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ORIGINAL PAPER

A hesitant group emergency decision making method based on prospect theory

Zi-Xin Zhang[1,2] · Liang Wang[1,2] · Rosa M. Rodríguez[3] · Ying-Ming Wang[1] · Luis Martínez[2]

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Abstract Group emergency decision-making (GEDM) problems have drawn great attention in past few years due to its advantages of dealing with the emergency events (EEs) effectively. Due to the fact that EEs are usually featured by lack of information and time pressure, decision makers (DMs) are often bound rational and their psychological behaviors are very crucial to the GEDM process. However, DM's psychological behaviors are neglected in current GEDM approaches. The assessments representing the individual wisdom provided by each expert are usually aggregated in the GEDM process. Nevertheless, the aggregation process always implies summarization of data that can result in loss of information. To overcome these limitations pointed out previously, this paper proposes a new GEDM method that considers the DM's psychological behaviors in the decision process using prospect theory and replaces the aggregation process by a fusion method with hesitant fuzzy set, which keeps the experts' information as much as possible. A case study is provided to illustrate the validity and feasibility of the proposed method.

Keywords Group emergency decision making · Hesitant fuzzy sets · Prospect theory

✉ Ying-Ming Wang
ymwang@fzu.edu.cn

Zi-Xin Zhang
zhangzixin768@outlook.com

Liang Wang
wangliangg322@hotmail.com

Rosa M. Rodríguez
rosam.rodriguez@decsai.ugr.es

Luis Martínez
martin@ujaen.es

1 Decision Science Institute, Fuzhou University, Fuzhou 350116, People's Republic of China

2 Department of Computer Sciences, University of Jaén, Jaén 23071, Spain

3 Department of Computer Science and A.I., University of Granada, Granada 18071, Spain

Introduction

Emergency event (EE) is defined as "events which suddenly take place, causing or having the possibility to cause intense death and injury, property loss, ecological damage and social hazards" [18], such as earthquakes, air crash, hurricanes, terrorist attacks, etc. When an EE occurs, it must be dealt with some measures to mitigate the losses of properties and lives, the process of selecting the measures is defined emergency decision making (EDM). EDM has received increasing attention and became a very active and important research field in recent years [12,15,20,35–37] because it plays a crucial role in mitigating the losses of properties and lives caused by EE.

Because EDM is typically characterized by time pressure and lack of information [10,17], it is difficult for a decision maker (DM) to predict its evolution and make comprehensive judgments under emergency situations. Therefore, EDM requires multiple experts from diverse professional backgrounds (such as hydrological, geological, meteorological, sociological, demographic, etc.) who help the DM to make a decision; this leads to GEDM problems. Usually, the GEDM consists of two processes [22,26]: (i) the aggregation process, where the individual information provided by experts is aggregated, and (ii) a selection process, in which an alternative is obtained as the solution to response the EE (see Fig. 1).

Current GEDM studies [41–43] have made significant contributions to emergency management; however, there are still two key issues that have not been well addressed yet:

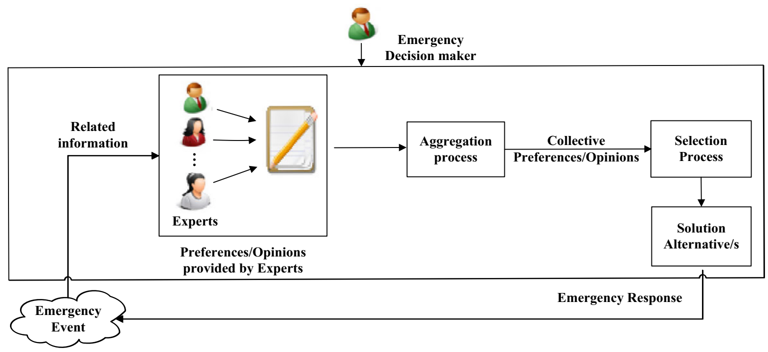


Fig. 1 The general scheme of GEDM process

1. *Losing information in aggregation process*. Current GEDM studies [41–43], use aggregation process that may imply loss of useful information for the decision process from the very beginning. Therefore, an important challenge for GEDM is to keep as much information as possible about the group, avoiding such a lost.
2. *DM's psychological behaviors in selection process*. Different studies [7,8] have shown that the DM is bounded rational under risk and uncertainty and his/her psychological behavior plays an important role in GEDM processes. However, such an important issue has been neglected in current GEDM methods.

Therefore, to overcome such limitations, this paper aims at developing a new GEDM method based on hesitant fuzzy set (HFS) with the following main contributions:

1. It considers different experts' opinions as the group hesitancy and fuses them into HFSs.
2. At the same time, it takes into account the DM's psychological behaviors using prospect theory (PT) in the selection process, because of its advantages of capturing human beings psychological behaviors under risk and uncertainty [16].

The remainder of this paper is organized as follows: Sect. 2 briefly introduces the basic knowledge about HFS and PT that will be used in the proposed method. Section 3 presents a new GEDM method that includes a fusion method by using HFS to keep the experts' information as much as possible and considers the DM's psychological behaviors using PT in the selection process. In Sect. 4, a case study is provided. Section 5 offers the conclusions.

Preliminaries

This section provides a brief review of different concepts about HFS and PT that will be utilized in the proposed method and to make it understood easily.

