Econ Theory Bull (2013) 1:3–10
DOI 10.1007/s40505-013-0010-8

SHORT PAPER

Incentive compatibility of rational expectations equilibrium in large economies: a counterexample

Yeneng Sun · Lei Wu · Nicholas C. Yannelis

Received: 10 March 2013 / Accepted: 19 March 2013 / Published online: 4 April 2013

Abstract This note provides a counterexample for a large economy in which a rational expectations equilibrium (REE) does not possess the desirable property of incentive compatibility for each agent. The key point here is that the REE equilibrium price depends on the private information of every individual agent. Thus, we propose to focus on those REE prices that depend only on the macro states and are not influenced by individual agents' private information. Such a REE will be incentive compatible.

Keywords Asymmetric information · Rational expectations equilibrium · Efficiency · Incentive compatibility · Fubini extension · Exact law of large numbers

JEL Classification C70 · D50 · D82

Y. Sun (✉)
Department of Economics, National University of Singapore,
1 Arts Link, Singapore 117570, Singapore
e-mail: matsuny@nus.edu.sg

L. Wu
J.P. Morgan, 168 Robinson Road, 18th Floor, Capital Tower,
Singapore 068912, Singapore
e-mail: lei.wu.web@gmail.com

N. C. Yannelis
Department of Economics, Henry B. Tippie College of Business,
University of Iowa, Iowa City, IA 52242, USA
e-mail: nicholasyanneli@gmail.com

N. C. Yannelis
Economics-School of Social Sciences, The University of Manchester,
Oxford Road, Manchester M13 9PL, UK

1 Introduction

Radner (1979) and Allen (1981) extended the finite agent Arrow–Debreu–McKenzie economy to allow for asymmetric information, where each agent is characterized by a random utility function, random initial endowment, and private information with a prior. The equilibrium notion that Radner puts forward is called rational expectations equilibrium (REE), which is an extension of the deterministic Walrasian equilibrium of the Arrow–Debreu–McKenzie model. According to the REE, each individual maximizes her interim expected utility conditioned on her own private information as well as the information generated by the equilibrium price.

In a REE, the private information possessed by individuals can influence the equilibrium price. This leads to the incentive compatibility issue as individuals may misreport their own private information in their favor. One would imagine that this issue should not arise in a large economy (an economy with an atomless measure space of agents) as each individual is negligible in size compared to the whole population. This note shows the contrary. We provide a counterexample for a REE in a large economy, where every agent can manipulate her private information for a higher utility.

The key point is that even if each individual is negligible in terms of size, her influence on the equilibrium price can nevertheless be significant if we allow the REE price to depend on all the information at the individual level. This suggests that considering an arbitrary REE in a large economy may not lead to desirable results. One should pay attention to those REE prices that capture the meaning of perfect competition, for example, those prices that depend only on the macro states and are not influenced by individual agents' private information. We may also point out that it may be too much to require an equilibrium price to reveal all the private information in a large economy. Our counterexample in Sect. 4 precisely has this problem.

This note is organized as follows. In Sect. 2, we present the economic model, and the notions of REE and incentive compatibility. The main result is stated in Sect. 3 with its proof in Sect. 4. Some concluding remarks are provided in Sect. 5.

2 The economic model

In this section, we define the notions of a private information economy, REE and incentive compatibility by following the model presented in Sun et al. (2012).

2.1 Private information economy

We consider an atomless probability space[1] $(I, \mathcal{I}, \lambda)$ as the space of agents. Each agent receives a *private signal* $q \in T^0 = \{q_1, q_2, \ldots, q_L\}$. $\mathcal{T}^0$ is the power set of T^0. A *signal profile* t is a function from I to T^0. For $i \in I$, $t(i)$ (also denoted by t_i) is the private signal of agent i while t_{-i} is the restriction of t to the set $I \setminus \{i\}$. Let $(T, \mathcal{T}, P)$ be a probability space that models the uncertainty associated with the private signal

[1] We use the convention that all probability spaces are countably additive.

profiles for all the agents.[2] For simplicity, we shall assume that $(T, \mathcal{T})$ has a product structure so that T is the product of T_{-i} and T^0, while $\mathcal{T}$ is the product σ-algebra of $\mathcal{T}^0$ and a σ-algebra $\mathcal{T}_{-i}$ on T_{-i}. For $t \in T$ and $t_i' \in T^0$, we shall adopt the usual notation (t_{-i}, t_i') to denote the signal profile whose value is t_i' for i and t_j for $j \neq i$.

