Int J Energy Environ Eng (2015) 6:245–254
DOI 10.1007/s40095-015-0172-2

ORIGINAL RESEARCH

Unification perspective of finite physical dimensions thermodynamics and finite speed thermodynamics

Stoian Petrescu[1] · Michel Feidt[2] · Vlad Enache[1] · Monica Costea[1] · Camelia Stanciu[1] · Nicolae Boriaru[1]

Received: 24 November 2014 / Accepted: 26 March 2015 / Published online: 15 April 2015

Abstract The paper studies the possibility of unifying the two branches of the irreversible engineering thermodynamics, namely finite physical dimensions thermodynamics (FPDT) and finite speed thermodynamics (FST), aiming to take into account their benefits and successes and to eliminate as much as possible their disadvantages. Actually, the two branches have the same goal, that of optimizing the performance of thermal machines and they were developed almost in parallel. Analysis of thermal machines cycles using the FPDT is based on the first and second law of thermodynamics, in the presence of the external irreversibility generated by the heat transfer at finite temperature difference at the thermal reservoirs and internal irreversibility, using the internal source of entropy considered as parameter or function to be specified. The FST is based on the mathematical expression of the first law for process with finite speed that involves three causes of internal irreversibility, namely the finite speed of the piston, internal friction and throttling. The direct method is used in the analysis of thermal machines cycles to provide analytical expression of the machine performance (efficiency and power) as a function of the speed of the process. The significant progress of these two branches of irreversible engineering thermodynamics makes their unification a desirable outcome. We hope that the new model yielded from this study will provide an even more important tool for engineers that will help their attempt to a better design and optimization of thermal machines.

✉ Vlad Enache
vlad.enache.email@gmail.com

[1] Department of Engineering Thermodynamics, Engines, Thermal and Refrigeration Equipments, University Politehnica of Bucharest, Splaiul Independenţei 313, 060042 Bucharest, Romania

[2] University of Lorraine, LEMTA, URA CNRS, 7563, 2, avenue de la Forêt de Haye, TSA 60604, 54518 Vandœuvre Cedex, France

Keywords Finite physical dimensions thermodynamics · Finite speed thermodynamics · Internal and external irreversibility · Source of internal entropy · Thermal machines

List of symbols

A_p	Piston area (m^2)
B	Coefficient
C_T	Total cost (cumulated cost of heat exchangers H and L) ($)
K_A	Constant dealing with static friction (bar)
K_C	Constant dealing with dynamic friction and throttling (bar)
L	Lagrange function
m	Mass of the gas (kg)
O	Objective function
p_1	Maximum pressure (bar)
$\dot{Q}_H$	Heat flux transferred from the source (W)
$\dot{Q}_L$	Heat flux transferred to the sink (W)
$\dot{W}$	Mechanical power output (W)
$\dot{S}_H$	Hot heat exchanger's entropy source (W/K)
$\dot{S}_L$	Cold heat exchanger's entropy source (W/K)
$\dot{S}_M$	Internal entropy source (W/K)
$\dot{S}_T$	Total entropy source (W/K)
T_{SH}	Source temperature (K)
T_{SL}	Sink temperature (K)
T_H	High temperature (for the internal system) (K)
T_L	Low temperature (for the internal system) (K)
V_1	Minimum volume (m^3)
w	Piston's average speed (m/s)

Greek symbols

α Coefficient
ε_H Isothermal volumetric compression ratio (–)
γ Specific heat ratio (–)
η Efficiency (–)

Introduction

Two branches of the engineering irreversible thermodynamics, namely finite physical dimensions thermodynamics (FPDT) and finite speed thermodynamics (FST), have been developed almost in parallel in recent years. They have the same goal, to model and optimize the performance of thermal machines.

The analysis and optimization of thermal machine cycles by FPDT are based on the first and second law of thermodynamics, where the external irreversibility generated by the heat transfer at finite temperature difference is considered, together with internal irreversibility introduced by the internal source of entropy. The finite dimension considered in the models could be thermal conductance, heat transfer area in the machine's heat exchangers, or cost of heat transfer area. Several models have been developed for direct and reverse cycle thermal machines as Stirling, Carnot, Otto, Diesel, Ericsson, Brayton–Joule, etc. [1–7].

FST is based on the expression of combined first and second law of thermodynamics for processes with finite speed that involves internal irreversibilities due to finite piston speed, internal friction and throttling of the gas in valves, and also external irreversibilities due to heat transfer at finite temperature difference at the source and the sink. Analysis of thermal machine cycles is made by the direct method that is able to derive analytical relationships of machine performance (efficiency/COP and power) depending on the speed of the process and other variables [8–14]. Also, the entropy generation in the cycle and system can be expressed as a function of speed and many constructive and operational parameters, such as temperature of the thermal reservoir, the compression ratio and the dimensions.

The significant progress of these two branches of engineering irreversible thermodynamics justifies an investigation into the feasibility of their unification. The present paper is aimed at establishing the framework of this unification tentative, highlighting the contribution of each branch.

Analyzing the number of variables and the relations between them, we find that the unification between FPDT and FST is not possible through a single "connection point" (the internal entropy source), but it has to be done through two "connection points" (the two heat fluxes, which have to be equal between the two theories).

The model

As in Ref. [4], either other processes inside the engine are adiabatic (Carnot engine) or the heat exchanges are entirely recuperated inside the engine (Stirling engine with perfect thermal regeneration).

The following analysis depends crucially on the number of parameters, the number of variables and the number of relations, so it is important to enumerate them clearly. The quantities of interest are given as follows:

1. The heat flux $\dot{Q}_{\mathrm{H}}$ transferred from the source,
2. The heat flux $\dot{Q}_{\mathrm{L}}$ transferred to the sink,
3. The mechanical power $\dot{W}$,
4. The efficiency $\eta = \dot{W}/\dot{Q}_{\mathrm{H}}$,
5. The internal entropy source $\dot{S}_{\mathrm{M}}$,
6. The total entropy source $\dot{S}_{\mathrm{T}}$, and
7. The total cost C_T (the cumulated cost of the heat exchangers H and L).

