



Why are Roman-period dice asymmetrical? An experimental and quantitative approach

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Abstract

Roman-period six-sided dice are common in archaeological sites across Europe. While some dice approach true cubes, many are visibly non-cubic (i.e., asymmetric/lopsided) and favor certain rolls, especially the numbers 1 and 6. It is unclear if such dice were intentional and distinctive “types” used in specific games or activities, represent “cheaters” dice, or are simply part of a continuum of variation in die shape and configuration. To explore this issue, we examine shape distribution of 28 well-dated Roman-period dice from modern-day Netherlands. Results show that Roman die asymmetry varies in a continuous fashion from true cube to highly parallelepiped, where the long side is over 50% longer than the short side. We then conduct replication experiments to examine how naïve producers configure pips across a range of shapes. Our results show a production bias, where makers place the 6 on the largest die face, not to favor certain rolls, but due to space limitations and/or the order in which they place the pips. Overall, we interpret asymmetrical Roman dice as part of a single but highly variable artifact category, not distinct types. We argue that such extreme variation was acceptable because makers and users understood roll outcomes as the product of fate, rather than chance or probability. Conformity to a true symmetrical cube was not perceived as essential to die function, and asymmetrical forms were tolerated as simply part of the acceptable range in shape variation.

Keywords Experimental archaeology · Production bias · Cubic dice · Netherlands · Roman period

Introduction

Human visual perception can distinguish difference in size when two objects vary in magnitude by approximately 5% or more (Eerkens 2000; Norwich 1983). These limits are relatively stable in all humans, and though training and experience can slightly increase sensitivity, ultimately visual perception is limited by neurobiological properties of the human body. Below this threshold, it is difficult for humans to reliably state whether one object or dimension is longer than another, while above, the eye/brain can more easily distinguish a difference in size. Importantly, it is not the

absolute difference in size that is key in this regard, but relative difference (Coren et al. 1994).

Roman-period six-sided dice are notable for the high rates of visually non-cubic, or asymmetrical, forms (Duggan 2015:37; Eerkens and de Voogt 2017; Feugère and Picod 2014; Greep 1983; Kovač 2011; Kruger 1982; Poplin 2004; Schmid 1978:58; Swift 2017a, 2017b). These objects stand out to us today because in western culture six-sided dice are highly symmetrical, and because we expect the die to roll “fairly,” wherein each side has equal probability of being rolled. Three main explanations have been offered to explain the presence of highly asymmetrical forms in Roman times. Each of these explanations has theoretical implications since they make assumptions about how broader Roman culture and worldview is reflected in material objects (e.g., Hodder 2012; Knox 2016), and how people learn and transmit information (Coto-Sarmiento et al. 2018; Harush et al. 2020; Roux 2015; Varma and Menon 2017).

The first explanation assumes that die asymmetry was intentional and had function in Roman society. One interpretation is that dice were purposefully shaped to be

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asymmetrical such that they favored particular throws that had importance in divinatory practices or gaming. In some sense, such dice could be considered “cheaters” dice that helped the owner/user to *better* predict the throws, though all throws were still possible. For instance, Swift (2017a: Appendix 5, 2017b:163–164) showed that asymmetrical dice are not equal in the probability of different rolls. She suggested asymmetry may have made game play more exciting, or may have been exploited during gambling by individuals with greater experience using dice. In this regard, it is possible that asymmetrical dice are a particular “type” of die, distinct in their function from cubic forms (i.e., used for particular games or divination).

A second explanation suggests that asymmetry was still intentional, but that cultural transmission factors mainly account for the presence of asymmetrical forms. For instance, some suggest that highly elongated parallelepiped or lozenge-shaped dice from pre-Roman Iron Age cultures may have influenced Roman-period die making in Southern Britain (Feugère and Picod 2014:38; Swift 2017b:161), and pre-Roman asymmetrical dice have also been attested for Etruria (Schädler 2019). In this respect, asymmetrical dice were consciously part of the Roman cultural repertoire, but functioned to elicit a sense of nostalgia or cultural affiliation with more ancient and non-Roman culture(s).

The third explanation suggests that production of Roman die asymmetry was largely unintentional. Some see asymmetrical dice as unintentional byproducts of the types of raw materials used to make dice (Greep 1983:247; Poplin 2004:62; Swift 2017a:210). Animal long bone, in particular, was a popular medium for dice, and could have imposed constraints on die size and shape (Swift 2017a:203). While plausible, this argument does not explain why the longer sides were not shaved/ground down to match the shorter sides, nor why dice fashioned from other materials such as clay and metal, with no obvious constraints on shape, are also often asymmetrical. Alternatively, some interpret asymmetrical dice as the outcome of unskilled die-makers who could not, or did not care to, carefully control the morphology of the object (Schmid 1978). Both interpretations suggest that users did not see conformity to a cubic shape as instrumental to die function.

In all cases, there is an assumption about the thoughts and ideas, or worldviews, of the makers and users of dice. For example, in the first, it is that some people actively manipulated technology to gain an advantage in game play or divination, presumably at the expense of people who lacked experience and knowledge of dice rolling, while in the third the lack of shape conformity suggests people were largely unaware of rolling probabilities for non-cubic forms. Whatever the explanation, it is unlikely they are idiosyncratic to particular people or places. As we discuss below the patterns in asymmetry exist over centuries of time, including in

preceding Iron Age periods, and are found within and across the borders of the multi-cultural Roman Empire.

