



An anthropological point of view: exploring the Chinese and Japanese issues of translation about teaching resources

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Abstract

The aim of this study is to advance understanding of teachers' and researchers' work, in particular its cultural specificities, from a resource perspective by exploring the issues and challenges faced during the translation of a theoretical framework, the *Documentational Approach to Didactics* (DAD), from Western (English and French) to East Asian languages (Chinese and Japanese). A basic assumption is that the challenges encountered while translating are driven by the cultural and linguistic differences that exist between the West and East. Adopting the perspective of the *Anthropological Theory of the Didactic* (ATD), we frame the translation work as a transposition process of research praxeology (a model of the researchers' practices and knowledge) from a Western to an Eastern institution. We investigate this process to identify cultural elements at different levels (school, society, civilization, etc.) using the ATD framework. After translating a DAD article into Chinese and Japanese separately, we then worked collaboratively to identify similar or diverse translation issues and investigated their origins. Consequently, the results revealed a considerable difference between the West and East, and ample similarities between China and Japan, especially in terms of the researchers' work and its relationship with the teachers' work.

Keywords Documentational Approach to Didactics · Translation · Comparative study · Research praxeology · Transposition

1 Introduction

Mathematics teaching is considered a cultural activity (Stigler & Hiebert, 1999). It concerns not only classroom activities but also teachers' practices outside the classroom, such as teachers' interactions with resources. The *Documentational Approach to Didactics* (DAD) is a theoretical framework to “understand teachers' professional development by studying their interactions with the resources they use and design in/for their teaching” (Trouche et al., 2020, p. 237). The cultural specificities of resources as well as teachers' work and its diversity, within DAD, have been a research

interest since the early stages of the theoretical development in this regard (Pepin et al., 2013).

The increasing cultural diversity of researchers working on DAD has, recently, further directed their attention to the cultural specificities of teachers' work with resources and consequently led to the *DAD Multilingual Project* (Trouche, 2021)¹. This project was launched in 2020 to gather translations of an English article that introduced the DAD framework. The goal of this project was not only to disseminate DAD but to also deepen the concepts of DAD and diversify the theoretical framework by adapting it to different contexts. The authors of this paper were involved in this project and worked on Chinese and Japanese translations.

The translation of scientific literature can be found in different countries and in diverse fields of the sciences. This is also the case for literature in mathematics education. The translation work entails various difficulties, including not only the problem of translating word by word, but also that of finding a properly shared understanding of concepts

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¹ <https://hal.archives-ouvertes.fr/DAD-MULTILINGUAL>.

in the context where they will be adapted (Arcavi et al., 2016; Balacheff, 2018). In the field of translation studies, the mechanism of translation has been investigated, and the ideas behind translation have been conceptualized (e.g., Pym, 2014). In China and Japan, there is a long tradition of the translation work on the sources from the West across disciplines (e.g. Sato-Rossberg & Wakabayashi, 2012; Yang, 2015). However, we consider that the challenges of translating a framework have not been sufficiently and explicitly discussed as an issue to tackle in mathematics education research.

In this special issue of *ZDM*, some articles deal with the translation issue (Ruthven, 2022; Shao et al., [this issue](#); Baştürk-Şahin et al., [this issue](#)). We view the processes of translation and interpretation of DAD concepts as uncovering new perspectives on the relationships between language and culture and raising new questions about how teaching practices can be studied, described, and understood (Trouche et al., [this issue](#)). In line with this idea, our research aims to advance understanding of teachers' and researchers' work, in particular its cultural specificities, from a resource perspective, through the investigation of the issues and challenges arising from the translation of DAD, a theoretical framework developed in Western languages (English and French), into East Asian languages (Chinese and Japanese).

To accomplish this aim, we first characterize the critical objects and practices related to the translation from the perspective of mathematics education to clarify what is translated, how it relates to the teachers' work, what affects the translation, and so forth. In this regard, we adopt the *Anthropological Theory of the Didactic* (ATD; Chevallard, 2019), which seems pertinent to capture the overall system that entails translation work in relation to the teachers' work. By exploring the issues that arise during the translation process, we discuss, from the ecological perspective of ATD, the different conditions and constraints that may explain the distance between the West and the East and/or between China and Japan.

