

Fixed income strategies based on the prediction of parameters in the NS model for the Spanish public debt market

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Abstract Using different econometric models, Diebold and Li (J Econom 130:337–364, 2006) addressed the practical problem of forecasting the yield curve by predicting the factors *level*, *slope* and *curvature* in the Nelson–Siegel framework. This paper has two main aims: on the one hand, to investigate the predictive possibilities of the yield curve for the Spanish public debt market, using the methodology proposed by Diebold and Li (J Econom 130:337–364, 2006); and on the other hand, to study the capability of generating profits by transforming these yield curve predictions into technical trading strategies. The Sharpe ratios of our strategies outperform the hedging strategy benchmarks for long (1 year) horizons in our prediction period (2000–2010) and also for the current crisis period (2008–2010). Nevertheless, these strategies do not outperform their benchmarks for short (1 month) horizons. The introduction of non-parametric models improves the profitability of the strategies in terms of the Sharpe ratio, especially in the 1-year-ahead predictions. This finding is in line with Diebold and Li (J Econom 130:337–364, 2006), whose forecasts for long horizons are much more accurate than those of several standard benchmark models.

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1 Introduction

The study of the Spanish public debt is of special interest in the panorama of the recent sovereign debt crisis in May 2010 that troubled European economies and threatened stability and unity in the Eurozone.

In 1997 the Spanish treasury prioritized the achievement of a more liquid and efficient public debt market, undertaking a set of initiatives aimed at attracting investor savings within the new capital market, providing greater depth and liquidity, decreasing bond yield volatility and increasing pricing efficiency. This was done through two channels: first, an appropriate exchange policy ensured an adequate tradable supply of bonds priced near par (at the expense of premium bonds, which some classes of investors avoid). Second, debt exchanges increased the outstanding amounts of stripable bonds that were critical in supporting bond dealer stripping and reconstitution operations in the new strips market (see [Díaz et al. 2006](#) for details).

However, since summer 2010 the Spanish debt market, as in other Eurozone periphery countries, suffered a sharp escalation of risk premia. During the recent sovereign debt crisis in the Eurozone, Spain reflected investors' perceptions of risks or uncertainties about the Spanish economy and raised important concerns about the possibilities of contagion to the global financial system, due to the size and importance of its economy (see [Gómez-Puig and Sosvilla-Rivero 2014](#)). Although prior to the crisis Spain had a low level of debt, in comparison with other developed economies, from late 2009 fears of a sovereign default developed among investors in Spanish public debt, in view of the growing volume of private debt, arising in turn from a property bubble. In a disruption scenario within the European interbank market, this situation deteriorated banking system balance sheets and provoked a downgrading of government debt by the international rating agencies. In these circumstances, the escalating yields paid on Spanish public debt increased the interest margin of these securities relative to the interest cost of bank deposits, making them a very attractive fixed income asset and providing additional option value for Spanish banks before and after the beginning of the financial crisis ([Pérez-Montes 2013](#)).

Thus, although term structure literature mainly focuses on the government bond markets of the most highly developed European economies, the analysis and better comprehension of a peripheral country of the Economic and Monetary Union, such as Spain, may be of considerable interest for policymakers, academic researchers and, especially, for international investors, who need to be aware of potential profit opportunities in the Spanish debt market.

The yield curve or term structure of interest rates is the relation between the (level of) interest rate (or cost of borrowing) and the time to maturity of the debt for a given borrower in a given currency. The yield curve forms the basis for the valuation of all fixed income instruments, because the price of a fixed income security can be

calculated as the net present value of the stream of cash flows, and each cash flow has to be discounted using the zero coupon interest rate for the associated term to maturity. The term structure of interest rates as the interception of macroeconomics and finance has increasingly been employed as a means of explaining the upward slope of the yield curve and the bond premium puzzle [see [Gürkaynak and Wright \(2012\)](#) for a survey on macroeconomics and the term structure or [Lange \(2013\)](#) and [Pericoli and Taboga \(2012\)](#) for recent specific examples]. Accordingly, fixed income portfolio managers, central bankers and market participants apply econometric models to achieve a better representation of the evolution of interest rates, in the view that these models are useful decision-orienting tools for their purposes.

The [Nelson and Siegel \(1987\)](#) model (NS hereafter) is an exponential component framework with four parameters by which the yield curve can be estimated parsimoniously; these parameters have the economic interpretation of *level*, *slope*, *curvature* and *speed of convergence to long term rates*. The NS model provides parametric curves that are flexible enough to describe a whole family of observed term structure shapes and is consistent with a factor interpretation of the term structure ([Litterman and Scheinkman 1991](#)). In addition to the factors present in the NS model, the [Svensson \(1994, 1996\)](#) model contains a second hump/trough factor which allows for an even broader and more complicated range of term structure shapes.

The NS model has been extensively used by central banks and monetary policy makers ([Bank for International Settlements 2005](#); [European Central Bank 2008](#)) for more than two decades. It is also used by fixed-income portfolio managers that wish to immunize their portfolios ([Barrett et al. 1995](#); [Hodges and Parekh 2006](#)).

[Diebold and Li \(2006\)](#), taking an explicit out-of-sample forecasting perspective of the term structure of interest rates, showed that the three-factor NS model, where the factor measuring the speed of convergence is fixed beforehand, can also be used to construct accurate term structure forecasts by considering it as a dynamic yield curve model that is capable of capturing its time-varying shape. These authors considered the practical problem of forecasting the yield curve by studying variations on the NS framework to model the entire yield curve, period by period, as a three-dimensional parameter evolving dynamically. By using a straightforward two-step estimation procedure, they generated term-structure forecasts for both the short and the long term, observing that their forecasts appear to be much more accurate for long horizons than are several standard benchmark forecasts.

The present study has two main goals: on the one hand, to examine the predictive possibilities of the yield curve for the Spanish public debt market, using the methodology proposed by [Diebold and Li \(2006\)](#). On the other hand, to consider the capability of generating profits from yield curve predictions, transforming them into technical trading strategies. The main contribution of our paper is that the trading strategies presented outperform benchmark hedging strategies for long (1 year) horizons in our prediction period (2000–2010) and specifically during the current crisis period (2008–2010). Nevertheless, these strategies do not outperform the benchmarks for short (1 month) horizons. A further point of interest is that the introduction of non-parametric models improves the profitability of the level, slope and curvature strategies in terms of Sharpe's ratio, especially in 1-year-ahead predictions. This finding is in line with [Diebold and Li \(2006\)](#), whose forecasts are

much more accurate for long horizons than are those of several standard benchmark models.

2 Methodology and data

The Nelson and Siegel (1987) (NS) model describes the yield curve through the four parameters $\{\beta_0, \beta_1, \beta_2, \tau\}$ which represent the level β_0 , the slope $-\beta_1$, the curvature β_2 and the speed of convergence to long term rates τ , together with the maturity, which is represented by t . In this model the forward rate curve is given by the expression

$$f_t = \beta_0 + \beta_1 e^{-t/\tau} + \beta_2 \frac{t}{\tau} e^{-t/\tau}$$

and the spot rate curve by

$$R_t = \beta_0 + (\beta_1 + \beta_2) \frac{\tau}{t} (1 - e^{-t/\tau}) - \beta_2 e^{-t/\tau}. \quad (1)$$

These parameters are estimated by weighted nonlinear least squares¹ where, in order to obtain homogeneous errors in the regression, the errors in prices are weighted by the inverse of the modified duration of the corresponding bond (see Bank for International Settlements 2005). Therefore, the objective function for estimating the NS model is

$$\text{Min}_{\beta_0, \beta_1, \beta_2, \tau} \sum_{i=1}^k \frac{\varepsilon_i^2}{MD_i} = \text{Min}_{\beta_0, \beta_1, \beta_2, \tau} \sum_{i=1}^k \frac{(P_i - \hat{P}_i)^2}{MD_i} \quad (2)$$

with the restrictions $\beta_0 > 0$, $\beta_0 + \beta_1 > 0$, $\tau > 0$, where k represents the number of bonds in the sample, P_i is its market price, \hat{P}_i is its theoretical price following the NS model and MD_i is the modified duration.

Data on secondary market operations of Spanish Public Debt were downloaded from the Bank of Spain website www.bde.es/banota/series.htm. In the construction of the yield curve, we followed step by step the methodology proposed by Díaz (<http://www.uclm.es/area/aef/etti.asp>) and employed in Díaz et al. (2009, 2011). This data sample includes daily mean prices of 29–39 treasury bills and bonds traded on the secondary market and ranks from January 2, 1995 to February 8, 2010 (3813 days) a period which includes interesting events and periods, such as the dotcom bubble, the Lehman Brothers collapse, financial turmoil, European Central Bank monetary policy decisions, and the beginning of the sovereign debt crisis. Following Bank for International Settlements (2005) and Díaz et al. (2011), to obtain a good adjustment at the short end of the yield curve, we also include, for each day, four bond repos (termed *operaciones simultáneas* in Spain) with a maturity of one, seven, fifteen and 30 days. In order to avoid the presence of illiquid assets, we eliminated from the portfolios all assets with a traded nominal value of <3 million euros in a single day

¹ Following Bank for International Settlements (2005), we estimated the Spanish yield curve using the Levenberg–Marquardt algorithm.

and bills and bonds with a maturity of <15 days or more than 15 years. [Díaz et al. \(2011\)](#) argue that transactions involving bonds with <15 days of maturity remaining usually conceal speculative trading. Bonds with maturities longer than 15 years in the estimation of the yield curve are discarded for two reasons: on the one hand, very few bonds have such a maturity (there are only two 30-year bonds in our database). And furthermore, the long duration of these bonds forces an over fitting of the long end of the curve.

The estimation of parameter τ in the model (1) always presents special difficulties. The highly nonlinear nature of the optimization problem in estimating the NS model parameters is originated by the parameter τ . This parameter determines the shape of the yield curve since the maturity at which the curvature factor is maximized and the speed of decay of the slope parameter depend only on τ . Although the simultaneous estimation of the four NS parameters improves the fitting quality of the yield curve and is convenient for pricing proposals, the objective function in term structure estimation with price errors is not only non-linear but also non-convex in parameters. This makes the final results sensitive both to the choice of the optimization routine and also to the starting values (initial guess) used in the optimization algorithm (see [Virmani 2013](#), among others). Alternative proposals to deal with this issue can be found, for instance, in [Bolder and Strélski \(1999\)](#) who suggest several global optimization algorithms based on grid search; [Gimeno and Nave \(2006\)](#) use genetic algorithms to find the values for the initial conditions and to reduce the risk of false convergence in the Spanish bond market, and [Annaert et al. \(2013\)](#) incorporate a ridge regression approach to the grid search. Finally, other authors have fixed τ as a specific value congruent with observed data; [Diebold and Li \(2006\)](#) propose fixing the value of τ at 1.37, approximately, with annualized data, implying that the maximal value of the curvature component in the spot rate function will be reached at 2.5 years to maturity, as they observed throughout their original sample, while [Fabozzi et al. \(2005\)](#) fixed τ at 3 with annualized data whose maximal curvature component was situated at 5.38 years.

In this paper, all the parameters $\{\beta_0, \beta_1, \beta_2, \tau\}$ in the NS model were initially estimated using an algorithm of nonlinear least squares as in (2), where the starting values were those obtained by [Gimeno and Nave \(2006\)](#) in their estimation of the Spanish yield curve, in order to reduce the risk of false convergence. These estimations are termed τ -free estimations and are very interesting from the point of view of improving the fitting quality.