Because humans share culture and worldviews, and use similar raw materials and techniques to make artifacts, examining large and diverse assemblages of artifacts provides greater insight into these processes at a societal (rather than individual) level. By looking towards variation in artifacts across different dimensions we can gain insight into which components of material culture were, apparently, free to vary, and which were tightly constrained. How an artifact was intended to function will affect different design components and variation therein. When a particular attribute is critical to the functioning of an artifact, producers will act to minimize variation along this dimension, winnowing out forms that are non- or sub-optimal in their performance. When they are less crucial to function, they are free to vary. In this respect, analysis of variation in artifact shape and size provides important insight into a range of issues, including artifact function, specialization, and production processes (Eerkens and Bettinger 2001, 2008; Lycett 2015; O’Brien and Bentley 2011; Schiffer et al. 2001; Stevens and McElreath 2015). In the case of dice, it is necessary to determine whether asymmetry is a property that varies freely or constitutes a distinctive type of die, constrained in its own way. While several previous studies have noted the presence of asymmetrical dice, and occasionally speculated on their meaning, we aim to test some of these assumptions through systematic experimental and quantitative analyses.

Dice as a source for understanding material culture practices

Relative to other artifact categories, gaming pieces, and especially dice, play a minor role in our understandings of ancient societies. Dice are not uncommon in sites, but are rarely studied as insight into more general technological or cultural practices, other than to tell us that people played games (e.g., Austin 1934) or used them for other purposes, such as divination (e.g., Graf 2005; Klingshirn 2005). However, dice have shown to be of particular interest because many of the economic forces that guide evolutionary processes in other material technologies, such as desires to increase efficiency (e.g., more aerodynamic projectiles, faster computer chips) or to increase strength/resilience (e.g., metals used in weaponry, clays used in ceramics), are unlikely to apply to dice.

Nevertheless, previous studies show that die design and form are not static, but vary over space and cultural groups (see Artioli et al. 2011 for Etruscan dice; de Voogt and Eerkens 2018 for Tibetan dice) and time (Eerkens and de Voogt 2017 in Northwest Europe). Within this variation, several scholars have noted the presence of clearly non-cubic,

parallelepiped forms of six-sided dice, especially in Roman-period archaeological contexts (Duggan 2015:37; Eerkens and de Voogt 2017; Feugère and Picod 2014; Greep 1983; Kovač 2011; Kruger 1982; Poplin 2004; Schmid 1978:58; Swift 2017a, 2017b). Detailed measurements show that 80–90% of Roman dice are visibly asymmetrical (Schmid 1978:58; Swift 2017b:159), compared to less than 50% of later Medieval and post-Medieval dice (Eerkens and de Voogt 2017). While asymmetry is to be expected in handmade objects dating to antiquity, such high rates suggest it is not simply due to chance or random manufacturing errors, especially when post-Roman handmade dice are much more symmetrical.

Our study addresses three main issues regarding asymmetrical dice. First, we summarize how asymmetry affects die function, and more specifically, the probability of rolling different numbers, by consulting the applied mathematical literature. We also tabulate empirical data from a rolling study with actual Roman-period dice reported by Swift (2017a: Appendix 5). Second, we examine whether asymmetrical forms comprise a distinctive “type” of die, or are part of a continuum of die shapes from cubic to rectangular parallelepipeds. We collected data from six-sided Roman dice found at a range of sites in the modern-day Netherlands, both within and across the northern border of the Roman Empire. Third, we examine whether the numbers, or pips, on asymmetrical dice were placed intentionally to favor certain rolls. By way of experiment we determine to what extent this placement is guided by production bias, a type of bias that is found in several other aspects of dice production (de Voogt et al. 2015). We then compare our set of dice from the Netherlands to examine how pips were placed on the dice with respect to shape. Finally, we discuss these three results to re-interpret what such objects tell us collectively about Roman-period societies on the northern fringe of the empire and about our understanding of the presence of asymmetrical dice in general.

Asymmetry and roll frequency

A number of mathematical models and experiments with non-cubic dice demonstrate that asymmetrical dice favor numbers that are placed on sides with a greater surface area (Bronson and Bronson 1990; Singmaster 1981). This result is not surprising, of course, and can be used by unscrupulous game players to alter the odds of rolling different pips on a die (Heilbronner 1984; Levin 1983). Based on physical properties of objects, Mungan and Lipscombe (2013) developed a mathematical formula to predict the probability of a side being rolled for dice that are elongated or shortened along *one* axis. That formula is given below:

$$p = \frac{2}{2 + \left(\frac{x}{y}\right)^n} \left[1 - \frac{2}{\pi} \tan^{-1} \left(\frac{x}{y} \right)^n \right]$$

where p represents the probability of one of the two numbers associated with the largest side rolling up, x/y represents the ratio of the shortened/lengthened axis to the other two equal axes, and n represents an empirically derived constant that can vary with a number of factors, including the speed at which a die is cast, the hardness of the die relative to the surface on which it is cast, and beveling of the edges of the die, among others. They suggest a value of $n = 3 \pm 0.5$ for most practical applications.

The Mungan and Lipscombe equation, with $n = 3$, is plotted in Fig. 1. When the axes are equal ($x/y = 1$), the probability of rolling any set of opposite sides is 1/3 (0.33%), in other words, a “fair” die with equal chance of all six sides. As the elongated axis increases with respect to the remaining two, the probability of rolling one of the two faces with a larger surface area increases. The curve is approximately linear close to the origin, where a 10% increase in length results in approximately 11–12% increase in probability of rolling one of the two larger faces (or 5.5–6% per face). Substituting smaller values for n results in a curve with a gentler slope that falls lower within the plotted area, while larger values for n increase the slope of the curve.