Hesitant fuzzy sets

HFS was introduced by Torra [30] as an extension of fuzzy sets to model the hesitancy in quantitative contexts reviewed in depth [23,25]. It is defined as below:

Definition 1 [30] Let $M = \{\mu_1, \ldots, \mu_n\}$ be a set of n membership functions. The HFS associated to M, h_M, is defined as:

$$h_M : X \to \wp([0, 1])$$

$$h_M(x) \to \bigcup_{\mu \in M} \{\mu(x)\} \quad (1)$$

where X is a reference set, $x \in X$.

This definition was extended and formalized with the concept of hesitant fuzzy element (HFE) by Xia and Xu [40]. In their proposal, the HFS was expressed by following mathematical representation, i.e.,

$$E = \{\langle x, h_E(x)\rangle : x \in X\}$$

where $h_E(x)$ is a set of values in [0,1], denoting the possible membership degrees of the element $x \in E$ to the set E. For convenience, they defined $h = h_E(x)$ as the HFE and $H = \cup h(x)$ as the HFSs, a HFE is a subset of HFSs (see [40] for further details).

Torra introduced in [30] the concept of envelop of a HFE and proved that is a intuitionistic fuzzy value (IFV) according to the following definition:

Definition 2 [30] Let h be a HFE, the IFV $A_{\text{env}}(h)$ is the envelop of h, in which $A_{\text{env}}(h)$can be represented as $\left(h^-, 1 - h^+\right)$ being $h^- = \min\{\sigma|\sigma \in h\}$ and $h^+ = \max\{\sigma|\sigma \in h\}$.

Different operations and properties has been defined for HFSs [30] such operations together the managing of intuitionistic fuzzy sets and intervals [14,30] allow us to interpret HFEs like an interval.

Table 1 Summary of the related works on hesitant fuzzy set

Authors	Contributions	Year
Torra [30]	Hesitant fuzzy set (HFS)	2010
Bedregal et al. [3]	Typical hesitant fuzzy set (THFS)	2014
Xia and Xu [40]	Hesitant fuzzy element (HFE)	2011
	Hesitant fuzzy weighted average operator	2011
	Hesitant fuzzy power average operator	2011
Yu [45]	Hesitant fuzzy Choquet integral operator	2011
Zhou [48]	Distance measures for hesitant fuzzy set	2012
Wei [38]	Entropy measures for hesitant fuzzy set	2016
Rodriguez et al. [25]	Hesitant fuzzy linguistic term set (HFLTS)	2014
Cevik Onar [9]	Multi-criteria decision making AHP method	2014
Wei [39]	Multi-criteria decision making VIKOR method	2014
Xue [44]	Group decision making	2017
Yu [46]	Personal evaluation	2013
Aliahmadipour [2]	Clustering	2016

Many researchers have paid attention on HFSs because it is a useful approach to model experts' hesitation. Therefore, different proposals have been introduced in the literature. Bedregal et al. [3] presented a special case of HFS named Typical Hesitant Fuzzy Set, that introduces some restrictions, because a HFS should be a finite and nonempty set. Many aggregation operators for HFSs have been defined such as, hesitant fuzzy weighted average [40], hesitant fuzzy power average [40], hesitant fuzzy choquet integral [45] and so on [23,25]. Distance measures are widely used in different fields such as, machine learning and decision making, for this reason some of them have been extended to deal with HFS [48]. Some entropy measures have been also defined for HFS [38]. And there are many applications based on HFS such as, multi-criteria decision making [9,39], group decision making [44], evaluation [46], and clustering approaches [2].

Recently, Rodriguez et al. [27] proposed the concept of Hesitant Fuzzy Linguistic Term Set (HFLTS), which not only keeps the basis on the fuzzy linguistic approach [47], but also extends the idea of HFS to linguistic contexts [24]. It has drawn great attention since it has been applied to solve different decision problems [4,19,26].

For sake of clarity, we make a summary of the related works on hesitant fuzzy set, see the following Table 1.

Prospect theory

PT was firstly proposed by Kahneman and Tversky [16] in 1979, which describes the human beings behavioral characteristics and provides a way to compute gains, losses, and prospect values, which has been widely used to solve various decision making problems considering human beings psychological behaviors [13,28,29,32,33].

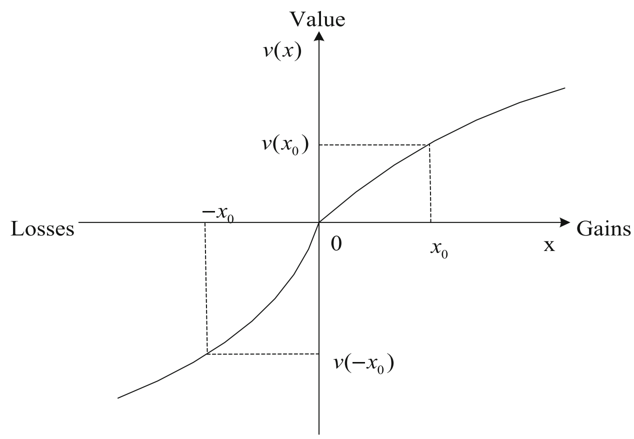


Fig. 2 S-shape value function of PT

Reference point (RP) is one key element in PT, which is defined as a neutral position asset or expectation value of people who wants to obtain or not loss, and decides the feeling of gains or losses based on the actual amounts to people; the location of the RP can be affected by the expectations of the people [16].