The terms 'West' and 'East (Asia)' in this study refer to cultural demarcations rather than geographic divisions. As Leung et al. (2006) stated, we acknowledge that 'neither of these "poles" is well defined, as with any label given to any culture' (p. 4) as well as various traditions that exist both across and within western European and east Asian countries. However, discussing and covering all important diversities is beyond the scope of this study. We use the terms not to essentialize these two distinctions, but to investigate cultural specificities in translating the theoretical framework developed in the Western European context, using pertinent examples from the Chinese and Japanese contexts.

2 Theoretical perspective and methodology

We find ATD to be valuable to investigate the activities around the translation of DAD and to better understand the cultural aspects of mathematics education. ATD adopts an institutional approach that allows us to understand the position of translation within a global system, including the teachers' and researchers' practices as well as the translation work. In this section, we introduce the theoretical elements, research questions, and methodology of this study.

2.1 Translation work from an anthropological perspective

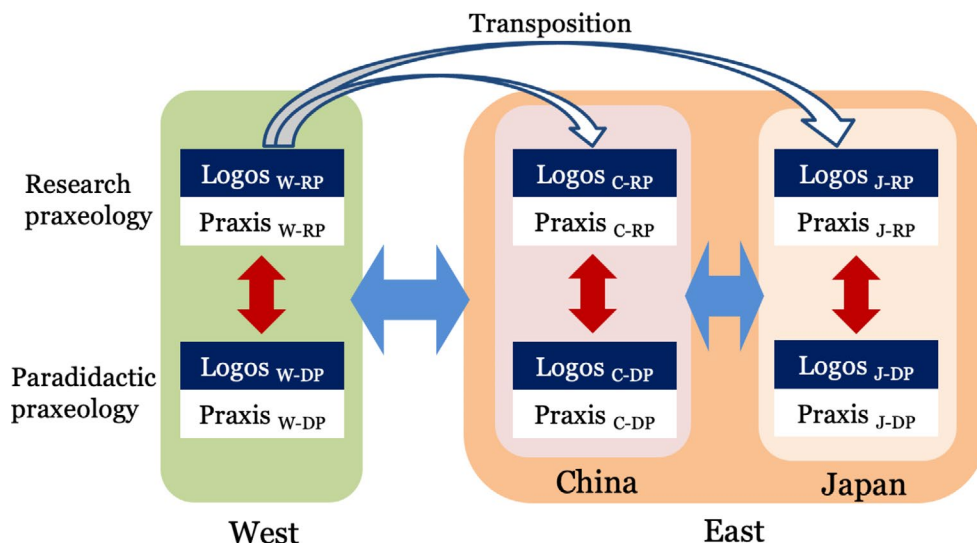
Within ATD, any human activity (not only those related to mathematics education) is modeled by the notion of *praxeology* (Chevallard, 2019; Gascón & Nicolás, 2019a), which consists of two blocks, namely: (1) a *praxis block*, which includes the *type of tasks* and a *technique* that describe the practice; and (2) a *logos block*, which includes the theoretical elements underpinning the practice, specifically *technology*, which explains and justifies the technique and *theory* that in turn justifies the technology. In general, praxeology is a tool that is used to model the knowledge and practice related to teaching and learning, such as *mathematical praxeology*, a model of mathematical knowledge and practice, and *didactic praxeology*, a model of teachers' knowledge and practice in the classroom.

From the perspective of ATD, this study is related to the translation of DAD and can be characterized and explained by the structure shown in Fig. 1.

The notion of praxeology can be applied to various practices. We may consider a *research praxeology* that models researchers' practices and knowledge in mathematics education research (Artigue & Bosch, 2014). This praxeology consists of a praxis block that describes the research questions (tasks) and the methods (techniques) to tackle them, and a logos block that provides a rationale for research questions that explains and justifies the methods. The DAD is considered part of the logos block of the research praxeology (Logos W-RP in Fig. 1), which is developed in the West. In addition, teachers' practices, which are the object of the study for DAD, can be modeled by the didactic praxeology, in particular by *the paradidactic praxeology* (Praxis W-DP and Logos W-DP in the case of West) when emphasizing the practices *about* teaching outside the classroom like documentation work (Miyakawa & Winsløw, 2019).

