

1765: A year in the correspondence between d’Alembert and Lagrange

Pierre Crépel · Guillaume Jouve

Published online: 29 March 2014
© Centro P.RI.ST.EM, Università Commerciale Luigi Bocconi 2014

Abstract The year 1765 was a crucial year for both Lagrange and d’Alembert, but in very different ways. The two scientists had begun their correspondence in 1758, they grew closer after meeting in Paris in 1763. D’Alembert played an important role in Lagrange’s decision to leave Turin in August 1765 for Berlin. In this article, we follow the course of the year 1765 through the letters exchanged by Lagrange and d’Alembert, as well as in their relations with notable personages of the day, including Frederick II, Clairaut, Euler, Daniel Bernoulli, Condorcet, Cesare Beccaria, Frisi and Caracciolo, which will help us understand more about the personality of Lagrange and the reasons behind his departure for Berlin.

Keywords Lagrange · D’Alembert · Euler · Frederick II · Cesare Beccaria · Vibrating strings · *Destruction des Jésuites* · Differential calculus · Partial differential equations

Lagrange turned 30 on 25 January 1766. In August he left Turin for good in order to move to Berlin, but the motive behind this decision was born some years earlier, and 1765 was a crucial year. Instead, for d’Alembert 1765 was a terrible year. In the first months of it he had published, in Geneva, his *Destruction des jésuites* [8], which caused him several problems, including the refusal of the Ministry to

assign him the position he was due following the death of Clairaut. During the summer, he came very close to dying, and left the “garret” of Madame Rousseau to move to the house of Julie de Lespinasse. In the meantime, Frederick II of Prussia had once again (in vain) done his best to attract him to the Berlin Academy of Sciences. And again that same year, the France of philosophers discovered *Dei Delitti e delle Pene (On Crimes and Punishment [1])*, which Cesare Beccaria had just published.

Here we will follow the course of this crucial year for d’Alembert and Lagrange. In particular, we will discuss the correspondence between the two,¹ but we will also examine their relationships with Clairaut, Euler, Daniel Bernoulli, Condorcet, Beccaria, Frisi and Caracciolo.

The correspondence between Lagrange and d’Alembert had begun by at least 1758, but the earliest letters have been lost, and thus there remain only five letters dating between 27 September 1759 and 1 October 1763. All of these concern the topics of the publications of the two scholars, those of Lagrange having appeared in the *Miscellanea Taurinensia* and those of d’Alembert in the *Opuscules*.

A significant event in the relationship between the two men was their meeting in Paris, in the course of the journey, from November 1763 to June 1764, on which Lagrange accompanied his friend Domenico Caracciolo, then the Kingdom of Naples’ ambassador to Turin and who would shortly become the ambassador first to London, in 1764–1771, and then to Paris, in 1771–1781. Unfortunately, we have very few documents about this trip. While it is clear that Lagrange was able to meet all of the French

P. Crépel
17, rue Marcel Achard, 69120 Vaulx en Velin, France
e-mail: crepel@math.univ-lyon1.fr

G. Jouve (✉)
LML, Laboratoire de Mathématiques de Lens (EA 2462),
Faculté des Sciences Jean Perrin, Université d’Artois, Rue Jean
Souvraz SP, 18, 62307 Lens Cedex, France
e-mail: guillaume.jouve@espe-inf.fr

¹ The correspondence between Lagrange and d’Alembert is found in the *Oeuvres* [16, vol. XIII]. All correspondence quoted here is taken from that volume.

scientists and academicians, it is not clear whether he remained in contact with them, not even with Clairaut, who wrote to Daniel Bernoulli on 27 December 1763:

M. de la Grange [Lagrange] est ici maintenant, et j'ai été charmé de le voir et de causer avec lui. C'est un jeune homme très singulier tant par ses talents que par sa modestie. Son caractère est doux et mélancolique. Il ne connaît d'autre plaisir que l'être de et semble plus s'y livrer par indolence que par ambition. Nous avons été sur le point de le perdre d'une fièvre putride, et il n'est point encore hors d'affaire. Ce serait en vérité dommage.

(“Mr Lagrange is here, and I am happy to have seen him and spoken with him. This is a quite singular young man of great talent and modesty. His character is sweet and melancholy. He appears to know no other pleasure than just being and that he is motivated more by indolence than ambition. We were at the point of losing him to a putrid fever, and he is not yet out of danger. It would be a serious loss.”)