We can evaluate the Mungan and Lipscombe prediction empirically by plotting data published by Swift (2017a). She received permission to physically roll a sample of 50 parallelepiped Roman dice of various shapes, and recorded the number of times different sides were rolled. Swift only reported the longest and shortest sides on a die, not the second longest. Figure 1 also plots her data, showing that as the longest side increases relative to the shortest, that the probability of the two sides with the largest surface areas increases. The data fit reasonably well to the Mungan and Lipscombe predicted line with $n = 3$ (reducing n to 2.8 results in the best fit to the data, but provides only a marginally better correlation coefficient).

A few points are worth noting about Fig. 1. First, the Mungan and Lipscombe line is based on parallelepiped shapes that have only one axis different in size from the other two. Archaeological dice are likely to be unequal along all three axes. Much of the variation around the line in Fig. 1 could be due to variation in the relative proportion of the second longest (unreported) side relative to the longest. Second, Swift’s dice that are closest to cubic (i.e., closest to $x/y = 1$) still seem to be rolling in notably “unfair” ways. Here, the larger axis is only 3% longer than the shortest axis, yet the two most common rolls still occurred 8 to 16% more often than predicted. This

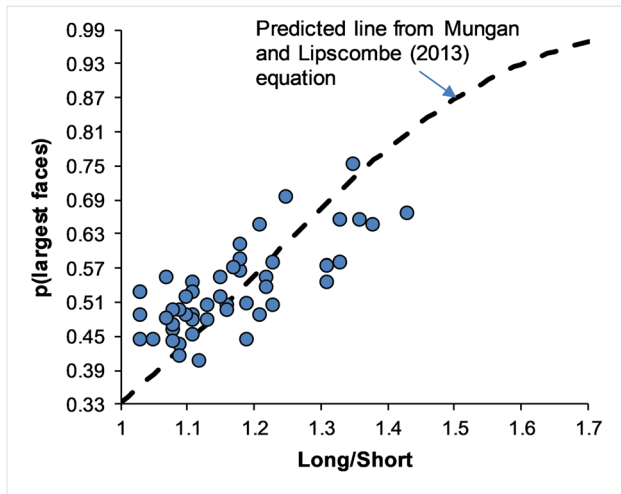


Fig. 1 Probability of rolling one of the two larger faces/sides vs. die proportions (longest side/shortest side) showing the Mungan and Lipscombe equation and empirical roll data from a sample of Roman dice (data from Swift 2017a)

suggests there are still other factors in real dice, unaccounted for, that result in variation in the rolls of dice.

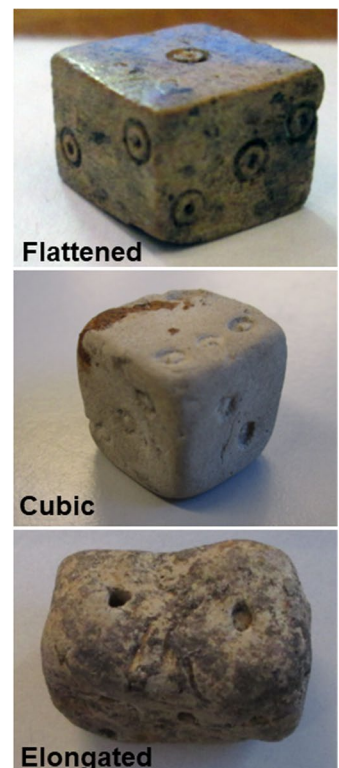
In our analysis of Roman dice from the Netherlands below, we were not able to roll dice as Swift did to record actual probabilities of different sides rolling up. Instead, we rely on the predictions from the physical model.

Distinctive asymmetrical types?

If die asymmetry so markedly affects die function (i.e., the probability of making a particular roll), why are such dice so common in Roman contexts? To address this question, we sought to contextualize shape by examining the distribution of shapes within an empirical set of Roman-period dice. We reasoned that if asymmetrical dice are a distinctive “type” (Ford 1954; Steward 1954), with a unique set of shapes set apart from other more cubic shapes, that their production and use was probably specialized. For example, certain games may have called for the use of dice with different roll probabilities than cubic forms. Alternatively, if asymmetrical forms are simply an endpoint within a continuous range of shape variation, they may not comprise a distinctive “type” at all, but may have been used for the same set of functions as other dice. In that case, what we view as a hindrance to proper die function (i.e., unequal probability of rolls), was not considered important and was acceptable to Roman die users.

To contextualize die shape, we measured a sample of 28 securely dated Roman-period dice that have been found across 13 sites in the Netherlands (Fig. 2). Our choice of the Netherlands was partly to generate a new and independent dataset to test hypotheses about Roman-period dice production, and to examine a dataset that straddles the borders of the ancient Roman Empire incorporating

Fig. 2 Map of modern-day Netherlands showing location of Roman sites included in this study (number corresponds to number of dice measured at each location) along with three examples of dice on right



both Roman and non-Roman cultures. During the Roman Period, the Rhine River formed the northern boundary of the empire (Wheeler 1954; Whittaker 2004:7), often referred to as the *Limes Germanicus* (Galestin 2010; Suharoschi et al. 2020). A series of forts and watchtowers were established along this border in the first century B.C.E. Dice from several of these forts are included in the dataset below. The Frisii, a Germanic tribe, generally occupied the region north of the Rhine border (Todd 1987). Rather than a hard boundary, the area around the Limes should be seen as a region of interaction between Romans and Germanic groups (de Bruin 2019; Galestin 2010; Suharoschi et al. 2020; Wells 1999). Indeed, many Frisii were recruited by the Romans, served in the army, gained Roman citizenship, and continued to trade and interact with other Germanic peoples further north from the Limes (Galestin 2007). That the modern-day Netherlands region falls on both sides of the border allows us to examine how information about die design, and by extension game play and

divination, was transmitted across a major cultural boundary (e.g., de Voogt et al. 2013).

