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Calculation of transient flow model for long-distance transportation pipeline

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Abstract Based on the elastic theory of transient flow, the motion equation and continuity equation of the transient flow are established, and the transient flow model of long-distance pipeline is derived. This paper lays a solid theoretical foundation for the further study of the small leakage of long-distance pipeline.

Keywords Long-distance pipeline · Transient flow · Model calculation

Introduction

The pipeline transients can cause tube of liquid or gas unsteady flow, which has already caused people's attention, and the motion equation and continuity equation to describe, using different algorithms get the propagation equation of the flow and pressure in the pipeline. Pipeline leakage is also a transient process, the method of transient flow analysis is applied to the leak detection, the formation of a new pipeline detection method, that is, transient flow detection method. Transient flow detection method is a hot spot in domestic and foreign research in recent years. It is a kind of method to detect the leakage of long-distance pipeline at present (Wylie and Streeter 1983). Because in transient condition, even tiny leakage in pipeline leak occurred before and after the pressure waveform will exist some differences and comparing this method can better determine the leak location.

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Transient flow model for long-distance transportation pipeline

The theory of transient flow in the pipeline is usually divided into rigid theory and elastic theory. Rigid theory is based on the assumption of incompressible fluid and the pipe wall cannot be deformed as the foundation theory, the basic equations of ordinary differential equations, easy to obtain theoretical solution, but its accuracy is limited, usually only for some simple preliminary calculation of liquid pipeline. Elastic theory is based on the assumption that the fluid can be compressed and the tube wall is elastic, the basic equations are partial differential equations, the solution is more complex, but the theory is more in line with the objective reality. Therefore, elastic theory is widely used in the study of transient flow phenomena (Zhang 1994).

Motion equation of transient flow

From the transient flow of the pipeline liquid in the selection of a control body, we apply Newton's second law to establish the equation of motion. Equations to the center line pressure $P(x,t)$ and the mean velocity $V(x,t)$ to indicate that the final transform into the liquid energy head $H(x,t)$ to indicate that the independent variable is the distance from the upstream boundary of the pipeline x and time t. The following assumptions are made before the equation is derived (Press et al. 1992): (1) the fluid in the tube is uniform and one-dimensional flow, which means that the pressure, flow rate and density of the distribution along the cross section of the pipeline are uniform. (2) The elastic deformation of the pipe wall and fluid are both linear deformation.

Figure 1 shows a fluid control body with a cross-sectional area of A thickness of δx, which is taken out of the

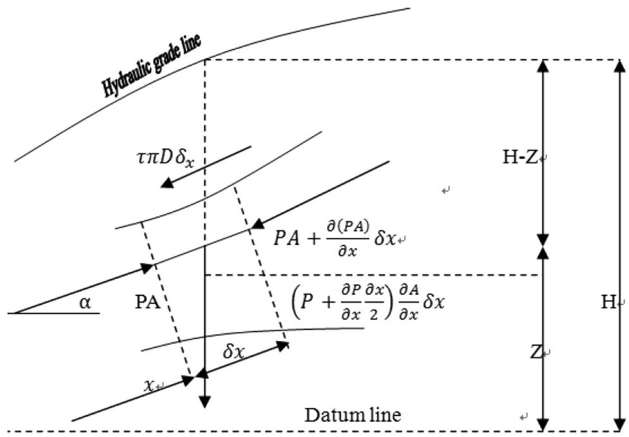

Fig. 1 Schematic diagram of control body

transient process. In general, the area of A is a function of x. The fluid flows along the x direction, and the angle between the horizontal axis and the horizontal axis is α. From the cross section at x area is A, two cross section spacing is δx, another cross-sectional area changes are $\frac{\partial A}{\partial x}\delta x$. The force acting on the x direction of the control body is: The positive pressure on the two cross section, the pressure on the annular surface and the shear stress on the side of the cylinder. In addition, the fluid self-gravity of the control body also has a component in the x direction. Acting on a small section of fluid along the x direction of the control body applied Newton's second law, the sum of the force should be equal to the product of its mass and acceleration.

The pressure acting on the two cross section is: $PA-\left[PA+\frac{\partial(PA)}{\partial x}\delta x\right]$.

The pressure acting on the annular surface is: $\left(P+\frac{\partial P}{\partial x}\frac{\partial x}{2}\right)\frac{\partial A}{\partial x}\delta x$.

Fluid gravity along the x direction for the component is: $-\rho g\left(A+\frac{\partial A}{\partial x}\frac{\delta x}{2}\right)\delta_x \sin\alpha$.