Given the erratic values obtained for the parameter τ , in order to obtain smooth parameters in the NS model and also to reduce problems of multicollinearity in their estimation, it is common in the literature on yield curve forecasting to fix τ at a constant value over the whole sample of the term structure and to estimate the beta parameters by ordinary least squares (OLS) fitting interest rates, as in [Diebold and Li \(2006\)](#). Given that our database is composed of coupon bonds, together with bills and repos, we still apply weighted nonlinear least squares, but fix τ at a constant value, to obtain beta parameters by minimizing (2) (see [Bank for International Settlements 2005](#) for the Spanish yield curve). In any case, the dimensionality of the optimization problem is reduced when τ is fixed at a constant value, and this also avoids the need to forecast the highly erratic parameter τ . We term these τ -fixed estimations and find them very interesting from the standpoint of predictive quality.

The τ parameter was fixed at 3, a value considered reasonable for several reasons: first, the prior inspection of the τ estimated by nonlinear least squares during the first 5-year window of our training period (January 2, 1995–December 31, 1999) ranked from 1.00 to 7.00 during 90 % of the days, with a mean value close to 3. Second, numerous authors have observed that the factors (level, slope and curvature) extracted from the NS model are insensitive to the choice of τ (see, among others, Nelson and Siegel 1987; Barrett et al. 1995; Willner 1996; Dolan 1999; Czaja et al. 2009; Favero et al. 2012). More recently, Annaert et al. (2013) estimated the parameters of the NS model for τ fixed at 1.37 (as in Diebold and Li 2006), and for τ fixed at 3 (as in Fabozzi et al. 2005) for Euro spot rate curves, obtaining very similar results and being unable to conclude which of the two τ values is more suitable. We also estimated our results with τ fixed at 1.37 and they were found to be robust.² Finally, we also considered the argument mentioned by Fabozzi et al. (2005) that when the shape parameter is fixed at 3, the correlation between slope and curvature will not cause severe problems of estimation; in our case this correlation is very low: -7% .

Figure 1 shows the daily evolution of the parameters $\{\beta_0, \beta_1, \beta_2, \tau\}$ estimated from the NS model for the Spanish public debt market from January 2, 1995 to February 8, 2010.³ As mentioned above, two kinds of estimation were considered: τ -free estimations where the four parameters $\{\beta_0, \beta_1, \beta_2, \tau\}$ are estimated using an algorithm of nonlinear least squares, and τ -fixed estimations for $\{\beta_0, \beta_1, \beta_2\}$. Figure 1 shows two aspects of interest. On the one hand, the estimations of β_1 and β_0 are similar in both procedures and the main differences are observed in the β_2 estimations; on the other hand, the τ -free estimations are very erratic, which is in line with the branch of literature advising τ -fixed estimations for predictive purposes.

These graphs reflect the evolution of the Spanish economy from 1995 to 2010. Until the establishment of the euro, interest rates fell steadily, due to Spain's application of the convergence criteria established in the Maastricht Treaty in 1992. The controls imposed on inflation, public debt and public deficit, together with exchange rate stability and convergence of interest rates, all contributed to reducing the rate of interest paid by the treasury. The evolution of the slope of the yield curve shows the effects of the Spanish business cycle, where low slopes correspond to the start of recessions and high ones to expansions after a cyclical trough. The parameter β_1 reflects the long expansion of the economy from 2005 to 2007 toward the end of the Spanish property bubble and the subsequent deep recession starting in 2008. The behaviour of the curvature is more erratic and has a more complex relation with the business cycle.

Figure 2 shows the goodness of fit of the NS model in our whole sample for τ -fixed estimations, with the evolution of the daily in-sample root mean squared pricing error (RMSPE) when fitting the NS model:

² Detailed results are available upon request.

³ Esteve et al. (2013), studying the period January 1974–February 2010, have detected two regimes in the Spanish term structure of interest rates, having the structural break in June 1979, 16 years before the beginning of our sample data so our sample does not suffer of structural breaks.

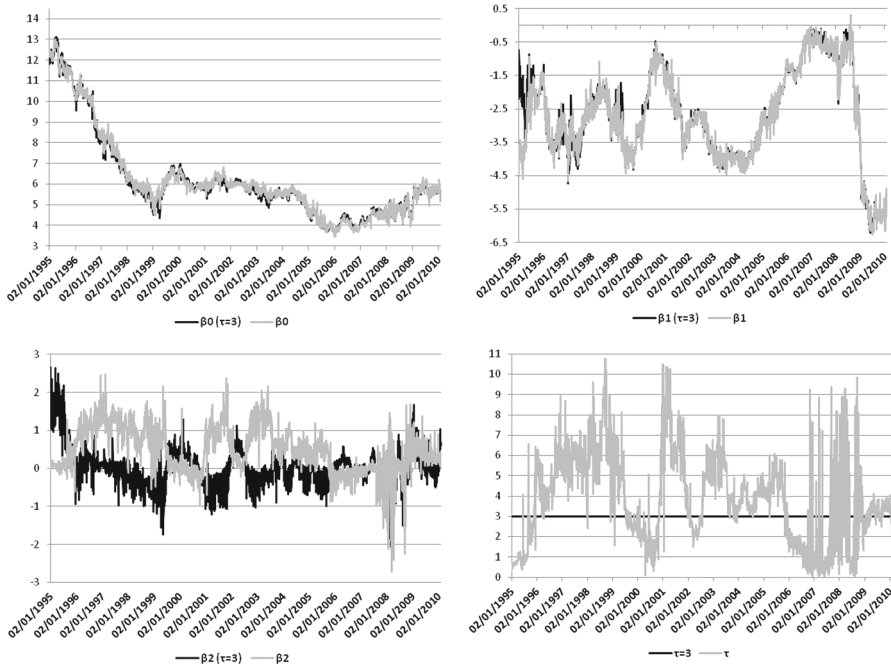


Fig. 1 Daily evolution of the level β_0 , slope β_1 , curvature β_2 and the speed of convergence to long term rates τ of the Nelson and Siegel model for the Spanish public debt market from January 2, 1995 to February 8, 2010. Two kind of estimation have been considered: the τ -free estimations where the four parameters $\{\beta_0, \beta_1, \beta_2, \tau\}$ are estimated using an algorithm of nonlinear least squares and the τ -fixed estimations for $\{\beta_0, \beta_1, \beta_2\}$ for a τ fixed at 3

$$RMSPE_t = \sqrt{\sum_{i=1}^k \frac{(P_i - \hat{P}_i)^2}{k}}$$

Thus, the evolution of RMSPE was fairly stable, with a mean value is 0.23 and minimum and maximum values of 0.02 and 0.72, respectively. This confirms the good in-sample fit of the NS model for τ -fixed in the Spanish bond market.

Following Diebold and Li (2006), we used various econometric procedures to forecast the beta parameters $\{\beta_0, \beta_1, \beta_2\}$ with horizons of 1 year and 1 month (corresponding to 260 and 22 trading days, respectively). To prevent possible structure breaks in the time series of the NS parameters, for the case of the 1 year horizon, the daily predictions were conducted recursively using a 5-year overlapping rolling window. The starting rolling window was from January 2, 1995 to December 31, 1999 and advanced, 1 day at a time, until February 8, 2009, i.e., the year before the end of our sample. Thus, our predicting period extended from January 2, 2000 to February 8, 2010. The 1-month-ahead recursive predictions were implemented in a similar way.

The econometric models used in these predictions are those developed in Diebold and Li (2006) and Rezende and Ferreira (2013) for forecasting the Brazilian zero-coupon data, and in other non-parametric models successfully employed in the prediction of exchange rates by Fernández-Rodríguez et al. (1999).

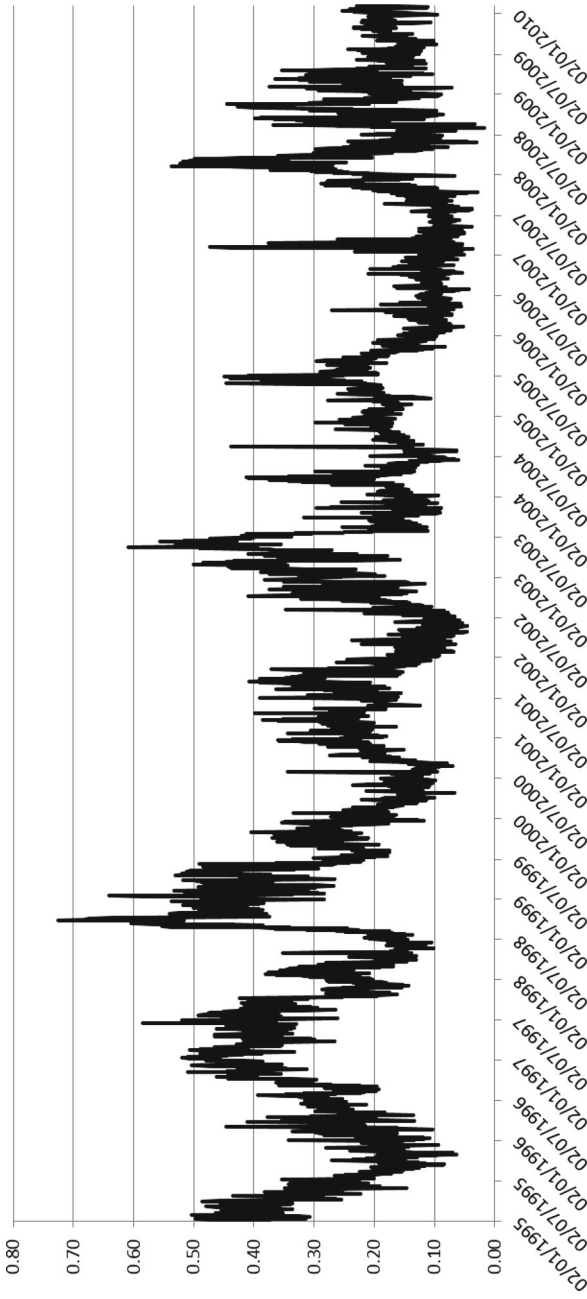


Fig. 2 Daily evolution of the goodness of fit, through the in-sample RMSPPE, when fitting the Nelson and Siegel model with τ -fixed at ξ

Using the [Diebold and Li \(2006\)](#) notation, $\hat{\beta}_{i,t}$, $i = 1, 2, 3$ represents the beta corresponding to day t , and $\hat{\beta}_{i,t+h/t}$ represents the prediction of $\hat{\beta}_{i,t}$ h days ahead. The econometric models considered are the following

(1) AR(1) model:

$$\hat{\beta}_{i,t+h/t} = \hat{c}_i + \hat{\gamma}_i \hat{\beta}_{i,t}, \quad i = \{0, 1, 2\}. \tag{3}$$

The coefficients \hat{c}_i and $\hat{\gamma}_i$ are recursively obtained by OLS and $h = 260$ or $h = 22$, for the case of 1 year and 1-month-ahead predictions, respectively.

(2) Random walk model:

$$\hat{\beta}_{i,t+h/t} = \hat{\beta}_{i,t}, \quad i = \{0, 1, 2\} \tag{4}$$

(3) VAR(1) on beta levels:

$$\hat{\beta}_{i,t+h/t} = \hat{c} + \hat{\Gamma} \hat{\beta}_t, \quad i = \{0, 1, 2\} \tag{5}$$

where $\hat{\beta}_t = [\hat{\beta}_{0,t}, \hat{\beta}_{1,t}, \hat{\beta}_{2,t}]$, and the \hat{c} and $\hat{\Gamma}$ are recursively obtained by OLS.