Besides these, we also consider four absolute temperatures: T_{SH} and T_{SL} for the sources (constant parameters) and T_{H} and T_{L} for the internal system (variables). FST introduces another variable, the piston's average speed w.

The quantities should satisfy these inequalities:

$$0 < T_{\mathrm{SL}} < T_{\mathrm{L}} < T_{\mathrm{H}} < T_{\mathrm{SH}},\ \dot{Q}_{\mathrm{H}} > 0,\ \dot{Q}_{\mathrm{L}} < 0,\ \dot{W} > 0,\ \dot{S}_{\mathrm{M}} > 0 \quad (1)$$

The first law of thermodynamics

The systems H and L do not exchange mechanical work and do not change state, so for them the first law is simply $\dot{Q}_{\mathrm{in}} = \dot{Q}_{\mathrm{out}}$. For the system M (the engine), the first law is given as follows:

$$\dot{W} = \dot{Q}_{\mathrm{H}} + \dot{Q}_{\mathrm{L}} \quad (2)$$

We use the general sign convention: the work $\dot{W}$ done by the system is positive, the received heat $\dot{Q}_{\mathrm{H}}$ is positive and the released heat $\dot{Q}_{\mathrm{L}}$ is negative.

The second law of thermodynamics

We use the same convention for entropy: the entropy going into the system is positive, the entropy going out of the system is negative. We define the following entropy sources:

$$\text{for the system H: } \overbrace{\frac{\dot{Q}_{\mathrm{H}}}{T_{\mathrm{SH}}}}^{\text{positive}} + \overbrace{\frac{-\dot{Q}_{\mathrm{H}}}{T_{\mathrm{H}}}}^{\text{negative}} + \dot{S}_{\mathrm{H}} = 0, \quad \dot{S}_{\mathrm{H}} > 0 \quad (3)$$

$$\text{for the system M: } \overbrace{\frac{\dot{Q}_{\mathrm{H}}}{T_{\mathrm{H}}}}^{\text{positive}} + \overbrace{\frac{\dot{Q}_{\mathrm{L}}}{T_{\mathrm{L}}}}^{\text{negative}} + \dot{S}_{\mathrm{M}} = 0, \quad \dot{S}_{\mathrm{M}} > 0 \quad (4)$$

for the system L: $$\overbrace{\frac{-\dot{Q}_{\mathrm{L}}}{T_{\mathrm{L}}}}^{\text{positive}}+\overbrace{\frac{\dot{Q}_{\mathrm{L}}}{T_{\mathrm{SL}}}}^{\text{negative}}+\dot{S}_{\mathrm{L}}=0,\quad \dot{S}_{\mathrm{L}}>0 \tag{5}$$

In the above equations $\dot{S}_{\mathrm{M}}$ is the internal entropy source—it represents the entropy generated due to all the irreversibilities inside the engine because of finite speed (friction, throttling, spatial non-uniformity of pressure, etc.). $\dot{S}_{\mathrm{H}}$ and $\dot{S}_{\mathrm{L}}$ are the external entropy sources, associated with the heat transfer in finite time and through finite area surfaces. $\dot{S}_{T}$ is the total entropy source of the whole system, which accounts for all the irreversibilities.

For the entropy sources we obtain immediately the formulae:

$$\dot{S}_{\mathrm{M}}=-\left(\frac{\dot{Q}_{\mathrm{H}}}{T_{\mathrm{H}}}+\frac{\dot{Q}_{\mathrm{L}}}{T_{\mathrm{L}}}\right),\ \dot{S}_{T}=-\left(\frac{\dot{Q}_{\mathrm{H}}}{T_{\mathrm{SH}}}+\frac{\dot{Q}_{\mathrm{L}}}{T_{\mathrm{SL}}}\right),$$
$$\dot{S}_{\mathrm{T}}=\dot{S}_{\mathrm{M}}+\underbrace{\dot{Q}_{\mathrm{H}}\left(\frac{1}{T_{\mathrm{H}}}-\frac{1}{T_{\mathrm{SH}}}\right)}_{\dot{S}_{\mathrm{H}}}+\underbrace{\dot{Q}_{\mathrm{L}}\left(\frac{1}{T_{\mathrm{L}}}-\frac{1}{T_{\mathrm{SL}}}\right)}_{\dot{S}_{\mathrm{L}}} \tag{6}$$

The cost

We consider that the cost of a heat exchanger is a function of: the heat flux $\dot{Q}$ that the heat exchanger must provide, the temperature T that the heat exchanger must provide, and the temperature T_{S} of the source to which the heat exchanger is connected. In fact, we consider that the cost is directly proportional to the heat flux:

$$C=\dot{Q}\cdot f(T,T_{\mathrm{S}}) \tag{7}$$

This hypothesis is compatible with any of the heat transfer laws considered in [4]:

$$f(T,T_{\mathrm{S}})=\frac{K}{T_{\mathrm{S}}^{n}-T^{n}}\quad \text{for radiation,}$$
$$f(T,T_{\mathrm{S}})=\frac{K}{(T_{\mathrm{S}}-T)^{n}}\quad \text{for convection, or}$$
$$f(T,T_{\mathrm{S}})=\frac{K}{T_{\mathrm{S}}-T}\quad \text{for conduction } (n=1)$$
$$f(T,T_{\mathrm{S}})=\frac{kA}{T^{-1}-T_{\mathrm{S}}^{-1}}\quad \text{for phenomenological law } (n=-1),$$

where K is a constant.

Remark The cost might be a financial cost and could also be any other parameter that is additive and should be kept constant, e.g., it could be the area of the heat exchanger. This means that the present approach unifies all the conditions that before were treated separately: constant thermal conductance, constant heat transfer area, or constant cost of heat transfer area. Any of these can be treated in the same manner, just by changing the function f.