ATD was originally developed from the *Theory of Didactic Transposition* (Chevallard, 1985/1991; Chevallard, 2019), which questions the nature of mathematical knowledge (praxeology) as an object of teaching in schools and the mechanism of handling and shaping it. This idea could

Fig. 1 The overall structure of the study in terms of ATD



be adapted to research praxeology. *The translation work of a didactic theory is considered a process of transposing a logos block of research praxeology from one research institution to another*: in our case, the transposition from the French or English research institution to the Chinese and Japanese ones (the curved arrows in Fig. 1), so that the DAD could be used to study paradidactic praxeologies in the East (C-DP and J-DP).

The term *institution* here should be understood in a broader sense as: “any created reality of which people can be members” (Chevallard & Bosch, 2019, p. xxxi). One of the critical hypotheses of ATD is that any *praxeology* cannot survive in an empty society but in an institution; that is, it cannot be understood without considering different social or institutional elements that surround it. In the institution, praxeology is always subject to *conditions* supporting it and *constraints* hindering it. These conditions and constraints constitute the *ecology* of praxeology, which is one of the main objects of study within ATD (Gascón & Nicolás, 2019b). In the case of research praxeology, a theoretical framework, part of praxeology, is developed within a research institution, in our case a French one, with some specificities shaped by its ecology (the rounded rectangle of the West in Fig. 1). In the Chinese or Japanese institutions of mathematics education research, research praxeologies also exist and are shaped by the ecology in those institutions. The transposition of the research praxeology is therefore the process of putting it under the effects of such ecology, which presumably produces difficulties during translation.

For the ecological analysis, ATD proposes the following classification of diverse conditions and constraints that affect a didactic or mathematical praxeology, which is called *the scale of levels of didactic co-determinacy* (Chevallard, 2019):

Humanity ⇔ *Civilization* ⇔ *Society* ⇔ *School* ⇔ *Pedagogy* ⇔ *Didactic system*.

This scale is a theoretical model that directs our attention to different elements beyond the classroom and sheds light on the specificities of an institution, which are often discussed in terms of *culture*. In this study, we discuss the conditions and constraints that caused the difficulties during translation process, in particular those that are often classified into the upper levels, such as civilization² (East Asian) and society (Chinese or Japanese), and those related to linguistic issues, which have not been fully addressed in the comparative perspective and were recently brought to the fore in the Lexicon Project (Mesiti et al., 2022).

2.2 Research questions

Within the structure shown in Fig. 1, we expanded the research questions to the following two questions from a comparative perspective:

- RQ1: What are the common and different institutional conditions and constraints that cause the difficulties in the translation work of DAD, between the East and West and between China and Japan?
- RQ2: What are the critical cultural specificities highlighted through exploring the transposition process of the research praxeology of DAD?

² In ATD, the term *civilization* does not mean the process of civilization but it “applies to a set of societies [...]. The concept is functional in pointing to conditions and constraints present in a whole range of societies” (Chevallard & Bosch 2019, p. xix). As the civilization level “does not refer to a globality: it has a *local* meaning” (ibid., p. xxxv), we consider East Asian societies at this level.

2.3 Methodology

This study consisted of two parts; the first was the translation work, which was carried out in the DAD Multilingual Project. The second was the retrospective analysis of the translation work from a cross-cultural perspective to elucidate the specificities of the teachers' work (C-DP, J-DP) and the researchers' work (C-RP, J-RP). We specifically investigated the difficulties or issues faced during the translation work as a methodological tool to shed light on the conditions and constraints underlying the transposition process. To this end, we also try to make sense of these difficulties in terms of some of the concepts that have been introduced in the field of translation studies, in particular the idea of *translation equivalences* between the source text (French/English) and the target text (Chinese/Japanese) (e.g., Pym, 2011, 2014; Ruthven, 2022).

In the Multilingual Project, the source text was the English article of the *Encyclopedia* (Trouche et al., 2020). The French version was consulted when necessary to ensure the meanings of the terms or phrases were maintained. The translation process was similar on both the Chinese and Japanese sides. First, one of the two authors translated the text, and the other reviewed the translation. In this process, both authors arrived at a consensus on the translation of the technical terms and invited other native speakers to read and confirm the work for final refinement. After the translation, the authors drafted a report on the rationale of the translation and the difficulties encountered. Thereafter, owing to the close language origin and relationship between the Chinese and Japanese, the authors were invited to form a team and study the cultural and linguistic aspects related to the teachers' work with resources.