(4) QAR(1) model:

$$\hat{\beta}_{i,t+h/t}^\tau = \hat{c}_i^\tau + \hat{\gamma}_i^\tau \hat{\beta}_{i,t}, \quad i = \{0, 1, 2\}. \tag{6}$$

The coefficients \hat{c}_i^τ and $\hat{\gamma}_i^\tau$ are recursively obtained using quantile autoregression (QAR) estimated at the median ($q = 0.5$); and $h = 260$ or $h = 22$, for the case of 1 year and 1-month-ahead predictions, respectively. So, we follow [Rezende and Ferreira \(2013\)](#), which use QAR method while comparing the predictive power of NS class of models with Brazilian zero-coupon data. Contrasting with QAR model is interesting due to its robustness to the presence of outliers and extreme values. For details regarding the quantile autoregression model see [Koenker and Xiao \(2002, 2004, 2006\)](#).

(5) VAR(1) on beta changes:

$$\begin{aligned} \hat{z}_{i,t+h/t} &= \hat{c} + \hat{\Gamma} z_t, \\ \hat{\beta}_{i,t+h/t} &= \hat{\beta}_{i,t} + \hat{z}_{i,t+h/t}, \quad i = \{0, 1, 2\}, \end{aligned} \tag{7}$$

where $z_t = [(\hat{\beta}_{0,t} - \hat{\beta}_{0,t-1}), (\hat{\beta}_{1,t} - \hat{\beta}_{1,t-1}), (\hat{\beta}_{2,t} - \hat{\beta}_{2,t-1})]$, $\hat{z}_{i,t+h/t}$ is the prediction h days ahead of the latter, and the \hat{c} and $\hat{\Gamma}$ are recursively obtained by OLS.

(6) Error correction mechanism [ECM(1)] with one common trend (level):

$$\begin{aligned} \hat{z}_{i,t+h/t} &= \hat{c} + \hat{\Gamma} z_t, \\ \hat{\beta}_{i,t+h/t} &= \hat{\beta}_{i,t} + \hat{z}_{i,t+h/t}, \quad i = \{0, 1, 2\} \end{aligned} \tag{8}$$

where $z_t = [(\hat{\beta}_{0,t} - \hat{\beta}_{0,t-1}), (\hat{\beta}_{1,t} - \hat{\beta}_{0,t}), (\hat{\beta}_{2,t} - \hat{\beta}_{0,t})]$, $\hat{z}_{i,t+h/t}$ is the prediction h days ahead of the latter, and the \hat{c} and $\hat{\Gamma}$ are recursively obtained by OLS.

(7) ECM(1) with two trends (level and slope):

$$\begin{aligned}\hat{z}_{i,t+h/t} &= \hat{c} + \hat{\Gamma}z_t, \\ \hat{\beta}_{i,t+h/t} &= \hat{\beta}_{i,t} + \hat{z}_{i,t+h/t}, \quad i = \{0, 1, 2\}\end{aligned}\quad (9)$$

where $z_t = [(\hat{\beta}_{0,t} - \hat{\beta}_{0,t-1}), (\hat{\beta}_{1,t} - \hat{\beta}_{1,t-1}), (\hat{\beta}_{2,t} - \hat{\beta}_{0,t})]$, $\hat{z}_{i,t+h/t}$ is the prediction h days ahead of the latter, and the \hat{c} and $\hat{\Gamma}$ are recursively obtained by OLS.

(8) Nearest neighbours NN(h): Non-parametric regression model by [Fernández-Rodríguez et al. \(1999\)](#).

$$\hat{\beta}_{i,t+h/t} = \hat{a}_{0,i}\hat{\beta}_{i,t} + \hat{a}_{1,i}\hat{\beta}_{i,t-1} + \cdots + \hat{a}_{d-1,i}\hat{\beta}_{i,t-(d-1)} + \hat{a}_{d,i}, \quad i = \{0, 1, 2\}.\quad (10)$$

This approach enables direct prediction of the betas in the NS model for the horizons $h = \{22, 260\}$. NN(h) is a non-parametric forecasting approach for betas in the NS model that relies on the idea of predicting their future values in a time series using the subsequent observations of past histories which are similar to the current one.

The fact that NN is non-parametric means that it requires minimal assumptions: (i) the process under study can be non-stationary; (ii) it sidesteps the need to specify a functional form for the conditional mean and conditional variance of the process; (iii) it does not require us to choose an innovation distribution driving the volatility process; and (iv) it is robust to the presence of possible structural breaks in the time series, so that recursive windows may be employed instead of rolling ones. Hence, the resulting forecasts are not bedevilled by the *model misspecification* risk. The NN regression models are developed in the Appendix.

(9) Simultaneous nearest neighbours (SNN)(h): Non-parametric regression model by [Fernández-Rodríguez et al. \(1999\)](#)

$$\hat{\beta}_{i,t+h/t} = \hat{a}_{0,i}\hat{\beta}_{i,t} + \hat{a}_{1,i}\hat{\beta}_{i,t-1} + \cdots + \hat{a}_{d-1,i}\hat{\beta}_{i,t-(d-1)} + \hat{a}_{d,i}, \quad i = \{0, 1, 2\}\quad (11)$$

generalizing the nearest neighbours procedure to the multivariate case. This is also explained in the Appendix.

Besides the econometric models considered above, we also studied another system of predictions based on the implicit forward rates obtained every day from the yield curve. Following expectation theory, the implicit forward rates are those expected by the agents at a future horizon (1 year, 1 month) under the present information of the yield curve, considered a predictor of future spot rates. In this case, once we have estimated the spot rates that the market expects in the future through actual forward

rates,⁴ it is also possible to obtain the future level, slope and curvature expected by the market.

3 Statistical assessment of our predictions

As observed by [Gürkaynak et al. \(2006\)](#), it is possible to find fairly similar yield curve shapes over most of the maturity range considered, through different combinations of parameters in a NS framework. Thus, if two econometric models predict different betas from the NS model, these differences might not imply differences in interest rates but rather that a different combination of parameters will produce similar estimated interest rates. Taking this into account, we assessed the prediction with the parameters $\{\beta_0, \beta_1, \beta_2\}$ obtained under the different econometric models mentioned above, and the implicit forward rates model of our database of bond prices, in three ways. First, we use the root mean squared error (RMSE) statistic to compare the predicted interest rates (with τ -fixed) at 3-month, 1, 3, 5 and 10-year expirations for the date $t + h$ with the real interest rates obtained by estimating the NS model (with τ -free) at the date $t + h$, where h is 1 month or 1 year.

Second, we assessed the point prediction for the empirical components of the yield curve. The empirical level is taken as the mean of all former interest rates; the empirical slope is obtained as the difference between the 10-year interest rates and the 3-month interest rate; the empirical curvature is obtained as twice the yield where the maximum curve is situated (in our case, 5 years), minus the 3-month interest rates and the 10-year interest rates.

Finally, we assessed the point prediction of the yield to maturity for quoted bills and bonds, comparing the predictions made by the different econometric models with the real values given by the market. The predictions of the yields to maturity are obtained through the predicted parameters from the NS model (with τ -fixed), which enables us to obtain the theoretical price of the bonds. Having obtained these theoretical prices, the yield to maturity is derived using the information on coupons, maturity and date for coupon payments.

Table 1 shows the goodness of fit of the different econometric models used to predict the interest rates (“Yield” panel, left), the empirical components of the yield curve (“Term structure interest rates” panel, centre) and the yield to maturity (“Yield to maturity” panel, right). In all cases the RMSE is shown for 1-month and 1-year horizons, in the predicting period from January 2, 2000 to February 8, 2010; bold values correspond to the model with lowest RMSE. Besides, in order to compare the predictive accuracy of all available models with the random walk model, we indicate significance at the 10, 5, and 1 % level by *, **, and ***, respectively, if the RMSE of model i th is statistically lower than random walk using [Diebold and Mariano \(1995\)](#)

⁴ The usual forward rates for compounding interest rates at 1 year horizon are

$$f(0, 1, j) = \left[\frac{(1+R(0, j))^j}{(1+R(0, 1))} \right]^{1/(j-1)} - 1$$

where $f(0, 1, j)$ is the implicit forward rate which starts within 1 year and matures within $j > 1$ years, and $R(0, j)$ is the actual spot rate which matures within $j > 1$ years. Therefore, the spot rates that the market expects within 1 year are $f(0, 1, 2) = R(1, 2)$; $f(0, 1, 3) = R(1, 3)$; $f(0, 1, 4) = R(1, 4) \dots$

Table 1 Goodness of fit of the different econometric models used to predict the interest rates (“Yield” panel, left), the empirical components of the yield curve (“Term structure interest rates” panel, centre) and the yield to maturity (“Yield to maturity” panel, right)

Models	Yields					Term structure of interest rates				Yield to maturity	
	3 months	1 year	3 years	5 years	10 years	Level	Slope	Curvature	Mean	SD	
AR(1)											
1 year	1.4589	1.3062	1.0176	0.8651	0.7539	1.0108	1.2775	0.5368**	0.9354	0.5177	
1 month	0.3937	0.3803	0.3649	0.3497	0.3173	0.3357	0.3514	0.2498	0.3304	0.1741	
VAR(1) levels											
1 year	1.4822	1.3073	0.9520	0.7389	0.5480*	0.9436	1.3524	0.5597	0.8830	0.5693	
1 month	0.3144	0.2789	0.2331	0.2131	0.2098	0.2152	0.3457	0.2391	0.2933	0.1522	
QAR(1)											
1 year	1.5590	1.3909	1.0696	0.8893	0.7388	1.0675	1.3192	0.5504*	0.9927	0.5781	
1 month	0.3130	0.2986	0.2970	0.2908	0.2649	0.2654	0.3185	0.2517	0.2930	0.1479	
VAR(1) changes											
1 year	1.5948	1.4057	1.0521	0.8564	0.6933	1.0490	1.4218	0.6477	0.9463	0.6943	
1 month	0.2861	0.2582	0.2294	0.2140	0.2001	0.2106	0.2958	0.2167	0.2785	0.1484	
ECM(1) 1 common trend											
1 year	1.6872	1.5258	1.1706	0.9218	0.5974	1.1468	1.3358	0.5074***	0.9786	0.7502	
1 month	0.3201	0.2924	0.2590	0.2372	0.2097	0.2358	0.3220	0.2255	0.2842	0.1486	
ECM(1) 2 common trends											
1 year	1.5535	1.3741	1.0307	0.8213	0.5926	1.0113	1.3867	0.6312	0.8790	0.6920	
1 month	0.2883	0.2522	0.2283	0.2162	0.2001	0.2065	0.3038	0.2559	0.2790	0.1475	
NN											
1 year	2.1946	2.0358	1.7253	1.5458	1.3662	1.7304	1.3613	0.5503	1.6232	0.8332	
1 month	0.5070	0.4669	0.3979	0.3547	0.3035	0.3840	0.4011	0.2445	0.3576	0.2758	

Table 1 continued

Models	Yields					Term structure of interest rates			Yield to maturity	
	3 months	1 year	3 years	5 years	10 years	Level	Slope	Curvature	Mean	SD
SNN										
1 year	2.3694	2.1599	1.7596	1.5282	1.2902	1.7658	1.6470	0.6901	1.6767	0.8663
1 month	0.5989	0.5431	0.4606	0.4146	0.3603	0.4533	0.4461	0.2779	0.3940	0.3513
Random walk										
1 year	1.4167	1.2521	0.9396	0.7570	0.5692	0.9303	1.2435	0.5730	0.8126	0.5946
1 month	0.2770	0.2498	0.2242	0.2114	0.1994	0.2053	0.2894	0.2155	0.2706	0.1442
Forward										
1 year	1.6652	1.4871	1.1451	0.9346	0.6938	1.1413	1.3128	0.5761	0.9737	0.6663
1 month	0.2988	0.2696	0.2352	0.2181	0.2053	0.2194	0.3027	0.1992***	0.2435	0.1366

Bold values denote the best econometric model for each interest rate and horizon

In all cases the RMSE is shown for 1-month and 1-year horizons, in the predicting period from January 2, 2000 to February 8, 2010. We indicate significance at the 10, 5, and 1 % level by *, **, and ***, respectively, if the RMSE of model *i*th is statistically lower than random walk using [Diebold and Mariano \(1995\)](#) test

test for comparing predictive accuracy. The results obtained show that, in most cases and in terms of the RMSE, the random walk model gives the best point prediction for the Spanish yield curve. In general, these results demonstrate the difficulty of achieving a good point forecast for the Spanish yield curve; none of the econometric models was capable of outperforming the random walk, during most of the period 2000–2010. Thus, our findings advise against the use of point forecasting in the Spanish yield curve for either policy-oriented as trading goals with the econometric models employed in our study. As it will be shown, from the point of view of our trading strategies, directional prediction is more interesting than point prediction.