The *private signal process* f is a function from $I \times T$ to T^0 such that $f(i, t) = t_i$ for any $(i, t) \in I \times T$. For each $i \in I$, let $\tilde{t}_i$ be the projection mapping from T to T^0 with $\tilde{t}_i(t) = t_i$.

We also would like to include another source of uncertainty in our model—the macro level uncertainty. Let $S = \{s_1, s_2, \ldots, s_K\}$ be the set of all possible macro states of nature and $\mathcal{S}$ the power set of S. The S-valued random variable $\tilde{s}$ on T models the macro level uncertainty. For each macro state $s \in S$, denote the event $(\tilde{s} = s) = \{t \in T : \tilde{s}(t) = s\}$ that s occurs by C_s. The probability that s occurs is $\pi_s = P(C_s)$. Without loss of generality, assume that $\pi_s > 0$ for each $s \in S$. Let P_s be the conditional probability measure on $(T, \mathcal{T})$ when the random variable $\tilde{s}$ takes value s. Thus, for each $B \in \mathcal{T}$, $P_s(B) = P(C_s \cap B)/\pi_s$. It is obvious that $P = \sum_{s \in S} \pi_s P_s$.

The private signal process f is essentially pairwise independent conditioned on the macro state of nature $\tilde{s}$. In other words, for λ-almost all $i \in I$, $\tilde{t}_i$ and $\tilde{t}_j$ are independent conditioned on $\tilde{s}$ for λ-almost $j \in I$. With this independence assumption, we need to work with a joint agent-probability space $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes P)$ that extends the usual measure-theoretic product $(I \times T, \mathcal{I} \otimes \mathcal{T}, \lambda \otimes P)$ of the agent space $(I, \mathcal{I}, \lambda)$ and the probability space $(T, \mathcal{T}, P)$, and retains the Fubini property, which is called a Fubini extension.[3] The process f is assumed to be $\mathcal{I} \boxtimes \mathcal{T}$-measurable.

The common *consumption set* for all the agents is the positive orthant $\mathbb{R}_+^m$. Let u be a function from $I \times \mathbb{R}_+^m \times T$ to $\mathbb{R}_+$ such that for any given $i \in I$, $u(i, z, t)$ is the *utility* of agent i at consumption bundle $z \in \mathbb{R}_+^m$ and signal profile $t \in T$. For any given $(i, t) \in I \times T$, we assume that $u(i, z, t)$ (also denoted by $u_{(i,t)}(z)$ or $u_i(z, t)$)[4] is continuous, monotonic in $z \in \mathbb{R}_+^m$.[5] For each $z \in \mathbb{R}_+^m$, $u_z(\cdot, \cdot)$ is a $\lambda \boxtimes P$-integrable function on $I \times T$.

---

[2] Thus T is a space of functions from I to T^0.

[3] See Sun (2006, Definition 2.2) for a formal definition. The existence of non-trivial, independent and measurable processes in a rich Fubini extension is shown in Sun (1998, Theorem 6.2) for general atomless Loeb product spaces. Sun (2006, Proposition 5.6) provides another construction of a rich Fubini extension with the unit interval [0, 1] as the agent space and an extended continuum product probability space as the sample space. The main results of Sun and Zhang (2009) and Podczeck (2010) show, respectively, that the agent space can be taken as an extended Lebesgue unit interval or a general saturated probability space. As noted in Wang and Zhang (2012, Corollary 1), Podczeck (2010, Theorem 1) and Sun (2006, Theorem 4.2) imply a characterization of saturation through rich Fubini extension. Sun (2006, Corollary 2.9) shows that the exact law of large numbers holds in *any* Fubini extension, and hence automatically in the rich Fubini extensions as constructed in Sun (2006), Sun and Zhang (2009) and Podczeck (2010). The previous three constructions of rich Fubini extensions do not cover the case of rich Fubini extensions based on Loeb product spaces. It remains a question to find a more general construction of rich Fubini extensions to cover all the four cases.