In order to fully understand the scientific content of the exchanges between Lagrange and d'Alembert, it is first of all necessary to say something about the polemic about vibrating strings that saw d'Alembert and Euler on opposite sides, beginning at the end of the 1740s, and Lagrange's intervention in the dispute.

1 The controversy over vibrating strings

In 1747, d'Alembert published his *Réflexions sur la Cause générale des vents* [3] and presented his work “Recherches sur la courbe que forme une corde tendue mise en vibration” [4] at the Paris Academy of Sciences. Today historians [10] acknowledge that d'Alembert, in these two papers, was the first to model a problem of mathematical physics with a partial derivative equation,² and was also the first to solve the resulting equation.

In his memoir on vibrating strings, d'Alembert established that the small movements of a string of length a fixed at both ends are governed by the equation $\frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$, where t represents time, and x and y the abscissa and ordinate respectively of the points of the string. Having found a general solution with the aide of differential expressions, he considered cases in which the ends were fixed and there were several different types of initial

conditions. In the case of a plucked string (taken out of its state of equilibrium and released without initial speed), he obtained the expression of the solution:

$$y(t, x) = \varphi(x + t) + \varphi(x - t)$$

The function φ is odd and $2a$ -periodic and corresponds to the shape of the string at the initial instant. This memoir of d'Alembert's was at the origin of a heated argument with Euler over the arbitrary nature of the functions used to integrate the partial differential equation. In 1750, d'Alembert explained that in order for the solution to make sense, it was necessary for function φ not to change its shape, that is, that it be everywhere defined by the same expression. Euler, who had in the meantime published a memoir [11] that was rather similar to that of d'Alembert, had however maintained that some of the restrictions placed on function φ were unnecessary, including those of being odd and periodicity.

The question raised by this controversy would occupy d'Alembert until his death, to the extent that it occupied a significant part of his research work, while Euler was concerned with it to a lesser degree. The polemic was still ongoing when, in 1759, Lagrange published his “Recherches sur la nature et la propagation du son” [12] in the first volume of the *Miscellanea Taurinensia*. Here he treated vibrating strings and provided an approach based on what we today call “discretization”. He considered a string loaded with n weights and expressed it as a system of ordinary differential equations, which he solved, and then he extended n to infinity. This method allowed him to avoid the use of partial differential equations and consequently also the arguments about the regularity of function φ . As to the form, Lagrange was insured of no criticism from either Euler or d'Alembert, but his aims were clear: he was seeking a solution that would be safe from the criticisms of Euler mentioned above.

In 1761, d'Alembert published the first volume of his *Opuscles mathématiques* [6], which were introduced by a short memoir on vibrating strings, the fruit of his reflections over the course of the preceding 10 years. He also added a supplementary text in response to Lagrange's memoir of 1759, in which he pointed to several disputed passages to the limit in the work of the Turinese scientist. This is how the correspondence between the two scholars began, with an exchange of the publications containing their respective works.

Let us go back for a moment to the “Premier Mémoire” in vol. I of d'Alembert's *Opuscles* [6], dedicated to vibrating strings, where he explained that any function φ could represent the initial shape of the string, on the condition that there be no *sauts de courbure* (abrupt changes in curvature), that is, in modern terminology, that its second derivative be continuous. In particular, this criterion implies that when $x = 0$, then $\frac{d^2\varphi}{dx^2} = 0$.

² At the time, partial differential equations were simply defined as “differential equations” without distinguishing them from ordinary differential equations. The term “partial differential equations” did not appear until around 1770. For convenience, we will use the modern expression.

Shortly after, in 1762, there appeared the second volume of the *Miscellanea Taurinensia*, which contained two memoirs regarding our topic. In particular, Lagrange published a very lengthy (over 160 pages) memoir entitled “Nouvelles recherches sur la nature et la propagation du son” [13], which focussed primarily on the propagation of sound, vibrating strings, and the nature of the functions involved in the partial differential equations. This time partial differential equations occupied a central place in his work. For the whole of 1750, Euler, d’Alembert and Lagrange devised instruments that could be used to address a vast range of mathematical-physical problems: the propagation of sound in a column of air,³ propagation of spherical waves, the motion of vibrating strings subject to a coefficient of resistance, the flow of liquids and more. Many of these problems appear in this memoir of Lagrange, in which he also proposed the method of integration.

In the same volume of the *Miscellanea Taurinensia* Lagrange presented his “Addition aux premières recherches sur la nature et la propagation du son” [14], in which he responded to the criticisms that d’Alembert had expressed in his supplement to his “Premier Mémoire” of 1761 and more generally, to the positions developed by the French scientist. After rather technical considerations on the passages to the limit and the convergence of the series used in the previous memoir, Lagrange addressed the question of what functions were admissible in the process of integration of partial differential equations. It is clear in this phase of their correspondence that Lagrange’s position is very close to that of Euler, who did not accept the “absence of jumps in curvature” required by d’Alembert.