Considering that the total cost C_{T} regards only the heat exchangers H and L, we have:

$$C_{\mathrm{T}}=\dot{Q}_{\mathrm{H}}\cdot f_{\mathrm{H}}(T_{\mathrm{H}})+\dot{Q}_{\mathrm{L}}\cdot f_{\mathrm{L}}(T_{\mathrm{L}}), \tag{8}$$

where

$$f_{\mathrm{H}}(T_{\mathrm{H}})=f(T_{\mathrm{H}},T_{\mathrm{SH}}),\quad f_{\mathrm{L}}(T_{\mathrm{L}})=f(T_{\mathrm{L}},T_{\mathrm{SL}}). \tag{9}$$

Remark When the temperature is higher than that of the source (as it is the case for T_{L}), the heat flux changes its sign; for the cost to remain positive, the function f also needs to change its sign. Therefore, f_{L} (T_{L}) is always negative.

Degrees of freedom

The system is completely characterized by 11 variables: seven quantities of interest plus four temperatures. The sources' temperatures are constant, so nine variable quantities remain (Fig. 1).

There are five relations linking them: the power formula (the first law), the definition of efficiency, the definition of the internal entropy source (the second law for the internal system), the definition of the total entropy source (the second law for the total system) and the definition of the cost. This means that each quantity can be expressed as a function of four others (usually the two internal temperatures plus two other quantities).

To these relations one can add relations concerning the heat transfer (which relate heat fluxes to temperatures).

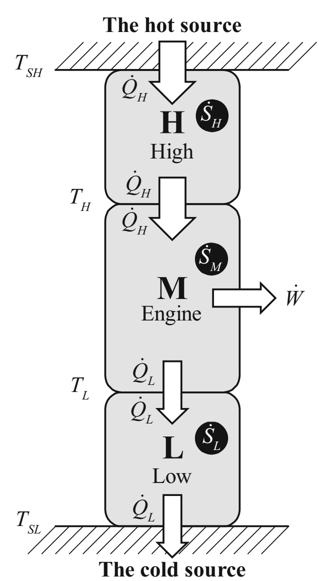

Fig. 1 The studied model

The total system

In Fig. 2, we show the dependencies between quantities of interest for the total system. In the total system we have only five variable quantities (the internal temperatures, the internal entropy source and the total cost vanish) and three relations (the definitions of the internal entropy source and of the total cost vanish). It follows that any quantity can be expressed as a function of other two. We choose one of these two to be the total entropy source $\dot{S}_{\mathrm{T}}$.

Independently of other conditions, from the relations valid for the total system follow the dependencies on the total entropy source shown in Table 1.

These formulae allow one to study the influence of the total entropy source when one of the quantities is held constant. For instance, in the first row one can see that at constant η the power $\dot{W}$ is directly proportional with $\dot{S}_{\mathrm{T}}$, both reaching synchronized maximum/minimum values. In the second row, one can see that at constant $\dot{W}$ the efficiency η decreases with $\dot{S}_{\mathrm{T}}$, reaching a maximum when $\dot{S}_{\mathrm{T}}$ is minimum and vice versa. How each quantity varies with $\dot{S}_{\mathrm{T}}$ is synthesized in Table 2.

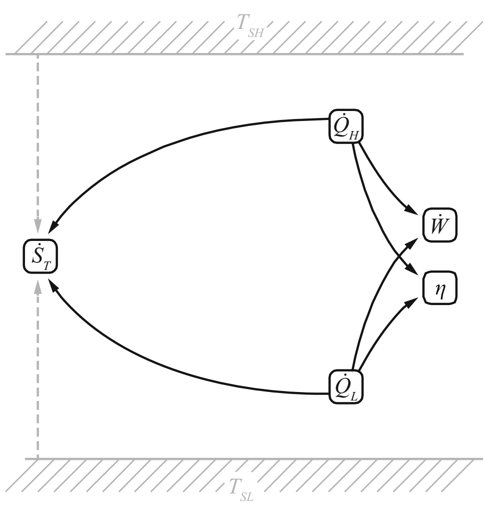

Fig. 2 Relations between quantities for the total system

Remark In this analysis, the internal temperatures were not taken into account: all the above results regarding the maximization/minimization of some quantities in relation to the total entropy source are valid generally, regardless of any other restrictions and of their influence on the internal temperatures. Particularly, it is true that at constant η, the maxima/minima of $\dot{W}$ is synchronized with those of $\dot{S}_{\mathrm{T}}$; while at constant $\dot{W}$, η is maximum when $\dot{S}_{\mathrm{T}}$ is minimum (and vice versa)—regardless of other restrictions (such as a cost restriction, for example) and regardless of the heat exchange laws.

The internal system

Figure 3 illustrates the dependencies between the quantities of interest and the variables (in black circles) for the internal system. The novelties are the internal temperatures, together with the quantities depending on them: the internal entropy source $\dot{S}_{\mathrm{M}}$ and the total cost C_{T}.

On the diagram there is no direct connection between the entropy sources; in fact, this connection exists, but just as a consequence of the fact that they both depend on the heat fluxes. We cannot eliminate the heat fluxes between them, but we can obtain a relation that contains only their ratio (expressed through the efficiency η):

$$\frac{T_{\mathrm{SL}}\dot{S}_{\mathrm{T}}}{1-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}-\eta}=\frac{T_{\mathrm{L}}\dot{S}_{\mathrm{M}}}{1-\frac{T_{\mathrm{L}}}{T_{\mathrm{H}}}-\eta} \qquad (10)$$

In this formula one can verify that if the internal entropy source is zero, then the engine efficiency has to be the Carnot efficiency between the internal temperatures. And if the total entropy source is zero, then the engine efficiency has to be the Carnot efficiency between the sources' temperatures.

Besides the efficiency, the formula (10) contains both the internal temperatures. As a consequence, all the relations regarding the internal entropy source also contain them. With the help of formula (10) we get the formulae relating $\dot{S}_{\mathrm{M}}$ to other quantities when keeping one parameter constant, presented in Table 3.