[4] In the sequel, we shall often use subscripts to denote some variable of a function that is viewed as a parameter in a particular context.

[5] The utility function $u(i, \cdot, t)$ is monotonic if for any $y, z \in \mathbb{R}_+^m$ with $y \leq z$ and $y \neq z$, $u(i, y, t) < u(i, z, t)$.

In our model, the *initial endowment* of an agent depends on her private signal. The *initial endowment profile* e is a function from $I \times T^0$ to $\mathbb{R}^m_+$ such that for $(i, q) \in I \times T^0$, $e(i, q)$ is the initial endowment of agent i when her private signal is q. We assume that for each $q \in T^0$, $e(\cdot, q)$ is λ-integrable over I, and $\int_I e(i, q)d\lambda$ is in the strictly positive cone $\mathbb{R}^m_{++}$.[6]

Formally, the private information economy is denoted by

$$\mathcal{E} = \left\{I \times T, u, e, f, (\tilde{t}_i, i \in I), \tilde{s}\right\}.$$

2.2 Rational expectations equilibrium

As usual, a price is a normalized nonnegative vector p in Δ_m, where Δ_m is the unit simplex of $\mathbb{R}^m_+$. A *price process* $\tilde{p}$ is a measurable function from T to Δ_m. For each $t \in T$, $\tilde{p}(t)$ is the price when the signal profile is t. For notational simplicity, the letter p will be used both for a price and a price process. The terms "price" and "price process" are used synonymously in this note.

An *allocation* x to be a measurable mapping from $I \times \Delta_m \times T^0$ to $\mathbb{R}^m_+$. For each $(i, p, q) \in I \times \Delta_m \times T^0$, $x(i, p, q)$ is the consumption bundle of agent i when the price is p and her derived signal is q.

Since an agent's initial endowment is contingent on her private signal $q \in T^0$, we denote the *budget set* for agent i by $B_i(p, q)$ when the price is p and her private signal is q. Hence, $B_i(p, q) = \{z \in \mathbb{R}^m_+ : p\, z \le p\, e(i, q)\}$.

Given a consumption bundle $z \in \mathbb{R}^m_+$, a signal $q \in q^0$ and a price p, the *interim (conditional) expected utility* of agent i is defined as follows:

$$U_i(z|p, q) = E\{u(i, z, t)|\tilde{p} = p, \tilde{t}_i = q\}. \tag{1}$$

In the REE, an agent updates her belief on the distribution of signal profiles based on her signal and observation of the equilibrium price. She computes her expected utility with the updated belief and aims to maximize the interim expected utility subject to her budget constraint. The formal definition of the REE is given below.

Definition 1 (*Rational Expectations Equilibrium* (*REE*)) A REE for the private information economy $\mathcal{E} = \{I \times T, u, e, f, (\tilde{t}_i, i \in I), \tilde{s}\}$ is a pair of an allocation and a price process (x^*, p^*) such that:

1. x^* is *feasible*, i.e., $\int_I x^*(i, p^*(t), t_i)d\lambda = \int_I e(i, t_i)d\lambda$ for P-almost all $t \in T$;
2. for λ-almost all $i \in I$ and for P-almost all $t \in T$, $x^*(i, p^*(t), t_i)$ is a maximizer of the following problem:

$$\begin{array}{ll} \max & U_i(z|p^*(t), t_i) \\ \text{subject to} & z \in B_i(p^*(t), t_i). \end{array}$$

[6] A vector z is in $\mathbb{R}^m_{++}$ if and only if all of its components are positive.

The following notion of incentive compatibility says that an agent cannot increase her interim expected utility by mis-reporting her private signal.

