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EDITORIAL

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Preface

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This is the first part of a special issue of our journal dedicated to the topic of singular perturbation theory and its various aspects, in particular analytic, geometric, and probabilistic.

The main concern of perturbation theory in an analytic context is to study unbounded operators acting in infinite dimensional spaces (e.g., differential operators acting on functions belonging to some spaces of square integrable functions) and to treat them by relating them with some simpler operators for which essential properties (e.g., spectral ones) are known. Such problems arise naturally in mathematical physics (quantum mechanics, electromagnetism, hydrodynamics...) where singular forces, configurations, and interactions are to be taken into account. Very often these problems are intimately related to those arising in the study of stochastic processes (it suffices to think of diffusion processes as related to elliptic and parabolic partial differential equations). In geometrical contexts, singular perturbation problems arise in the study of natural geometric operators on non-necessarily smooth manifolds or on objects like graphs, which can be looked upon as limits of manifolds. In recent years, new mathematical techniques have been developed to handle such problems. These techniques are coming from different areas, including scattering theory, analysis (PDEs, functional analysis, non-standard analysis, generalized functions...), asymptotics, probability theory, statistical mechanics, as well as from the study of manifolds, measured metric spaces, and natural operators on them. The main aim of this issue is to provide an overview of recent developments of singular perturbation theory in these areas and possible synergies between ideas and methods developed there.

The present issue consists of four contributions. The one of G. Carinci, A. De Masi, C. Giardinà, and E. Presutti has the title "Hydrodynamic limit in a particle system with topological interactions". It concerns systems of a large number of classical particles confined to move on a bounded part of a one-dimensional lattice as symmetric independent random walks, with reflections on the boundary. Simultaneously new particles are injected at the left boundary point at a certain rate and removed at the right most occupied site at the same rate. The authors prove that the system has a hydrodynamic limit. The limit density at time t is characterized as weak solution of a limit parabolic free boundary problem with barriers.

The contribution by Dr. Mugnolo has the title "Asymptotics of semigroups generated by operator matrices". It gives a survey of results concerning operator semigroups generated by operator matrices and their long-time asymptotic behaviors. It covers characterizations of well-posedness and ill-posedness of evolution equations with dynamic boundary conditions, both on domains in Euclidean space and on metric graphs.

The contribution by A. Posilicano has the title "Direct sums of trace maps and self-adjoint extensions". It concerns a basic problem in the theory of self-adjoint extensions of symmetric operators in Hilbert spaces. The

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main tool is a new result by the author providing a criterion for the direct sum of trace maps to be again a trace map. Applications include, e.g., one-dimensional Schrödinger operators with δ and δ' interactions supported by discrete subsets of the real line and to the problem of finding all self-adjoint extensions of a minimal Laplace–Beltrami operator corresponding to a singular Riemannian metric on a two-dimensional cylinder.

The contribution by K. Yasuda carries the title "On the order of divergence of the sum of p-adic-valued independent identically distributed random variables". The author has pioneered the study of p-adic versions of the classical results on the weak limit of sequences of normalized sums of independent and identically distributed random variables. She has shown in particular that for rotation symmetric, bounded non-degenerate random variables the behavior in the p-adic valued case is very different from the one in the Euclidean-valued case. In the unbounded case p-adic semi-stable laws arise. In this issue, she shows that no stronger sense convergence holds and she determines the precise order of divergence of the normalized sums, providing thus an analogue in the p-adic case of the classical law of the iterated logarithm.