The shear stress acting on the side of the cylinder is $-\tau_0\pi D\delta x$, By using Newton's second law, we can get

$$PA-\left[PA+\frac{\partial(PA)}{\partial x}\delta x\right]+\left(P+\frac{\partial P}{\partial x}\frac{\partial x}{2}\right)\frac{\partial A}{\partial x}\delta x - \rho g\left(A+\frac{\partial A}{\partial x}\frac{\delta x}{2}\right)\delta_x\sin\alpha-\tau_0\pi D\delta x=\rho A\delta x\frac{\mathrm{d}V}{\mathrm{d}t} \tag{1}$$

Ignore higher-order infinitesimal, and control the quality of the $\rho A\delta x$ divided by the various:

$$\frac{1}{\rho}\frac{\partial P}{\partial x}+g\sin\alpha+\frac{4\tau_0}{\rho D}+\frac{\mathrm{d}V}{\mathrm{d}t}=0 \tag{2}$$

It is very troublesome to accurately calculate the viscous force of unstable flow medium. For the convenience of analysis, the coefficient of frictional resistance is assumed to be constant during the process of instability, and the value is equal to the value of stable flow. By using Darcy Bach Weiss (Darcy–Weisbach) formula:

$$\Delta p=\frac{\rho f L}{D}\frac{V^2}{2} \tag{3}$$

The f is Darcy friction coefficient, and L is the length of the pipeline. In addition, according to the force-balance condition of steady flow, we can get:

$$\Delta p\frac{\pi D^2}{4}=\tau_0\pi DL \tag{4}$$

ΔP will be deleted by formulas (3) and (4).

$$\tau_0=\frac{\rho f V|V|}{8} \tag{5}$$

The absolute value of the velocity terms in the formula can ensure that the tangential stress direction is always opposite to the velocity direction. Handle (5) into Eq. (2), we can get:

$$\frac{\mathrm{d}V}{\mathrm{d}t}+\frac{1}{\rho}\frac{\partial p}{\partial x}+g\sin\alpha+\frac{f}{2D}V|V|=0 \tag{6}$$

The above formula is the general form of the equation of motion, which is applicable to the compressible fluid and the incompressible fluid, the horizontal tube and the inclined tube. The liquid energy head H can be replaced by P, which can be obtained from Fig. 1:

$$P=\rho g(H-z) \tag{7}$$

z is the distance from the x center to the reference line. x do not change over time, and $\mathrm{d}z/\mathrm{d}x=\sin\alpha$. So for liquids:

$$\frac{1}{\rho}\frac{\partial p}{\partial x}=g\left(\frac{\partial H}{\partial x}-\frac{\mathrm{d}z}{\mathrm{d}x}\right)=g\frac{\partial H}{\partial x}-g\sin\alpha \tag{8}$$

The formula (8) into the formula (6):

$$g\frac{\partial H}{\partial x}+V\frac{\partial V}{\partial x}+\frac{\partial V}{\partial t}+\frac{f}{2D}V|V|=0 \tag{9}$$

Continuous equation of transient flow

As shown in Fig. 2, along the inner wall of the tube to take a section of the control body, in the time t its length is δx. According to the law of conservation of mass, the quality of the body is controlled by the rate of change of body mass with time.

According to the principle of continuity (Wylie and Streeter 1993):

$$-\frac{\partial(PAV)}{\partial x}\delta x=\frac{\partial}{\partial t}(\rho A\delta x) \tag{10}$$

In the equations, the quality of the control body is divided by the $\rho A\delta x$:

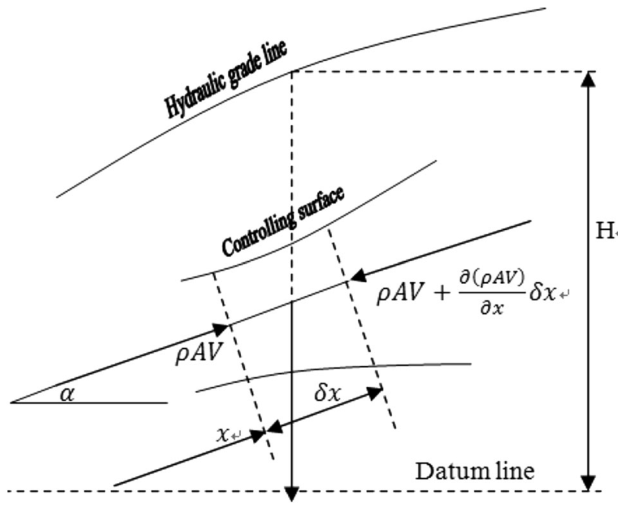

Fig. 2 Schematic diagram of control volume of continuous equation

$$\frac{V}{A}\frac{\partial A}{\partial x}+\frac{1}{A}\frac{\partial A}{\partial t}+\frac{V}{\rho}\frac{\partial \rho}{\partial x}+\frac{1}{\rho}\frac{\partial \rho}{\partial t}+\frac{\partial V}{\partial x}=0 \tag{11}$$