2 The reconciliation between d’Alembert and Lagrange

Following Lagrange’s stay in Paris, from November 1763 to May 1764,⁴ the correspondence between the two scientists intensified. Their exchanges became much more regular starting in autumn 1764, and the question of vibrating strings was at the centre of many of their letters. Moreover, the letter than Lagrange sent to d’Alembert on

13 November 1764 marked the definitive reconciliation between their two positions on this question. Lagrange acknowledged that “when the initial curve can be expressed by an equation”, its even derivatives must be null in $x = 0$, which is equivalent to admitting the absence of jumps in curvature defending by d’Alembert in that particular case. However, Lagrange declared that this condition is not met when the “initial curve cannot be expressed by an equation”.

D’Alembert’s fragile health did not permit him to answer until 12 January 1765. In this letter, he reacted favourably to Lagrange’s change in stance. He also tried to turn the situation to his advantage: for d’Alembert, even in the case in which the “curve cannot be described with an equation”, the derivative $\frac{d^m y}{dx^m}$ must not have any “jumps”. His reasoning was based on the idea that the partial differential equation $\frac{d^2 y}{dt^2} = \frac{d^2 y}{dx^2}$ must be verified at each point and in each instant, which meant declaring that a “jump in curvature” corresponds to a situation in which there would be a problem in determining at least one of the two members of this equation. However, d’Alembert did not succeed in obtaining a full agreement with these considerations of his, as shown by Lagrange’s letter of 26 January. These considerations regarding curves that can or cannot be described by an equation might leave a modern reader rather perplexed, but they effectively reflect the difficulties of the scientists to agree on a general definition of the concept of function.⁵ The discussions regarding the notion of the absence of “jumps” shows the constant evolution of the concept of the continuity of a function. In effect at that time what we today call “in parts” were still defined as functions. Beginning with his memoir of 1762, d’Alembert began a more frequent use of the term “jump”, which he associated with a more geometric and less formal view of the regularity of a function; this corresponds to our modern definition of discontinuity. For decades, the two approaches of continuity/regularity would co-exist until the former finally prevailed over the latter.

3 Other scientific topics

The letters exchanged between the two scientists in the early months of 1765 addressed many other scientific questions as well; celestial mechanics played a particularly important role. The topics dealt with were primarily the methods for integrating the equations for the precession of

³ In 1747 d’Alembert had observed that the propagation of sound in a (one-dimensional) air column can be modelled by the same partial differential equation used to express the problem of vibrating strings $\frac{d^2 y}{dt^2} = \frac{d^2 y}{dx^2}$, with y in this case representing the longitudinal excursions of the particles of air.

⁴ The end of the Seven Years’ War in 1763 made it possible for scientists to travel more easily throughout Europe. In the summer of 1763 d’Alembert made a trip to Berlin, which resulted in an improvement in his relationship with Euler, which had been seriously damaged some dozen years earlier. He also planned to travel to Italy in 1765, but that fell through.

⁵ Some historians maintain that Euler provided a first general definition of the concept of function in his *Institutiones calculi differentialis* of 1755. This assertion is often refuted by other historians who note that the definition in question was not correctly used in the practice of Euler and his contemporaries.

the equinoxes, the three-body problem, the libration of the moon, and the equations for fluid motion. The reflections, discoveries and methods of Lagrange in mechanics constitute a fundamental moment in his scientific work, and here it is impossible to develop all of the topics, thus we limit ourselves at present to referring the reader to other papers in this same journal. We will content ourselves with saying a few words about the mathematical aspects of hydrodynamics.

D'Alembert, following Euler, discovered shortly before 1750 that partial differential equations could be applied to the study of the flow of a fluid. This topic was especially interesting to him. In fact, he presented a memoir on it in vol. I of his *Opuscles mathématiques* [6, Memoire 4], and prepared many others that were later published in vol. V in 1768. Instead, Lagrange's research appeared in a memoir entitled "Solution de différents problèmes de calcul intégral" [15], published in the third volume of the *Miscellanea Taurinensia*. The difficulty and problems that the two scientists discussed were never unrelated to the problem of the curves whose initial shape cannot be represented by a vibrating string. After the integration of the partial differential equation that describes the circulation of fluids, they obtained an equation of this form: $\varphi(x + y\sqrt{-1}) - \varphi(x + y\sqrt{-1}) = 2M\sqrt{-1}$, where φ is tied to the arbitrary functions affected by the integration and M is a constant.⁶ Thus the problem was limited to understanding if it was possible to determine φ , and if so, how to do it. The two shared their ideas and the methods applied (the development of a logarithmic series, etc.) without, however, arriving at a definitive solution. D'Alembert remained ever sceptical and concluded that this was another problem that could not be solved in all cases.