Table 1 Total system's quantities dependence on the total entropy source

Const. =	$\eta(\dot{S}_T)=$	$\dot{W}(\dot{S}_T)=$	$\dot{Q}_H(\dot{S}_T)=$	$\dot{Q}_L(\dot{S}_T)=$
η		$\frac{\eta}{1-\eta-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}T_{\mathrm{SL}}\dot{S}_T$	$\frac{1}{1-\eta-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}T_{\mathrm{SL}}\dot{S}_T$	$-\frac{(1-\eta)}{1-\eta-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}T_{\mathrm{SL}}\dot{S}_T$
$\dot{W}$	$\frac{1-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}{1+\frac{T_{\mathrm{SL}}\dot{S}_T}{\dot{W}}}$		$\frac{\dot{W}+T_{\mathrm{SL}}\dot{S}_T}{1-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}$	$-\frac{\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}\dot{W}+T_{\mathrm{SL}}\dot{S}_T}{1-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}$
$\dot{Q}_{\mathrm{H}}$	$1-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}-\frac{T_{\mathrm{SL}}\dot{S}_T}{\dot{Q}_{\mathrm{H}}}$	$\left(1-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}\right)\dot{Q}_{\mathrm{H}}-T_{\mathrm{SL}}\dot{S}_T$		$-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}\dot{Q}_{\mathrm{H}}-T_{\mathrm{SL}}\dot{S}_T$
$\dot{Q}_{\mathrm{L}}$	$1-\frac{\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}{1-\frac{T_{\mathrm{SL}}\dot{S}_T}{-\dot{Q}_{\mathrm{L}}}}$	$\frac{\left(1-\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}\right)(-\dot{Q}_{\mathrm{L}})-T_{\mathrm{SL}}\dot{S}_T}{\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}$	$\frac{-\dot{Q}_{\mathrm{L}}-T_{\mathrm{SL}}\dot{S}_T}{\frac{T_{\mathrm{SL}}}{T_{\mathrm{SH}}}}$	

Table 2 Quantities variation with the total entropy source

Const. =	$\eta(\dot{S}_T)$	$\dot{W}(\dot{S}_T)$	$\dot{Q}_H(\dot{S}_T)$	$\lvert\dot{Q}_L(\dot{S}_T)\rvert$
η		↑ with S_T	↑ with S_T	↑ with S_T
$\dot{W}$	↓ with S_T		↑ with S_T	↑ with S_T
$\dot{Q}_H$*	↓ with S_T	↓ with S_T		↑ with S_T
$\dot{Q}_L$**	↓ with S_T	↓ with S_T	↓ with S_T	

* S_T cannot grow unbounded, but is limited to $S_{Tmax} = Q_H\ (1/T_{SL} - 1/T_{SH})$

** S_T cannot grow unbounded, but is limited to $S_{Tmax} = |Q_L|\ (1/T_{SL} - 1/T_{SH})$

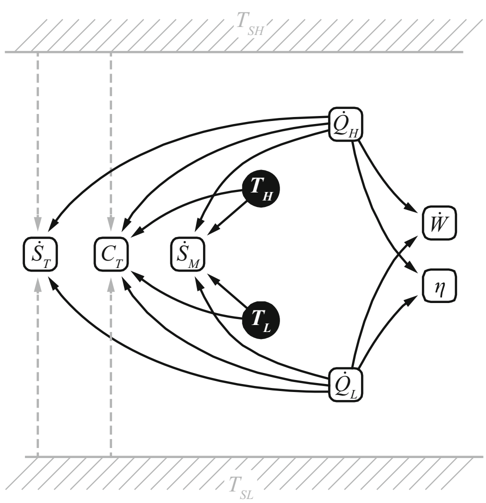

Fig. 3 Dependencies between quantities in the internal system

We can also write formulae for the total cost (they would make an additional column, which do not fit in the table):

$$\text{at } \eta = \text{const.: } C_T(T_L, T_H, \dot{S}_M) = \frac{T_L\dot{S}_M}{1 - T_L/T_H - \eta}\cdot f_H(T_H) - \frac{(1-\eta)T_L\dot{S}_M}{1 - T_L/T_H - \eta}\cdot f_L(T_L) \tag{11}$$

$$\text{at } \dot{W} = \text{const.: } C_T(T_L, T_H, \dot{S}_M) = \frac{\dot{W} + T_L\dot{S}_M}{1 - T_L/T_H}\cdot f_H(T_H) - \frac{T_L/T_H\dot{W} + T_L\dot{S}_M}{1 - T_L/T_H}\cdot f_L(T_L) \tag{12}$$

$$\text{at } \dot{Q}_H = \text{const.: } C_T(T_L, T_H, \dot{S}_M) = \dot{Q}_H\cdot f_H(T_H, T_{SH}) - \left[T_L/T_H\dot{Q}_H + T_L\dot{S}_M\right]\cdot f_L(T_L) \tag{13}$$

$$\text{at } \dot{Q}_L = \text{const.: } C_T(T_L, T_H, \dot{S}_M) = \frac{-\dot{Q}_L - T_L\dot{S}_M}{T_L/T_H}\cdot f_H(T_H) - \left(-\dot{Q}_L\right)\cdot f_L(T_L) \tag{14}$$

From these we obtain the formulae for constant total cost (they would have made an additional row in the table, but it does not fit there):

$$\eta(T_L, T_H, \dot{S}_M) = 1 - \frac{T_L/T_H C_T + f_H(T_H)T_L\dot{S}_M}{C_T + f_L(T_L)T_L\dot{S}_M} \tag{15}$$

$$\dot{W}(T_L, T_H, \dot{S}_M) = \frac{(1 - T_L/T_H)C_T - [f_H(T_H) - f_L(T_L)]T_L\dot{S}_M}{f_H(T_H) - T_L/T_H f_L(T_L)} \tag{16}$$

$$\dot{Q}_H(T_L, T_H, \dot{S}_M) = \frac{C_T + f_L(T_L)T_L\dot{S}_M}{f_H(T_H) - (1 - T_L/T_H)f_L(T_L)} \tag{17}$$

$$\dot{Q}_L(T_L, T_H, \dot{S}_M) = -\frac{T_L/T_H C_T + f_H(T_H)T_L\dot{S}_M}{f_H(T_H) - T_L/T_H f_L(T_L)} \tag{18}$$

The presence of the internal temperatures in all the above formulae prevents us from studying the quantities' variation with the internal entropy source $\dot{S}_M$, as it was possible for the total system.