Of the 28 dice in our study, 10 were found to the north of the Limes, and 18 on or to the south. The location of the Limes with respect to our sites is given in Fig. 2, while the first ten entries in Table 1, up until Dieserstraat 82–106, separates these subsamples of dice. Each die was measured for length, width, and thickness, with the measurement taken across the center of the die (i.e., the middle of a side) to the nearest tenth of a millimeter using digital calipers. For example, the right side of Fig. 2 shows three dice from our study, one symmetrical (middle) and two clearly asymmetrical (top and bottom). Each die was also examined for material type, configuration (i.e., the arrangement of numbers on the six faces, see de Voogt et al. 2015), and pip style. Table 1 shows the database of dice used in this study, along with metrics.

Dice represent a range of raw materials. Our sample of 28 included four ceramic, three metal (bronze), and 21 bone or antler examples. Measurements show a range of shapes

Table 1 Database of Roman-period dice from the Netherlands

Site/location	Province	Cat #	Mat	Dimensions	Config	Pips
Wierhuizen	Groningen	1975-X-36(1)	Bone/ivory	21 × 17.6 × n/a	Sevens	D-R-R
De Druppels*	N. Holland	8997–01	Ceramic	17.4 × 12.9 × 11.6	Irregular	Dots
Uitgeest, UB 18*	N. Holland	8367–02	Ceramic	15.8 × 15.2 × 15.2	Sevens	D-R
Uitgeest, UB 18	N. Holland	8367–03	Ceramic	15 × 14.5 × 14.5	Sevens	D-R
Assendelft	N. Holland	8325–09	Ceramic	15.8 × 15.6 × 15.7	Irregular	Dots
Velsen 1	N. Holland	G2008/6.1990–16	Bone/ivory	12.9 × 12 × 11.6	Sevens	D-R
Velsen 1	N. Holland	G2008/6.19-zn46	Bone/ivory	14.9 × 14.6 × 12.7	Sevens	D-R
Velsen 1	N. Holland	G2008/6.19-zn47	Bone/ivory	13.4 × 13.1 × 12.5	Sevens	D-R
Velsen 1	N. Holland	G2008/6.19-zn48	Bone/ivory	12.3 × 12 × 11.6	Sevens	D-R
Dieserstraat 82–106	Gelderland	368	Bone/ivory	6.6 × 5.6 × 5.6	Primes	D-R
Ulpia Noviumagus	Gelderland	MP1.6.594	Bone/ivory	16.8 × 16.3 × 15.4	Sevens	D-R-R
Ulpia Noviumagus	Gelderland	MP1.31.744	Bone/ivory	14.2 × 13.7 × 10	Sevens	D-R-R
Ulpia Noviumagus	Gelderland	WW.1.10.925	Bone/ivory	27.1 × 26.1 × 18.5	Sevens	D-R-R
Castra	Gelderland	1598-Dob 1	Bronze	10.4 × 10.1 × 9.7	Irregular	D-R
Castra	Gelderland	1324-Dob 2	Bronze	13.9 × 12 × 11.3	Sevens	D-R
Castra	Gelderland	3105-Dob 3	Bronze	13.5 × 13.3 × 12.7	Sevens	Dots
Valkenburg	Z. Holland	922	Bone/ivory	12.3 × 11.7 × 11	Sevens	D-R-R
Valkenburg	Z. Holland	9157	Bone/ivory	13.5 × 12.7 × 12	Sevens	D-R-R
Roomburg	Z. Holland	9290	Bone/ivory	11.2 × 11.1 × 9.1	Irreg/false	D-R-R
Woerden/Laurium*	Utrecht	n/a	Bone/ivory	14.2 × 13.7 × 9.4	Sevens	D-R
Woerden/Laurium	Utrecht	n/a	Bone/ivory	13.2 × 12.8 × 10	Sevens	D-R
Vechten	Utrecht	VES 10	Bone/ivory	18 × 17.7 × 17.2	Sevens	D-R-R
Vechten	Utrecht	VES 11	Bone/ivory	12 × 11.1 × 11	Sevens	D-R
Vechten	Utrecht	VES 12	Bone/ivory	13.4 × 12.1 × 9.3	Sevens	D-R-R
Vechten	Utrecht	VES 439	Bone/ivory	13.2 × 11.8 × 9.7	Sevens	D-R-R
Vechten	Utrecht	VES 555a	Bone/ivory	14.4 × 12.7 × 9.9	Sevens	D-R
Vechten	Utrecht	VF 438	Bone/ivory	17.2 × 14.4 × 12.9	Sevens	D-R-R
Borgharen	Maastricht	93	Bone/ivory	7.3 × 7.2 × 6.2	Sevens	D-R

Config., configuration (see de Voogt et al., 2015); *D-R*, dot surrounded by a single ring or a “dot and circle” motif; *D-R-R*, dot surrounded by two rings or a “double circle and dot” motif, *pictured in Fig. 2

as well. Some dice are close to true cubes, including three of the four ceramic items. However, the majority (24 of the 28) are visibly non-cubic, where the longest side is more than 5% longer than the shortest side. In extreme cases, the longest side is over 50% larger than the shortest side. As shown in Fig. 1, this is expected to shift the probability of rolling the favored sides from an odds of 1 in 6 (17% for a true cube) to odds of better than 1 in 2.4 (43% for the most asymmetrical die).