4 Vol. III of the *Miscellanea Taurinensia*

We know that Lagrange, together with a group of friends, had founded the *Miscellanea Taurinensia*, the publishing venue of a "private society" that would be recognised as a "royal society" in 1760, and only much later, in 1783, as a "royal academy". Charles Emmanuel III, Duke of Savoy and King of Sardinia, had no great interest in science, and thus treated scientists with indifference. For this reason Lagrange was extremely motivated to seek help and contributions from other countries in order to assure the publication's prestige. In his letter of 26 January 1765, he announced to d'Alembert that vol. III of the *Miscellanea Taurinensia* was then in preparation, and invited him to

⁶ The modern reader will undoubtedly understand that the two scholars manipulated the imaginary quantities with little care.

"enrich this volume with his name". In a letter dated 2 March, d'Alembert responded to this invitation:

C'est un honneur auquel je suis très-sensible et auquel je désire fort de pouvoir répondre. Mais comme je veux éviter les tracasseries avec les Académies, où je ne donne point de Mémoires par les raisons que je vous ai dites, ... voici ce que je pourrais faire : ce serait de vous écrire une grande Lettre où je traiterais fort sommairement différentes matières Vous pourriez donner à cet écrit le titre d'Extrait de différentes Lettres M. d'Alembert à M. de la Grange; ce serait comme une espèce d'analyse des principales choses que je dois traiter dans le quatrième Volume de mes Opuscles.

(It is an honourable offer to which I am quite sensitive, and to which I am quite anxious to respond. But as I wish to avoid quarrels with the Academies, to which I give no memoirs for the reasons I have told you,⁷ ... here is what I would do: I would write you a long letter in which I would treat very sketchily different matters.... You could give this writing the title of "Extract of Various Letters of Mr d'Alembert to Mr de la Grange [Lagrange]; it would be like a kind of analysis of the principal things I have to deal with in the fourth volume of my *Opuscles*.)

Thus, in the third volume of the *Miscellanea Taurinensia*, which did not appear until 1766, there is a memoir in the form of a *lettre factice* entitled "Extraits de différentes lettres de M. D'Alembert à M. de la Grange écrites pendant les années 1764–1765" [9]. This paper reprises the themes discussed by the two scholars in those years, as well as those investigated by d'Alembert in volumes IV and V of his *Opuscles mathématiques*.

5 The edition of Leibniz

Lagrange also solicited d'Alembert regarding another topic. Louis Dutens (1730–1812), born in Tours to Protestant parents and forced into exile in London, spent about a decade in Turin at the end of the 1750s, mainly as chargé d'affaires for Great Britain. This enterprising man undertook an edition of the works of Leibniz, published in seven volumes in Geneva in 1768. Naturally, this was not a "complete" edition, which would have required many

⁷ Subsequent to his quarrels with the European academies throughout the 1750s, d'Alembert decided not to publish his studies in the collections of these societies. Rather, he decided to go directly to a publisher, reserving his scientific papers for the new volumes entitled *Opuscles Mathématiques* [7] that he himself edited from 1761 until his death in 1783 (one would remain unpublished). He would, however, make a few exceptions to this decision.

more volumes (and which still today in the twenty-first century does not exist), but it was in any case a serious and innovative edition. Of course, Dutens tried to engage Lagrange for the mathematical parts. He met with him in Turin, and commissioned him to write a preface about the invention of infinitesimal calculus, putting him, whoever, in an uncomfortable position. Lagrange thus turned to d’Alembert for help. Reluctant to take on supplementary work, d’Alembert proposed that Lagrange refer to the entry “Calcul différentiel” in the *Encyclopédie* [5], which he himself had written and in which Leibniz and Newton were placed on equal footing regarding the invention of infinitesimal calculus. He also sent Lagrange an autograph letter from Leibniz to Varignon “found among his papers”. In the end, however, the preface requested of Lagrange never saw the light of day.

