

Determination of geopotential value W_0^L at Polish tide gauges from GNSS data and geoid model

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Abstract Results of three campaigns of Baltic Sea Level Project and further studies reveal the GPS and spirit levelling data possibly contain errors which affect SST and W_0^L computations. For that reason, the old data were revised and additionally, in spring 2015, the new GNSS campaign was carried out at tide gauges in Swinoujscie, Ustka and Wladyslawowo. The study concerns computation of the local average geopotential value W_0^L using water level data at the three tide gauges, geoid undulations from a global geopotential model EGM2008 and ellipsoidal heights from GNSS observations, which were obtained using revised data from three campaigns of Baltic Sea Level Project and from the new campaign conducted in 2015. Results obtained indicate that the best estimation of W_0^L was achieved from the campaign carried out in 2015, where the mean value of W_0^L calculated for three investigated tide gauge stations is equal to $62636857.45 \text{ m}^2 \text{ s}^{-2}$.

Keywords Geoidal geopotential · Height system unification · Sea surface topography · Tide gauge · Vertical datum

1 Introduction

At present there are more than 100 height systems realized globally. Most of them refer to a local mean sea level at a reference tide gauge. The current development of space geodetic techniques enables unification of local vertical datums into a single global vertical datum. In practice, this involves determination of the geopotential value W_0 of the level surface which globally represents the ideal ocean surface or the so-called Gauss–Listing geoid. Here we define the local or regional datum by W_0^L of the surface which best fits the mean

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sea level in a defined region at a given epoch. In this case W_0^L can be determined by constructing points on the geoid and estimating the geopotential values at these points using a high resolution global Earth gravitational model (GGM) e.g. Heck and Rummel (1990) and Heck (2004).

Work on the definition and creation of global vertical datum is underway in the “Vertical Datum Standardisation” working group which directly depends on the GGOS Theme 1 and is supported by the IAG Commissions 1 (reference frames) and 2 (gravity field), as well as by the International Gravity Field Service (IGFS). The main purpose of the working group is to provide a reliable geopotential value W_0 to be introduced as the conventional reference level for the realization of a Unified Global Height System. The value of the most commonly accepted W_0 (McCarthy and Petit 2004) presents discrepancies of about $2 \text{ m}^2 \text{ s}^{-2}$ with respect to recent computations, therefore work on new estimations are continuing (Sánchez 2012; Sánchez et al. 2014).

In Europe, the attempt to unify the national height systems was carried out by Rülke et al. (2013). The gravity field approach, more than 1300 physical heights given in European national height reference frames, and GNSS derived ellipsoidal heights have been used. Different gravity field models have been selected. The study reveals that the gravimetric approach is the preferred method for the realization of a World Height System.

Investigations relating to unification of a vertical datum in the region of Baltic Sea were conducted in Poland by the Department of Planetary Geodesy SRC PAS between 1990 and 1997 within the three campaigns of Baltic Sea Level Project (BSLP) (Kakkuri 1990, 1995; Poutanen and Kakkuri 1999).

The results of the first campaign did not allow researchers to get enough credible information about the topography of the Baltic sea as the accuracy of one ellipsoidal height was found to be no better than $\pm 4.6 \text{ cm}$ (Kakkuri 1994).

Determination of the topography of the Baltic Sea, among others, was the result of the second campaign (Kakkuri 1995). The sea surface topography (SST) was computed in respect to the regional geoid model computed by Vermeer (1995) based on a method of buried masses and OSU91A geopotential model. SST calculated at the tide gauge station in Swinoujscie was—30.0 cm, Ustka—34.4 cm and Wladyslawowo—17.4 cm, as it is shown in Fig. 1. We suspect that SST value at the tide gauge Wladyslawowo has not been computed correctly since it does not agree with the SST computed at the neighboring stations in Swinoujscie and Ustka.

Based on the results of the European Vertical Reference Network (EUVN) campaign conducted in 1997 (Wöppelmann et al. 2000), SST obtained in Swinoujscie equals—3 cm, in Ustka—11 cm and in Wladyslawowo—11 cm. Computed values are referred to European geoid model EGG97 calculated by Denker et al. (2009).

