5.3689	5.5048	3,864,500
50	2.8030	5.4223	5.5596	3,819,800
55	2.7316	5.4773	5.6160	3,745,100
60	2.6628	5.5341	5.6742	3,670,300

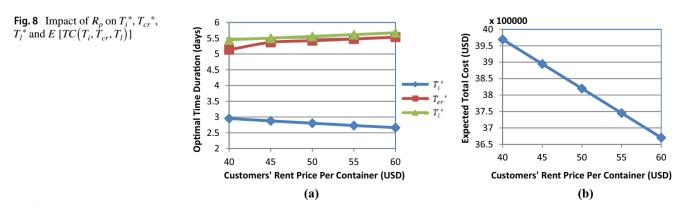




 Table 4 Impacts of carrying charge of the returned containers and the repairable containers on the optimal solutions

Parameter	Value of the parameter (USD)	T_i^* (days)	$E\left[TC\left(T_{i}, T_{cr}, T_{l}\right)\right]$ (USD)	6]
	1	3.0665	3,814,800	$+\frac{1}{2}$
	2	2.8030	3,819,800	2
H_u	3	2.5976	3,824,400	L
	4	2.4316	3,828,800	$F[\tilde{\lambda}](a)$
	5	2.2938	3,832,800	
	1	2.9798	3,816,300	+
	2	2.8873	3,818,100	
H_r	3	2.8030	3,819,800	$\eta \left(rE \right) \rho$
	4	2.7257	3,821,500	+
	5	2.6545	3,823,100	

$$\frac{\partial E \left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_i} = \frac{-nE[\tilde{\lambda}](\alpha - \beta R_p)(K_i + K_r)}{\eta T_i^2} + \frac{1}{2} \left[\left(\eta - E[\tilde{\lambda}](\alpha - \beta R_p) \right) H_u + \frac{E[\tilde{\lambda}](\alpha - \beta R_p) \left(\eta E[\tilde{\rho}^2] - rE[\tilde{\rho}] \right) H_r}{r} + \frac{\eta \left(rE[\tilde{\rho}] - E[\tilde{\lambda}]E[\tilde{\rho}^2](\alpha - \beta R_p) \right) H_s}{r} \right],$$

$$T_{i}^{*} = \sqrt{\frac{2nrE[\tilde{\lambda}](\alpha - \beta R_{p})(K_{i} + K_{r})}{\eta r \left(\eta - E[\tilde{\lambda}](\alpha - \beta R_{p})\right)H_{u} + \eta E[\tilde{\lambda}](\alpha - \beta R_{p})\left(\eta E[\tilde{\rho}^{2}] - rE[\tilde{\rho}]\right)H_{r} + \eta^{2}\left(rE[\tilde{\rho}] - E[\tilde{\lambda}]E[\tilde{\rho}^{2}](\alpha - \beta R_{p})\right)H_{s}}}$$

$$(19)$$

$$T_{cr}^{*} = \sqrt{\frac{2nK_{cr}}{\tilde{\lambda}' \left(1 - E\left[\tilde{\lambda}\right]E[\tilde{\rho}]\right) \left(\alpha - \beta R_{p}\right)H_{cr}}}$$
(20)

and

$$T_l^* = \sqrt{\frac{2nK_l}{\left(1 - E\left[\tilde{\lambda}'\right]\right)\left(1 - E\left[\tilde{\lambda}\right]E[\tilde{\rho}]\right)(\alpha - \beta R_p)H_l}}.$$
 (21)

Proof Differentiating Eq. (14) partially with respect to T_i , T_{cr} and T_l , it is obtained as

$$\frac{\partial E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_{cr}} = \frac{-nK_{cr}}{T_{cr}^2} + \frac{E\left[\tilde{\lambda}'\right]\left(1 - E\left[\tilde{\lambda}\right]E\left[\tilde{\rho}\right]\right)(\alpha - \beta R_p)H_{cr}}{2}$$

 \sim

$$\frac{\partial E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_l} = \frac{-nK_l}{T_l^2} + \frac{\left(1 - E\left[\tilde{\lambda}'\right]\right)\left(1 - E\left[\tilde{\lambda}\right]E\left[\tilde{\rho}\right]\right)(\alpha - \beta R_p)H_l}{2}.$$

Fig. 9 Impact of
$$H_u$$
 and H_r on
 T_i^* and $E[TC(T_i, T_{cr}, T_l)]$
 $Impact of H_u and H_r on
 T_i^* and $E[TC(T_i, T_{cr}, T_l)]$
 $Impact of H_u$ and H_r on
 T_i^* and $E[TC(T_i, T_{cr}, T_l)]$
 $Impact of H_u$ and H_r on
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Parameter	Value of the parameter	$E\left[TC\left(T_{i}, T_{cr}, T_{l}\right)\right]$ (USD)
	3	3,506,400
k_h	4	3,663,100
	5	3,819,800
	6	3,976,500
	7	4,133,300
	6	3,663,100
	7	3,741,400
k _t	8	3,819,800
	9	3,898,200
	10	3,976,500
	10	3,819,800
	12	3,968,900
k_l	14	4,118,000
	16	4,267,100
	18	4,416,100
	20	4,565,200
	(0.25, 0.30, 0.40)	3,574,800
	(0.35, 0.40, 0.50)	3,697,400
$\widetilde{\lambda'}$	(0.45, 0.50, 0.60)	3,819,800
	(0.55, 0.60, 0.70)	3,942,000
	(0.65, 0.70, 0.80)	4,064,000