6 The Destruction des jésuites

In the second half of the eighteenth century, the Jesuits were not viewed with favour. They were not only targeted by philosophers, but had enemies within the Catholic Church as well. In particular, they had been involved in a dispute with the Jansenists for over a century. After many vicissitudes, the Jesuit colleges were closed in 1762, the Society of Jesus was suppressed in France by royal decree in 1764, and their property was confiscated.⁸

D’Alembert, who esteemed neither the “*canaille janséniste*” (Jansenist rascals), nor the “*polissons jésuites*” (naughty Jesuits), composed a work on this topic. He wrote to Lagrange on 2 March 1765 to announce the publication of the volume, entitled *Sur la destruction des jésuites en France* [8]:

Vous recevrez, à ce que j’espère, bientôt l’Histoire de la destruction des Jésuites ..., non pas celle que je vous ai lue, mais le même fond avec beaucoup d’adoucissements. J’ai tâché d’y mettre en finesse ce que j’avais mis en force dans l’autre, et je crois que le diable et la Société, et tous les fanatiques, jansénistes, molinistes, augustinistes, congruistes et autres fous en istes, n’y perdront rien.

(You will receive, soon I hope, the *History of the Destruction of the Jesuits* ..., it is not the one that you have read, but reprises the same topics in a milder manner. I have tried to be more diplomatic where elsewhere I was harsher, but I think that the devil and the Society, and all the fanatics, Jansenists, Molinists,

Augustinists, Congruists and the other lunatics and ists will lose nothing by it.)

The phrase used here “*je vous ai lue*”, the one that you have read, shows us that a first draft of the book (unfortunately lost) had been completed by the early months of 1764, when Lagrange was in Paris. Printed in Geneva, this book was presented as though penned by an *auteur désintéressé*, a disinterested author. D’Alembert spared neither the Jesuits nor the Jansenists, and condemned the Society’s *esprit philosophique*.

As everybody knows, Lagrange was neither ostentatious nor boisterous, nor was he one to throw his weight around or become involved in politics, but his ideas on religion and justice transpire at least in part in his letters. He appears to have received with favour the work announced by d’Alembert. In a letter of 20 March 1765 he wrote:

Je brûle de voir votre Histoire de la destruction des Jésuites. Il n’y en a encore ici qu’un exemplaire ...; il est entre les mains du cardinal delle Lanze; mais nos libraires ne seront pas longtemps sans en recevoir, pourvu néanmoins qu’ils ne tombent pas entre les griffes d’une certaine bête qui guette toujours avec une extrême vigilance tous les Livres nouveaux.

(I am impatient to see your *History of the Destruction of the Jesuits*. There is still but one copy here ... it is in the hands of Cardinal Delle Lanza⁹; but our booksellers will receive it before long, provided, however, they do not fall into the clutches of a certain beast that lurks always with extreme vigilance around all new books.)

Fascinated by the book, a few months later Lagrange wrote to d’Alembert about its reception in Italy:

Les fanatiques d’ici l’ont déchirée comme de raison; mais le petit nombre de ceux qui pensent l’a regardée comme l’un des meilleurs Ouvrages qui soient sortis de votre plume.

(Fanatics here have ripped it apart as expected, but the small number of those who think have regarded it as one of the best things to come out of your pen.)

As we will see, the *Destruction des jésuites* caused a few problems for its author.

Condorcet’s debut and the question of Clairaut’s pension

Spring 1765 was marked by many important events. One young scientist at the time arose into the firmaments of analysis: Condorcet. He was not yet a member of the Paris Academy of Sciences and spent much of this time in Ribemont in Picardy. He presented his book *Du Calcul*

⁸ Pope Clement XIV suppressed the Society of Jesus universally in 1773. The order was re-established by Pius VII in 1814.

⁹ Carlo Vittorio Amedeo Ignazio Delle Lanze (1712–1784).

intégral [2] to the Academy, and the commissioners (d’Alembert and Étienne Bezout) approved it on 22 May 1765. On 18 June d’Alembert wrote Lagrange to say he had sent him the book. In his reply of 6 July, Lagrange wrote that he judged it worthy of the praise that d’Alembert had honoured it with in his report to the Academy, albeit with some interrogative remarks. However, a late remark of Baron Maurice, an old friend of Lagrange, published in *Le Moniteur Universel* of 26 February 1814, appears to reverse or at least strongly qualify this positive judgment:

Le vrai talent obtenait toujours son suffrage; j’ai presque dit son hommage, tant il avait de modestie. Il ne parlait de ses prédécesseurs dans la carrière, et de ceux qui de nos jours s’y sont fait un nom mérité, qu’avec la plus haute estime et les plus grands égards. Le seul Condorcet ne pouvait trouver grace devant lui; il disait que cet académicien n’avait fait de passable que son premier ouvrage, que ses autres productions étaient médiocres ou mauvaises, et qu’il n’avait de toute sa vie intégré une équation nouvelle; qu’il aurait gâté l’analyse si on l’eût laissé faire, et qu’elle devenait entre ses mains un barbarisme complet.

