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EDITORIAL

Subject Matter Didactics in Mathematics Education

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1 Introduction

What is the role of mathematics within mathematics education research? This question is at the heart of mathematics education as a science. Every individual researcher within the mathematics education community and even the community as a whole has to take a stance regarding this question. Forty years ago Kirsch presented his particular German view regarding the role of mathematics within mathematics education research at the ICME 3 in Karlsruhe in his paradigmatic plenary lecture “Aspects of simplification in mathematics teaching” (Kirsch 1977; Kirsch and Scherk 2000). On the occasion of IMCE 13 we pick up the threads since the theme has not lost any of its relevance – especially, since recent publications in the field might evoke the impression that mathematics disappears in mathematics education research (Jahnke 2010). The aim of this special issue is to contribute to the discussion about the role of mathematics within mathematics education research.

Stoffdidaktik has been a dominant approach to mathematics education research within the German speaking countries, which puts the analysis of the mathematical subject matter at its heart. It has been the prominent approach to research until the 1980s. Nowadays it still influences research in mathematics education in German speaking countries. Due to its focus on the subject matter it puts an emphasis on

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a core of the discipline. Thus, a discussion of *Stoffdidaktik* is not just a retrospect of a historical strand of research in mathematics education in German speaking countries, which might be interesting from a cultural and historical point of view. Our hope is that the discussion of *Stoffdidaktik* and its recent developments contributes to the discussion about the identity of the discipline (Sierpinska and Kilpatrick 1998). In fact, mathematics is the unifying element in mathematics education research beyond the diversity of theories in the field. Thus, a minor role of mathematics within mathematics education research will result in a loss of identity. Therefore, it is important to reflect upon the role of mathematics within mathematics education research. *Stoffdidaktik* has taken a particular stance regarding this issue. The idea of a discussion of *Stoffdidaktik* is not to adopt this point of view, but to learn from it in terms of its affordances and its constraints in order to develop a contemporary and reflected answer to the question of the role of mathematics within mathematics education research.

2 Attempt of a Definition

Generally, *Stoffdidaktik* is an approach to mathematics education and research on teaching and learning mathematics (i. e. didactics of mathematics) in German speaking countries, which focuses on the mathematical contents to be taught aligning with disciplinary mathematics. A major aim is to make mathematics accessible and understandable to the learner based on an analysis of the subject matter with mathematical means. Kirsch (1969) may be regarded as a prototypical analysis of this kind. Hefendehl-Hebeker (in this issue) gives an overview about the historical development and the main ideas of *Stoffdidaktik*.

The very term '*Stoffdidaktik*' – originally used as an expletive (c.f. Hefendehl-Hebeker, this issue) – combines the two nouns 'Stoff' and 'Didaktik'. 'Stoff' denotes the mathematical topic, the mathematical subject matter or content. The composite designates didactics, which takes the mathematical topic as the starting point and the object of the analysis as opposed to didactics, which centres on the negotiable character of mathematical knowledge in mathematics classrooms. Different attempts have been made to translate the German term '*Stoffdidaktik*' into English: Sträßer uses "subject matter didactics" (Sträßer 2013), Steinbring (2011, p. 44) suggests "content based didactics" or "didactically oriented content analysis" (p. 45).

In a narrow sense, *Stoffdidaktik* is restricted to a scientific elaboration of the mathematical content by mathematical means. However, it is not clarified by the representatives of *Stoffdidaktik* how mathematical methods are selected and applied in order to analyse the subject matter. Furthermore, there is some ambiguity about the role of the learner: On the one hand the learner is only taken into account as a passive receiver of the clearly expounded subject matter (Steinbring 2011) while on the other hand the analysis of the subject matter serves the goal of making mathematics accessible to learners and thus has to take learners into account in order to define the goal of the analysis. However, this seems to be done from a normative point of view in the sense that it is taken for granted that a clearly and coherent expounded subject matter from the researcher's point of view will also be clear and comprehensible to the learner. As soon as the psychology of the learner or psychological or discursive aspects of learning are taken into account the traditional notion of *Stoffdidaktik* is already widened.

The fundamental problem of writing about *Stoffdidaktik* is that an important part of its prominent representatives have not entered into a scientific discourse on characterising this approach to research in mathematics education. Consequently, there are a lot of examples of 'stoffdidactical' analyses, but comparable few meta-level elaborations on the main ideas and principles of *Stoffdidaktik* (for rare examples see Griesel 1974 or Kirsch 1977). Due to this tendency it appears that the editors of this special issue are no prominent representatives of *Stoffdidaktik* themselves.