Assuming all corners maintain right angles to one another, there are two main ways to modify a cube. First, one of the axes can be lengthened while holding the others constant. This results in a stretched or “elongated” cube. Alternatively, one of the axes can be shortened while holding the others constant, resulting in a squat or “flattened” cube. Figure 3 plots the longest/shortest side vs. the middle/shortest side for the sample of Roman dice in our study, with symbols coded by raw material type. In Fig. 3, artifacts closer to the origin are most like true cubes, those along the X-axis represent items that are elongated along one axis, while artifacts along the 1:1 diagonal represent items that are flattened. Both asymmetrical parallelepiped shapes appear within the sample, though it is clear that more of the non-cubic dice fall above the dotted diagonal line, that is, more are flattened than elongated. The data also show that elongated and flattened shapes occur both north and south of the Limes.

At the same time, the degree of asymmetry in non-cubic dice is not standardized, as would be expected of shapes that comprise true functional “types.” Fig. 4 shows a histogram of longest side relative to the shortest side, which does not reveal a clear bi- or tri-modal distribution. Instead,

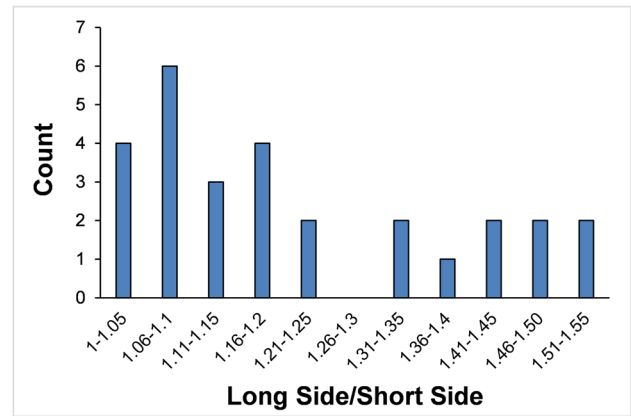


Fig. 4 Histogram of long side relative to short side for the 28 dice in this study

the distribution of longest side/shortest side is more-or-less continuous from close to a true cube to highly asymmetrical. Such a near-continuous distribution in morphological shape is also shown in datasets accumulated by Schmid (1978) for a Roman legionary camp in Switzerland, and by Swift (2017a) across a large number of sites in Roman Britain and Roman Egypt.

Raw material type does not obviously explain variation in parallelepiped dice form. Nearly cubic and non-cubic forms are found in all three media, including raw materials that would not be constrained in forming a die, such as clay and metal. However, it does seem that organic dice depart from near-cubic form more often than clay or metal dice. Thus, the average value for long-side/short-side of organic dice (1.24) is larger than the average value for clay (1.15) and metal (1.12) dice. Constraints by the shape of antler or bone, then, may account for some of the variation in the side proportions in dice, but certainly not all. This pattern seems to hold more broadly across the Roman world. Swift (2017a) also found that bone/antler dice were more often asymmetrical in Britain and Egypt, but that dice of glass and stone were also occasionally asymmetrical.

Given that many of the dice are non-cubic, we can examine whether the pips were consistently placed to favor rolling certain numbers over others. Because of their shape, flattened dice are more likely to favor rolls for two of the six sides (with the other four sides much less likely), while elongated dice make two of the smaller sides less likely to fall in the “up” position (with the other four sides more likely). An examination of which numbers appear on the more likely vs. less likely sides reveals a bias, but only among the flattened dice. Thus, of the 17 flattened dice, 10 have the “1” and “6” pips opposite one another on sides that are most favored to be rolled. In other words, flattened dice were produced in a manner that made rolls of 1 and 6 much more likely. Discounting the one die in a non-standard configuration

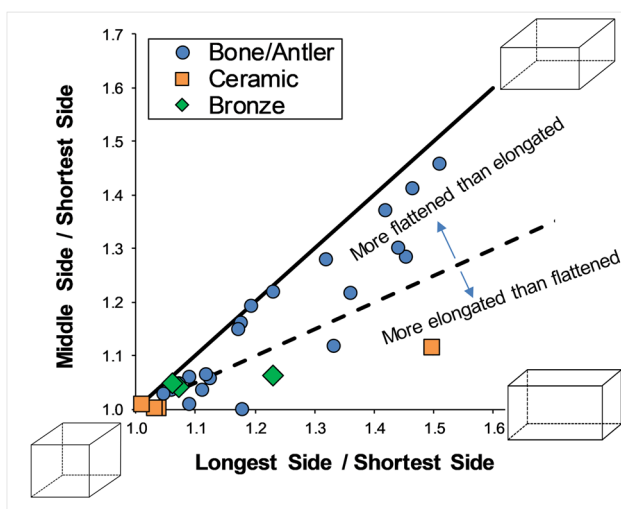


Fig. 3 Idealized cubic and non-cubic forms, showing the longest/shortest side vs. middle/shortest side, for the sample of 28 Roman dice

(see Table 2 footnote), a multinomial distribution suggests the probability of this outcome by chance is approximately 0.6%. This result mimics other studies of Roman-period dice, which show a similar favoring of 1–6 rolls on asymmetrical dice (Greep 1983:243; Poplin 2004:62; Schmid 1978:58; Swift 2017a:138).