Table 5 Effect of leasing cost, terminal handling charge, transportation charge and the fuzzy reposition rate on expected total cost

Again, differentiating partially with respect to T_i , T_{cr} and T_l are as follows,

$$\frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_i^2} = \frac{2n E[\lambda] (\alpha - \beta R_p) (K_i + K_r)}{\eta T_i^3},$$

$$\frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_{cr}^2} = \frac{2nK_{cr}}{T_{cr}^3}$$

and

$$\frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_l^2} = \frac{2nK_l}{T_l^3}.$$

Also,

$$\frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_i \partial T_{cr}} = \frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_{cr} \partial T_i}$$
$$= \frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_l \partial T_i} = \frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_i \partial T_l}$$
$$= \frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_{cr} \partial T_l} = \frac{\partial^2 E\left[TC(T_i, T_{cr}, T_l)\right]}{\partial T_l \partial T_{cr}} = 0.$$

Therefore, the Hessian matrix for $E[TC(T_i, T_{cr}, T_l)]$ is

10

9

0.9

x 100000 x 100000 42 40 Expected Total Cost (USD) 41 39.5 Expected Total Cost (USD) 40 39 39 38.5 38 38 37 37.5 36 37 35 36.5 34 36 7 4 5 6 3 7 8 6 Terminal Handling Charge for ECR (USD) Transportation Charge for ECR (USD) (a) **(b)** x 100000 x 100000 41 46 Expected Total Cost (USD) 40 39 38 37 36 35 38 0.3 0.5 0.7 10 12 14 16 18 20 Expected Fuzzy Proportion of Repositioned Rent Price Per Leased Container (USD) Containers (USD) (c) (d)

Fig. 10 Effect of k_h , k_l , k_l and λ' on $E[TC(T_l, T_{cr}, T_l)]$



$$H = \begin{bmatrix} \overbrace{\partial^2 E[TC(T_i, T_{cr}, T_l)]}^{\sim} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_{cr} \partial T_i} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_l \partial T_i} \\ \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_i \partial T_{cr}} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_{cr} \partial T_{cr}} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_{cr} \partial T_l} \\ \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_i \partial T_l} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_l \partial T_{cr}} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_l^2} \\ \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_i^3} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_{cr}^3} & \frac{\partial^2 E[TC(T_i, T_{cr}, T_l)]}{\partial T_l^2} \end{bmatrix} \\ = \begin{bmatrix} \frac{2n E[\lambda] (\alpha - \beta R_p) (K_i + K_r)}{\eta T_i^3} & 0 & 0 \\ 0 & \frac{2n K_{cr}}{T_{cr}^3} & 0 \\ 0 & 0 & \frac{2n K_l}{T_l^3} \end{bmatrix}.$$

It is clear that, the above Hessian matrix is positive. Thus, the expected total cost function $E[TC(T_i, T_{cr}, T_l)]$ is strictly convex.

By setting, $\frac{\partial E[TC(T_i, T_{cr}, T_i)]}{\partial T_i} = 0$, $\frac{\partial E[TC(T_i, T_{cr}, T_i)]}{\partial T_{cr}} = 0$, and $\frac{\partial E[TC(T_i, T_{cr}, T_i)]}{\partial T_i} = 0$, the optimal length of the screening period, the optimal duration for ECR cycle and the leasing cycle of the containers are obtained.

Thus,

repairable units is given as TFN by considering "about 0.95, not more than 1.00, not less than 0.925", that is $\rho = (0.925, 0.95, 1.00)$ and the fraction of repositioned units is given as TFN by considering "about 0.50, not more than 0.60, not less than 0.45", that is $\lambda' = (0.45, 0.50, 0.60)$.

From Eqs. (19), (20), (21) and (14), the optimal time length for screening process, the optimal repositioning cycle length and the optimal lease cycle length are $T_i^* = 2.8030$ days, $T_{cr}^* = 5.4223$ and $T_l^* = 5.5596$ days, the expected total cost is $E [\text{TC}(T_i, T_{cr}, T_l)] = \$3,\$19,\00 . By substituting the value of T_i^* , the idle time is obtained as 2.1118 days. The impact of fuzziness of return rate, repairable percentage and repositioned percentage on the optimal solution is examined and they are given in Table 2.