(True talent always got his vote, I would almost have said his tribute, such was his modesty. He spoke of his predecessors in his career, and those of our days who have made a name they deserved, with the highest esteem and the greatest respect. Only Condorcet could not find favour with him; he said that the academician had made nothing that was passable except his first book, that his other productions were mediocre or bad, and he had never in his whole life integrated a new equation; that he would have spoiled analysis if he had been allowed to do so, and that in his hands it would have become a complete barbarism.)

The month of May was marked by the death of Clairaut, who died on the 17th. According to the by-laws of the Paris Academy, Clairaut’s pension should have passed to d’Alembert because of his seniority in the Academy. But this did not happen, and d’Alembert, in a letter dated 18 June, wrote to Lagrange about the “clear and absurd” injustice of which he was the victim: *Clairaut, qui vient de mourir, avait 9000 à 10000 francs de pension sur différents objets: on ne m’en a pas donné un sol* (Clairaut, who had just died, had a pension of 9,000–10,000 francs for different things: to me has not been given a single one).

The situation became clear in November. The reason behind the decision lay in the publication of the *Destruction des Jésuites*, which had irritated the Duke of Choiseul.¹⁰ The Academy had requested the promotion of d’Alembert on 18 May, but the final decision did not rest

with the Academy and the Ministry had blocked it. If d’Alembert chose Lagrange as a witness to this, it was above all so that this injustice would become known on the other side of the Alps.

Beccaria’s on crimes and punishment

Ailing, d’Alembert did not write to Lagrange for almost three months in the summer 1765. Still convalescent, in his letter of 28 September he described what had befallen him:

Je vois par votre Lettre, mon cher et illustre ami, que vous avez ignoré la maladie dangereuse qui m’a mis aux portes du tombeau. C’était une inflammation d’entrailles, qui m’était comme annoncée depuis longtemps par le dérangement de mon estomac.

(I see by your letter, my dear and illustrious friend, you have not heard about the dangerous disease that brought me to death’s door. It was an inflammation of the bowels, which had long been announced to me as the fault of my stomach.)

In July 1764, the Milanese marquis Cesare Beccaria published his treatise *Dei delitti e delle pene* [1] in Livorno. This short book, written in a scientific, almost mathematical style, also arrived at just the right time to extend an ongoing debate and propose a revolution in the law. It is known as the first great offensive against the death penalty, but its purpose and impact are much broader and quickly met the opposition of Voltaire and the encyclopaedists.

The first mention by d’Alembert of this book is in a letter of 9 July 1765 addressed to Paolo Frisi, a Milanese geometer and thus a compatriot of Beccaria. He says that he has read it and that he is enthusiastic, and he also tells of its translation by Morellet. D’Alembert spoke of Beccaria’s work for the first and only time in his letter of 28 September: *Connaissez-vous un Livre italien, Dei delitti e delle pene, fait un gentilhomme milanais qui me paraît penseur et vertueux ?* (Do you know an Italian book, *On Crimes and Punishment*, written by a Milanese gentleman who appears to me to be a thinker and virtuous?)

Unfortunately, we don’t have Lagrange’s reply to this question. We also do not know when Lagrange learned about this book. Yet it is difficult to believe that Lagrange had neither read nor approved of this famous work from this period; it seems inconceivable that neither Frisi or Caracciolo brought his attention to the subject. Although Milan was under Austrian domination, and Turin was not at the time part of the same country, all the cities of Italy were tied by a common language and information circulated quickly.

¹⁰ Étienne-François, comte de Stainville, duc de Choiseul (1719–1785).

7 Lagrange's departure for Berlin

Before 1766, Frederick II of Prussia had more than once invited both d'Alembert and Lagrange to move to Berlin. The encyclopaedist had declined the offer and suggested the name of his Turinese friend. But Lagrange, even though he was on good terms with Euler, did not relish a "cohabitation" with him in the Berlin Academy. The situation changed at the beginning of 1766, and in a letter of 6 March 1766 d'Alembert spoke of Lagrange about it in these terms:

M. Euler s'en va, dit-on, à Pétersbourg pour quelque mécontentement qu'il a eu à Berlin. Je lui ai écrit pour l'en dissuader. S'il s'en va, et que vous vouliez le remplacer, vous n'avez qu'à m'écrire un mot et je ferai de mon mieux pour vous servir.