By contrast, among the elongated dice, rolls that would be least favored are equally distributed across the six numbers. That is, elongated dice were not systematically produced in a manner that would favor particular rolls. The sample size is smaller, but a multinomial distribution suggests the probability of this outcome by chance is approximately 60%.

In short, Roman dice do not show a standardized “type” of asymmetrical die. Elongated dice show no favoritism towards particular configurations that prefer certain rolls. However, flattened forms tend to have a configuration of the pips that intentionally favor rolls of 1 and 6. The presence of this bias in flattened dice could be taken to suggest that dice makers intentionally placed certain numbers on certain sides to manipulate die function, for example as cheater’s dice (Greep 1983:247; Swift 2017a:142–145). Alternatively, as Swift (2017a:203–211) shows, bone grain direction also seems to correlate to the orientation of the dots that comprise a 6 on many flattened bone dice, and preference to engrave particular numbers along certain axes of the dice could lead to preferences to placing 6 and 1 on the largest sides. She further suggests that cultural transmission processes may have extended this preference to non-bone dice (Swift 2017a:211). We explore these hypotheses further below.

Experimental approach to die shape

Production bias refers to a process wherein a particular artifact shape or configuration is produced in higher frequencies, not due to intentional design or cultural preferences, but due

Table 2 Favored sides for flattened dice, and unfavored sides

Side	Flattened (sides favored)	Elongated (sides not favored)
1–6	10	2
2–5	1	2
3–4	5	3
Other: 2–4*	1	

*Die with non-standard configuration where opposite sides do not add to seven. Note: two of the elongated dice are also in non-standard configurations; however, on one die, the 1 and 6 are still opposite to one another and least favored to be rolled, and on another, the 3 and 4 are opposite to one another and least favored to be rolled. As well, one of the flattened dice is a “false” die with two sides with 2 and two sides with 3 (but the 1 and 6 are in the favored positions)

to an interaction between the human body or mind and the raw materials used to make artifacts. Previous experimental research shows that dice are subject to a production bias in the configuration of how pips are placed on a cube (de Voogt et al. 2015). Many readers will be familiar with the “sevens” configuration of pips in a modern die, where opposites sides of the die add to seven (i.e., the 1 is opposite 6, 2 opposite 5, and 3 opposite 4). It is clear that this configuration is not the only one possible, and many ancient dice are in the “primes” configuration (i.e., 1 opposite 2, 3 opposite 4, and 5 opposite 6). However, a naïve die manufacturer with no knowledge of the “proper” configuration of pips will produce neither of these forms. Instead, they will produce a “turned” configuration, wherein numbers are placed sequentially on the cube while turning the object in their hand (i.e., 1 opposite 3, 2 opposite 4, and 5 opposite 6). As we previously speculated, this is likely because it is most economical for the producer to place numbers sequentially to ensure that each appears only once on the die surface, in other words, an interaction between the mind and the raw material.

Could a production bias account for the placement of 1 and 6 on the larger surfaces of flattened dice? Or did Roman die makers intentionally place 1 and 6 on the larger surfaces to favor these rolls, for example, to better predict and/or manipulate the outcome in games or divination? To explore this question, we conducted an experiment with naïve die producers to see how they configure pips on blank cubes with different degrees of asymmetry. If there is a strong tendency to configure pips in a certain way that is different from what we see in the archaeological record then the configuration on the archaeological dice is more likely to be intentional. However, if naïve die-producers show a strong production preference that is the same as the pattern in archaeological finds, then the predisposition may be explained by a production bias.

Naïve dice makers and production bias

Twenty-three psychology majors from an American university volunteered to participate in an experiment for college credit. The Institutional Review Board of Drew University approved the study and a consent form was signed by each participant prior to participating. Each experiment consisted of two conditions in which participants were asked to apply dots on four clay dice (for a total of eight dice per person). In the first condition, the participant was sitting at a desk and given a HB No.2 black pencil, while in the second condition they were given a broad-point Fabiano black pencil. The conditions were chosen to maximize the number of dice produced by the participants and prevent them from anticipating the purpose of the experimental design.

In each condition, the participant was requested to replicate the dots on a cubic die on a clay die with the pencil

provided. They were instructed to carefully copy the pattern. For instance, if a participant placed the three dots of the number three in an orthogonal (i.e., vertical or horizontal) instead of a diagonal fashion, they were asked to correct this on the subsequent die. Each condition consisted of four blank dice ranging from an almost perfect cube to a highly parallelepiped one, where the long side was over 33% longer than the short side, mimicking examples found in the archaeological record. Although the measures of the dice slightly differed from one person to the next, the sizes of the four dice were identical in each condition for a single participant. As a result, each participant produced two dice of each size that was offered, one made with a regular and one with a large pencil.

One group of twelve participants was asked to also replicate the configuration of modern dice, i.e., placing the numbers so that opposites make seven. This so-called Sevens configuration is also found on most Roman dice (Eerkens and de Voogt 2017). The other group was given no instructions as to the configuration and although most of them made a so-called Turned configuration—a configuration that appears if numbers one through four are placed sequentially when turning the die on a vertical axis (see de Voogt et al. 2015)—there was some additional variation. For each die, the experimenter observed in which order the participant placed the numbers. After all dice were made, the four dice of the second condition were placed in front of the participant and they were asked to what extent they perceived them as cubic or non-cubic.