Based on the translation and report, the team organized several online meetings from 2021 to 2022, through which we shared and discussed the details of the study. We specifically selected some key terms of the DAD (see Table 1), discussed the common and divergent issues while translating,

investigated their origins to elucidate the cultural specificities of the teachers' and researchers' work, and compared them between the East and West as well as between China and Japan. The DAD is based on the *Instrumental Approach*, developed in cognitive ergonomics and then integrated into *didactics* of mathematics (Trouche et al., 2020). As a result, some terms related to the *instrument* were selected. The main idea of DAD is to consider teachers' use and/or design of the resources like their use of tools or instruments. A *document* consists of *resources* and their *scheme* of usage (including *operational invariant*), as an instrument consists of an artifact and its scheme of usage. Within DAD, the process of developing the document has been modeled as *documentational genesis*, which allows us to understand teachers' interactions with different kinds of resources and their learning over time.

It should be noted that the translation work cannot be done without our own meaning making, which is affected by the translators' previous experiences working with researchers in Europe and their knowledge of DAD. In our case, all the authors were familiar with DAD and had research experience in Europe. In adopting ATD perspective, however, we were also aware of the importance of taking an 'external' position (called the *detachment principle*; see Bosch, 2015) to consider the translation work as an object of study.

3 East Asian contexts

In this section, we introduce civilization and education in China and Japan. These East Asian contexts constitute a part of the ecology that shapes both teachers and researchers' work in Eastern countries.

Table 1 Translations of the key terms of DAD

Key terms	Chinese translation	Japanese translation
DAD	文献纪录教学论 (wén-xiàn jì-lù jiào-xué-lùn)	教授文書活動研究法 (kyōju bunsho katsudō kenkyūhō)
didactics	教学论 (jiào-xué lùn)	教授学 (kyōjugaku)
document	文献 (wén-xiàn)	文書 (bunsho)
documentation	文献纪录 (wén-xiàn jì-lù)	文書活動 (bunsho katsudō)
documentation work	文献纪录工作 (wén-xiàn jì-lù gōng-zuò)	文書活動 (bunsho katsudō)
resource	资源 (zī-yuán)	リソース (risōsu)
resource system	资源系统 (zī-yuán xì-tǒng)	リソースシステム (risōsu shisutemu)
instrument	工具 (gōng-jù)	道具 (dōgu)
instrumentation	工具性 (gōng-jù xìng)	用具化 (yōguka)
instrumentalization	工具化 (gōng-jù huà)	道具化 (dōguka)
scheme	图式 (tú-shì)	スキーム (sukīmu)
operational invariant	操作不变量 (cāo-zuò bú-biàn-liàng)	操作不变量 (sōsa fuhenyō)

3.1 East Asian civilization and education

3.1.1 Civilization

At the level of civilization, East Asian countries, including China, Japan, Korea, and Vietnam, share a large set of cultures, which is often called *the East Asian cultural sphere* (Choi, 2010). Although there are different societies across countries, this sphere has linguistic commonality, characterized especially by the usage of Chinese characters. Historically, the writing system with Chinese characters was principally used in East Asian countries, while the spoken language may have differences according to the area or country (e.g., Miyake, 2003).

The Chinese characters were locally adapted with some evolution in different written languages (e.g., Simplified Chinese, Traditional Chinese, and Japanese *Kanji*). In some countries, new writing systems and letters have been developed in addition to or in place of Chinese characters (e.g., Japanese, Korean, and Vietnamese alphabets). Currently, the official use of Chinese characters in everyday written language exists only in China and Japan.

Furthermore, the East Asian cultural sphere shares a tradition known as *Confucianism* (see some chapters in Leung et al. (2006)). Beyond the linguistic aspect, this tradition may characterize East Asian civilization, society, and education (Leung et al., 2015).

3.1.2 Educational culture

In China and Japan, one may find similarities in the actual circumstances of school education when it is contrasted with the West (e.g., Wu & Zhang, 2006). The two countries adopt a similar school system, and students perform relatively well in international assessments (e.g., PISA and TIMSS). In both countries, there is a critical role and status of textbooks. Although there are several textbook series from different publishers, they should be approved by the government based on the national curriculum. In principle, the teachers in both countries have an obligation to use textbooks in the classrooms. Furthermore, the teachers' work is also similar: school-based professional development through collaboration with teachers, such as the Lesson Study in Japan and the Teaching Research Group (TRG) in China (Miyakawa & Xu, 2019). Proximity between the teachers (practitioners) and the researchers is also common, and makes mathematics education researchers often get involved in the implementation of professional development (e.g., Huang et al., 2014; Takahashi, 2014).