Uncertainties in the data from the Polish tide gauge stations can also be found in Dayoub (2010). The author presents the results of calculations of the gravity potential for tide gauge stations for the first, second and the third campaign of the Baltic Sea Level Project (Dayoub 2010, p. 78). In all these cases, values of W_0^L obtained at the tide gauge stations in Swinoujscie and Ustka significantly differ from the other values and were not taken by the author for further calculations.

Having access to the original records and the levelling data kindly released by the Department of Geodesy Space Research Centre PAS, the W_0^L from the revised data were computed once more at the three main Polish tide gauges. The W_0^L for each campaign was computed using mean sea level estimated from the tide gauge data received from the Institute of Meteorology and Water Management Maritime Branch in Gdynia and EGM2008 geopotential model up to degree 2190 and order 2159 (Pavlis et al. 2012).

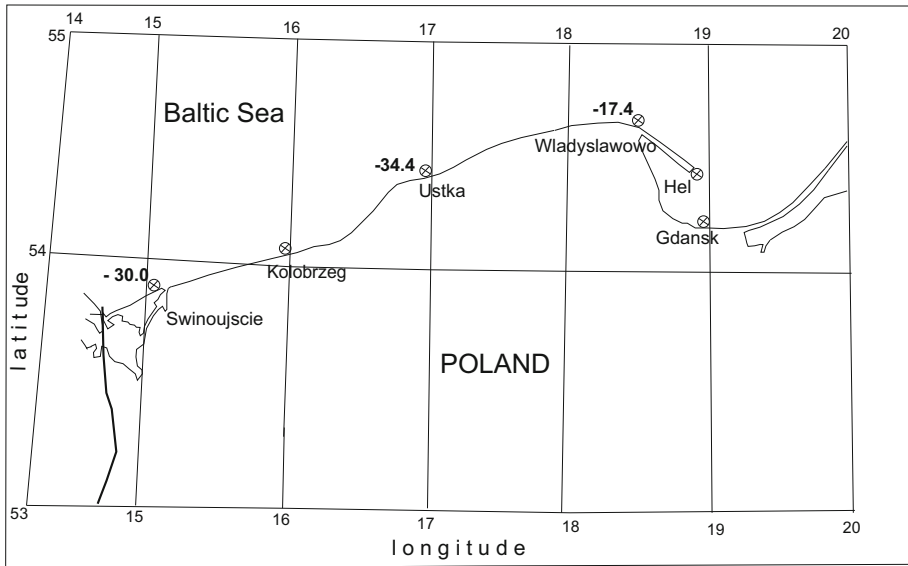


Fig. 1 SST computed from the observations of the second *Baltic Sea Level Project* campaign (Kakkuri and Poutanen 1997)

Results of this calculation are given in Table 1. The values of W_0^L differ for each campaign since they are computed for different ITRF solutions and epochs.

Many factors have influenced the accuracy of W_0^L computations, including the accuracy of geopotential model EGM2008. Conducted investigations e.g. Lyszkowicz (2009) indicate that this model on the territory of Poland has an accuracy about $0.2 \text{ m}^2 \text{ s}^{-2}$ ($\sim 2 \text{ cm}$), which is yet to be accepted.

Therefore the study concerning the calculation of SST and W_0^L at Polish tide gauge stations in Wladyslawowo, Ustka and Swinoujscie were undertaken again and the fourth GNSS campaign was conducted.

The theoretical foundations of determining SST and W_0^L are given in the Chapter 2. The next chapter contains results obtained from the new 4th GNSS campaign at the tide gauge stations realized in April 2015. The last chapter contains conclusions.