It appears that it would be necessary to know how the internal entropy source $\dot{S}_M$ is related to the internal temperatures—not through the heat fluxes (the definition of $\dot{S}_M$, which we already have), but independently of them.

This is the motivation of the present study: researching the possibility that FST and FPDT can provide a formula for the internal entropy source $\dot{S}_M$—a formula dependent on the internal temperatures and on other variables or parameters, but independent of the heat fluxes.

Table 3 Internal system's quantities dependence on the internal entropy source

Const. =	$\eta(T_L, T_H, \dot{S}_M) =$	$\dot{W}(T_L, T_H, \dot{S}_M) =$	$\dot{Q}_H(T_L, T_H, \dot{S}_M) =$	$\dot{Q}_L(T_L, T_H, \dot{S}_M) =$
η		$\frac{\eta}{1-\eta-\frac{T_L}{T_H}}T_L\dot{S}_M$	$\frac{1}{1-\eta-\frac{T_L}{T_H}}T_L\dot{S}_M$	$-\frac{(1-\eta)}{1-\eta-\frac{T_L}{T_H}}T_L\dot{S}_M$
$\dot{W}$	$\frac{1-\frac{T_L}{T_H}}{1+\frac{T_L\dot{S}_M}{\dot{W}}}$		$\frac{\dot{W}+T_L\dot{S}_M}{1-\frac{T_L}{T_H}}$	$-\frac{\frac{T_L}{T_H}\dot{W}+T_L\dot{S}_M}{1-\frac{T_L}{T_H}}$
$\dot{Q}_H$	$1-\frac{T_L}{T_H}-\frac{T_L\dot{S}_M}{\dot{Q}_H}$	$\left(1-\frac{T_L}{T_H}\right)\dot{Q}_H - T_L\dot{S}_M$		$-\frac{T_L}{T_H}\dot{Q}_H - T_L\dot{S}_M$
$\dot{Q}_L$	$1-\frac{\frac{T_L}{T_H}}{1-\frac{T_L\dot{S}_M}{-\dot{Q}_L}}$	$\frac{\left(1-\frac{T_L}{T_H}\right)(-\dot{Q}_L)-T_L\dot{S}_M}{\frac{T_L}{T_H}}$	$\frac{-\dot{Q}_L-T_L\dot{S}_M}{\frac{T_L}{T_H}}$	

Finite speed thermodynamics

Approaching the problem from the inside of the engine, FST focused on the internal irreversibilities: spatial non-uniformity of pressure, various frictions and throttling in valves. All these phenomena are related to the average speed of the engine's piston. Evaluating these losses for each transformation and adding them, FST succeeds in providing an analytical formula for the engine's power and efficiency, both as functions of the piston's average speed. In [15], an analytical formula for the internal entropy source is provided, which seems to be exactly what we are looking for.

The calculation scheme is in each step, the internal temperatures T_H and T_L are considered given, and based on them, on the speed w and on other constructive parameters, FST computes the heat; then, it computes the cycle duration and thus finds the heat fluxes $\dot{Q}_H$ and $\dot{Q}_L$; from them, the power $\dot{W}$ and the efficiency η follows immediately (Fig. 4). Knowing the heats and the temperatures, the entropy source $\dot{S}_M$ follow from its definition.

Coupling the two diagrams (Figs. 3, 4), we get Fig. 5. Now we can compute the entropy sources, based on the heat fluxes provided by FST. For each triple of values given to the variables (w, T_H, T_L) we get the heat fluxes, the power, the efficiency, the two entropy sources and the total cost. The determining factor is not one (the internal entropy source), but two: the heat fluxes—which, once computed, determine uniquely all the other quantities of interest.

FST is not only able to provide the internal entropy source, but in the process also determines uniquely the heat fluxes, which cannot be varied any more by FPDT. We introduced a new variable (w) and two relations (the heat fluxes provided by FST). If before the unification we could express each quantity as a function of four quantities (two temperatures and two other quantities—Table 3), now we can express each quantity as a function of three variables: the speed w and the two internal temperatures. But this was anyway done by FST alone, so we gained nothing—we did nothing more than to "unblock" the temperatures, which were constant in FST and now are variable.

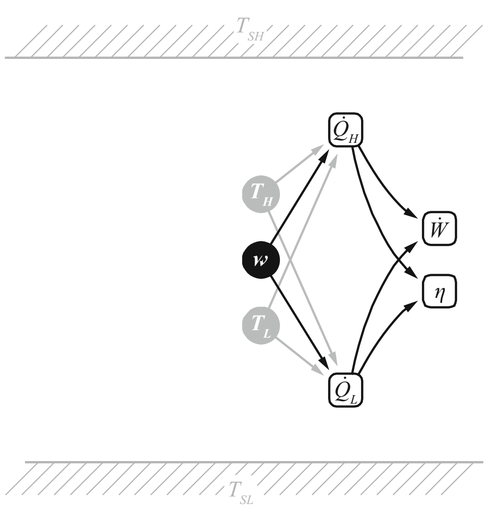

Fig. 4 Dependecies between quantities and variables in FST

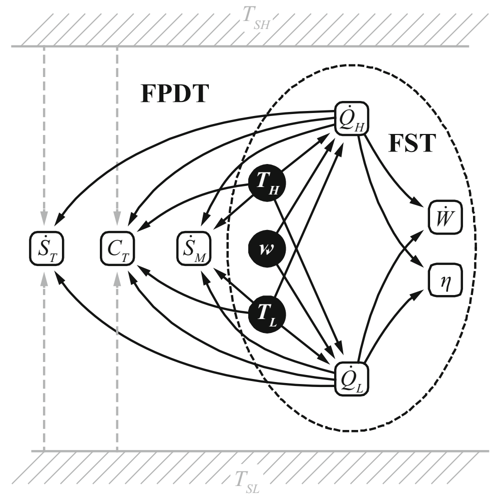

Fig. 5 The role of FST

Unfortunately, this is not enough for optimization. Because there will be no optimum point—in any regime, if the internal temperatures get closer to the external ones we will always get more power. FST does find an optimum speed w at which the power is maximum, but this result is relative to the internal temperatures—increasing their ratio even by a tiny bit will increase the power, so that we will always discover that "we can do better". In other words, the absolute maximum of power will be that calculated by FST when the internal temperatures equal the sources' temperatures.