During the expansion process, the control body is fixed on the wall of the pipe, and the δx has nothing to do with the practice, for the fluid particles, using the full derivative $(\mathrm{d}())/\mathrm{d}t = V\cdot(\partial())/\partial x + (\partial())/\partial t$, the formula (11) is changed to:

$$\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t}+\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}+\frac{\partial V}{\partial x}=0 \tag{12}$$

Because there is no assumption in the derivation of the formula, whether it is for the expansion tube or contraction tube, this equation is established.

According to the definition of liquid volumetric elastic coefficient K:

$$K=\frac{\Delta P}{\Delta\rho/\rho} \tag{13}$$

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}=\frac{1}{K}\frac{\mathrm{d}P}{\mathrm{d}t} \tag{14}$$

Take the formula (14) into the formula (12), we can get:

$$\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t}+\frac{1}{K}\frac{\mathrm{d}P}{\mathrm{d}t}+\frac{\partial V}{\partial x}=0 \tag{15}$$

The first term in the formula indicates the elasticity of the wall, that is, the rate of deformation of the tube wall with the pressure and the change rate of the transverse section of the pipe along with the axial direction. The second item indicates the compressibility of the liquid. The cross-sectional area of pipeline, if only considered the area change caused by stress, formula (14) can be written as:

$$\frac{\mathrm{d}P}{\mathrm{d}t}\left(\frac{K}{A}\frac{\mathrm{d}A}{\mathrm{d}P}+1\right)+K\frac{\partial V}{\partial x}=0 \tag{16}$$

The propagation velocity of the pressure wave in the transient flow of the pipeline is determined by the following formula a:

$$a^2=\frac{K/\rho}{1+(K/A)(\Delta A/\Delta P)} \tag{17}$$

Liquid density ρ divided by formula (16), bring into formula (17):

$$\frac{\mathrm{d}P}{\mathrm{d}t}+\rho a^2\frac{\partial V}{\partial x}=0 \tag{18}$$

This equation is only suitable for uniform tube incompressible fluid. According to the formula (7):

$$\frac{\mathrm{d}P}{\mathrm{d}t}=\rho g\frac{\mathrm{d}H}{\mathrm{d}t} \tag{19}$$

Bring formula (19) into the formula (18):

$$V\frac{\partial H}{\partial x}+\frac{\partial H}{\partial t}+\frac{a^2}{g}\frac{\partial V}{\partial x}=0 \tag{20}$$

Now we conclude that the pipeline transient flow of two basic differential Eqs. (9) and (20), but the two equations apply only to uniform tube of incompressible fluid.

Transient flow model for long-distance transportation pipeline

Figure 3 schematic diagram of a section of the control body which is taken when the pipeline is leaking (Press et al. 1992). Assuming that the pipeline is in a horizontal state, leakage occurs at $x = x_{\mathrm{L}}$, the leakage rate is Q_{L}. According to the law of conservation of mass, the formula (10) is modified:

$$-\frac{\partial(PV)}{\partial x}\Delta x-\rho Q_{\mathrm{L}}=\frac{\partial}{\partial t}(\rho A)\Delta x \tag{21}$$

On both sides of the equation is divided by Δx, and tends to be 0:

$$\frac{\partial}{\partial t}(\rho A)+\frac{\partial(PVA)}{\partial x}=-\rho Q_{\mathrm{L}}\delta(x-x_{\mathrm{L}}) \tag{22}$$

The definition of the function $\delta(x - x_{\mathrm{L}})$ in the formula is as follows:

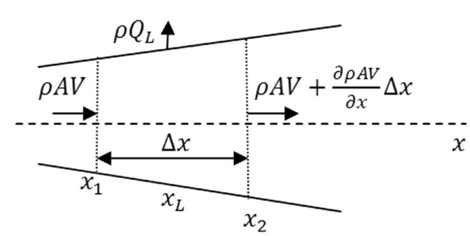

Fig. 3 Schematic diagram of the control body when the pipeline leakage

$$\delta(x - x_{\mathrm{L}}) = \begin{cases} \infty & x = x_{\mathrm{L}} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

$$\lim_{\varepsilon \to 0} \int_{x_L-\varepsilon}^{x_L+\varepsilon} \delta(x - x_{\mathrm{L}}) \mathrm{d}x = 1 \tag{24}$$