Table 1 W_0^L computed from the improved data of Baltic Sea Level Project campaigns

| Campaign | Mean value of W_0^L ($\text{m}^2 \text{ s}^{-2}$) | Discrepancies from the mean value ($\text{m}^2 \text{ s}^{-2}$) | |
|----------|---|---|------|
| | | Min | Max |
| 1 | 62636857.24 | -0.44 | 0.35 |
| 2 | 62636857.21 | -0.90 | 0.55 |
| 3 | 62636858.23 | -0.01 | 0.01 |

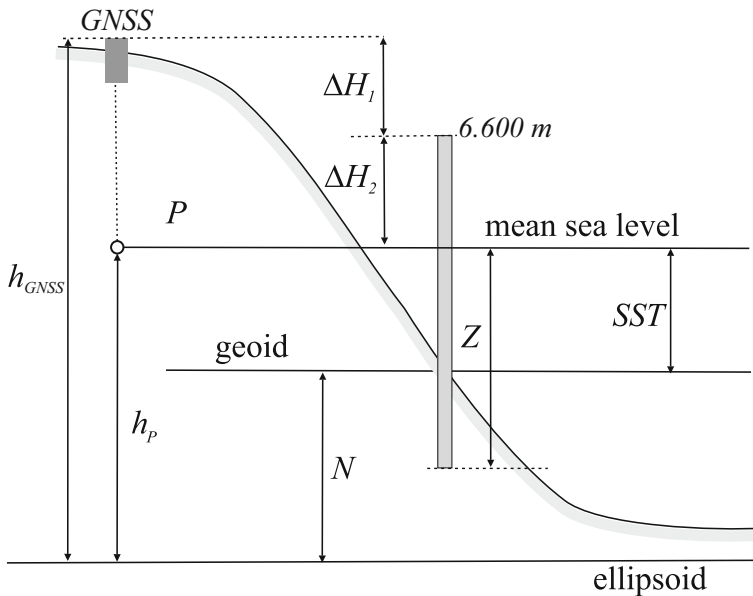


Fig. 2 SST and ellipsoidal height h_P

2 Theoretical foundations

Figure 2 shows that a sea surface topography SST can be computed from the equation:

$$SST = h_{GNSS} - N - \Delta H_1 - \Delta H_2 \quad (1)$$

where h_{GNSS} is an ellipsoidal height from GNSS observations, N is a geoid-ellipsoid separation, ΔH_1 is a height difference from spirit levelling and ΔH_2 in the case of Polish tide gauges usually is

$$\Delta H_2 = 6.600m - Z \quad (2)$$

where Z (in meters) is computed from the tide gauge data at the epoch of GNSS observations. Equation (1) can be evaluated using either a tide-free system, a mean-tide system, or a zero-tide system. In our computations the tide-free system has been adopted for the sake of convenience as h_{GNSS} and N were given in a tide-free system.

In order to determine the gravity potential on the geoid, the ellipsoidal height of the point P at the mean sea level should be known (Fig. 2):

$$h_P = h_{GNSS} - \Delta H_1 - \Delta H_2 \quad (3)$$

The point P is a virtual point, where its ellipsoidal coordinates $(h_P, \lambda_P, \varphi_P)$ are obtained from GNSS measurements, where $\lambda_P = \lambda_{GNSS}$, $\varphi_P = \varphi_{GNSS}$ and h_P is computed from the Eq. (3). Indispensable to the calculation of the gravity potential W_0^L , spherical coordinates $(r_P, \lambda_P, \varphi_P)$ can be calculated by transforming ellipsoidal coordinates into spherical coordinates according to the formulas given, e.g. in Hofmann-Wellenhof and Moritz (2006).

Having spherical coordinates, the gravity potential W_0^L of the mean sea level can be determined using the equation (Hofmann-Wellenhof and Moritz 2006):

$$W_0^L(r_P, \lambda_P, \varphi_P) = \frac{GM}{r_P} \left[1 + \sum_{n=2}^{\infty} \sum_{m=0}^n \left(\frac{R}{r_P} \right)^n (\bar{C}_{nm} \cos m\lambda_P + \bar{S}_{nm} \sin m\lambda_P) \bar{P}_{nm}(\sin \varphi_P) \right] + \frac{\omega^2 r_P^2}{2} \cos^2 \varphi_P \quad (4)$$

where \bar{C}_{nm} , \bar{S}_{nm} are fully normalized spherical harmonic coefficients of degree n and order m , GM is the geocentric gravitational constant, R is the radius of the Earth, \bar{P}_{nm} are the fully normalized associated Legendre functions and ω is the angular velocity of the Earth.