Gains and losses are defined with regards to the RP; the DM's psychological behaviors are exhibited risk-averse tendency for gains and risk-seeking tendency for losses. For measuring the magnitude of gains and losses, a value function is used in PT, which is defined on deviations from the RP with a concave and convex S-shape for losses and gains, respectively (see Fig. 2), and it is expressed in form of a power law according to the following expression [16].

$$v(x) = \begin{cases} x^{\alpha}, & x \geq 0 \\ -\lambda(-x)^{\beta}, & x < 0 \end{cases} \tag{2}$$

where x denotes the gains or losses, with $x \geq 0$ or with $x < 0$ respectively. αandβ are power parameters related to gains and losses, respectively, $0 \leq \alpha, \beta \leq 1$. λ is the risk aversion parameter, which represents a characteristic of being steeper for losses than for gains, $\lambda > 1$. The values of α, β and λ are determined through experiments [1,5,6,34].

A hesitant group emergency decision making dealing with DM's behaviors

This section introduces a novel hesitant GEDM method based on PT that aims at keeping experts' information as much as possible during the decision process and taking into account the DM's psychological behaviors during the selection process.

The proposed method consists of three main phases and graphically in Fig. 3:

1. Definition framework;
2. Information fusion based on HFS;
3. Alternative selection based on PT.

These phases are further detailed in the following subsections.

Definition framework

The basic notations that will be used in our proposal are given.

- $A = \{a_1, \ldots, a_i, \ldots, a_I\}$: set of alternatives, where a_i denotes the *i-th* alternative, $i = 1, 2, \ldots, I$.
- $C = \{c_1, \ldots, c_j, \ldots, c_J\}$: set of criteria, where$c_j$denotes the *j-th* criterion, $j = 1, 2, \ldots, J$.
- $S = \{s_1, \ldots, s_m, \ldots, s_M\}$: set of emergency situations, where s_m denotes the *m-th* emergency situation, $m = 1, 2, \ldots, M$.
- $W = \left(w_{c_1}, \ldots, w_{c_j}, \ldots, w_{c_J}\right)$: vector of criteria weights, where w_{c_j}denotes the weight of the *j-th* criterion, $j = 1, 2, \ldots, J$.
- $E = \{e_1, \ldots, e_h, \ldots, e_H\}$: set of experts, where$e_h$denotes the *h-th* expert, $h = 1, 2, \ldots, H$.
- $C^h = \left\{c_j^h(a_i)\right\}$: set of opinions provided by expert e_h, where $c_j^h(a_i) \in R$ denotes the preference over the$i$-th alternative regarding to the$j$-th criterion, $i = 1, 2, \ldots, I$; $h = 1, 2, \ldots, H$; $j = 1, 2, \ldots, J$.
- $\bar{C}^h = \left\{\bar{c}_j^h(a_i)\right\}$: denotes the normalization of C^h, where $\bar{c}_j^h(a_i) \in [0, 1]i = 1, 2, \ldots, I; h = 1, 2, \ldots, H; j = 1, 2, \ldots, J$.
- $h_M(a_i) = \{\bar{c}_1(a_i), \ldots, \bar{c}_J(a_i)\}$: denotes the HFS of experts' preference, where $\bar{c}_j(a_i)$ is the hesitant fuzzy element (HFE) and $\bar{c}_j(a_i) = \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\}$, $i = 1, 2, \ldots, I$; $h = 1, 2, \ldots, H$; $j = 1, 2, \ldots, J$.
- $E_{ij} = \left[E_{ij}^L, E_{ij}^U\right]$: be an interval value, where E_{ij} denotes the effective control scope [37] over the i-th alternative with respect to the j-th criterion.
- $R_j = \left[R_j^L, R_j^U\right]$: be an interval value, where R_j^L, R_j^U are preferences, and $R_j$denotes the RP provided by the DM with respect to the j-th criterion.
- $\bar{R}_j = \left[\bar{R}_j^L, \bar{R}_j^U\right]$: denotes the normalization of R_j, where $\bar{R}_j \in [0, 1] j = 1, 2, \ldots, J$.

Information fusion based on HFS

As it was pointed out in the introduction, the aggregation always implies a summarization of original experts' opin-