3 Characteristics of *Stoffdidaktik*

From the perspective of traditional *Stoffdidaktik*, mathematics is understood as a uniform, homogeneous and universal body of knowledge. *Stoffdidaktik* "specifically proceeds to prepare the pre-given mathematical disciplinary knowledge for instruction as a mathematical content, to elementarise it and to arrange it methodically" (Steinbring 2011, p. 45). As one influential protagonist of subject matter didactics, Griesel has identified the following features of 'didactically oriented content analysis' – as he prefers to name the approach: "The research methods of this area are identical to those of mathematics, so that outsiders have sometimes gained the impression that, here, mathematics (particularly elementary mathematics) and not mathematics education is being conducted" (Griesel 1974, p. 118; translation from Steinbring 2011, p. 45). In terms of research methodology, this is a very clear and somehow very restricted preference, which – at least in terms of research methods – makes it difficult to distinguish *Stoffdidaktik* from mathematics. The main difference seems to be the goal of the application of mathematical methods. Whereas in mathematics the major goal is to develop and verify mathematical knowledge, the goal of subject matter analysis from a didactical perspective is to make (verified) mathematics accessible for learners (Kirsch 1977). According to Kirsch "making accessible" is achieved by different means:

1. making accessible by concentration on the mathematical heart of the matter,
2. making accessible by including 'surroundings' of mathematics,
3. making accessible by recognizing and activating pre-existing knowledge,
4. making accessible by changing the mode of representation (Kirsch 1977).

Nevertheless, the mathematical content is what is "made accessible" and thus remains the starting point and the heart of didactical analyses. The very term "surroundings" already indicates what is focal and and what is peripheral. According to Steinbring (2011), this view has consequences for the epistemological status of the content itself, the vision of the learner and of teaching and learning respectively:

"[T]here clearly is the conception that ultimately a uniform, objective and unchangeable content of teaching and learning is to be elaborated in didactics according to the paradigm of scientific mathematics. The teaching, learning and understanding processes of the participating persons (teacher and students) are orientated around the rigid subject matter structures: the teacher is the 'conveyor' of the didactically prepared content to the students who are regarded as passive receivers. The relations between the three elements of the didactical triangle are of an essentially linear nature: the mathematical knowledge arrives by means of the preparation and transfers from the teacher to the students. In the research paradigm of *Stoffdidaktik*, the scientific elaboration of mathematical knowledge is the central and crucial means practiced for steering and optimizing mathematical instruction, learning and understanding processes" (Steinbring 2011, p. 46).

Furthermore: "The goal of 'didactically oriented content analysis' is to give a better foundation for the formulation of content-related learning goals and for the development, definition and use of a differentiated methodical set of instruments" (Griesel 1974, p. 118; translation from Steinbring 2011, p. 45). The practice of 'didactically oriented content analysis' up to the 1960s suggests that implicitly *Stoffdidaktik* starts from the assumption that after a decent mathematical analysis, one will find one and only one best way to teach a certain content matter, which then should be incorporated into mathematics textbooks (for a critical description of this feature of *Stoffdidaktik* see Jahnke 1998, p. 68).

In the preface of a book series, which Griesel himself identified as a prototypical example of *Stoffdidaktik*, Griesel (1971, p. 7) named six areas, which are important for the progress of didactics of mathematics. The first two are of utmost importance, especially the first one: research into the content, the methods and the applications of mathematics; and didactical ideas and insights, "which make it possible to attend better, or at all, to a subject area within instruction". For him, the first area was most successful at that time. The other four influential factors are: general experience, statistically based evidence about instruction, insights into the mathematical learning process and the developmental-psychological and sociological conditions (translations from Steinbring 2011, p. 45). With these statements, Griesel identified some limitations of 'didactically oriented content analysis' using mathematical methods. He even went as far as calling content analysis meaningless if it is not complemented by empirical investigations, which show that the results of *Stoffdidaktik* are meaningful for the actual teaching.