Definition 2 (*Incentive Compatibility*) A REE (x^*, p^*) is said to be incentive compatible if for λ-almost all $i \in I$,

$$U_i(x^*(i, p^*(t), t_i)|p^*(t), t_i) \geq U_i(x^*(i, p^*(t_{-i}, t_i'), t_i)|p^*(t_{-i}, t_i'), t_i)$$

holds for P-almost all $t \in T$ and for all $t_i' \in T^0$.

3 Incentive compatibility: a counterexample

The following result shows that it is not true that under a REE in a large economy, the agents will automatically report their signals truthfully. The key idea in such a result is that the REE price p^* can be influenced by the private signal of each individual agent; the details are given in the next section.

Proposition 1 *There exists a REE (x^*, p^*) in a large private information economy $\mathcal{E}^p$, where the private signals are independent of each other, and the macro state function $\tilde{s}$ can be regarded as constant, such that every agent has an incentive to mis-report her signal.*

4 Proof of Proposition 1

In this section, we need to use some very basic nonstandard analysis. The reader can refer Loeb et al. (2000). Fix an infinitely large even hyperinteger $n \in {}^*\mathbb{N}_\infty$. Let I be $\{1, 2, \ldots, n\}$ with its internal power set $\mathcal{I}_0$ and internal counting probability measure λ_0 on $\mathcal{I}_0$ with $\lambda_0(A) = |A|/|I|$ for any $A \in \mathcal{I}_0$, where $|A|$ is the internal cardinality of A. Let $(I, \mathcal{I}, \lambda)$ be the Loeb space of the internal probability space $(I, \mathcal{I}_0, \lambda_0)$, which will serve as the space of agents for the large private information economy considered here.

Let $T^0 = \{0, 1\}$ be the signals for individual agents, and T the set of all the internal functions from I to T^0 (the space of signal profiles). Let $\mathcal{T}_0$ be the internal power set on T, P_0 the internal counting probability measure on $(T, \mathcal{T}_0)$ (i.e., the probability weight for each $t = (t_1, t_2, \ldots, t_n) \in T$ under P_0 is $1/2^n$), and $(T, \mathcal{T}, P)$ the corresponding Loeb space.

Let $(I \times T, \mathcal{I}_0 \otimes \mathcal{T}_0, \lambda_0 \otimes P_0)$ be the internal product probability space of $(I, \mathcal{I}_0, \lambda_0)$ and $(T, \mathcal{T}_0, P_0)$. Let $(I \times T, \mathcal{I} \boxtimes \mathcal{T}, \lambda \boxtimes P)$ be the Loeb space of the internal product $(I \times T, \mathcal{I}_0 \otimes \mathcal{T}_0, \lambda_0 \otimes P_0)$, which is indeed a Fubini extension of the usual product probability space by Keisler's Fubini Theorem (see, for example, Loeb et al. 2000). Let $f(i, t) = t_i$. Then, the private signals f_i are independent of each other. In other words, we can assume that the macro state function $\tilde{s}$ is constant.

The common consumption space is the nonnegative orthant $\mathbb{R}^2_+$. For agent $i \in I$ with i odd, let her endowment $e_i = (1, 0)$, and her utility function $u_i(x_1, x_2)$ be $\left(x_1^{-2} + \left(\frac{12}{37}\right)^3 x_2^{-2}\right)^{-\frac{1}{2}}$ for positive x_1 and x_2 and zero otherwise. For agent $i \in I$

with i even, let her endowment $e_i = (0, 1)$, and her utility function $u_i(x_1, x_2)$ be $\left(\left(\frac{12}{37}\right)^3 x_1^{-2} + x_2^{-2}\right)^{-\frac{1}{2}}$ for positive x_1, x_2, and zero otherwise.[7] Given a price system (p_1, p_2), agent i's demand is $\left(p_1\left[p_1 + \frac{12}{37}p_2(\frac{p_1}{p_2})^{1/3}\right]^{-1}, \frac{12}{37}(\frac{p_1}{p_2})^{1/3}p_1\left[p_1 + \frac{12}{37}p_2(\frac{p_1}{p_2})^{1/3}\right]^{-1}\right)$ if i is odd, and $\left(\frac{12}{37}(\frac{p_2}{p_1})^{1/3}p_2\left[p_2 + \frac{12}{37}p_1(\frac{p_2}{p_1})^{1/3}\right]^{-1}, p_2\left[p_2 + \frac{12}{37}p_1(\frac{p_2}{p_1})^{1/3}\right]^{-1}\right)$ if i is even.