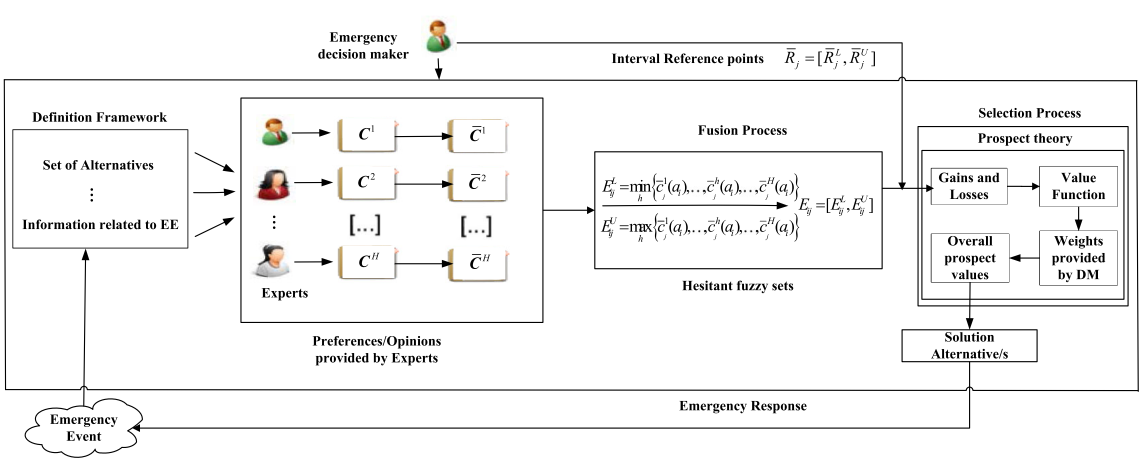


Fig. 3 General framework of proposed method

ions that can imply loss of information from different points of view such as distribution, diversity of data, etc. This loss of information can either bias or lead to wrong decisions regardless the aggregation operator. To overcome such a limitation, the experts' preferences in the GEDM problem are considered as the group hesitation about the alternatives and they will be fused by utilizing a HFS to keep as much information as possible.

Keeping in mind this idea, the effective control scopes of alternatives and the RP must be determined. The effective control scope of alternatives can be obtained by considering the group hesitation about it as a HFS, which is introduced in detail as follows:

Step 1: The experts involved in the GEDM problem provide the related information $c_j^h(a_i)$ about the emergency alternative with regarding to different criteria through analyzing the emergency alternatives

Step 2: Based on information $c_j^h(a_i)$ provided by experts, the preference of experts $\bar{c}_j^h(a_i)$ of effective control scope of the *i-th* alternative with respect to the *j-th* criterion can be calculated by Eq. (3):

$$\bar{c}_j^h(a_i) = \frac{c_j^h(a_i)}{\max\left\{\max_h c_j^h(a_i)\right\}}, \quad j = 1, 2, \ldots J \tag{3}$$

Step 3: From $\bar{c}_j^h(a_i)$ calculated by Eq. (3), the HFEs $\bar{c}_j(a_i)$ for the *j-th* criterion with respect to the *i-th* alternative and the HFS $h_M(a_i)$ can be formed and managed according to their envelops as interval values.

Step 4: Based on step 3, the lower bound E_{ij}^L and upper bound E_{ij}^U of the effective control scope E_{ij} can be calculated by Eqs. (4), (5) [31].

$$E_{ij}^L = \min_h \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\} \tag{4}$$

$$E_{ij}^U = \max_h \left\{\bar{c}_j^1(a_i), \ldots, \bar{c}_j^h(a_i), \ldots, \bar{c}_j^H(a_i)\right\} \tag{5}$$

The interval value E_{ij} is the result of fusion information that can avoid the loss of information and keep the experts' opinions as much as possible.
In order to facilitate the computations, the preferences R_j need to be transformed into $\bar{R}_j$ by utilizing the Eqs. (6), (7):

$$\bar{R}_j^L = \frac{R_j^L}{\max\left\{\max_h c_j^h(a_i)\right\}} \tag{6}$$

$$\bar{R}_j^U = \frac{R_j^U}{\max\left\{\max_h c_j^h(a_i)\right\}} \tag{7}$$

Alternative selection based on PT

Due to the fact that DM's psychological behaviors play an important role in GEDM process, this proposal uses PT to address such an important issue, because of its advantages to capture the psychological behaviors.

Calculation of gains and losses

According to the RP $\bar{R}_j$ and the effective control scope E_{ij} of emergency alternatives, gains and losses can be obtained.

Due to the fact that, we are dealing with interval values, before obtaining the gains and losses, the relationship between $\bar{R}_j$ and E_{ij} should be determined. There are six possible cases of positional relationship between $\bar{R}_j$ and E_{ij} as shown in Table 2.

To obtain the gains and losses with respect to each alternative, the following definition is provided.

Definition 3 For the effective control scope E_{ij} of alternatives, let x be an arbitrary value in interval number $[E_{ij}^L, E_{ij}^U]$, regarded as a random variable with uniform distribution [11]. The probability density function of x is

$$f(x) = \begin{cases} \frac{1}{E_{ij}^U - E_{ij}^L}, & E_{ij}^L \leq x \leq E_{ij}^U \\ 0, & \text{otherwise} \end{cases}, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{8}$$

where $\int_{E_{ij}^L}^{E_{ij}^U} f(x)\mathrm{d}x = 1$ and $f(x) \geq 0$ for all $x \in [E_{ij}^L, E_{ij}^U]$.

From Table 2, the calculation of gains and losses is discussed. In general, the criteria can be classified into two types: benefit and cost [21]. A benefit criterion means the higher the better while a cost criterion the higher the worse. Note that for cost criteria, if $E_{ij}^U < \bar{R}_j^L$, the expert feels gains, and if $E_{ij}^L > \bar{R}_j^U$, the expert feels losses. The following discussion is for cost criterion only.

