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CORRECTION

Correction to: Harmonic cubic homogeneous polynomials such that the norm-squared of the Hessian is a multiple of the Euclidean quadratic form

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- Although Lemma 6.14 is correct as stated, the proof given for it is trivially wrong because the first inequality in (6.50) has direction opposite to that given by Lemma 4.6. Fortunately, what matters is the inequality, not its direction, and the spirit of the argument is correct. The fallacious proof should be replaced by:

Lemma 6.14 *For $m, n \geq 2$ the orbits $[\![P_m \otimes P_n]\!]$ and $[\![P_{mn}]\!]$ are not $CO(mn)$ equivalent.*

Proof If v is a critical point of the restriction of P_n to the h unit sphere, then $r^{-1}v$ is a critical point of $r^2 P_n$ to the $r^2 h$ unit sphere, so if $P_n(v) \neq 0$, then $e = \frac{v}{6P_n(v)}$ and $c(v) = |(\text{Hess } P_n)(e)|_h^2$ depend only on the homothety class of h. By Corollary 6.11, e has the form $\frac{1}{n+1-2k}\sum_{i\in I} v_i$ for $v_i \in \mathbb{M}(P_n)$ and $1 \leq k \leq n+1$ such that $2k \neq n+1$, and the possible values of $c(v)$ are $|(\text{Hess } P_n)(e)|^2 = n(n-1)|e|^2 = \frac{k(n+1-k)(n-1)}{(n+1-2k)^2} = f_n(k)$ for the same range of k. Because $f_n(n+1-k) = f_n(k)$ in what follows it suffices to consider $1 \leq k < (n+1)/2$. Let u and v be critical points of the restrictions of P_m and P_n to the unit spheres such that $P_m(u) \neq 0$, $P_n(v) \neq 0$, $c(u) = \frac{m}{m-1}$, and $c(v) = \frac{n}{n-1}$. Then $u \otimes v$ is a critical point of the restriction of $P_m \otimes P_n$ to the unit sphere such that $c(u \otimes v) = \frac{mn}{(m-1)(n-1)}$. Were the orbits $[\![P_m \otimes P_n]\!]$ and $[\![P_{mn}]\!]$ $CO(mn)$-equivalent, there would be a critical point w of the restriction of P_{mn} to the unit sphere such that $P_{mn}(w) \neq 0$ and $c(w) = \frac{mn}{(m-1)(n-1)}$. It will be shown that this is impossible. By the preceding, the possible values of $c(w)$ are $f_{mn}(k) =$

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$\frac{k(mn+1-k)(mn-1)}{(mn+1-2k)^2}$ for $1 \leq k < (mn+1)/2$. Because in this range $f_{mn}(k)$ is increasing in k, to complete the proof it suffices to check that $f_{mn}(1) = \frac{mn}{mn-1} < \frac{mn}{(m-1)(n-1)}$ and $f_{mn}(2) = \frac{2(mn-1)^2}{(mn-3)^2} > \frac{mn}{(m-1)(n-1)}$ for $m, n \geq 2$. The first inequality is immediate. For the second, if $m \geq 3$ and $n \geq 4$, then $\frac{mn}{(m-1)(n-1)} = 1 + \frac{1}{m-1} + \frac{1}{n-1} + \frac{1}{(m-1)(n-1)} \leq 2$ while $f_{mn}(2) > 2$; the remaining cases $(m, n) \in \{(2, 2), (2, 3), (2, 4), (3, 3)\}$ can be checked by direct evaluation. $\square$

- The part of Remark 4.5 implicitly referring to the fallacious proof of Lemma 6.14 should be suppressed.
- The last line of claim (11) of Corollary 6.11 is missing a k and should read *There holds* $6P_n(v) = (n+1-2k)\sqrt{\frac{n}{k(n+1-k)}}$. (The correct value is given in the proof.)