To be able to optimize something, we have to introduce either some more relations or a restriction. They have to depend on the internal temperatures, but in such a way that they introduce a new relation, not already present. There are two options, both based on heat transfer:

(a) We add specific relations for the heat transfer (thus linking heat fluxes to temperatures through known, fixed conductances), or
(b) We impose a fixed total cost of the heat exchangers (the cost being based on heat transfer laws, but with unknown conductances).

Both these options lead to a problem that can be solved through the Lagrange multipliers method: find the extrema of some objective function O with the restrictions stated in (a) or (b). But unfortunately, since these restrictions involve the very complicated functions $\dot{Q}_{H,FST}(w, T_H, T_L)$ and $\dot{Q}_{L,FST}(w, T_H, T_L)$ provided by FST (see Sect. 6), the resulting systems of equations cannot be solved analytically. Yet, they can be solved by choosing an appropriate numerical algorithm.

Heat transfer laws

Unfortunately, in our case it is not simple to add the heat transfer laws. Because we already have connections between the temperatures and the heat fluxes: those given by FST. If we add formulae for heat transfer, we get the situation in Fig. 6.

The arrows marked with "!" show a double determination: starting from the same temperatures, FST determines some heat fluxes, and the heat transfer laws determine other heat fluxes.

We can overcome the difficulty by considering the two additional relations as restrictions and using the Lagrange multipliers method.

Let us consider that the heat transfer is described by one of the following formulae:

$$\dot{Q} = K_Q\left(T_S^n - T^n\right) \quad \text{for radiation,}$$
$$\dot{Q} = K_Q(T_S - T)^n \quad \text{for convection,}$$
$$\dot{Q} = K_Q(T_S - T) \quad \text{for conduction } (n = 1),$$
$$\dot{Q} = \frac{K_Q}{T_S - T} \quad \text{for phenomenological law } (n = -1),$$

where the constant K_Q encapsulates the properties and the area of the heat exchange surface.

In general we can write

$$\dot{Q} = K_Q f(T). \tag{19}$$

Denoting by O the chosen objective function, we write the Lagrange function:

$$\begin{aligned} L(w, T_H, T_L) = O(w, T_H, T_L) \\ - \lambda\left[\dot{Q}_{H,TVF}(w, T_H, T_L) - K_{QH} f(T_H)\right] \\ - \mu\left[\dot{Q}_{L,TVF}(w, T_H, T_L) - K_{QL} f(T_L)\right] \end{aligned} \tag{20}$$

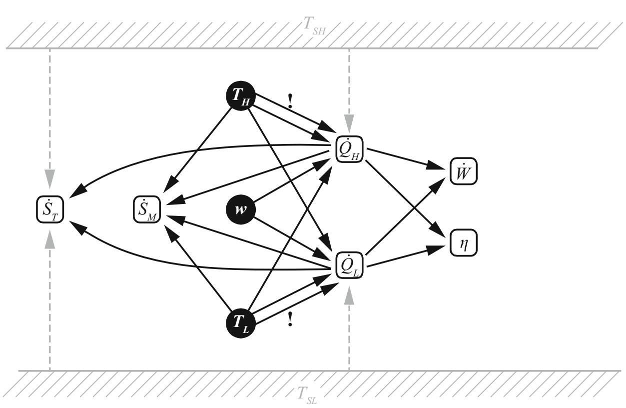

Fig. 6 Introducing the heat transfer equations leads to a double determination of the heat fluxes

We nullify the partial derivatives:

$$\begin{cases} \dfrac{\partial L}{\partial T_H} = 0 & \Rightarrow \quad \ldots \text{partial derivatives equation, with } \lambda \text{ and } \mu\ldots \\ \dfrac{\partial L}{\partial T_L} = 0 & \Rightarrow \quad \ldots \text{partial derivatives equation, with } \lambda \text{ and } \mu\ldots \\ \dfrac{\partial L}{\partial w} = 0 & \Rightarrow \quad \ldots \text{partial derivatives equation, with } \lambda \text{ and } \mu\ldots \\ \dfrac{\partial L}{\partial \lambda} = 0 & \Rightarrow \quad \dot{Q}_{H,TVF}(w, T_H, T_L) = K_{QH} f(T_H) \\ \dfrac{\partial L}{\partial \mu} = 0 & \Rightarrow \quad \dot{Q}_{L,TVF}(w, T_H, T_L) = K_{QL} f(T_L) \end{cases} \tag{21}$$

From the first two equations we find the Lagrange multipliers, which we introduce in the third equation. We eventually get a system of three equations with three unknowns:

$$\begin{cases} \ldots \text{partial derivatives equation}\ldots \\ \dot{Q}_{H,TVF}(w, T_H, T_L) = K_{QH} f(T_H) \\ \dot{Q}_{L,TVF}(w, T_H, T_L) = K_{QL} f(T_L) \end{cases} \tag{22}$$

The last two equations are the initial heat equalities. One cannot solve them analytically, because the temperatures are too deeply buried into the formulae—they show up simultaneously in exponentials, in logarithms and under the radical (see Sect. 6).

The system can be solved numerically by choosing a suitable algorithm. Now let us explore solution (b), choosing the total cost as a constant parameter.

Constant total cost

We introduce a cost function as shown in Fig. 7.