3.1.3 Mathematics education research

Both countries have been influenced by the educational research in the West. When writing a research paper in their own language, many researchers often refer to literature from international journals or books either in the original or the translated version and try to adapt some theoretical frameworks developed by Western researchers to their study. Some well-known books that have been translated into Japanese and/or Chinese for instance, are: A. Bishop's *Mathematical Enculturation* (2011); and P. Ernest's *The Philosophy of Mathematics Education* (2015).

Another shared characteristic of mathematics education research in China and Japan is that most research can be conceived as developmental, design-oriented, or interventional (such as action research) (e.g., Sriraman et al., 2015). Empirical studies often aim to investigate an effective teaching approach, task design, or curriculum development, in contrast to research in the West that tends to be more theoretically oriented. This research tradition is crucial to understanding the specificities of research praxeologies in both China and Japan.

3.2 Chinese language and education

3.2.1 Chinese language

The Chinese characters are a type of logogram that are written symbols that represent words instead of sounds. In Chinese, there are many monosyllabic and homophonic morphemes. In layman terms, the same character can carry different meanings, properties, and pronunciations. An example is, 数 (shù) can be a noun when forming a word like mathematics (数学) and number (数字) with an appropriate character, but it can also be a verb meaning counting (shǔ), when forming a word such as 数数 (shǔ-shù), which means to count numbers. There is definitely no one-to-one relationship between the characters and words. The total number of Chinese characters is approximately 90,000, but students are not required to learn all of them. They need to know and be able to write at least 3,500 characters by the end of Grade 9, which is the end of compulsory education (MOE, 2011).

3.2.2 Chinese education

The Chinese school system before higher education has three levels: primary school, lower secondary, and upper secondary. A characteristic of Chinese mathematics education is the emphasis on “two basics” and “four basics”. The “two basics” refer to basic knowledge and skills, while the “four basics” involve two more elements: basic

mathematical thoughts and methods, as well as basic activity experiences (Fan et al., 2004). Since 2016, China has been implementing the mathematics curriculum reform oriented by subject competency (MOE 2018). The formation of these curriculum goals can be partly attributed to the trends in the international mathematics education community, but it is principally a result of the historical development and inheritance of the Chinese mathematics curriculum.

The implementation of *The Teacher Act* in 1994 resulted in teaching being officially recognized as a profession in Chinese culture. It is believed that the knowledge required for teaching can be developed from examples and by doing (Li et al., 2011). The teacher preparation programs focus, not only on the learning of the subject knowledge, but also on the teaching skills and teaching perceptions.

3.3 Japanese language and education

3.3.1 Japanese language

Japanese written texts currently consist of a combination of Chinese characters, called *Kanji*, and two kinds of Japanese letters, called *Hiragana* and *Katakana*. These Japanese letters are, like those in European alphabets, symbols that represent the unit of sound of the Japanese spoken language and do not have a specific meaning in each letter, unlike Chinese characters. There are 50 Hiragana and 50 Katakana. A one-to-one correspondence exists between them. For example, the word “mathematics” can be translated into Japanese by using these three kinds of expressions: 数学 (*Kanji*), すうがく (*Hiragana*), and スウガク (*Katakana*) – which are all pronounced as “sūgaku”.

Kanji is used to express main parts of speech such as nouns, verbs, etc. In a sentence, Hiragana are used for particles, auxiliary verbs, etc., to complement the sentence consisting of terms written in *Kanji*. Conversely, *Katakana* is usually used to express terms imported from other languages or onomatopoeic words, such as テニス (tennis pronounced as *tenisu*) and コーヒー (coffee pronounced as *kōhī*). Therefore, *Katakana* allows us to produce a phonetic expression of a foreign word.

3.3.2 Japanese education

The education system in Japan is outlined in the national curriculum published by the Ministry of Education (MEXT) for each school level (elementary, lower secondary, and upper secondary), which determines the number of hours that each subject should be taught for, the objectives, and the contents to teach in each grade. The MEXT seeks to ensure that a certain level of quality education is delivered throughout Japan, by the national curricula and the teacher

education system. Like many Western countries’ curricula, the current Japanese curriculum emphasizes “competency” in all school subjects at all grades.