Figure 3 compares the random walk model with the other models in terms of the daily evolution of the RMSE obtained by predicting the yields to maturity (real against predicted) 1 year ahead. As can be seen, although the random walk is the best model from the RMSE standpoint, there are periods when other models outperform it. In particular, the implicit forward rates give better predictions than random walk after January 2006 and during almost the whole crisis period, proving to be a good predictor of future yields to maturity for this period. As expected, the highest peak in RMSE occurred during the current crisis period (2008–2010), when trading activity on the debt markets of the peripheral European countries, especially in Spain, fell dramatically, rapidly increasing yields and the country risk, i.e., the differential on long yields against the German bond yields. As shown in Fig. 3, this produced difficulties in accurately fitting the yield to maturities, and accordingly the yield curves, and worsened the forecasts of all the econometric models.

For additional statistical evidence on the performance of the econometric models and the implicit forward rates, we also studied their rate of directional success in predicting the rise and fall of empirical components of the yield curve (from the point of view of our trading strategies, directional prediction is more interesting than point prediction). Table 2 shows the percentage of directional success with respect to the empirical level, slope and curvature, comparing the predictions made by the econometric models (parameters of the NS model with τ -fixed) with their real empirical values (with τ -free), as described above. Bold values correspond to the best econometric model for each empirical component of the yield curve and horizon. For instance, if the empirical slope predicted by an econometric model at date $t + h$ is increased (obtained as the difference between the empirical slope predicted at date $t + h$ minus the real empirical slope at date t with τ -free) in line with the real empirical slope at date $t + h$ (obtained as the difference between the real empirical slope at date $t + h$ minus the real empirical slope at date t both with τ -free), the econometric model is considered successful in the directional prediction of the empirical slope. The random walk model is not reported in Table 2 because this model, by definition, does not predict any movement in the empirical components of the yield curve.

As shown in Table 2, not all models attain a rate of success of 50 % in predicting the level and the slope, and higher success rates are achieved for the curvature. In the 1-month-ahead directional prediction, ECM(1) is the most successful model, with two common trends in predicting the level and the slope, and one common trend in predicting the curvature. In the 1-year-ahead directional prediction, the most successful models are the ECM(1) with two common trends in predicting the level, the NN in predicting the slope, and ECM(1) with one common trend in predicting the curvature.

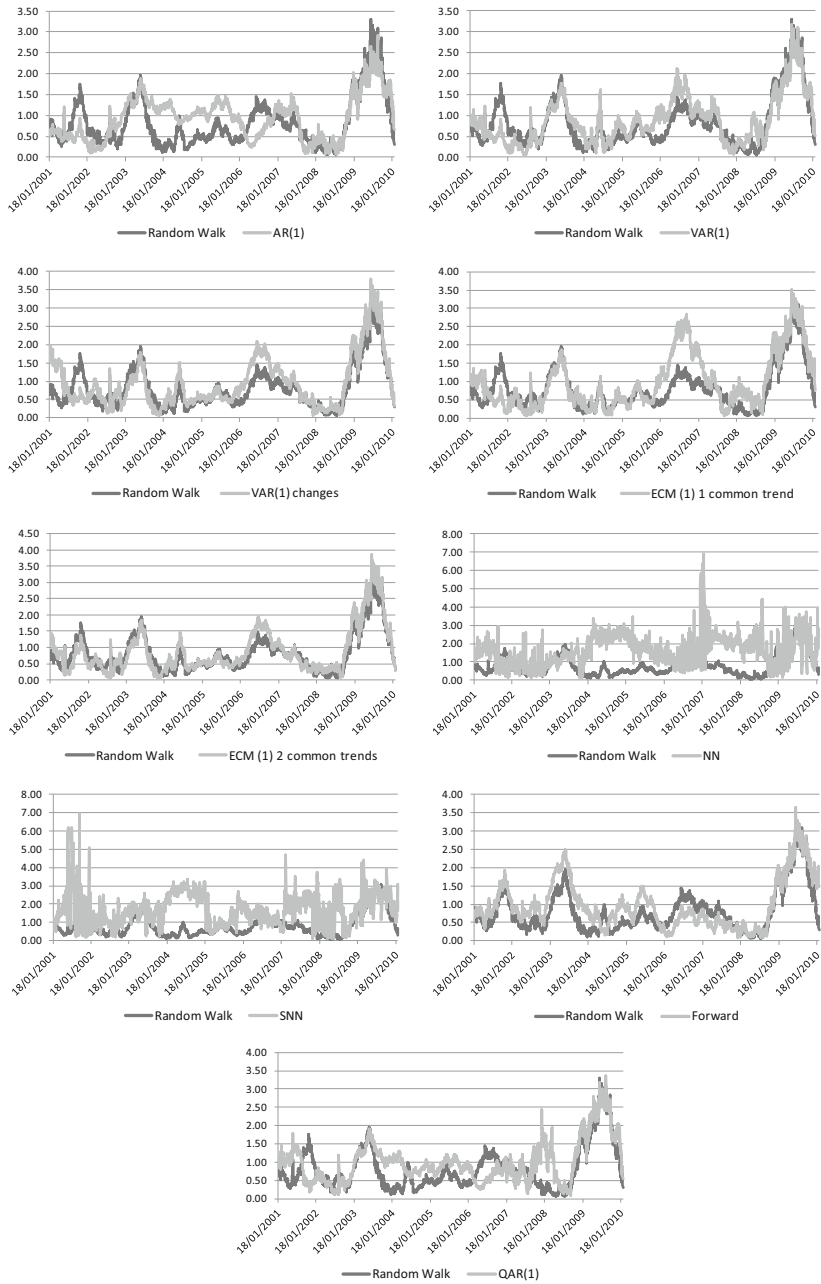


Fig. 3 Comparison of the random walk model with the other models in terms of the daily evolution of the RMSE obtained by predicting the yields to maturity (real against predicted) 1 year ahead

Table 2 Percentage of directional success with respect to the empirical level, slope and curvature, comparing the predictions made by the econometric models (parameters of the Nelson and Siegel model with τ -fixed at 3) with their real empirical values (with τ -free), as described above

Models	Level	Slope	Curvature
AR(1)			
1 year	0.5219	0.4946	0.6736
1 month	0.5339	0.5646	0.6233
VAR(1) levels			
1 year	0.4863	0.5180	0.6332
1 month	0.5441	0.5445	0.6214
QAR(1)			
1 year	0.5654	0.4420	0.6810
1 month	0.5362	0.5496	0.6225
VAR(1) changes			
1 year	0.5511	0.3133	0.5332
1 month	0.5114	0.5753	0.5961
ECM(1) 1 common trend			
1 year	0.4515	0.4376	0.7175
1 month	0.5016	0.5607	0.6517
ECM(1) 2 common trends			
1 year	0.5837	0.4181	0.6280
1 month	0.5670	0.6198	0.6068
NN			
1 year	0.5632	0.5598	0.6080
1 month	0.4890	0.5879	0.6131
SNN			
1 year	0.4976	0.4602	0.5511
1 month	0.4811	0.5642	0.5859
Forward			
1 year	0.3546	0.4588	0.5612
1 month	0.4764	0.5460	0.5932

Bold values denote the best econometric model for each empirical component of the yield curve and horizon. For instance, if the empirical slope predicted by an econometric model at date $t + h$ is increased (obtained as the difference between the empirical slope predicted at date $t + h$ minus the real empirical slope at date t with τ -free) in line with the real empirical slope at date $t + h$ (obtained as the difference between the real empirical slope at date $t + h$ minus the real empirical slope at date t both with τ -free), the econometric model is considered successful in the directional prediction of the empirical slope.

4 Economic assessment of our predictions and technical strategies

This section addresses our second objective, and the main contribution of this paper, which consists of using the econometric predictions on the parameters of the yield curve for the NS model to develop trading strategies in the Spanish bond market [see [Martellini et al. \(2003\)](#) as a general reference in fixed-income strategies]. Once the

yield curve is predicted, on the basis of the parameters of the NS model with τ -fixed, three active trading strategies are defined to bet on yield curve changes by comparing the empirical level, slope and curvature of the implicit forward interest rates and those of the interest rates predicted by our 1-year and 1-month-ahead econometric models.

Every day we start with 29–39 fixed-income assets (Spanish treasury bills and bonds) quoted that day on the secondary market. Note that the portfolios employed in the trading strategies are composed of the same repos, bill and bonds used to estimate the NS model in Sect. 2, i.e., we have eliminated from the portfolio all assets with a traded nominal value of <3 million euros in a single day, as well as bills and bonds with a maturity of <15 days or more than 15 years.

The only exception to this is that, in all the strategies described below, except for immunizations and bets on the empirical level, the maturities of the traded assets are >18 months for the 1 year and 1 month investment horizons. This selection of assets that do not mature within our investment horizon avoids the need for rebalancing the portfolios.

In accordance with the expectations theory, the forward curve reflects the agent's expectations on the future evolution of interest rates. Therefore, and following this theory, in order to take positions in the assets of the Spanish bond market, we compare the expectations of market agents for the future yield curve (1 year, 1 month) reflected in the forward curve, with the predictions of our econometric models. More specifically, we estimate the empirical components (level, slope and curvature) resulting from the current estimated NS parameters with τ -free, from the forward yield curve and compare it with the empirical predicted components resulting from the forecast NS parameters with τ -fixed. Hence, there are three kinds of strategies, as explained below.

4.1 Strategies betting on the future evolution of the empirical level

This strategy bets on the future evolution of the empirical level of the yield curve for an investment horizon of 1 year or 1 month, i.e., it is betting that all the interest rates move in parallel. When our econometric models predict a rise (fall) in the empirical level of the yield curve with respect to the empirical level of today's forward curve for this horizon, we decrease (increase) the [Macauley \(1938\)](#) duration (henceforth, simply duration) of our portfolio with respect to the investment horizon, buying assets of adequate duration. In this strategy, only long positions are employed.

For the 1-year horizon, we consider a strategy consisting of reducing (increasing) by 1 month the duration of the portfolio when a rise (fall) in the empirical level of the yield curve is predicted. For the case of an investment horizon of 1 month, the strategy consists in increasing or reducing the duration of the portfolio by 15 days.⁵ To that end, we estimate the empirical level (the mean of all the interest rates) expected by the market, resulting from the current estimated NS parameters with τ -free, from the forward yield curve, $Level_{t+h}^{Forward}$, at a horizon $h = \{22, 260\}$ days, and compare it with the empirical level, $Level_{t+h}$, resulting from the forecast NS parameters with

⁵ The periods employed for increasing or reducing the duration of the portfolio with respect to the investment horizon were considered arbitrarily; nevertheless the results are robust to other selections.