It can be checked that $p^1 = (\frac{1}{2}, \frac{1}{2})$ and $p^2 = (\frac{64}{91}, \frac{27}{91})$ are equilibrium prices of such a large deterministic economy.[8] Under the equilibrium price p^1, agent i's demand and utility are, respectively, $x_i^1 = (\frac{37}{49}, \frac{12}{49})$ and $u_i^1 = (\frac{37}{49})^{3/2}$ when i is odd, and $x_i^1 = (\frac{12}{49}, \frac{37}{49})$ and $u_i^1 = (\frac{37}{49})^{3/2}$ when i is even. Under the equilibrium price p^2, agent i's demand and utility are, respectively, $x_i^2 = (\frac{148}{175}, \frac{64}{175})$ and $u_i^2 = (\frac{148}{175})^{3/2}$ when i is odd, and $x_i^2 = (\frac{27}{175}, \frac{111}{175})$ and $u_i^2 = (\frac{111}{175})^{3/2}$ when i is even. It is clear that $\frac{111}{175} < \frac{37}{49} < \frac{148}{175}$.

Now, for the corresponding private information economy, define a price process

$$p^*(t) = \begin{cases} p^1 & \text{if } \sum_{j=1}^n t_j \text{ is odd,} \\ p^2 & \text{if } \sum_{j=1}^n t_j \text{ is even.} \end{cases} \quad (2)$$

Let $x^*(i, p, q)$ be x_i^1 if $p = p^1$, and x_i^2 if $p = p^2$. Then (x^*, p^*) is a REE.

Fix an agent $i \in I$ with i odd. Consider the signal profile $t \in T$ with $\sum_{j=1}^n t_j$ odd (the probability of such an event is half). Then, $x^*(i, p^*(t), t_i) = x_i^1$ with utility $u_i^1 = (\frac{37}{49})^{3/2}$. However, if agent i mis-reports her type from t_i to $t_i' = 1 - t_i$, then $1 - t_i + \sum_{j\neq i}^n t_j$ is even, and her consumption would become $x^*(i, p^*(t_{-i}, t_i'), t_i) = x_i^2$ with a strictly higher utility $u_i^2 = (\frac{148}{175})^{3/2}$. Since u_i does not depend on t, the interim (conditional) expected utility $U_i(z|p, q)$ as in Eq. (1) is always $u_i(z)$. Hence, for $t \in T$ with $\sum_{j=1}^n t_j$ odd and $t_i' = 1 - t_i$, we have

$$U_i(x^*(i, p^*(t), t_i)|p^*(t), t_i) = u_i(x^*(i, p^*(t), t_i)) = \left(\frac{37}{49}\right)^{3/2},$$

which is strictly less than

$$U_i(x^*(i, p^*(t_{-i}, t_i'), t_i)|p^*(t_{-i}, t_i'), t_i) = u_i(x^*(i, p^*(t_{-i}, t_i'), t_i)) = \left(\frac{148}{175}\right)^{3/2}.$$

Next, fix an agent $i \in I$ with i even. Consider the signal profile $t \in T$ with $\sum_{j=1}^n t_j$ even (the probability of such an event is half). Then, $x^*(i, p^*(t), t_i) = x_i^2$ with utility $u_i^2 = (\frac{111}{175})^{3/2}$. However, if agent i mis-reports her type from t_i to $t_i' = 1 - t_i$, then $1 - t_i + \sum_{j\neq i}^n t_j$ is odd, and her consumption would become $x^*(i, p^*(t_{-i}, t_i'), t_i) = x_i^1$

[7] These utility functions, which are continuous, are taken from Mas-Colell et al. (1995, 15.B.6, p. 541).