(Mr Euler is leaving, he says, for St. Petersburg because of some unhappiness he has had in Berlin. I wrote to him to dissuade him. If he leaves, and you want to replace him, you have only to write me and I will do my best to serve you.)

D'Alembert also wrote to Frederick II in praise of Lagrange, and in his letter of 26 April 1766, he announced to the erudite Turinese scholar:

Mon cher et illustre ami, le roi de Prusse me charge de vous écrire que, si vous voulez venir à Berlin pour y occuper une place dans l'Académie, il vous donne 1500 écus de pension qui sont 6000 livres argent de France; ... M. Euler mécontent pour des raisons dont je ne sais pas bien le détail, mais dans lesquelles je vois que tout le monde lui donne le tort, demande son congé et veut s'en aller à Pétersbourg. Le roi, qui n'a pas trop d'envie de le lui accorder, le lui donnera certainement si vous acceptez la proposition qu'on vous fait;....

(My dear and illustrious friend, the king of Prussia has charged me to write you that, if you would like to come to Berlin to occupy a place in the Academy, he would give you a pension of 1,500 crowns, which are 6,000 French pounds ... Mr Euler, unhappy for reasons of which I do not know the details, but in which I see that everyone thinks him wrong, requests permission to leave and wants to go to St. Petersburg. The king, who was not too anxious to grant it, would definitely give it to him if you accept the proposition that he has made;)

Reading the epistolary exchanges between the two scientists shows that they shared a common sentiment: they felt they were treated with less favour in their own country than they would have been in a foreign country. In retrospect, when one reads a little between the lines in the

correspondence of 1764–1765, one can feel the rise in Lagrange's bitterness that will lead him into exile. As Marco Segala [17] shows, a low, stagnant salary forced him to teach in a rather hostile environment, and uninterested in theory, the contempt of Piedmont authorities for science and remoteness of European intellectual centres led Lagrange to leave his country and accept the offer of d'Alembert and Frederick II in Berlin. He regrets that in Turin, *on regarde la science dont [il s']occupe comme très-inutile et même ridicule* (one regards the science with which I am occupied as quite unnecessary and ridiculous, letter of 10 May 1766). Fearing that his letters to d'Alembert might be intercepted, he also used roundabout methods to deliver correspondence to Paris. But the King of Sardinia did not allow him to leave very easily. He had made up his mind to accept Frederick II's offer, but he was not granted permission to leave Turin until early July 1766.

Lagrange left Turin definitively on 21 August 1766 and arrived to Berlin at the end of October, after a short time spent in Paris and London. Thus began for Lagrange a new period of intense scientific production.

(Translated from the French by Kim Williams)

Appendix: The new edition of the works of d'Alembert by Pierre Crépel

Editions of complete works (all told, more or less 'complete'!) of Lagrange and Laplace have existed since the nineteenth century. Those of Euler and the Bernoullis were begun in the twentieth: Euler's *Opera omnia* is almost at an end, while the *Gesammelte Werke* of the Bernoullis are being published now. However, there is no edition of complete works thus far for either d'Alembert or Clairaut. Rather, to be more precise, editions of d'Alembert's work were produced at the beginning of the nineteenth century, but these were limited to his literary writings. Why? There are at least four reasons. (1) D'Alembert is much more difficult to read for a mathematician or a physicist today than Euler or Lagrange, where the style is modern and the writings much better structured. (2) D'Alembert, above all in the last 30 years of his life, scarcely applied himself to editing his scientific memoirs. (3) The connections between text and figures are somewhat outdated and puzzling for us. (4) The vocabulary vacillates between a terminology that is well defined and everyday language; for example, words such as 'force', 'power' and 'convergence' assume different meanings, sometimes within the same sentence.

Publishing d'Alembert's scientific works is thus more difficult than publishing it is for his contemporaries. On the other hand, as Lacroix remarked, already by the beginning of the nineteenth century, his works were hardly read, since it was preferred to study his discoveries by means of the

works of scientists such as Lagrange who had digested and transformed them.

The edition of d'Alembert's *Oeuvres Complètes* now underway (six volumes published since 2002) thus faces a tricky challenge: to give the original texts, their variants, to situate them in context; explain them thoroughly, without confounding the author and the interpretations. It comprises five series: (1) mathematics up to 1756; (2) the texts of the *Encyclopédie*; (3) mathematics after 1756; (4) literature, history and philosophy; (5) correspondence. The effort is so much more worthwhile since d'Alembert's work contains many ideas that are incomplete and stimulating, and shows an author at work, not only his final products.