Results

Twenty-three participants completed 184 dice in total. In one case, the participant made a mistake and had added one number twice. This person was given an additional die but the original die was left in the study as it had the correct measurements and the repeated number did not affect the results. In each set of four dice, the first had sides that differed less than 1 mm in size. The second die had a size differential between 1.5 and 3 mm, the third between 3 and 6 mm, and the final between 4 and 7.5 mm. The maximum side of a die varied in size between 8 and 17 mm.

In 60 cases (32.6%), the dots were applied starting with 1 then 2, 3, 4, 5, and 6. One person applied the numbers in reverse order (6, 5, 4, 3, 2, 1), but for one condition only. In 66 cases (35.9%), they were applied starting with 1, followed by 6, then 2 and 5, and then 3–4 or 4–3, to complete a Sevens configuration. One person started with 6–1, then added 5–2 followed by either 3–4 or 4–3 for seven dice. Indeed, all other orders of numbers occurred only four times or less in the entire dataset and are associated with a single individual only. The vast majority (68.5%) of the dice were produced starting with the number 1.

If we discount the first die in each condition, which was perceived as almost or close to cubic by 21 participants—the remaining dice were considered clearly non-cubic—we have 138 dice with a visibly larger side. This larger side holds two numbers (front and back). Out of 138, the two numbers on the larger side are 1 and 6 in 63 (45.7%) dice, 1 and 3 in 19 (13.8%) dice, and 5 and 6 in 16 (11.6%) dice. In one case 1 and 2, and in three cases 1 and 4 were found on the largest sides. In other words, of those dice with a Sevens configuration ($n=72$), 63 (87.5%) had the 6 on one of the two largest sides. In 62.3% cases, the lowest number 1 was (also) found on a larger side. By contrast, of those with a Turned or other freely chosen configuration ($n=66$) only 19 (29.9%) had the highest number on the largest side. In one case, a participant was asked why they changed the order of putting on the numbers and replied that they wanted to make sure that the 4 and 5 were on the largest sides as “they take up the most space.” Three dice in the dataset had 4 and 5 on the larger sides, all produced by this one individual.

In sum, the results from the experiment show that naïve die producers strongly favor the number 6 on one of the two larger sides of flattened dice, especially when they are constricted by the Sevens configuration. This is at least partly explained by a tendency to start with the number 1 on the largest side (with six appearing on the opposite side), and partially by favoring the pip that takes up the most space (6 dots) on the largest side.

The preference for 6 appearing on one of the larger surfaces is so pronounced that the presence of 1–6 on the larger side of flattened Roman dice is no longer surprising. Rather than representing an intentional placing of the pips to influence rolls favoring 1 and/or 6, we suggest the result is better explained as the by-product of a production bias. Roman die makers probably reserved one of the largest sides on a die for the pip that took up the most space (i.e., the 6), or simply started with a 1 on the largest face and followed a Sevens configuration such that the 6 also appeared on a larger face. Although this production bias will favor 1 and 6 during a throw, there is no reason to assume that this effect was intended or consciously manipulated by the die-maker. Similarly, the Etruscan asymmetrical dice reported by Schädler (2019) have a primes configuration, and yet, the larger surfaces are mostly taken by the 6 and 5. This would further support the suggestion that die makers reserved the largest sides on a die for the pips that took up the most space.

Discussion

Previous research shows that human visual perception can distinguish difference in size when two objects differ in magnitude by approximately 5% or more (Eerkens 2000). Our survey of Roman-period dice from the Netherlands shows

that over 80% of dice exceed this threshold. Thus, the majority of Roman-period dice were visibly asymmetrical.

Modeling by Mungan and Lipscombe (2013) shows that even a small increase in one axis of a cube shifts the probability of rolling one of the larger sides markedly, for example, from approximately 33% for a true cube, to nearly 40% for a die that is 5% larger on one axis. Empirical data by Swift (2017a) supports this finding, where dice differing by just 5% on their sides roll one of the larger sides as much as 53% of the time (rather than 33% for a true cube).

Our analyses of asymmetrical Roman-period dice show several patterns that, together, provide insight about the production of material culture that is *not* governed by an evolutionary process of increasing economic efficiency (e.g., Bamforth 1986; Basalla 1988; Bettinger et al. 2006; Kuhn 2004; Schiffer et al. 2001). First, if die asymmetry was the result of attempts to alter probability outcomes of certain rolls, without being noticeably and visibly asymmetrical, we would expect greater percentages of dice at or close to the 5% visibility threshold. Instead, nearly half the asymmetrical dice are obviously lopsided, being 20% or more larger along one axis. This pattern is repeated across many regions, including Roman-period dice in Britain, Egypt, Central Europe, and Switzerland. Clearly, such asymmetrical dice were a normal part of most game play or divination throughout Europe during the Roman period.

Second, die shape varies along a continuum, with no clear divisions in shape type. Both flattened and elongated cubes are present, with various shapes in between. As well, we did not find bi- or tri-modality in the degree of cube asymmetry. Although there are other differences in die design north vs. south of the *Limes Germanicus* (e.g., dice north of the Limes are more often in non-Sevens configuration and typically have only one circle or ring around a central dot rather than two circles), the pattern in asymmetry holds in both regions. This suggests that whatever factors promoted the production of asymmetrical dice, that this held for both Frisii and Roman populations. In short, both symmetrical and asymmetrical dice were part of an accepted range in shape variation broadly within Roman-period societies.