[8] A third equilibrium price is $(\frac{27}{91}, \frac{64}{91})$.

with a strictly higher utility $u_i^1 = (\frac{37}{49})^{3/2}$. Hence, for $t \in T$ with $\sum_{j=1}^{n} t_j$ even and $t_i' = 1 - t_i$, we have

$$U_i(x^*(i, p^*(t), t_i)|p^*(t), t_i) = u_i(x^*(i, p^*(t), t_i)) = \left(\frac{111}{175}\right)^{3/2},$$

which is strictly less than

$$U_i(x^*(i, p^*(t_{-i}, t_i'), t_i)|p^*(t_{-i}, t_i'), t_i) = u_i(x^*(i, p^*(t_{-i}, t_i'), t_i)) = \left(\frac{37}{49}\right)^{3/2}.$$

Therefore, every agent can mis-report her private signal to obtain a strictly higher utility under an event with probability 1/2. This means that the incentive compatibility condition fails for every agent.

5 Concluding remarks

A REE in a large economy is naturally expected to be incentive compatible as each individual in such an economy is intuitively negligible. This note provides a counterexample showing the contrary. In the counterexample, the equilibrium price flips as an individual switches from one private signal to another unilaterally, resulting in a better utility for the individual. We believe that it may be too much to require an equilibrium price to reveal the private information of every individual agent in a large economy. Therefore, we propose that one should restrict the attention to those equilibria whose prices capture the meaning of perfect competition, for example, prices that depend only on macro state of nature, as in Sun et al. (2012) and Sun and Yannelis (2008). When the REE price depends only on the macro states, incentive compatibility is not an issue since an individual agent's private signal cannot influence the macro states.[9]

References

Allen, B.: Generic existence of completely revealing equilibria with uncertainty, when prices convey information. Econometrica **49**, 1173–1199 (1981)

de Castro, L., Pesce, M., Yannelis, N.C.: Core and equilibria under ambiguity. Econ. Theory **48**, 519–548 (2011)

Condie, S., Ganguli, J.V.: Informational efficiency and ambiguous information. Econ. Theory **48**, 229–242 (2011)

Loeb, P.A., Wolff, M. (eds.): Nonstandard Analysis for the Working Mathematician. Kluwer Academic Publishers, Dordrecht (2000)

Mas-Colell, A., Whinston, M.D., Green, J.R.: Microeconomic Theory. Oxford University Press, Oxford (1995)

Podczeck, K.: On existence of rich Fubini extentions. Econ. Theory **45**, 1–22 (2010)

Radner, R.: Rational expectation equilibrium: generic existence and information revealed by prices. Econometrica **47**, 655–678 (1979)

[9] In the recent papers of de Castro et al. (2011) and Condie and Ganguli (2011), the Bayesian conditional expected utility as used here is replaced by the maximin expected utility in the setting of ambiguity aversion for a finite-agent economy with finitely many states. It will be interesting to know if one can consider ambiguity aversion in our setting with a continuum of agents and states to derive new insights.

Sun, Y.N.: A theory of hyperfinite processes: the complete removal of individual uncertainty via exact LLN. J. Math. Econ. **29**, 419–503 (1998)
Sun, Y.N.: The exact law of large numbers via Fubini extension and characterization of insurable risks. J. Econ. Theory **126**, 31–69 (2006)
Sun, Y.N., Wu, L., Yannelis, N.C.: Existence, incentive compatibility and efficiency of the rational expectations equilibrium. Game Econ. Behav. **76**, 329–339 (2012)
Sun, Y.N., Yannelis, N.C.: Exante efficiency implies incentive compatibility. Econ. Theory **36**, 35–55 (2008)
Sun, Y.N., Zhang, Y.C.: Individual risk and Lebesgue extension without aggregate uncertainty. J. Econ. Theory **144**, 432–443 (2009)
Wang, J., Zhang, Y.C.: Purification, saturation and the exact law of large numbers. Econ. Theory **50**, 527–545 (2012)