Case 1: obviously, there is no loss to the expert, since $E_{ij}^U < \bar{R}_j^L$,

$$L_{ij} = 0, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{9}$$

According to Definition 3, the gain to the expert is given by

$$G_{ij} = \int_{E_{ij}^L}^{E_{ij}^U} (\bar{R}_j^L - x) f(x)\mathrm{d}x, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{10}$$

Table 2 Possible cases of positional relationship between $\bar{R}_j$ and E_{ij}

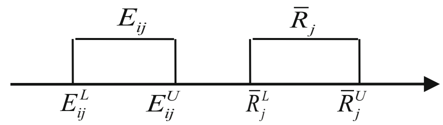
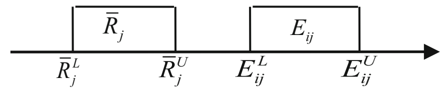
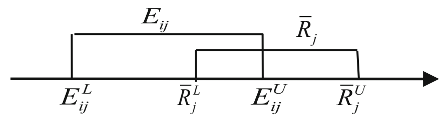
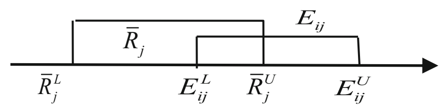
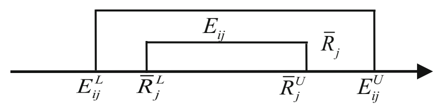
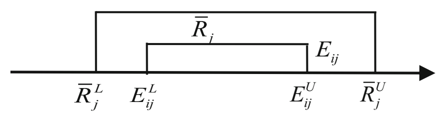
Cases		Positional relationship between $\bar{R}_j$ and E_{ij}
Case 1	$E_{ij}^U < \bar{R}_j^L$	
Case 2	$\bar{R}_j^U < E_{ij}^L$	
Case 3	$E_{ij}^L < \bar{R}_j^L \le E_{ij}^U < \bar{R}_j^U$	
Case 4	$\bar{R}_j^L < E_{ij}^L \le \bar{R}_j^U < E_{ij}^U$	
Case 5	$E_{ij}^L < \bar{R}_j^L < \bar{R}_j^U < E_{ij}^U$	
Case 6	$\bar{R}_j^L \le E_{ij}^L < E_{ij}^U \le \bar{R}_j^U$	

Table 3 Gains and losses for all possible cases (cost criteria)

Cases		Gain G_{ij}	Loss L_{ij}
Case 1	$E_{ij}^U < \bar{R}_j^L$	$\bar{R}_j^L - 0.5(E_{ij}^L + E_{ij}^U)$	0
Case 2	$\bar{R}_j^U < E_{ij}^L$	0	$\bar{R}_j^U - 0.5(E_{ij}^L + E_{ij}^U)$
Case 3	$E_{ij}^L < \bar{R}_j^L \le E_{ij}^U < \bar{R}_j^U$	$0.5(\bar{R}_j^L - E_{ij}^L)$	0
Case 4	$\bar{R}_j^L < E_{ij}^L \le \bar{R}_j^U < E_{ij}^U$	0	$0.5(\bar{R}_j^U - E_{ij}^U)$
Case 5	$E_{ij}^L < \bar{R}_j^L < \bar{R}_j^U < E_{ij}^U$	$0.5(\bar{R}_j^L - E_{ij}^L)$	$0.5(\bar{R}_j^U - E_{ij}^U)$
Case 6	$\bar{R}_j^L \le E_{ij}^L < E_{ij}^U \le \bar{R}_j^U$	0	0

Obviously, by Eqs. (9), (10) can be rewritten as:

$$G_{ij} = \bar{R}_j^L - 0.5(E_{ij}^L + E_{ij}^U), \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{11}$$

Similar to Case 1, the rest cases can be calculated respectively. The calculation formulae of gain and loss for all possible cases are summarized in Table 3, which shows the gain and loss for all possible cases for "cost criteria".

Similar to cost criterion, the calculation formulae of gain and loss for all possible cases with respect to the benefit criterion are summarized in Table 4:

Furthermore, based on Tables 3 and 4, the gain and loss matrix GM and LM can be constructed, which are used to calculate prospect values using value function.

Calculation of overall prospect values

Let $\mathrm{GM} = (G_{ij})_{I\times J}$ be the gain matrix, $\mathrm{LM} = (L_{ij})_{I\times J}$ be the loss matrix, $\mathrm{VM} = (v_{ij})_{I\times J}$ be the value matrix, where

$$v_{ij} = (G_{ij})^{\alpha} + \left[-\lambda\left(-L_{ij}\right)^{\beta}\right], i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{12}$$

different values can be used for the parameters of Eq. (12) according to [32] we will use, $\alpha = 0.88, \beta = 0.92, \lambda =$

Table 4 Gains and losses for all possible cases (benefit criteria)

Cases		Gain G_{ij}	Loss L_{ij}
Case 1	$E_{ij}^U < \bar{R}_j^L$	0	$0.5(E_{ij}^L + E_{ij}^U) - \bar{R}_j^L$
Case 2	$\bar{R}_j^U < E_{ij}^L$	$0.5(E_{ij}^L + E_{ij}^U) - \bar{R}_j^U$	0
Case 3	$E_{ij}^L < \bar{R}_j^L \le E_{ij}^U < \bar{R}_j^U$	0	$0.5(E_{ij}^L - \bar{R}_j^L)$
Case 4	$\bar{R}_j^L < E_{ij}^L \le \bar{R}_j^U < E_{ij}^U$	$0.5(E_{ij}^U - \bar{R}_j^U)$	0
Case 5	$E_{ij}^L < \bar{R}_j^L < \bar{R}_j^U < E_{ij}^U$	$0.5(E_{ij}^U - \bar{R}_j^U)$	$0.5(E_{ij}^L - \bar{R}_j^L)$
Case 6	$\bar{R}_j^L \le E_{ij}^L < E_{ij}^U \le \bar{R}_j^U$	0	0