τ -fixed of our econometric models. This is achieved as follows. The duration of the portfolio is reduced by a time K , holding short-term instruments until their maturity, if $Level_{t+h}^{Forward} < Level_{t+h}$, where K is 1 month for a 1-year investment horizon or 15 days for a 1-month investment horizon. We then rebalance the portfolio in order to make the duration equal to the time remaining until the end of the investment horizon. This strategy is known as rollover (see [Martellini et al. 2003](#)). Instead, the duration of the portfolio is increased by a time K , if $Level_{t+h}^{Forward} \geq Level_{t+h}$.

Each time the portfolio is altered because of the payment of coupons or the maturity of an asset, we must rebalance the portfolio by buying new assets to bring the duration into line with that advised by our strategy (i.e., the time until the end of the investment horizon $\pm K$, depending on the prediction). As the problem of selecting several assets, and their weightings, in order to create a portfolio with a specific duration has multiple solutions, we employ the weight-selecting criteria suggested by [Nawalkha et al. \(2005\)](#), thus making the duration of the portfolios equal to the 1-month or 1-year investment horizon $\pm K$, depending on the kind of prediction.

4.2 Strategies betting on the future evolution of the empirical slope

We now estimate the empirical slope (10-year interest rate minus 3-month interest rate) resulting from the current estimated NS parameters with τ -free, from the forward yield curve, $Slope_{t+h}^{Forward}$, at a horizon $h = \{22,260\}$ days, and compare it with the empirical slope, $Slope_{t+h}$, resulting from the forecast NS parameters with τ -fixed obtained from our econometric models. In this strategy, we choose a maturity of 5 years, employed as a pivot which corresponds to half of the maturities of the empirical slopes.

In order to limit the number of assets in our portfolios, we always use six fixed-income assets,⁶ three with maturities lower than the pivot (5 years) and three with maturities higher than the pivot.

A bet for the empirical slope to rise ($Slope_{t+h}^{Forward} < Slope_{t+h}$) means that we expect the yields of the three assets with maturities lower than the pivot (5 years) to fall and those of the other three assets to rise. Thus, we take a long position in the three assets with maturities lower than the pivot (their prices are expected to increase) and a short position in the three assets with maturities higher than the pivot (their prices are expected to decrease). These short positions are taken through a repo reverse.⁷

⁶ The choice of just six assets is subjective and intended only to set a limit on the number of assets in the portfolio. Nevertheless, the results are robust to other combinations of assets.

⁷ The returns in the short positions are divided into two components: the return generated by the short position itself (the difference between the price at which the asset was sold and the price at which it was bought) and the return earned by the repo reverse which is known from the beginning of the investment horizon. Instead, the returns in the long positions are composed of the capital gains (the differences in prices), the coupon paid and the reinvestment of the coupon at the risk free rate. As our point of view is a fund manager who takes both positions with the fund's endowment, neither long nor short positions need additional funding.

On the contrary, a bet for the empirical slope to fall ($Slope_{t+h}^{Forward} \geq Slope_{t+h}$) means we take a long position in the three assets with maturities higher than the pivot and a short one in the other three assets.

The positions advised by our econometric predictions with 1-month or 1-year horizons are held until the end of the strategy (1 month or 1 year, respectively). Thus, no rebalancing is made of strategies betting on the empirical slope; positions are taken at the beginning of the investment horizon and closed at the end of the investment horizon.

As 29–39 fixed-income assets are quoted in the market every day, and as our portfolio in this case has only six assets, we have the same problem as before, i.e., that of selecting those six assets, and the fact that they can be weighted in many ways. We decided to immunize the portfolio by employing the weighting criteria suggested by Nawalkha et al. (2005) and thus hedge the portfolio against the risks of parallel shifts in the yield curve. Specifically, we employed the criterion of maximizing the diversification, making the durations equal to the investment horizon of 1 month or 1 year (see Nawalkha et al. 2005, page 89, equation 4.19). In this case, we seek the combination of six assets that matches the duration objective and maximizes the convexity.

4.3 Strategies betting on the future evolution of the empirical curvature

A change in the empirical curvature is related to butterfly movements on the yield curve, that is, a rise (fall) of the short term and long term rates (the wings of the butterfly), complemented with a fall (rise) of the intermediate term rates (the body of the butterfly) [see Martellini et al. (2002) as a general reference to butterfly strategies].

Our strategy uses a portfolio of six assets, two for short term rates (maturities shorter than 3 years), two in the body of the butterfly (maturities between 3 and 8 years) and two for long term rates (maturities >8 years).

We now estimate the empirical curvature (twice the 5-year interest rate minus the 3-month interest rates and the 10-year interest rates), resulting from the current estimated NS parameters with τ -free, from the forward yield curve, $Curvature_{t+h}^{Forward}$, at a horizon $h = \{22, 260\}$ days, and compare this with the empirical curvature, $Curvature_{t+h}$, resulting from the forecast NS parameters with τ -fixed obtained from our econometric models. The strategy is now:

If a fall in the empirical curvature is predicted: $Curvature_{t+h}^{Forward} > Curvature_{t+h}$, we take a long position in assets on the body (these prices are expected to increase), and a short position in the wings (these prices are expected to fall).

If a rise in the curvature is predicted: $Curvature_{t+h}^{Forward} < Curvature_{t+h}$, we take a short position in assets on the body, and a long position in the wings.

As in the slope strategies, the positions advised by our econometric predictions with 1-month or 1-year horizons are held until the end of the strategy (1 month or 1 year, respectively). Furthermore, and as in the case of the slope strategies, we immunize the portfolio by employing the weight selection criterion suggested by Nawalkha et al. (2005) in order to hedge our portfolios against the risks of parallel shifts in the yield curve. To this end, the portfolios are immunized by making their durations equal to the investment horizon, and maximizing their convexities.

4.4 Our benchmarks

Various benchmarks are used to compare the strategies applied.

For the level bets, we take as a benchmark the immunization strategy in which the duration of the portfolio is equal to the investment horizon. This ensures that a small parallel shift in the yield curve will not affect an objective return (the 1-year or 1-month yield initially defined). In this strategy, only long positions are permitted. As there is an infinite number of combinations of portfolio weights to perform the immunization, once again the selection criteria suggested by [Nawalkha et al. \(2005\)](#) are employed.⁸

For the case of slope and curvature bets, two benchmarks are considered. On the one hand, the “riding the yield curve” strategy in which the benchmark selected is an equally-weighted portfolio, as in the ladder strategy, taking long positions in the same assets selected by each econometric model in the original strategy. This target strategy represents the bet made with the random walk model, that is, assuming that the yield curve will have the same shape in the future. In this case (provided the yield curve continues to present an upward slope, as is usually the case), it could be profitable to buy long-maturity assets and sell them later, benefiting from the fall in interest rates, and the consequent rise in prices, which represents a future yield curve with the same shape as the current one.

On the other hand, the benchmark portfolio for each empirical component of the yield curve, given by the NS model, should have a neutral sensitivity to movements of this component, and therefore the benchmark portfolio needs to be hedged against it. Therefore, the performance of the active strategy should be measured by comparing the results of two portfolios with the same components in which the weights would be provided by two programmes. One portfolio would be constructed to hedge against shape changes and the other portfolio would bet on the predicted movements. In this way, our active trading strategy based on the predictions of one of the econometric models should have a good performance whether or not we systematically beat this new benchmark portfolio. Thus, for the case of slope and curvature bets, we considered a second benchmark, provided by hedging strategies, consisting of a portfolio with the same components and positions as our strategy but with weights designed to hedge the portfolio against movements of the slope or curvature. Following [Martellini et al. \(2003, pages 196–197\)](#), these weights make the slope (or curvature) duration of the hedging portfolios equal to zero. In other words, the hedge strategies are hedged against slope or curvature movements, depending on the bet.

Finally, in order to discard spurious signals and excessively intensive trading strategies, we filter the trading signals using a trading threshold of {5 %}. For instance, if

$$\left| Slope_{t+h}^{Forward} - Slope_{t+h} \right| \leq threshold_j,$$

⁸ The reported results for level and immunization strategies are obtained by minimizing the M-Squared subject to the condition that the portfolio’s duration is equal to the target duration. The findings with the rest of the selection criteria of [Nawalkha et al. \(2005\)](#) are similar; although not reported here for reasons of space, they are available from the authors.

no position is taken for this day. In the above formula

$$threshold_j = [\max (Slope_{1:t}) - \min (Slope_{1:t})] .c_j,$$

where $c_j = \{0.05\}$ and $Slope_{1:t}$ is the in-sample empirical slope obtained by updating the strategy.

5 Empirical results

One of the most widely employed measures of performance used to compare strategies in terms of risk is the Sharpe ratio (SR). This consists in estimating the excess return of a strategy over the risk-free return (the repo rate) per unit of volatility. This risk-free asset represents the opportunity costs of investing in the repo rate at the beginning of each investment horizon and holding for 1 year or 1 month.

$$SR = \frac{\text{Mean} \left(R_{t+h}^{Portf} - R_{free} \right)}{\sigma_{Portf}}$$

where R_{t+h}^{Portf} is the annualized return of the portfolios for a given day and investment horizon of 1 month or 1 year⁹ which started on date t and ended on date $t + h$, with t from January 2, 2000 to February 8, 2009 for a 1-year investment horizon and from January 2, 2000 to January 8, 2010 for a 1-month investment horizon), presenting a standard deviation $\sigma_{Portf} = \text{Std} \left(R_{t+h}^{Portf} \right)$. As is very well known, the higher the Sharpe ratio of a strategy, the better its performance.

Recently, adjustments to the Sharpe ratio have been proposed to account for non-normality in the returns. Thus, Díaz et al. (2009) estimated alternative Sharpe ratios. Besides the traditional Sharpe ratio, they considered an “adjusted Sharpe ratio” (ASR) which accounts for non-normality in the distribution of returns by making use of the value at risk (VaR).

$$ASR = \frac{\text{Mean} \left(R_{t+h}^{Portf} - R_{free} \right)}{\text{VaR}}$$

where VaR is equal to N times σ_{Portf} and $N = 2.33$ is the number of standard deviations associated with a 1 % level of probability, assuming that returns are normally distributed.

In order to extend the tail risk to other moments of the distribution, these authors also considered the “modified Sharpe ratio” (MSR), which corrects the adjusted Sharpe ratio to include the impact of skewness and excessive kurtosis.

⁹ A new trading strategy is started each day of our prediction period and closed within $h = \{22,260\}$ days, obtaining a return for the whole investment horizon equal to R_{t+h}^{Portf} .

$$MSR = \frac{\text{Mean} \left(R_{t+h}^{Portf} - R_{free} \right)}{MVAR},$$

where the MVAR is measured as

$$MVAR = \left(N + \frac{1}{6} (N^2 - 1) S + \frac{1}{24} (N^3 - 3N) K - \frac{1}{36} (2N^3 - 5N) S^2 \right) \sigma_{Portf}$$

where S and K stand for the skewness and kurtosis of the returns, respectively.

In order to study the economic significance of our Sharpe ratios, we also considered the [Opdyke \(2007\)](#) test, which, under very general conditions of time-varying conditional volatilities, serial correlations and non-IID returns, allows us to test whether the Sharpe ratios for each of the predictions provided by the econometric models are statistically different from those for the benchmarks (riding the yield curve and hedging strategies).