4 Developing *Stoffdidaktik*

The understanding of *Stoffdidaktik* was controversially discussed by proponents (e. g. Holland 1974; Kirsch 1977). Not all of them pursued the target bringing all relevant concepts, theorems, and procedures into an adequate linear structure. For instance, Holland (1974) presented a deductively organized theory of mathematical concepts, but he emphasized that the deductive structure alone cannot be the fundamentum for the learning trajectory and Vollrath (1979) asked for flexible connections between the concepts and theorems as a base of a didactical analysis.

An extension of this kernel of *Stoffdidaktik* regarding especially the specific conditions of the learner is not only transferring the complex formal system into a simple one, but also strives for giving meaning to the concepts, theorems and procedures reaching beyond the purely mathematical meaning of the objects in order to make them accessible and understandable to the learner. Meaning could be provided in different ways, e. g. genetic structuring of learning processes (Wagenschein 1968; Freudenthal 1991; Winter 1991), the use of applications (Blum 1985), 'big ideas' (in German 'fundamentale Ideen', Bruner 1960; Schweiger 2006) and 'basic ideas' or 'basic mental models' (in German 'Grundvorstellungen', vom Hofe 1995).

All approaches have in common that they should give stable and essential ideas or intuitions of the mathematical objects. They concern the global and the local significance of the mathematical topic in relation to learning processes. Global ideas are understood in the sense that specific mathematical objects are linked with each other in order to be significant for the learner, whereas local ideas refer to the meaning of the certain concept, theorem, and procedure itself.

Already in the 1960s, Bruner drew red lines in a spiral curriculum for each topic and tried to identify the core of these topics. For this he introduced the notion of 'big ideas'. Since then a lot of different lists of big ideas have been proposed (for the German mathematics education community see e. g. Schweiger 2006; Schwill 1993; Tietze et al. 1997). However, a proof for the actual relevance for learning is still missing (Vohns 2010). In this special issue Vohns gives an overview of important stages in the specific development of the understanding of big ideas. Nevertheless, these ideas need to be specified regarding the particular concept and its relevance, for instance the idea of 'functional relationship' will be specified for exponential functions by using their proportional relationship in specific application situations like compound interest (Hußmann and Prediger in this issue).

Besides the emphasis on the global significance of topics for the learner, local meanings of topics are of particular importance for the learner to establish an access to the mathematical concept. The significance of the integration of local meanings into the didactical analysis has been emphasized in the German mathematics education tradition (Lietzmann 1916; Oehl 1962, 1970; Kühnel 1919). Local meanings are mostly concretized by the concept 'basic mental model' (in German 'Grundvorstellungen', Bender 1991; vom Hofe 1995 for a synopsis and further elaboration), and mediated by suitable representations (verbal, graphical, numerical or symbolic). A basic mental model is understood as the normative interpretation of a mathematical object, for instance the two models of subtraction 'taking away' and 'determining the difference' (van den Heuvel-Panhuizen and Treffers 2009). In this issue vom Hofe and Blum elaborate on the concept of basic mental models, particularly concerning its normative, descriptive, and constructive use. Vohns (in this issue) discusses relations between basic mental models and big ideas.

Hence, both aspects, the global sense and the local meaning, are crucial for a modern understanding of a didactical analysis of the mathematical topic. They are as important as the mathematical method of traditional *Stoffdidaktik*. Both facets of the didactical analysis of the content are purely normative, the individual perspective of the learner is still not included. Application-, problem- or context-orientation and genetic processes push the mathematical topic into the real world of learners. The subject-orientation, which came to the fore since the 1990s (e. g. Gallin and Ruf

1990 or the discussion on PCK as a competence for the teachers[1]) emphasize other aspects:

- experiences of the students and pre-existing knowledge,
- the rules of the real world, which determine the specific individual view on mathematics,
- individual and typical (mis-)conceptions,
- the real world language and the language of the students, which is used to explain the understanding of the mathematical objects and relations between them,
- the cultural and social context.

Empirical studies are required to gain insights into the field of individual thinking and learning, e. g. which typical misconceptions exist or how individuals with specific cultural background learn the concept of fractions. These empirical studies do not only produce findings about the learner, but also about the mathematical topics and theoretical approaches. The contributions of Hussmann and Prediger; Weber; Greefrath et al.; Witzke; Leuders in this issue give a variety of examples of subject matter content analyses with didactical orientation, which have included the learners' perspective in one or the other way and thus widened the traditional understanding of *Stoffdidaktik*. Nevertheless, they elicit the important role of the mathematical subject matter content in mathematics education research.