2.25; and v_{ij} denotes the value of the *i-th* alternative with respect to the *j-th* criterion. In PT, magnitude of gains and losses are measured by Eq. (12), the prospect values reflect the different feelings of DM, the higher v_{ij} is, the better DM feels. It means that the DM is satisfied with the decisions what he/she has done; otherwise, the DM feels regret or depressed with the decisions what he/she has done. By using PT, the psychological behaviors of DM can be described clearly and easily understood.

The attribute weights are provided by DM, using the simple additive weighting method, the Overall Prospect Value (OPV) of each alternative can be obtained, i.e.,

$$\text{OPV}_i = \sum_{j=1}^{J} v_{ij} w_{c_j}, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \tag{13}$$

Obviously, the bigger OPV_i, the better alternative a_i. Based on the OPV_i, the ranking of alternatives can be obtained. According to the ranking of alternatives, the DM can select the best alternative to cope with the EE.

Case study and comparison

Case study

To illustrate the validity and feasibility of the proposed method, this section presents an adapted real case about a barrier lake emergency caused by a huge earthquake that occurred in southwestern China.

A barrier lake, formed by fallen rocks and a landslide after a huge earthquake, threatened the lives and properties of thousands of people both upstream and downstream. When a barrier lake formed, the DM must obey the principles of immediate response, timeously rescue, evacuating and blasting, so as to control the situation effectively and prevent it from further deterioration. The following criteria are concerned in our proposal:

c_1: The cost of alternatives (10,000 RMB which is the acronym of "renminbi", the official currency of the People's Republic of China).

c_2: The number of casualties.

c_3: Property loss (10,000 RMB).

The emergency alternatives are described as follows:

a_1: Evacuate people from the most dangerous upstream and downstream areas of the barrier lake to safe areas, and inform people in potentially dangerous areas to prepare for evacuation. At the same time, combine repeated small batch quantities of artificial blasting and excavation of drain grooves to meet the requirements of the discharged barrier lake floods;

a_2: Based on a_1, increase the joint scheduling of the reservoir and hydropower station in the upstream and downstream areas to reduce the pressure of the barrier lake;

a_3: Based on a_2, mobilize large, heavy machinery and implement large-scale blasting to reduce the water level of the barrier lake as much as possible to lower the risk of dam break;

a_4: Based on a_3, increase the joint scheduling of the reservoir and hydropower station in the upstream and downstream areas. Meanwhile, mobilize large, heavy machinery and implement large-scale blasting to reduce the water level of the barrier lake as much as possible to lower the risk of dam break.

Analyzing by professional experts, the barrier lake might be evolve into four possible emergency situations in 72 h, the emergency situations are as follows:

s_1: The dam body of the barrier lake will not break;

s_2: $1/3$ of the dam body of the barrier lake will break;

s_3: $1/2$ of the dam body of the barrier lake will break;

s_4: The entire dam body of the barrier lake will break.

Assume that three experts are invited to participate in the decision process to help DM makes a final decision. First, they are asked to define the effective control scope of the four emergency alternatives mentioned above. Through analyzing these emergency alternatives, the preferences $c_j^h(a_i)$ and $\bar{c}_j^h(a_i)$ of the effective control scopes for alternatives are given (see Table 5), where $\bar{c}_j^h(a_i)$ is calculated by Eq. (3).

Table 5 $c_j^h(a_i)$ and $\bar{c}_j^h(a_i)$ of effective control scope for alternatives

Alternatives	Experts	Criteria (weights)					
		c_1 (0.3)		c_2 (0.4)		c_3 (0.3)	
		$c_1^h(a_i)$	$\bar{c}_1^h(a_i)$	$c_2^h(a_i)$	$\bar{c}_2^h(a_i)$	$c_3^h(a_i)$	$\bar{c}_3^h(a_i)$
	e_1	250	0.42	5000	0.59	3500	0.64
a_1	e_2	280	0.47	5000	0.59	3000	0.55
	e_3	300	0.50	4000	0.47	4000	0.73
	e_1	300	0.50	6500	0.76	4000	0.73
a_2	e_2	300	0.50	5500	0.65	4000	0.73
	e_3	350	0.58	5000	0.59	4500	0.82
	e_1	400	0.67	7000	0.82	4500	0.82
a_3	e_2	350	0.58	7500	0.88	4500	0.82
	e_3	400	0.67	6500	0.76	5000	0.91
	e_1	600	1.00	8000	0.94	5000	0.91
a_4	e_2	500	0.83	8500	1.00	5300	0.96
	e_3	550	0.92	7500	0.88	5500	1.00

Table 6 The HFEs $\bar{c}_j(a_i)$ and the effective control scope E_{ij} for the alternatives