We consider that the cost of a heat exchanger is a function of the constant K_Q from the heat transfer law:

$$C = g(K_Q) = g\left(\frac{\dot{Q}}{f(T)}\right) \tag{23}$$

"Cost" might not only mean a financial cost, but could also mean size, or any other parameter which we wish to maintain constant.

We consider that the total cost consists solely of the costs of the heat exchangers H and L:

$$\begin{aligned} C_T &= C_H + C_L \\ &= g\left(\frac{\dot{Q}_{H,TVF}(w, T_H, T_L)}{f(T_H)}\right) + g\left(\frac{\dot{Q}_{L,TVF}(w, T_H, T_L)}{f(T_L)}\right) \end{aligned} \tag{24}$$

We want to optimize an objective function O with the constant cost restriction, so we employ again the Lagrange multipliers method. The Lagrange function is

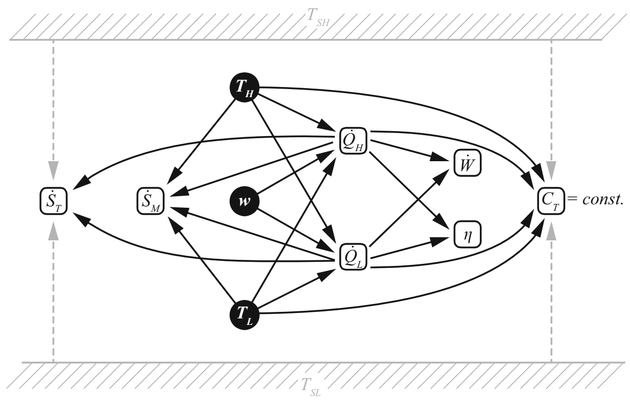

Fig. 7 Optimizing the speed and the temperatures at constant total cost

$$L(w, T_H, T_L) = O(w, T_H, T_L) - \lambda \times \left[g\left(\frac{\dot{Q}_{H,TVF}(w, T_H, T_L)}{f(T_H)}\right) + g\left(\frac{\dot{Q}_{L,TVF}(w, T_H, T_L)}{f(T_L)}\right) - B \right] \quad (25)$$

B is the budget available for the heat exchangers.

We nullify the partial derivatives:

$$\begin{cases} \frac{\partial L}{\partial w} = 0 \quad \Rightarrow \quad \ldots \text{partial derivatives equation with } \lambda \ldots \\ \frac{\partial L}{\partial T_H} = 0 \quad \Rightarrow \quad \ldots \text{partial derivatives equation with } \lambda \ldots \\ \frac{\partial L}{\partial T_L} = 0 \quad \Rightarrow \quad \ldots \text{partial derivatives equation with } \lambda \ldots \\ \frac{\partial L}{\partial \lambda} = 0 \quad \Rightarrow \quad g\left(\frac{\dot{Q}_{H,TVF}(w, T_H, T_L)}{f(T_H)}\right) + g\left(\frac{\dot{Q}_{L,TVF}(w, T_H, T_L)}{f(T_L)}\right) = B \end{cases} \quad (26)$$

From the first equation, we express λ and then plug it in into the other two. We obtain a system of three equations with three unknowns:

$$\begin{cases} \ldots \text{partial derivatives equation} \ldots \\ \ldots \text{partial derivatives equation} \ldots \\ g\left(\frac{\dot{Q}_{H,TVF}(w, T_H, T_L)}{f(T_H)}\right) + g\left(\frac{\dot{Q}_{L,TVF}(w, T_H, T_L)}{f(T_L)}\right) = B \end{cases} \quad (27)$$

The last equation is the constant cost condition. It cannot be solved analytically, again because of its complexity. The system can be solved numerically, if we choose a suitable algorithm.

The irreversible heat fluxes and the entropy source provided by FST

From Ref. [15] we have the formula of the internal entropy source for an irreversible Carnot engine, which was determined based on the heat fluxes. Adapting the formulae in [15] to the present notations and adding some new notations, the heat fluxes $\dot{Q}_{H,FST}$ and $\dot{Q}_{L,FST}$ depend on the average piston speed w, on the internal temperatures T_H and T_L, and on the following seven parameters:The formulae for the heat fluxes are

$$\dot{Q}_{H,FST}(w, T_H, T_L) = \frac{mR}{V_1}\frac{A_p}{2} \ln \varepsilon_H \cdot \frac{wT_H}{\varepsilon_H \left(\frac{T_H}{T_L}\right)^{\frac{1}{\gamma_{23}-1}} - 1} \cdot B_{T,H} \quad (28)$$

$$\dot{Q}_{L,FST}(w, T_H, T_L) = -\frac{mR}{V_1}\frac{A_p}{2} \ln \varepsilon_H \cdot \frac{wT_L}{\varepsilon_H \left(\frac{T_H}{T_L}\right)^{\frac{1}{\gamma_{23}-1}} - 1} \cdot B_{T,L}\left[1 - \alpha \frac{\ln\left(\frac{T_L}{T_H}\right)}{\ln \varepsilon_H}\right], \quad (29)$$

where

$$\gamma_{23} = 1 + (\gamma - 1)B_{ad,2-3ir}, \quad (30)$$

$$\gamma_{41} = 1 + (\gamma - 1)B_{ad,4ir-1}, \quad (31)$$

$$\alpha = \frac{1}{\gamma_{23} - 1} - \frac{1}{\gamma_{41} - 1}, \quad (32)$$

and the numerical coefficients B are

$$B_{ad,2-3ir} = 1 - \frac{aw}{\sqrt{3R\frac{T_H+T_L}{2}}} - \frac{2\left(K_A + K_C\frac{w}{100y}\right) \cdot 10^5}{\frac{p_1}{\varepsilon_H}\left[1 + \left(\frac{T_L}{T_H}\right)^{\frac{\gamma}{\gamma-1}}\right]} - \frac{2K_C\left(\frac{w}{100y}\right)^2 \cdot 10^5}{\frac{p_1}{\varepsilon_H}\left[1 + \left(\frac{T_L}{T_H}\right)^{\frac{\gamma}{\gamma-1}}\right]} \quad (33)$$