Tables 3 and 4 show the annual SR, the annual ASR and the annual MSR over overlapping portfolios for 1-year-ahead predictions (Table 3) and 1-month-ahead predictions (Table 4), obtained by our econometric models for the prediction period 2000–2010. Bold values correspond to the models with highest values for SR, ASR and MSR. These tables also show whether the null hypothesis of the Opdyke test of equal Sharpe ratios for the strategy and the benchmark is rejected, together with the size of the rejection and the acceptances of the Jarque and Bera test of normal distribution in R_{t+h}^{Portf} returns for the prediction period 2000–2010. For the sake of simplicity, only the hedging strategy was considered as a benchmark. Although not reported here, for reasons of space, the results for the riding-the-yield-curve benchmark are qualitatively similar; they can be obtained from the authors upon request. In order to avoid spurious signals, a filter of {5 %} was also applied.

Summarizing the data shown in Table 4, the trading strategies associated with the signals provided by the 1-month-ahead predictions do not improve on the hedging strategy benchmark, with the exception of slope bets with a 5 % trading threshold.¹⁰ On the contrary, it can be seen in Table 3 that most of the models for the 1-year-ahead predictions outperform the hedging strategy, with the exception of level bets, which fail to outperform the immunization strategy (this has a SR of 0.1429, an ASR of 0.0613 and a MSR of 0.0479 for the 1-year investment horizon). As shown in Table 3, for 1 year horizon, QAR(1) strategies obtain the highest SR, ASR, and MSR average for level, and NN and VAR(1) changes for betting for slope and curvature, depending on the threshold. As shown in Table 4, for a 1 month horizon, QAR(1) strategies obtain the highest SR, ASR, and MSR average for level, and NN and ECM(1) with 2 common trend, for betting for slope and curvature, depending on the threshold.

To gain a broader perspective of the findings of Table 3 and Fig. 4 plots the daily evolution of the returns for our best strategies, given their Sharpe ratios, on level [the QAR(1)], slope (the NN) and curvature (the NN) without trading threshold based on

¹⁰ None of the level strategies give a signal higher than the trading threshold for a 1 month horizon.

Table 3 Annual Sharpe ratios (SR), annual adjusted Sharpe ratio (ASR), and annual modified Sharpe ratio (MSR) over overlapping portfolios for trading strategies based in 1-year-ahead predictions obtained by different econometric models in the prediction period 2000–2010

Models	Betting for level		Betting for slope		Betting for curvature		Average
	Threshold at 5 %		Threshold at 5 %		Threshold at 5 %		
	Without threshold		Without threshold		Without threshold		
AR(1)							
SR	0.0743	–	0.0272	–	0.5195	+	0.4916
ASR	0.0319		0.0117		0.2229		0.2110
MSR	0.0231		0.0094		0.1645		0.1454
VAR(1) levels							
SR	0.0015	–	0.0256	–	0.6269	+	0.5060
ASR	0.0006		0.0110		0.2691		0.2172
MSR	0.0005		0.0082		0.1953		0.1446
QAR(1)							
SR	0.1436		0.0847	–	0.6337	+	0.5055
ASR	0.0616		0.0364		0.2720		0.2169
MSR	0.0462		0.0271		0.1975		0.1476
VAR(1) changes							
SR	0.0424	–	0.0230	–	0.5019	+	0.4721
ASR	0.0182		0.0099		0.2154		0.2026
MSR	0.0135		0.0073		0.1543		0.1361
ECM(1) 1 common trend							
SR	0.0747	–	0.0324	–	0.5660	+	0.4671
ASR	0.0320		0.0139		0.2429		0.2005
MSR	0.0243		0.0104		0.1741		0.1352

Table 3 continued

Models	Betting for level		Betting for slope		Betting for curvature		Average				
	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %					
ECM(1) 2 common trends											
SR	0.0626	–	0.0313	–	0.5180	+	0.7928	0.7115	+	0.4368	
ASR	0.0269		0.0134		0.2223		0.3403	0.3053		0.1875	
MSR	0.0202		0.0100		0.1564		0.2352	0.2129		0.1325	
NN											
SR	0.0603	–	0.0492	–	0.8540	+	0.9489	+	0.8749	+	0.5978
ASR	0.0259		0.0211		0.3665		0.4072	0.3755		0.2566	
MSR	0.0183		0.0151		0.2769		0.2682	0.2530		0.1819	
SNN											
SR	0.0779	–	0.0612	–	0.7232	+	0.9079	0.8657		0.5508	
ASR	0.0334		0.0263		0.3104		0.3896	0.3715		0.2364	
MSR	0.0243		0.0192		0.2250		0.2466	0.2339		0.1589	
Average SR	0.0671		0.0418		0.6533		0.8822	0.7925			
Average ASR	0.0288		0.0180		0.2804		0.3786	0.3401			
Average MSR	0.0213		0.0133		0.2036		0.2428	0.2230			

Bold values denote the best forecasting approach for each strategy

It is also signaled whether the null hypothesis of Opdyke test of equal Sharpe ratios for the strategy and the benchmark is rejected, which is the size of the rejection, and the acceptances of the Jarque and Bera test of normal distribution in the returns for the prediction period (2000–2010). In order to avoid spurious signals a trading threshold of { 5 % } has also been considered

* Signals that the null hypothesis of Jarque and Bera test normality of returns is accepted at the 5 % level

+ Null hypothesis of equal Sharpe ratios (two-tailed test of [Opdyke 2007](#)) rejected at the 5 % level. Case 1: Sharpe ratio of Benchmark is statistically lower than Sharpe ratio of model ith

– Null hypothesis of equal Sharpe ratios (two-tailed test of [Opdyke 2007](#)) rejected at the 5 % level. Case 2: Sharpe ratio of Benchmark is statistically higher than Sharpe ratio of model ith

Table 4 Annual Sharpe ratios (SR), annual adjusted Sharpe ratio (ASR), and annual modified Sharpe ratio (MSR) over overlapping portfolios for trading strategies based in 1 month-ahead predictions obtained by different econometric models in the prediction period 2000–2010

Models	Betting for level		Betting for slope		Betting for curvature		Average
	Without threshold		Without threshold		Without threshold		
	Threshold at 5 %		Threshold at 5 %		Threshold at 5 %		
AR(1)							
SR	0.0544	–	0.0378	–	0.0654	0.0653	0.0499
ASR	0.0234		0.0162		0.0281	0.0280	0.0214
MSR	0.0186		0.0084		0.0152	0.0136	0.0120
VAR(1) levels							
SR	–0.0574	–	0.0640	+	0.0663	0.0656	0.0413
ASR	–0.0246		0.0275		0.0285	0.0282	0.0177
MSR	–0.0202		0.0038		0.0156	0.0139	0.0048
QAR(1)							
SR	0.1110	–	0.0673		0.0694	0.0749	0.0754
ASR	0.0477		0.0289		0.0298	0.0321	0.0324
MSR	0.0365	*	0.0148		0.0163	0.0161	0.0185
VAR(1) changes							
SR	–0.0387	–	0.0640	+	0.0750	0.0790	0.0498
ASR	–0.0166		0.0275		0.0322	0.0339	0.0214
MSR	–0.0137		0.0038		0.0181	0.0157	0.0068
ECM(1) 1 common trend							
SR	0.0079	–	0.0394	–	0.0550	–0.0277	0.0256
ASR	0.0034		0.0169		0.0236	–0.0119	0.0110
MSR	0.0029		0.0088		0.0133	–0.0052	0.0054

Table 4 continued

Models	Betting for level		Betting for slope		Betting for curvature		Average
	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	
ECM(1) 2 common trends							
SR	-0.0134	-	0.0530	0.0980	0.0353	0.0359	0.0417
ASR	-0.0058	-	0.0228	0.0421	0.0152	0.0154	0.0179
MSR	-0.0048	-	0.0019	0.0103	0.0090	0.0079	0.0049
NN							
SR	0.0244	-	0.0722	0.0620	0.0757	0.0833	0.0635
ASR	0.0105	-	0.0310	0.0266	0.0325	0.0358	0.0273
MSR	0.0079	*	0.0165	0.0111	0.0181	0.0177	0.0143
SNN							
SR	0.0330	-	0.0614	0.0568	0.0750	0.0728	0.0598
ASR	0.0142	-	0.0264	0.0244	0.0322	0.0312	0.0257
MSR	0.0109	*	0.0025	0.0078	0.0181	0.0161	0.0111
Average SR	0.0152	-	0.0574	0.0611	0.0646	0.0561	
Average ASR	0.0065	-	0.0246	0.0262	0.0277	0.0241	
Average MSR	0.0048	-	0.0076	0.0088	0.0155	0.0120	

Bold values denote the best forecasting approach for each strategy
 It is also signaled whether the null hypothesis of Opdyke test of equal Sharpe ratios for the strategy and the benchmark is rejected, which is the size of the rejection, and the acceptances of the Jarque and Bera test of normal distribution in the returns for the prediction period (2000–2010). The 5 % trading threshold is not applied in level strategies because none give a signal higher than the trading threshold for a 1 month horizon
 * Signals that the null hypothesis of Jarque and Bera test normality of returns is accepted at the 5 % level.
 + Null hypothesis of equal Sharpe ratios (two-tailed test of [Opdyke 2007](#)) rejected at the 5 % level. Case 1: Sharpe ratio of Benchmark is statistically lower than Sharpe ratio of model *ith*
 - Null hypothesis of equal Sharpe ratios (two-tailed test of [Opdyke 2007](#)) rejected at the 5 % level. Case 2: Sharpe ratio of Benchmark is statistically higher than Sharpe ratio of model *ith*

1-year-ahead predictions. In contrast, Fig. 5 shows the daily evolution of the returns for our worst strategies on level [the VAR(1) levels], slope [the VAR(1) changes] and curvature [the ECM(1) 2 common trends].

Both figures show that our strategies obtain positive returns during the prediction period from January 2000 to February 2010, i.e., at any time it is profitable to apply our strategies for a 1-year investment horizon. However, the same bets do not always obtain the highest return in our sample; thus, there could be factors other than each empirical component of the yield curve which explain the profitability of our trading rules; however, seeking to determine other macroeconomic or financial factors that might explain the profitability of our strategies is a question that is beyond the scope of this paper.

On comparing Figs. 4 and 5 with the behaviour of slope parameter β_1 in Fig. 1, we can see how strategies for betting on empirical slope movements achieve the highest returns for periods close to changes in slope trends, i.e., 2001, 2003, 2005 and during the current crisis period; it is also interesting to note that the huge change in the slope trend during the crisis period in 2009 produced a positive peak in the slope strategies; we attribute this to the fact that the movements in the prices of fixed-income assets, when the slope trend changed in 2009, were more urgent and rapid than any other yield curve shape. Curvature strategies were fairly stable during our prediction period. Finally, the level strategies are the smoothest of all, following the actual evolution of the empirical level in the prediction period.

In the previous results, transaction costs were not considered because our strategies are not trading intensive and so transaction costs are very low. For instance, we used only six assets in our bets on slope and curvature, and these were only traded at the beginning and end of the investment horizon, while the bets on the level generally involved two assets, at most, with just a little rebalancing, and transaction costs were only paid twice, to open and close the positions.

As each financial institution which is able to sell bonds determines its own levels of commission, and given the difficulty of finding suitable transaction costs in the literature, in order to take a view on the transactions costs in our trading strategies, we report the break-even transaction cost for each strategy. This is defined as the transaction cost per trade which makes the mean return of the strategy equal to zero for our prediction period, 2000–2010. The higher the break-even level, the lower the trading intensity of the strategy and the higher the transaction costs required to eliminate its profitability.