More information about the *Oeuvres Complètes* of d'Alembert is available online at: <http://dalembert.obspm.fr/>

References

1. Beccaria, C.: *Dei Delitti e delle Pene*. Livorno (1764). Eng. trans. On crimes and punishment, David Young, trans. Hackett, New York (1986)
2. de Condorcet, N.: *Du Calcul intégral*. Didot, Paris (1765)
3. D'Alembert, J.L.R.: *Réflexions sur la Cause générale des Vents*. David l'aîné, Paris (1747)
4. D'Alembert, J.L.R.: Recherches sur la courbe que forme une corde tendue mise en vibration. *Mémoires de l'Académie royale des sciences de Berlin, année 1747*, pp. 214–249 (1749)
5. D'Alembert, J.L.R.: Calcul différentiel. *Encyclopédie ou dictionnaire raisonné des sciences, des arts et des métiers*, vol. IV, pp. 985a–989b (1754)
6. D'Alembert J.L.R.: *Opuscules mathématiques*, vol. I (1761). In: Crépel, P., Guilbaud, A., Jouve, G. (eds.) Rpt. in *Oeuvres complètes*, vol. III/1. CNRS éditions, Paris (2008)
7. D'Alembert, J.L.R.: *Opuscules mathématiques*, 8 vols. Paris (1761–1780)
8. D'Alembert, J.L.R.: *Sur la destruction des Jésuites en France*, Geneva (1765)
9. D'Alembert, J.L.R.: Extraits de différentes lettres de M. D'Alembert à M. de la Grange écrites pendant les années 1764–1765. *Miscellanea Taurinensia*, vol. III, 1762–1765, pp. 381–396 (1766)
10. Demidov, S.S.: Création et développement de la théorie des équations différentielles aux dérivées partielles dans les travaux de J. D'Alembert. *Revue d'Histoire des Sciences* **35**, 3–42 (1982)
11. Euler, L.: Sur la vibration des cordes. *Mémoires de l'Académie royale des sciences de Berlin IV*, pp. 69–85 (1750)
12. Lagrange, J.-L.: Recherches sur la nature et la propagation du son. *Miscellanea Taurinensia*, vol. I, pt. III, pp. 1–112 (1759). Rpt. in [16], vol. I, pp. 39–148, Gauthier-Villars, Paris (1867)
13. Lagrange, J.-L.: Nouvelles recherches sur la nature et la propagation du son. *Miscellanea Taurinensia*, vol. II, 1760–1761 (1762). Rpt. in [16], vol. I, pp. 151–316, Gauthier-Villars, Paris (1867)
14. Lagrange, J.-L.: Addition aux premières recherches sur la nature et la propagation du son. *Miscellanea Taurinensia*, vol. II, 1760–1761 (1762). Rpt. in [16], vol. I, pp. 319–332, Gauthier-Villars, Paris (1867)
15. Lagrange, J.-L.: Solution de différents problèmes de calcul intégral. *Miscellanea Taurinensia*, vol. III, 1762–1765, pp. 179–380 (1766). Rpt. in *Oeuvres*, vol. III, pp. 471–668, Gauthier-Villars, Paris (1867)
16. Lagrange, J.-L.: *Oeuvres de Lagrange* (14 vols). Joseph Alfred Serret, ed. Gauthier-Villars, Paris (1867–1892)
17. Segala, M.: Lagrange: 'Why I left Turin and never went back again'. *Nuncius* **25**(1), 23–40 (2010)



Pierre Crépel was a CNRS researcher in mathematics and history of mathematics at the Université Rennes 1 and the Université Lyon 1. His work focuses on the history of science and mathematics in the eighteenth century. Author of several publications on Condorcet, he was also one of the initiators of the project for the *Oeuvres Complètes de D'Alembert*. He is currently a member of the Academy of Sciences, Belles-Lettres et Arts de Lyon.



Guillaume Jouve is a lecturer in the history of mathematics at the University of Artois (Laboratoire de mathématiques de Lens). His work concerns the history of the physical-mathematical sciences and more particularly the history of analysis and its partial differential equations in the eighteenth and nineteenth centuries. In 2007 he defended a thesis on the problem of vibrating strings in the second part of the work of d'Alembert. He is also a member of the editorial board of the *Oeuvres Complètes de D'Alembert* and has directed several volumes of that edition.