Third, flattened asymmetrical dice, where two faces are larger in area than the other four, tend to have the 1 and 6 represented on the larger surfaces more often than expected by random chance alone. Here too, the pattern in the Netherlands is similar to other locations in Europe. At first glance, this might suggest intentional manipulation of dice to increase the probability of the 1 and 6 rolls. However, our experiments show that such bias is also produced among naïve die makers. This suggests that placing the 1 and 6 on the larger sides is likely a by-product of a production bias rather than intentional placement. We interpret the Roman pattern as either a perceived need to reserve a larger surface to fit all the dots of a 6, or simply because the numbering

process started with 1 on the largest surface and required a 6 on the opposite side.

Taking these outcomes together, our contention is that asymmetrical dice were not intentionally produced to manipulate die rolls (i.e., “cheaters” dice) nor a distinctive type of die. Instead, these dice are a byproduct of two processes. First, as others have argued (Greep 1983:247; Poplin 2004:62; Swift 2017a:210), many of the raw materials used to produce dice, especially bone and antler, are themselves not symmetrical, leading to objects that are longer across some axes. While it would have been possible to grind or shave down the longer sides, thereby reducing the overall size of the object but producing a true(r) cube, this was not seen as necessary due largely to the second process. As well, even ceramic and metal dice, where the raw material would not seem to impose any bias on final shape, are still often visibly asymmetrical.

Second, we argue that Roman worldview and cultural transmission processes consistently led to the production of asymmetrical dice, and these forms were not viewed as detrimental to overall die function. According to Swift (2017a:123), Romans generally believed that the outcome of die rolls was influenced by the will of supernatural entities, or gods. Rolling dice could be a means to communicate or engage with gods, for example, to provide guidance or reveal information about the outcome of a future event. As well, players of games often felt that gods who looked favorably upon them would control die roll outcomes to help win or provide fortune in a game (see Purcell 1995).

This Roman worldview ensured that die shape was free to vary across a wide range of forms because “fate” rather than “probability” dictated roll outcomes. While we can statistically estimate probabilities today among populations of rolls, single throws are still unpredictable (this partly explains why even today’s gambling casinos remain popular despite long-term odds heavily against an individual player). Producing an even probability of rolls across the different numbers 1 through 6, which is typically the main “function” of dice in modern gaming, was clearly not how Romans perceived the function of their dice, since fate made any particular roll unpredictable. Dice only had to have the ability to roll each face because die shape was not seen as related to a specific outcome. That is, the connection between the frequency with which particular numbers came up and die asymmetry was not part of the awareness of most dice users.

This does not mean that all individual Romans were oblivious to rolling frequencies. For example, much has been written about Cicero (106–43 BCE) and his views on probability (e.g., David 1962: 24–26; Meador 1968, 1970). Cicero’s writings indicate that he was frustrated by the more general Roman worldview that gods controlled all manner of human experience. He specifically uses die-rolling as an example to bring up the subject of chance and an albeit

limited notion of probability, questioning whether dice rolls were always determined by the gods. Other individuals may have made similar observations. This may have included frequent users of dice, who through experience may have come to similar tacit understandings regarding asymmetrical dice. They may have even used this understanding to gain an edge in game play.

At the same time, however, the writings of Cicero are unlikely to have reached a wide audience, especially in areas far from Rome and across its borders. The fact that Cicero's thoughts are only found in formal treatises suggests that his views were novel and not widespread in Roman society. What is most important in archaeological contexts, is whether the majority of individuals understood the world in this fashion, as those worldviews are most likely to influence the design of every-day artifacts we recover. In that respect, we note that our small sample of dice from north of the *Limes Germanicus*, despite being different in design in other ways, shows the same pattern favoring shape asymmetry. There are few historical records on the Frisii that would indicate how they viewed the supernatural influencing every-day human experience, much less specifically on rolling dice. Given the large number of asymmetrical forms, however, we suggest that Roman worldviews about the outcome of die rolls were probably parallel to that of the Frisii, or perhaps even transmitted across cultural boundaries.

Only in the Renaissance, in particular with the work of Blaise Pascal, did probability theory reach a mainstream audience (Hacking 2006). Not surprisingly, as ideas about chance and probability took hold, and European worldviews changed, die shape also changed, becoming more standardized with a much higher percentage approximating a true cube (Eerkens and de Voogt 2017).

Conclusions

This study shows that variation in Roman-period die shape in the Netherlands is consistent with what we know (1) about die shape in other areas of Europe, and (2) about Roman culture and understandings of the world. By contrast, variation in the placement of numbers on non-cubic dice is consistent with production bias. This further compels us to rethink die function in Roman society. Dice should not be viewed as “randomizing devices” in a mathematical sense, the way we generally understand them today. Instead, we should view Roman-period dice in a cultural sense as representing a belief in fate, and a material connection to the gods who controlled that fate. Since mathematics and probability theory are unable to predict the outcome of an individual throw, such a belief would not necessarily be altered or challenged by a die that was closer to, or even visually indistinguishable from, a true cube. Our findings hold on both sides of the

Roman border, across cultural group boundaries, indicating that ideas about dice design and function were likely similar between Germanic, Etruscan, and Roman societies.

In short, the shapes of six-sided dice in Roman times are part of a continuum of which the cubic form only became dominant in post-Roman periods. The placement of the pips may be explained by a production bias and does not necessarily indicate divinatory or gaming preferences, especially since probabilities were not yet widely understood and individual throws remain unpredictable. Today's preference for a cubic rather than an asymmetrical form, as well as a broad understanding of probability, contrasts that of the Roman world as well as that of some of its neighbors. Future research may show to what extent other non-Roman populations shared their preferences and understanding.

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Declarations

Conflict of interest The authors declare no competing interests.

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