Table 5 shows the break-even transaction cost for the different trading strategies adopted. As expected, these costs are higher for the trading threshold where no position is taken and the latter is not overcome. On average, the 1-year-ahead strategies produce a break-even transaction cost between 0.29 and 0.42 per asset trade without trading threshold. The models AR(1), and ECM(1) with two common trends require a higher and a lower transaction cost to eliminate their profitability, respectively, in the 1-year-ahead strategies. On the other hand, in the 1-month-ahead strategies, the econometric models with highest and lowest break-even transaction costs are the QAR(1) and the VAR(1) changes in the yield curve, respectively. These findings corroborate our

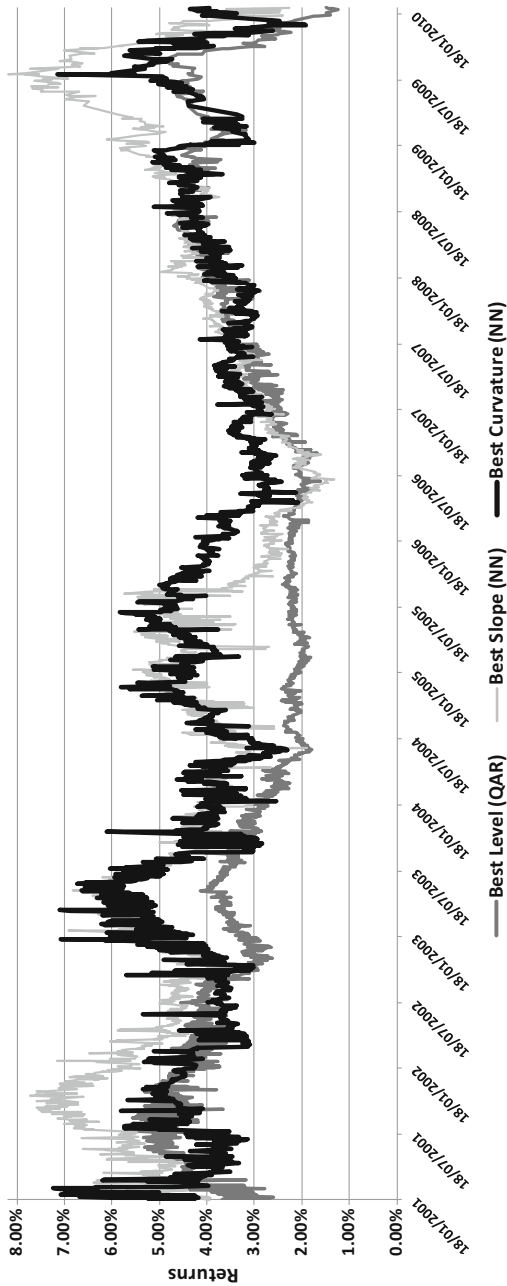


Fig. 4 Daily evolution of the returns for the best strategies, given their Sharpe ratios, on level [the QAR(1)], slope (the NN) and curvature (the NN) between 2000 and 2010, without trading threshold based on 1-year-ahead predictions

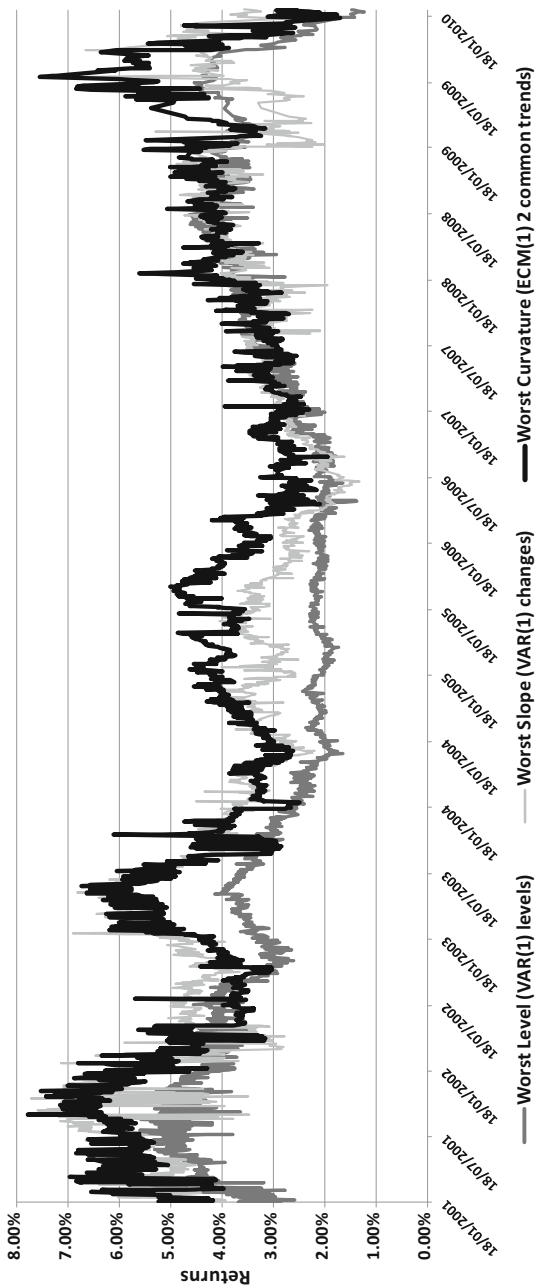


Fig. 5 Daily evolution of the returns for the worst strategies on level [the VAR(1) levels], slope [the VAR(1) changes] and curvature [the ECM(1) 2 common trends] between 2000 and 2010, without trading threshold based on 1-year-ahead predictions

Table 5 Break-even cost: transaction cost which makes the mean return of the strategy equal to zero for our prediction period, 2000–2010

Models	Betting for level		Betting for slope		Betting for curvature		Average
	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	
AR(1)							
1 year	0.3900	0.4000	0.3600	0.4200	0.3300	0.4000	0.3833
1 month	0.4800	–	0.2600	0.2500	0.2700	0.2600	0.3040
VAR(1) levels							
1 year	0.3300	0.3300	0.3500	0.3700	0.3300	0.3800	0.3483
1 month	0.3600	–	0.2800	0.2600	0.2700	0.2600	0.2860
QAR(1)							
1 year	0.3900	0.4100	0.3500	0.4200	0.3300	0.3900	0.3817
1 month	0.4900	–	0.2700	0.2600	0.2700	0.3200	0.3220
VAR(1) changes							
1 year	0.3400	0.2900	0.3300	0.3700	0.3400	0.4100	0.3467
1 month	0.3500	–	0.2800	0.2500	0.2700	0.2600	0.2820
ECM(1) 1 common trend							
1 year	0.3500	0.3200	0.3500	0.4000	0.3400	0.4100	0.3617
1 month	0.3800	–	0.2600	0.2500	0.2600	0.2300	0.2760
ECM(1) 2 common trends							
1 year	0.3400	0.2900	0.3300	0.3900	0.3500	0.4100	0.3517
1 month	0.3700	–	0.2800	0.2700	0.2500	0.2500	0.2840

Table 5 continued

Models	Betting for level		Betting for slope		Betting for curvature		Average
	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	
NN							
1 year	0.3800	0.3800	0.3700	0.3900	0.3400	0.3800	0.3733
1 month	0.4200	–	0.2700	0.2600	0.2700	0.2700	0.2980
SNN							
1 year	0.3800	0.3700	0.3600	0.3900	0.3500	0.4000	0.3750
1 month	0.4200	–	0.2800	0.2500	0.2700	0.2600	0.2960
Average 1 year	0.3586	0.3400	0.3500	0.3900	0.3400	0.3986	
Average 1 month	0.3971	–	0.2729	0.2557	0.2657	0.2557	

The higher the break-even level, the lower the trading intensity of the strategy and the higher the transaction costs required to eliminate its profitability. The break-even costs are paid twice to open and close the positions

conclusions that the trading strategies described are profitable even after accounting for high transaction costs per asset trade.¹¹

Finally, in order to determine the profitability of our trading strategies with respect to the current crisis period, Table 6 shows the annual SRs and the Opdyke test for the period January 2008 to February 2010. As before, bold values correspond to the models with highest values for SR. The crisis period (2008–2010) includes interesting events, such as Lehman Brothers' demise, financial turmoil, European Central Bank monetary policy decisions and the beginning of the sovereign debt crisis. During this period, trading activity on the debt markets of the peripheral European countries fell drastically, with increasing yields and risk premiums with respect to the German bond market. As mentioned above, this situation can make it difficult to accurately fit yield curves and so forecasts and performance worsened.

As in the prediction period 2000–2010 reported in Table 4, our 1-month-ahead trading strategies did not outperform the hedging strategies, except for ECM(1) with two common trends, VAR(1) changes in betting for slope and ECM(1) with one common trend in betting for level. Nevertheless, the bets for level based on the models ECM(1) with one and two common trends did outperform the level benchmark, which is the immunization strategy at the 1-year investment horizon. Moreover, all the slope and curvature strategies were capable of outperforming the benchmarks provided by hedging strategies at the 1-year investment horizon. Of all the strategies the NN model obtained the highest Sharpe ratios, on average, in the current crisis period for a 1 month horizon and QAR(1) for a 1-year horizon.

As mentioned before, on comparing the evolution of parameter β_1 in Fig. 1 with the situation shown in Figs. 4 and 5, we observe that the slope bets had higher Sharpe ratios, on average, during the current crisis period for the 1-year investment horizon; we attribute this to the urgent, and faster movements in the prices of fixed-income assets when slope trends were increasing drastically. However, the other bets presented slightly lower Sharpe ratios than those for the period as a whole.

To summarize our results, both with respect to the crisis period (2008–2010) and for the period as a whole (2000–2010), from the standpoint of statistical prediction, the NN model is a mediocre performer, and its percentage of success in predicting the direction of the level and curvature movements is not significantly higher than that achieved by the other models; nevertheless, the NN model does obtain more profitable strategies on average than the other econometric models for all the trading strategies at the 1-month and the 1-year investment horizons, obtaining Sharpe ratios that are significantly higher than their benchmarks for the Opdyke test in the slope and curvature strategies for the 1-year horizon. Nevertheless, in general, the NN model fails to improve on its hedging strategy benchmark for the 1-month investment horizon.

Finally, our findings suggest there is a mismatch between the statistical significance and the economic significance of forecasts, and indirectly that purely statistical loss functions may be of little value to market practitioners, as was highlighted by [Satchell and Timmermann \(1995\)](#). These authors were the first to explain in theoretical terms

¹¹ The average transaction costs on large stocks for a US institutional investor are estimated at 25–31 basis points per trade ([Peterson and Sirri 2003](#); [Bessembinder 2003](#)).