In the Formula ε represents a small distance away from the leak. Taking into account the compressibility of the fluid and the elasticity of the pipe wall, the formula (22) can be expressed in a similar form (20):

$$\frac{\partial H}{\partial t} + \frac{Q}{A}\frac{\partial H}{\partial x} + \frac{a^2}{gA}\frac{\partial Q}{\partial x} + \frac{a^2}{gA} Q_{\mathrm{L}} \delta(x - x_{\mathrm{L}}) = 0 \tag{25}$$

In the formula, H is the pressure head; Q is the pipeline flow; a is the pressure wave velocity in the fluid; g is the gravity acceleration. When the pipeline leak, according to Newton's second law can get a similar type (9):

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{Q}{gA^2}\frac{\partial Q}{\partial x} + \frac{fQ^2}{2DgA^2} - \frac{QQ_L \delta(x - x_{\mathrm{L}})}{gA^2} = 0 \tag{26}$$

In the formula, f is the Darcy friction coefficient; D is the diameter of the pipe. The formula (26) the last one is caused by the discontinuity of the leaking point of the mass flow. All these equations assume that the frictional coefficient of the pipeline is constant.

For pipeline leak quantity Q_{L} description can be used through the valve orifice equation to represent, and pressure pipe, vent holes size.

$$Q_{\mathrm{L}} = C_d A_{\mathrm{L}} \sqrt{2g\Delta H_{\mathrm{L}}} \tag{27}$$

In the formula $\Delta H_{\mathrm{L}} = H_{\mathrm{L}} - Z_{\mathrm{L}}$ for the leak point pressure difference; H_{L} as head of the leak point pressure; Z_{L} for leak point elevation; C_d is leakage flow coefficient; A_{L} leak area. In order to facilitate the calculation, non-dimensional treatment of the above equations is carried out.

$$H^* = \frac{H}{H_1}, \quad t^* = \frac{t}{L/a}, \quad x^* = \frac{x}{L}, \quad Q^* = \frac{Q}{Q_0}, \quad \delta(x_{\mathrm{L}}^*) = \delta(x - x_{\mathrm{L}})L \tag{28}$$

In the formula, H_1 is the head of the first end of the pipeline; L is the length of the pipeline; Q_0 is the steady flow of the pipeline. The formulas (27) and (28) were substituted into the formulas (25) and (26):

$$\frac{\partial H^*}{\partial t} + \frac{V_0}{t}\frac{Q^* \partial H^*}{\partial x^*} + \frac{aQ_0}{gAH_1}\frac{\partial Q^*}{\partial x^*} + \frac{aQ_0}{gAH_1}\frac{C_d A_L}{Q_0}\sqrt{2gH_1}\sqrt{\Delta H_{\mathrm{L}}^*}\,\delta(x^* - x_{\mathrm{L}}^*) = 0 \tag{29}$$

$$\frac{gAH_1}{Q_0 a}\frac{\partial H^*}{\partial x^*} + \frac{\partial Q^*}{\partial t^*} + \frac{V_0}{a} Q^* \frac{\partial Q^*}{\partial x^*} + \frac{fLQ_0}{2DAa}(Q^*)^2 - \frac{V_0}{a}\frac{C_d A_L \sqrt{2gH_1}}{Q_0} Q^* \sqrt{\Delta H_{\mathrm{L}}^*}\,\delta(x^* - x_{\mathrm{L}}^*) = 0 \tag{30}$$

Because $\frac{V_0}{a}$ is much less than 1, it can be ignored, so the formulas (29) and (30) can be simplified:

$$\frac{\partial H^*}{\partial t} + \frac{1}{F}\frac{\partial Q^*}{\partial x^*} + M\sqrt{\Delta H_L^*}\,\delta(x^* - x_{\mathrm{L}}^*) = 0 \tag{31}$$

$$F\frac{\partial H^*}{\partial x^*} + \frac{\partial Q^*}{\partial t^*} + R(Q^*)^2 = 0. \tag{32}$$

In the formula: $R = \frac{fLQ_0}{2aDA}$, $M = \frac{C_d A_{\mathrm{L}}}{A}\frac{2a}{\sqrt{2gH_1}}$, $F = \frac{H_1}{aV_0/g}$.

At this point, we have obtained the transient flow model of long-distance transmission pipeline, which can be very convenient to use R, M and F these dimensionless parameters to describe the leakage of the pipeline.

Conclusions

Based on the theoretical study of transient flow in long-distance pipeline, the transient flow model of long pipeline leakage is obtained, which lays a theoretical foundation for further research on the micro-leakage of long-distance pipeline. On the basis of this paper, other dynamic detection signals are introduced. The new method of pipeline leak detection is proposed, which can increase the accuracy of leakage detection and location.

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