Alternatives	Criteria					
	c_1		c_2		c_3	
	$\bar{c}_1(a_i)$	E_{i1}	$\bar{c}_2(a_i)$	E_{i2}	$\bar{c}_3(a_i)$	E_{i3}
a_1	$\langle \bar{c}_1(a_1), \{0.42, 0.47, 0.50\} \rangle$	[0.42, 0.50]	$\langle \bar{c}_2(a_1), \{0.59, 0.59, 0.47\} \rangle$	[0.47, 0.59]	$\langle \bar{c}_3(a_1), \{0.64, 0.55, 0.73\} \rangle$	[0.55, 0.73]
a_2	$\langle \bar{c}_1(a_2), \{0.50, 0.50, 0.58\} \rangle$	[0.40, 0.58]	$\langle \bar{c}_2(a_2), \{0.76, 0.65, 0.59\} \rangle$	[0.59, 0.76]	$\langle \bar{c}_3(a_2), \{0.73, 0.73, 0.82\} \rangle$	[0.73, 0.82]
a_3	$\langle \bar{c}_1(a_3), \{0.67, 0.58, 0.67\} \rangle$	[0.58, 0.67]	$\langle \bar{c}_2(a_3), \{0.82, 0.88, 0.76\} \rangle$	[0.76, 0.88]	$\langle \bar{c}_3(a_3), \{0.82, 0.82, 0.91\} \rangle$	[0.82, 0.91]
a_4	$\langle \bar{c}_1(a_4), \{1.00, 0.83, 0.92\} \rangle$	[0.83, 1.00]	$\langle \bar{c}_2(a_4), \{0.94, 1.00, 0.88\} \rangle$	[0.88, 1.00]	$\langle \bar{c}_4(a_4), \{0.91, 0.96, 1.00\} \rangle$	[0.91, 1.00]

Based on the data $\bar{c}_j^h(a_i)$ in Table 5, the HFEs $\bar{c}_j(a_i)$ can be obtained and the effective control scope for the alternatives with respect to different criteria can be calculated by Eqs. (4), (5). Table 6 shows the results.

According to the four possible emergency situations of the barrier lake, the DM provided the RP according to his/her professional knowledge and experience by using interval values. The R_j and $\bar{R}_j$ are shown in Table 7, where $\bar{R}_j$ are obtained by Eqs. (6), (7).

According to the effective control scope E_{ij} and the RP $\bar{R}_j$ in Tables 6 and 7, respectively, and the positional relationship between $\bar{R}_j$ and E_{ij} in Table 2, the gain matrix (GM) and loss matrix (LM) can be constructed based on the equations in Tables 3 and 4, respectively, the GM and LM are as follows,

$$\mathrm{GM} = \begin{bmatrix} 0.04 & 0 & 0.09 \\ 0 & 0 & 0.23 \\ 0 & 0.03 & 0.32 \\ 0 & 0.12 & 0.41 \end{bmatrix}, \quad \mathrm{LM} = \begin{bmatrix} 0 & -0.18 & 0 \\ 0 & -0.06 & 0 \\ -0.04 & 0 & 0 \\ -0.33 & 0 & 0 \end{bmatrix}$$

Table 7 The RP R_j and $\bar{R}_j$

RP	c_1	c_2	c_3
R_j	[300, 350]	[6000, 7000]	[2000, 3000]
$\bar{R}_j$	[0.5, 0.58]	[0.71, 0.82]	[0.36, 0.55]

For sake of clarity, the computation of G_{13} is detailed:

$\bar{R}_3 = [0.36, 0.55]$, the effective control scope on property loss of a_1 is [0.55, 0.73], based on Table 2, their positional relationship is case 2, then:

$$0.5(E_{13}^L + E_{13}^H) - \bar{R}_3 \Rightarrow G_{13}$$
$$= 0.5(0.55 + 0.73) - 0.55 = 0.09,$$

according to Table 4.

Based on the GM and LM, the value matrix (VM) can be obtained directly by Eq. (12), just because both the R_j and E_{ij} are dimensionless. The VM is

Table 8 Overall prospect values and the ranking of alternatives

Alternatives	a_1	a_2	a_3	a_4
OPV_i	−0.1278	0.0150	**0.0912**	−0.04823
Ranking	4	2	**1**	3

$$VM = \begin{bmatrix} 0.0611 & -0.4562 & 0.1212 \\ 0 & -0.1662 & 0.2715 \\ -0.1210 & 0.0450 & 0.3561 \\ -0.8191 & 0.1522 & 0.4554 \end{bmatrix}.$$

Following the details of computation for v_{13}, given that, $G_{13} = 0.09 (0.09 > 0)$, based on Eq. (12):

$$v_{13} = G_{13}^{0.88} = 0.09^{0.88} = 0.1212.$$

The OPV_i of each alternative can be obtained by Eq. (13) and the weighting vector provided by DM (see Table 5), the results of OPV_i and the ranking of alternatives based on OPV_i are shown in Table 8.

Following the details of computation for a_1, based on Eq. (13):

$$\begin{aligned} OPV_1 &= 0.3 * 0.0611 + (-0.4562) * 0.4 + 0.1212 * 0.3 \\ &= -0.1278. \end{aligned}$$

According to Table 8, the alternative a_3 with the highest OPV is the best one (values are in bold) for coping with the barrier lake emergency situation.