$$B_{ad,4ir-1} = 1 + \frac{aw}{\sqrt{3R\frac{T_H+T_L}{2}}} + \frac{2\left(K_A + K_C\frac{w}{100y}\right) \cdot 10^5}{p_1\left[1 + \left(\frac{T_L}{T_H}\right)^{\frac{\gamma}{\gamma-1}}\right]} + \frac{2K_C\left(\frac{w}{100y}\right)^2 \cdot 10^5}{p_1\left[1 + \left(\frac{T_L}{T_H}\right)^{\frac{\gamma}{\gamma-1}}\right]} \quad (34)$$

$$B_{T,H} = 1 - \frac{aw}{\sqrt{3RT_H}} - \frac{2\left(K_A + K_C \frac{w}{100y}\right) \cdot 10^5}{p_1\left(1 + \frac{1}{\varepsilon_H}\right)} - \frac{2K_C\left(\frac{w}{100y}\right)^2 \cdot 10^5}{p_1\left(1 + \frac{1}{\varepsilon_H}\right)} \quad (35)$$

$$B_{T,L} = 1 + \frac{aw}{\sqrt{3RT_L}} + \frac{2\left(K_A + K_C \frac{w}{100y}\right) \cdot 10^5}{p_3(1 + \varepsilon_L)} + \frac{2K_C\left(\frac{w}{100y}\right)^2 \cdot 10^5}{p_3(1 + \varepsilon_L)}. \quad (36)$$

The maximum pressure p_1 is

$$p_1 = \frac{mRT_H}{V_1}. \quad (37)$$

and the denominator $p_3(1 + \varepsilon_L)$ in the last formula is

$$p_3(1 + \varepsilon_L) = p_1 \frac{T_L}{T_H}\left[\frac{\left(\frac{T_L}{T_H}\right)^{\frac{1}{\gamma_{23}-1}}}{\varepsilon_H} + \left(\frac{T_L}{T_H}\right)^{\frac{1}{\gamma_{41}-1}}\right]. \quad (38)$$

Remark In [15], this pressure is considered reversible (as if $\gamma_{23} = \gamma_{41} = \gamma$); but this approximation is not necessary, since at this point we already know $B_{ad,2\text{-}3ir}$ and $B_{ad,4ir\text{-}1}$, so we know γ_{23} and γ_{41}.

In the expressions of B coefficients, y is the stroke of the respective transformation (isothermal or adiabatic). Taking all these strokes equal to the piston's semi-stroke, we approximate it reversibly:

$$y = \frac{V_1}{2A_p}\left[\varepsilon_H\left(\frac{T_H}{T_L}\right)^{\frac{1}{(\gamma-1)}} - 1\right] \quad (39)$$

The expressions (28) and (29) of the irreversible heat fluxes are needed for optimizing the speed w together with the temperatures T_H and T_L. In those formulae, the temperatures appear simultaneously in exponentials, in logarithms and under the radical (and combined). It seems clear that they are too complicated to give an analytical solution and the resulting systems of equations need to be treated numerically.

Applying the definition of the internal entropy source to the formulae for the irreversible heat fluxes $\dot{Q}_{H,FST}$ and $\dot{Q}_{L,FST}$, in [15] the following synthetic formula is obtained:

$$\dot{S}_{M,FST}(w, T_H, T_L) = \frac{mR}{V_1}\frac{A_p}{2} \cdot w \frac{(B_{T,L} - B_{T,H}) \ln \varepsilon_H - \alpha B_{T,L} \ln\left(\frac{T_L}{T_H}\right)}{\varepsilon_H\left(\frac{T_H}{T_L}\right)^{\frac{1}{(\gamma-1)B_{ad,2-3ir}}} - 1} \quad (40)$$

This formula, regarded by itself, hides the fact that the internal entropy source was calculated based on the heat fluxes and creates the illusion that it could be used with arbitrary heat fluxes. But doing so would be a mistake: the formula (40) is valid only for the heat fluxes $\dot{Q}_{H,FST}$ and $\dot{Q}_{L,FST}$ used to calculate it and is not valid for some arbitrary heat fluxes $\dot{Q}_H$ and $\dot{Q}_L$ that may happen to have the same weighted sum, $\dot{Q}_H/T_H + \dot{Q}_L/T_L$. When we write the formula (40) the heat fluxes are already determined, as well as all the other quantities in the system (power, efficiency, etc.).

Conclusions

FST can provide to FPDT a formula for the internal entropy source of the irreversible Carnot engine, but only through the heat fluxes. This means that the heat fluxes cannot be varied independently in FPDT: they are determined by FST and they determine all the other quantities. Analyzing the number of quantities, variables and relations between them, we conclude that the unification between FPDT and FST is not possible through a single connection (the relation for the internal entropy source provided by FST), but it needs to be done through two connections (the two relations for the heat fluxes provided by FST).

In this way, the average speed of the piston and the internal temperatures can be optimized simultaneously either by imposing heat transfer laws (conductive, convective, radiative, or a combination) or by maintaining the total cost constant. Even if obtaining analytical formulae is not possible, the resulting systems of equations can be solved using the Lagrange multipliers method and a suitable numerical algorithm.

Some of the studies at one constant parameter can be performed in a simpler way, independently of irreversibilities and of heat transfer laws. For instance, using only The First Law and The Second Law of Thermodynamics (and no other constraints, not even the heat transfer laws), we showed that:

– at constant efficiency, the regime that maximizes the power of the irreversible Carnot engine is the one that also maximizes the total production of entropy,
– at constant power, the regime that maximizes the efficiency of the irreversible Carnot engine is the one that minimizes the total production of entropy.

Acknowledgments This work was partially supported by the strategic grant POSDRU/159/1.5/S/137070 (2014) of the Ministry of National Education, Romania, co-financed by the European Social

Fund—Investing in People, within the Sectorial Operational Program Human Resources Development 2007–2013.

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