If one looks back on the history of *Stoffdidaktik*, this approach turns out to be a specific perspective on the object of research in mathematics education (or didactics of mathematics). By focussing on (disciplinary) mathematics, *Stoffdidaktik* offers a specific potential to take disciplinary mathematics into account, but obviously has certain shortcomings if understood in a narrow sense. A revised and modern understanding of *Stoffdidaktik* has to contain not only formal and epistemological aspects of the mathematical topic, but also semantic aspects as well as psychological, social and cultural aspects of learning processes. All aspects serve the understanding and preparation of the mathematical object.

5 An International Perspective

From an international perspective, the approach closest to *Stoffdidaktik* is the French approach of *ingénierie didactique* – didactical engineering – especially its 'a-priori analysis'. As a scientific approach, *ingénierie didactique* appeared in the 1980s (Artigue 1988). *Stoffdidaktik* shares with didactical engineering the focus on disciplinary mathematics, its history and its epistemology. Especially for the 'a-priori

[1] The notion of 'pedagogical content knowledge' (PCK), which was introduced into the debate on the professional knowledge of teachers by Shulman (1987), is also close to Stoffdidaktik. With PCK as "understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman 1987, p. 8), PCK shares a close link to subject matter knowledge with Stoffdidaktik. In contrast, Stoffdidaktik tends to be more authoritarian, looking for the best, one and only mathematical solution, but cares less for the personal aspects of the teaching and learning process – with Shulman's concept of "content knowledge" confirming the importance of disciplinary mathematics for the teaching and learning of the subject.

analysis', didactical engineering as well as *Stoffdidaktik* in its entirety heavily depend on a detailed analysis of the content. If taken as a preparation to a teaching experiment, the a-priori analysis of didactical engineering tends to enact the very same activities and methods as *Stoffdidaktik* would apply. A difference between these two approaches appears when the actual practice is taken into consideration: From the very beginning, didactical engineering is also interested in teachers and learners of the subject matter content under consideration, their pre-knowledge before teaching (experiments) and the consequences afterwards. Traditional *Stoffdidaktik* was neither interested in the human side of the teaching-learning process nor did it traditionally look into the consequences of a certain setup of the teaching-learning process (for a detailed comparison see Sträßer 1996). The reason for more or less neglecting these aspects may be the idea of the one and only best way to teach a certain subject matter content, which allows to forget about alternatives. Coming from the French tradition Colette Laborde offers "A view on subject matter didactics from the left side of the Rhine", widening the perspective of the German approach to subject matter orientation of didactics of mathematics.

6 Overview of the Special Issue

The special issue starts with a short historical sketch by *Lisa Hefendehl-Hebeker*, who gives an account of the development of "Subject-matter didactics in German traditions". While basically following the historical analysis of Schubring and mainly confirming the origin from Gymnasium teachers activities during the first half of the 20th century, she also identifies the influence of teacher trainers from the teacher training pedagogical academies as a major source of *Stoffdidaktik*.

The texts by *Stephan Hußmann and Susanne Prediger, Christof Weber, Gilbert Greefrath et al., Ingo Witzke and Susanne Spies*, and *Timo Leuders* give a variety of examples of subject matter analysis from a didactical perspective. Not surprisingly, they all relate to subject matter normally taught in secondary mathematics classrooms or "above" (with *Leuders* about teacher training) and elaborate on influences coming from outside mathematics as a scientific discipline. From the history of *Stoffdidaktik*, it does not come as a surprise that two texts treat subject matter from upper secondary mathematics teaching (see *Greefrath et al.* and *Witzke and Spies*). One of the two texts on subject matter from lower secondary mathematics (*Weber* on logarithms) can be read as a more or less classical example of *Stoffdidaktik*, while the text by *Hußmann and Prediger* takes exponential growth as a prototypical subject matter to illustrate the incorporation of classical *Stoffdidaktik* into a wider scope of research related to the design area of didactics of mathematics as a research domain. The text by *Rudolf vom Hofe* details the integration of the concept "Grundvorstellungen" as a category of subject matter didactics, while *Andreas Vohns* shows how big ideas can be a guiding category in subject matter oriented didactics of mathematics. *Colette Laborde* offers "A view on subject matter didactics from the left side of the Rhine", widening the perspective of the German approach to subject matter orientation of didactics of mathematics.

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