Comparison

To illustrate the validity and feasibility of the proposed method, a comparison between the new information fusion process using HFSs and the aggregation process using weighted average method is performed.

The weighted average method is widely used to aggregate the experts' opinion in the aggregation process of the group decision making problem. In the weighted average method, the weight is assigned to each expert. In this paper, we assume that three experts' opinions are equally important, i.e., $(1/3, 1/3, 1/3)$. Let $\tilde{E}_{ij}$ be the aggregated information of the effective control scope. In order to make a validity comparison between the results of the two different methods, the $\bar{c}_1^h(a_i)$ will be utilized to generate the effective control scope $\tilde{E}_{ij}$, where $\tilde{E}_{ij} = \frac{1}{3}\sum_{h=1}^{3} \bar{c}_j^h(a_i)$. The results are shown in Table 9.

According to the RP $\bar{R}_j$ and the effective control scope $\tilde{E}_{ij}$ in Tables 7 and 9, respectively, and the positional relationship between $\bar{R}_j$ and E_{ij} in Table 2, the gain matrix $\widetilde{GM}$ and loss matrix $\widetilde{LM}$ can be constructed based on the equations in Tables 3 and 4, respectively, the $\widetilde{GM}$and $\widetilde{LM}$ are given as follows,

$$\widetilde{GM} = \begin{bmatrix} 0.04 & 0 & 0.0909 \\ 0 & 0 & 0.2118 \\ 0 & 0 & 0.3027 \\ 0 & 0.1176 & 0.4116 \end{bmatrix},$$

$$\widetilde{LM} = \begin{bmatrix} 0 & -0.1565 & 0 \\ 0 & -0.0382 & 0 \\ -0.0558 & 0 & 0 \\ -0.3342 & 0 & 0 \end{bmatrix}$$

Table 9 The aggregated information of the effective control scope $\tilde{E}_{ij}$

Alternatives	Experts (weights)	Criteria (weights)					
		c_1 (0.3)		c_2 (0.4)		c_3 (0.3)	
		$\bar{c}_1^h(a_i)$	$\tilde{E}_{i1}$	$\bar{c}_2^h(a_i)$	$\tilde{E}_{i2}$	$\bar{c}_3^h(a_i)$	$\tilde{E}_{i3}$
	$e_1(1/3)$	0.42		0.59		0.64	
a_1	$e_2(1/3)$	0.47	0.4607	0.59	0.5494	0.55	0.6364
	$e_3(1/3)$	0.50		0.47		0.73	
	$e_1(1/3)$	0.50		0.76		0.73	
a_2	$e_2(1/3)$	0.50	0.5275	0.65	0.6676	0.73	0.7573
	$e_3(1/3)$	0.58		0.59		0.82	
	$e_1(1/3)$	0.67		0.82		0.82	
a_3	$e_2(1/3)$	0.58	0.6392	0.88	0.8235	0.82	0.8482
	$e_3(1/3)$	0.67		0.76		0.91	
	$e_1(1/3)$	1.00		0.94		0.91	
a_4	$e_2(1/3)$	0.83	0.9175	1.00	0.9412	0.96	0.9571
	$e_3(1/3)$	0.92		0.88		1.00	

Table 10 Overall prospect values and the ranking of alternatives

Alternatives	Our proposal		Weighted average method	
	OPV_i	Ranking	$\widetilde{\text{OPV}}_i$	Ranking
a_1	−0.1278	4	−0.1096	4
a_2	0.0150	2	0.0319	2
a_3	**0.0912**	**1**	**0.0573**	**1**
a_4	−0.0482	3	−0.0480	3

The bold values highlight the optimal results of the proposed method and the weighted average method

Based on the $\widetilde{\text{GM}}$ and $\widetilde{\text{LM}}$, the value matrix $\widetilde{\text{VM}}$ can be obtained directly by Eq. (12), i.e.,

$$\widetilde{\text{VM}} = \begin{bmatrix} 0.0580 & -0.4084 & 0.1212 \\ 0 & -0.1117 & 0.2552 \\ -0.1582 & 0 & 0.3494 \\ -0.8208 & 0.1521 & 0.4579 \end{bmatrix}.$$

The $\widetilde{\text{OPV}}_i$ of each alternative can be obtained by Eq. (13), similar to the calculation of OPV_i. The results of $\widetilde{\text{OPV}}_i$ and corresponding ranking of alternatives are shown in Table 10 from column 4–5.

As it can be seen that the ranking of alternatives obtained by different methods is the same, it verifies the validity and feasibility of our proposal.

The value obtained for the best alternative based on our proposal is greater than the value obtained based on the weighted average method because the proposal considers all the information provided by experts avoiding the loss of information.

Conclusion

Current GEDM approaches aggregate experts' individual assessments that may incur in loss of information that bias the decision process. Therefore, to take all the experts' opinions into account and also, DM's psychological behaviors during the GEDM process, this paper has introduced a new GEDM that considers DM's psychological behaviors using PT and the aggregation process is replaced by a fusion process using HFSs. Eventually, a case study and a comparison with the weighted average method about a barrier lake EE that happened in real world is provided to illustrate the validity and feasibility of the proposed method.

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