From Fig. 7a, it is noted that when λ raises, the optimal time length of screening period, repositioning cycle and leasing cycle period increase. Figure 7b shows that when ρ raises, the optimal time length of screening period and the repositioning cycle increase but the optimal leasing cycle period decreases. Figure 7c shows that when λ' raises, the repositioning cycle length decreases and the leasing cycle

$$T_{i}^{*} = \sqrt{\frac{2nrE[\tilde{\lambda}](\alpha - \beta R_{p})(K_{i} + K_{r})}{\eta r \left(\eta - E[\tilde{\lambda}](\alpha - \beta R_{p})\right)H_{u} + \eta E[\tilde{\lambda}](\alpha - \beta R_{p})\left(\eta E[\tilde{\rho}^{2}] - rE[\tilde{\rho}]\right)H_{r} + \eta^{2}\left(rE[\tilde{\rho}] - E[\tilde{\lambda}]E[\tilde{\rho}^{2}](\alpha - \beta R_{p})\right)H_{s}}}$$

$$T_{cr}^{*} = \sqrt{\frac{2nK_{cr}}{\tilde{\lambda'}\left(1 - E\left[\tilde{\lambda}\right]E[\tilde{\rho}]\right)\left(\alpha - \beta R_{p}\right)H_{cr}}}$$

and

$$T_l^* = \sqrt{\frac{2nK_l}{\left(1 - E\left[\tilde{\lambda}'\right]\right)\left(1 - E\left[\tilde{\lambda}\right]E[\tilde{\rho}]\right)(\alpha - \beta R_p)H_l}}$$

This completes the proof.

Discussions on numerical analysis

Some of the values of the parameters are adopted from [3]: $\alpha = 6000, \beta = 20, \quad R_p = \$50 / \text{u n it}, \quad \eta = 8000,$ $r = 6000, \quad s' = \$100 / \text{u n it}, \quad K_i = \$200,$ $K_r = \$400, K_{cr} = \$100, K_l = \$100, k_i = \$2/\text{unit}, \quad k_r = \4 /unit, $k_l = \$10, k_h = \$5/\text{unit}, k_t = \$8/\text{unit}, H_u = \$2/\text{unit},$ $H_r = \$3/\text{unit}, H_s = \$5/\text{unit}, H_{cr} = \$5/\text{unit}, H_l = \$5/\text{unit},$ n = 240 days.

The proportion of the returned units is given as TFN by considering "about 0.90, not more than 1.00, not less than 0.85", that is $\lambda = (0.85, 0.90, 1.00)$, the proportion of

length increases but the optimal screening period remains unchanged (Table 3).

From the above table, it is observed that the customer rent price per container increases, both the optimal screening period, optimal repositioning cycle length and the expected total cost decrease but the optimal duration of leasing cycle increases which is clearly displayed in Fig. 8 (Table 4).

From Fig. 9a, it is clear that the carrying charge of the returned containers and the repairable containers increase, the optimal screening period decreases. Figure 9b shows that the carrying charge of the returned containers and the repairable containers raise, the expected total cost increases (Table 5).

From the above table, it is noted that the leasing charge, terminal handling charge, transportation charge per container and the fuzzy reposition rate increase, and so, the expected annual cost also increases and they are clearly shown in Fig. 10.

When $\alpha = 10,000, \beta = 40, R_p = \$50/\text{unit}, \eta = 12,000, r = 10,000, s' = \$40/\text{unit}$, the optimal time length for screening process in this study is a better one when compared to the result of [3].



Conclusion

This study conferred a closed-loop supply chain inventory model for returnable containers. The expected maximum stock level for returned, repairable, serviceable, repositioned and leased containers are framed. Since the purchasing of new containers is more expensive, the ECR option and the lease option are preferred. As the proportion of repositioning units increases, the total cost also increases. Hence, when leasing of container is less expensive than repositioning of containers, leasing option may also be considered. The fixed cost, variable cost and the holding cost are formulated by scrutinizing the demand as price sensitive. The expected total cost in fuzzy view is developed based on the three main factors. They are the fraction of returned containers, the proportion of repositioned containers and the proportion of repairable from returned units which are considered as TFNs. The optimal screening length, the optimal repositioning cycle length and the optimal leasing cycle length that minimize the expected total cost are obtained and it is endorsed with the numerical illustration. The sensitivity analysis shows the impact of fuzzy return rate, fuzzy repair rate, fuzzy repositioning percentage, customer rent price, carrying charge of the returned containers, carrying charge of the repairable containers, terminal handling charge, transportation charge for ECR and the leasing cost of a container on the optimal solutions. In this study, it is also observed that, the optimal time length for screening process is a better one when it is compared to the result of [3]. Similar notions may be contributed by the researchers in future by considering different types of demands like stochastic, exponential, non-linear etc. In future, better results could be obtained by the researchers by adopting the different types of fuzzy numbers.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval The article does not contain any studies with human participants or animal performed by any of the authors.

Informed consent Informed consent was obtained from all individual participants included in the study.

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