The W_0^L at the tide gauge stations was computed using GrafLab software (Bucha and Janak 2013), with ellipsoidal heights h_P computed from Eq. (3).

3 Results of the campaign conducted in 2015

For the reasons given above (see Sect. 1), it was decided to conduct a new GNSS campaign and levelling measurements at the tide gauge stations in Wladyslawowo, Ustka and Swinoujscie. The GNSS benchmarks were established at an immediate vicinity of the tide gauges and were connected by a spirit levelling to the tide gauge stations and to the national precise levelling network. In 24–26 April 2015, the GNSS measurements in Ustka and Swinoujscie were carried out during the 48-h session using Topcon HiPer Pro receivers.

The tide gauge in Wladyslawowo is connected with WLAD reference station of the ASG-EUPOS system, which is the Polish national system of 130 stations conducting permanent GNSS observations (www.asgeupos.pl). Therefore we used GNSS observation which were carried out in the frame of ASG-EUPOS network.

The processing was carried out using the Bernese GNSS Software v.5.2 (Dach et al. 2007). For computations, the six EUREF Permanent Network (EPN) stations were used as reference stations. In general, the processing strategy followed the current EPN recommendations (http://www.epncb.oma.be/_documentation/guidelines/guidelines_analysis_centres.pdf). The collected dual frequency GPS data were processed in two 24-h sessions using IGS final precise satellite orbits and clocks, earth rotation parameters, absolute antenna phase center variations and offsets for receiver and satellite antennas.

Using the method described in the previous section, SST at the three tide gauge stations for the epoch of the new GNSS campaign were calculated. These values were compared with the SST computed in the frame of the second and the third Baltic Sea Level campaign. The results of computations are shown in Fig. 3.

As it is shown by Kakkuri and Poutanen (1997), the surface of the Baltic Sea should generally rise toward the north, while along the Polish part of the coast it is rather flat. Oceanographic studies show the same trend, e.g. Lisitzin (1974). The sea surface topography determined from the precise levelling of the tide gauges (Ekman 1993) also shows the same trend. This indicates that the results obtained during 2015 GNSS campaign are correct.

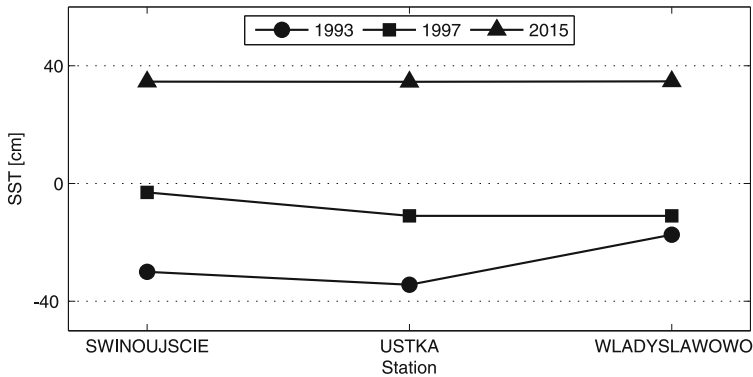


Fig. 3 SST for the Polish part of the Baltic Sea computed from the second (Kakkuri 1995), third (Wöppelmann et al. 2000) and 2015 campaign. SST are referred to different geoids

Table 2 W_0^L calculated for the spring 2015 GNSS campaign for the epoch 2015.31

| Tide gauge station | W_0^L ($\text{m}^2 \text{s}^{-2}$) |
|--------------------|--|
| Swinoujscie | 62636857.45 |
| Ustka | 62636857.45 |
| Wladyslawowo | 62636857.43 |
| Mean value | 62636857.44 |

The gravity potential W_0^L was calculated adopting the following values of constants: $3.986004415 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$ for GM and $6,378,137 \text{ m}$ for R . In the calculation EGM2008 geopotential model to maximum degree 2190 and reference ellipsoid GRS80 were used.