Table 6 Sharpe ratios and Opdyke test in the crisis period between January 2008 and February 2010, for our strategies betting for the level, slope and curvature

Models	Betting for level		Betting for slope		Betting for curvature		Average						
	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %							
	AR(1)												
1 year	-0.0067	-	-0.0088	-	0.9594	+	0.7711	+	0.8035	+	0.5425	+	0.5102
1 month	-0.0471	-	-	-	-0.1734	-	-0.2040	-	0.1845		0.1182		-0.0244
VAR(1) levels													
1 year	-0.1315	-	0.0020	-	0.9519	+	0.7435	+	0.7812	+	0.5505	+	0.4829
1 month	-0.1023	-	-	-	-0.0597	-	-0.1714	-	0.1604		0.1600		-0.0026
QAR(1)													
1 year	0.2088	+	0.1524	+	0.8428	+	0.5474	+	0.8952	+	0.3942	+	0.5664
1 month	-0.0077	-	-	-	-0.0594	-	-0.1319	-	0.1592		0.1419		0.0204
VAR(1) changes													
1 year	0.1263	-	0.0038	-	0.1790	+	0.2086	+	0.5603	+	0.5735	+	0.3050
1 month	-0.0121	-	-	-	0.0891		0.0261	+	0.1019		0.1621		0.0734
ECM(1) 1 common trend													
1 year	0.2789	+	0.0467	-	0.8336	+	0.6364	+	0.7993	+	0.6030	+	0.5330
1 month	0.0153	+	-	-	-0.1332	-	-0.1843	-	0.1604		0.1517		0.0020
ECM(1) 2 common trends													
1 year	0.2209	+	-0.0007	-	0.2365	+	0.2047	+	0.5885	+	0.5225	+	0.2954
1 month	0.0019	-	-	-	-0.0618	+	0.1413	+	0.1341		0.1692		0.0770

Table 6 continued

Models	Betting for level		Betting for slope		Betting for curvature		Average
	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	Without threshold	Threshold at 5 %	
NN							
1 year	-0.0605	-0.0801	1.1771	1.1370	0.5758	0.5442	0.5489
1 month	-0.0239	-	0.0858	0.1081	0.1223	0.1346	0.0854
SNN							
1 year	-0.0605	-0.0801	0.8137	0.7089	0.6762	0.6429	0.4502
1 month	-0.0293	-	-0.0119	-0.0017	0.0804	0.1322	0.0339
Average 1 year	0.0720	0.0044	0.7493	0.6197	0.7100	0.5467	
Average 1 month	-0.0256		-0.0405	-0.0522	0.1379	0.1462	

Bold values denote the best forecasting approach for each strategy and horizon

In order to avoid spurious signals a trading threshold of (5 %) has also been considered

+ Null hypothesis of equal Sharpe ratios (two-tailed test of [Opdyke 2007](#)) rejected at the 5 % level. Case 1: Sharpe ratio of Benchmark is statistically lower than Sharpe ratio of model ith

- Null hypothesis of equal Sharpe ratios (two-tailed test of [Opdyke 2007](#)) rejected at the 5 % level. Case 2: Sharpe ratio of Benchmark is statistically higher than Sharpe ratio of model ith

the possibility of a disconnection between statistical accuracy and profitability, and illustrated it through a US\$ trading rule. Thus, and as [Satchell and Timmermann \(1995\)](#) pointed out, for the case of our non-parametric and non-linear predictions, the standard criteria for statistical forecast accuracy do not have a direct mapping onto profitability.

6 Conclusions

In this study, we predict the yield curve for the Spanish public debt market, 1 month and 1 year ahead, using the methodology proposed by [Diebold and Li \(2006\)](#). The capability of generating profits by means of these yield curve predictions, transforming them into technical trading strategies, is also considered.

By re-interpreting the Nelson–Siegel yield curve as a dynamic model that achieves a reduction in dimensionality, the factors level, slope and curvature are predicted using different econometric models, namely the parametric ones suggested by [Diebold and Li \(2006\)](#) and [Rezende and Ferreira \(2013\)](#), and two non-parametric models suggested by [Fernández-Rodríguez et al. \(1999\)](#) for exchange rates. Our findings show that the random walk model is competitive from the standpoint of point predictions for Spanish yield curves. However, various econometric models are capable of obtaining a success rate in the direction of empirical level, slope and curvature exceeding 50 % at 1 year and 1 month ahead.

In addition, by converting the predictions of the econometric models into technical trading strategies for betting on the movements of the level, slope and curvature of the yield curve in the Spanish public debt market, we obtain Sharpe ratios which outperform the benchmarks of the hedging strategies for long (1 year) horizons both for the overall prediction period (2000–2010) and also for the current crisis period (2008–2010). Nevertheless, these strategies do not outperform their benchmarks for short (1 month) horizons. The Sharpe ratios for all the strategies are reasonably stable for different econometric models with the same investment horizon. This finding is in line with the conclusions of [Diebold and Li \(2006\)](#), whose forecasts appear to be much more accurate for long horizons.

With regard to the introduction of non-parametric models, although, from the standpoint of point predictions and the success in directional prediction, these models do not outperform the parametric models proposed by [Diebold and Li \(2006\)](#), the non-parametric models, and especially the NN model, are capable of obtaining more profitable strategies than the other econometric models, at both the 1-month and the 1-year investment horizons; in fact, the latter model obtains Sharpe ratios that are significantly higher than their benchmarks for the Opdyke test in the slope and curvature strategies for the 1-year horizon.

Thus, and as [Satchell and Timmermann \(1995\)](#) pointed out for the case of nonlinear predictions (as is the case of the NN model), we find that the standard criteria for statistical forecast accuracy do not have a direct mapping onto profitability.

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Appendix: Non-parametric regression models: nearest neighbours (NN) and simultaneous nearest neighbours (SNN)

In order to reformulate the k -nearest neighbour (k -NN) method, let us consider the problem of predicting the observation x_{T+h} generated from the stochastic process $x_{t+h} = f_t(x_t, \dots, x_{t-(d-1)}) + \varepsilon_t$, where ε_t is white noise, and $f_t(\cdot)$ is not constrained to belong to a specific class of functions (see [Hastie et al. 2001](#) for a review of NN methods).

Let the finite time-series $\{x_t\}_{t=1}^T$ represent the daily observations available to the forecaster and assume the goal is to obtain out-of-sample predictions \hat{x}_{T+h} . The first stage of the NN technique is to subsample data segments of equal length d from the available time series

$$\mathbf{x}_t^m \equiv (x_t, x_{t-1}, \dots, x_{t-(d-1)}) \in \mathbb{R}^d, \quad d \leq t \leq T - 1$$

where d is called the *embedding dimension*. These d -dimensional vectors of consecutive observations are also called m -histories and \mathbb{R}^d is called the *phase space*. The proximity of two m -histories in the phase space \mathbb{R}^d allows the notion of ‘nearest neighbour’. This non-parametric approach to prediction begins by finding the k nearest neighbours defined as the m -histories $\mathbf{x}_{t_i}^m \equiv (x_{t_i}, x_{t_i-1}, \dots, x_{t_i-(d-1)})$, $i = 1, \dots, k$, that represent the first k minima of the Euclidean distance function

$$\|\mathbf{x}_{t_i}^m - \mathbf{x}_T^m\|, \quad t_i = d, d + 1, \dots, T - 1 \tag{12}$$

where \mathbf{x}_T^m represents the last m -history observed, $\mathbf{x}_T^m \equiv (x_T, x_{T-1}, \dots, x_{T-(d-1)})$. Let the scalar x_{t_i+h} denote the h steps ahead observation to the m -history or neighbouring sequence $\mathbf{x}_{t_i}^m$. A simple way of obtaining the prediction of x_{T+h} consists in regressing the observations x_{t_i+h} , h steps subsequent to the nearest neighbours, on the k nearest neighbours $\mathbf{x}_{t_i}^m = (x_{t_i}, x_{t_i-1}, \dots, x_{t_i-(d-1)})$, $i = 1, \dots, k$, by OLS. By using \hat{a}_i to describe the values of a_i that minimize

$$\sum_{i=1}^k (x_{t_i+h} - a_0x_{t_i} - a_1x_{t_i-1} - \dots - a_{d-1}x_{t_i-(d-1)} - a_d)^2,$$

the predictions for x_{T+h} can be obtained from a linear autoregressive predictor with varying coefficients:

$$\hat{x}_{T+h} = \hat{a}_0x_T + \hat{a}_1x_{T-1} + \dots + \hat{a}_{d-1}x_{T-(d-1)} + \hat{a}_d. \tag{13}$$

This non-parametric regression model permits a direct prediction of the betas in the NN model for the horizons $h = \{22, 260\}$ as shown in Eq. 9.

Following Fernández-Rodríguez et al. (1999), it is also possible to generalize the local regression procedure employed in the NN non-parametric regression to the multivariate case using the information contained in several time series. This methodology is termed simultaneous nearest neighbours (henceforth) and avoids the problems that frequently occur with non-parametric multivariate methodologies, like the curse of dimensionality, which makes it necessary to use huge amounts of data to estimate the parameters of multivariate models (see Hastie et al. 2001).

In order to clarify these the ideas and simplify the notation, let us consider a set of two time series $\{x_t\}_{t=1}^T$ and $\{y_t\}_{t=1}^T$. The extension to the general case is similar. We are now interested in predicting one of the sample observations of these series (e.g. the observation x_{T+h}), also considering information for the past of both series. With this purpose, we embed each of these series in the vector space \mathbb{R}^{2d} . Thus, taking into account the vectors

$$(\mathbf{x}_{t_i}^m, \mathbf{y}_{t_i}^m) \in R^d \times R^d,$$

this gives us the available m -histories of dimension d for each time series. In order to establish SNNs for the last m -histories $(\mathbf{x}_T^m, \mathbf{y}_T^m)$, we search for the k points that minimize the function

$$\|\mathbf{x}_{t_i}^m - \mathbf{x}_T^m\| + \|\mathbf{y}_{t_i}^m - \mathbf{y}_T^m\|, \quad t_i = d, d + 1, \dots, T - 1. \tag{14}$$

In this way we obtain a set of k simultaneous m -histories in both series

$$(\mathbf{x}_{t_1}^m, \mathbf{y}_{t_1}^m), (\mathbf{x}_{t_2}^m, \mathbf{y}_{t_2}^m), \dots, (\mathbf{x}_{t_k}^m, \mathbf{y}_{t_k}^m).$$

Now, the predictions for x_{T+h} and y_{T+h} can be obtained from a linear autoregressive predictor with varying coefficients:

$$\hat{x}_{T+h} = \hat{a}_0 x_T + \hat{a}_1 x_{T-1} + \dots + \hat{a}_{d-1} x_{T-(d-1)} + \hat{a}_d \tag{15}$$

$$\hat{y}_{T+h} = \hat{b}_0 y_T + \hat{b}_1 y_{T-1} + \dots + \hat{b}_{d-1} y_{T-(d-1)} + \hat{b}_d. \tag{16}$$

The parameters in the linear model for the series x_{t_i} are estimated by regressing x_{t_i+h} on $\mathbf{x}_{t_i}^m = (x_{t_i}, x_{t_i-1}, \dots, x_{t_i-(d-1)})$, $i = 1, \dots, k$, by OLS. Therefore, the \hat{a}_i are the values of a_i that minimize

$$\sum_{i=1}^k (x_{t_i+h} - a_0 x_{t_i} - a_1 x_{t_i-1} - \dots - a_{d-1} x_{t_i-(d-1)} - a_d)^2.$$

Analogously, the \hat{b}_i are the values of b_i that minimize

$$\sum_{i=1}^k (y_{t_i+h} - b_0 y_{t_i} - b_1 y_{t_i-1} - \dots - b_{d-1} y_{t_i-(d-1)} - b_d)^2.$$

Therefore, the difference between the NN prediction \hat{x}_{T+h} of x_{T+h} given by (13) and the SNN prediction given by (15) is the way in which the nearest neighbours employed in (12) and (14) are selected to estimate the parameters.

In a practical situation, the embedding dimension (d) and the number of nearest neighbours (k) are estimated by minimizing the in-sample prediction errors, where $d \in \{2, 3, \dots, 8\}$ and $k \in \{1, 2, \dots, 20\}$ is the percentage of neighbours with respect to the sample size.

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