The mean value of W_0^L computed from GNSS observations of the campaign conducted in 2015 is equal to $62636857.44 \text{ m}^2 \text{ s}^{-2}$ (Table 2).

Dayoub (2010, p. 71) estimated the mean value of W_0^L from the all three Baltic Sea Level Project campaigns as $62636855.04 \pm 0.36 \text{ m}^2 \text{ s}^{-2}$ which refers to 1994.57 epoch. This value is slightly different from the value computed in this paper because it refers to a different epoch. Next in the study (Dayoub 2010) the UK, French and German datums were unified on the basis of their geopotential values. Using the high resolution Earth gravity field model EGM2008 the geopotential value, which defines the geoid at the tide gauge sites, was estimated as $62636857.88 \pm 0.33 \text{ m}^2 \text{ s}^{-2}$, which is very close to our results.

Furthermore, IAG Resolution (No. 1) for the definition and realization of an International Height Reference System (IHRs) adopted in Prague IUGG 2015 General Assembly, accepted the value $62635853.4 \text{ m}^2 \text{ s}^{-2}$ as realization of the potential value of the vertical reference level for the IHRs. This value significantly differs from the value adopted in IERS conventions: IERS 2003 Convention (McCarthy and Petit 2004) and IERS 2010 Convention (Petit and Luzum 2010), which is $62636856.0 \text{ m}^2 \text{ s}^{-2}$. In comparison to our estimation, the differences are 4 and $1.4 \text{ m}^2 \text{ s}^{-2}$ respectively (datum offset corresponds to about 40 and 14 cm).

The estimated gravity potential difference between the Kronstadt86 vertical datum in Poland and the global one defined by IERS Convention 2010 is shown in Lyszkowicz et al. (2015). The computation is based on the use of ellipsoidal heights from satellite observations, normal heights obtained from the levelling campaign and quasigeoid/ellipsoid separations computed from the EGM2008 geopotential model. Results vary depending on

the satellite network used from 0.158 to 0.606 m² s⁻², which correspond to about 2 and 6 cm respectively.

The offsets between the European Vertical Reference Frame 2007 (EUVN2007) and the respective national height systems are published on <http://www.bkg.bund.de>. For Poland it is equal to +17 cm. Results of the unification of European height system realizations based on gravimetric approach using global gravity models from the recent satellite gravity missions GRACE and GOCE are given in Rülke et al. (2013). The estimated offset for Kronstadt2006 using the filter-combined GOCO03S+EGG2008 gravity field model in reference to U_0 GRS80 (62636860.850 m² s⁻²) is equal to 49 cm. The comparison with our result is unjustified given inter alia the different epochs of computations and height system realizations.

4 Summary

Uncertainties in the quality of the data from the Polish tide gauge stations were found in e.g. (Kakkuri 1994, 1995; Dayoub 2010). Therefore the revised calculation of W_0^L and SST from the three Baltic Sea Level campaigns has been done using access to the original records and the levelling data.

The W_0^L for each campaign was computed for the mean sea level estimated from the tide gauge data and EGM2008 geopotential model (Table 1). Apparently the W_0^L value obtained as a mean value from three stations differs for each campaign and differences from station to station in every campaign were significant, which does not correspond to physical assumption. Therefore we decided to carry out the new GNSS campaign and new spirit levelling measurements.

The new GNSS campaign at the three main Polish tide gauges in Swinoujscie, Ustka and Wladyslawowo was carried out in spring 2015. The mean value of W_0^L computed from GNSS observations for this campaign is equal to 62636857.44 m² s⁻². Therefore the ellipsoidal coordinates of the tide gauge stations determined in the 2015 campaign together with the spirit levelling data can be recommended for further studies of SST and W_0^L in the region of the Baltic Sea. On the other hand, W_0^L can serve as the local reference value for geodynamic studies or can be used for unification of height systems.

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