Mathematics education in Japan is currently recognized internationally, thanks to the Lesson Study (e.g., Stigler & Hiebert, 1999). Japanese teachers’ lesson preparation work is called *kyōzai kenkyū* (e.g., Watanabe et al., 2008; Fujii, 2016). In *kyōzai kenkyū*, the Japanese teachers study different curricular resources (e.g., teaching guide of national curricula, textbooks and their supplementary materials, commercial books or magazines, and digital content). By this study, the teachers develop their professional and mathematical knowledge for teaching.

4 Exploring the issues of translation

4.1 Issues of linguistic origin

Several challenges, often intertwined, appeared at two levels, namely: linguistic and cultural. In this section, we investigate four main issues triggered by the Eastern languages: (1) the development of new terminology; (2) the multiple meanings and the candidates; (3) the phonetic expression; and (4) the rules of derived words.

4.1.1 Development of new terminology

A considerable difference in terms of language exists between the West and East. This can be mainly attributed to the non-use of letters of Western alphabet but also to the use of Chinese characters as well as other letters. A major issue when translating the DAD was to invent or create new terms for technical terminology, as a result of a lack of the “*natural*” *equivalence* between the source and target languages (in the sense of Pym, 2014).

Most of the translations listed in Table 1 do not make sense as a single term for people in other fields, however, the Chinese characters or the smaller units (i.e., words formed by characters) may be familiar to the local people in the way they were used regularly. This is because the translators created them as technical terms. For example, *documentation* in English is easy to translate into Romance languages such as French and Portuguese, without significant changes, such as *documentation*; and *documentação*, respectively. Most technical words used in English also exist as cognate words in most languages of Latin or Greek origin (with certain exceptions). However, this is not the case for the Chinese and Japanese. Our Chinese translation of *documentation* was 文献纪录 (*wén-xiàn jì-lù*), created using two terms that already exist in Chinese: 文献 (translated from *document*); and 纪录 (expressing both the noun and verb aspects of the

term *documentation*). This approach may produce a *translation shift*, which means a formal discrepancy between the source and target texts (e.g., Pym, 2014; Ruthven, 2022).

4.1.2 Multiple meanings and multiple candidates

A common issue faced during translation was the availability of multiple candidates that may be considered equivalent to the source text. More often than not, there is no one-to-one correspondence between the source and target terms. Our translations were therefore the result of active decisions made by ourselves, as there was a specific direction from the source language to the target language (the perspective called *directional equivalence* in translation studies; e.g., Pym, 2014).

In the case of *document*, the possible Chinese translation terms were 文献 (wén-xiàn) and 文档 (wén-dàng), and consequently, the former was chosen. A major issue when selecting possible translation terms for a specific term is considering the *literal* and *contextual meaning* of the original term; the *formal equivalence* and *dynamic equivalence* in terms of translation studies is considered (Nida, 1964 cited in Pym, 2014). The Chinese candidates given here were selected in terms of their literal meaning, but the final decision was made according to the contextual meaning using an exclusion method. Compared to 文档 (wén-dàng), 文献 (wén-xiàn) is less orally used in general, however, it is formally acceptable in the academic field (corresponding to *literature* or *references* in English). Another reason was that in the field of library and information, *document* (as 文献) is defined as a wider concept far more than the shape or form. A 文献 can be papers, articles, or books. In addition, it can refer to text/CD/photos/videos, that is, anything that provides information.

4.1.3 Phonetic expression

When translating a term from the West, a possible option is to use English or French phonetic expressions. The Chinese and Japanese languages both have such an option, which assists in achieving *formal equivalence*.

In Japanese, *Katakana* can express the phonetic elements of an imported term without using Chinese characters. This was the case for some Japanese translations. For example, our translation, リソース (risōsu), is a phonetic expression of the English term *resource*. In the Chinese language, one may also make use of a term with Chinese characters according to the pinyin pronunciation (Mandarin phonetic system). However, this is specifically used for expressing names (e.g., James is written as 詹姆斯 in Chinese), and not scientific concepts.

While it seems that a phonetic expression is a good option, as not much effort is required in finding the appropriate translation term, there are several constraints, and we cannot always make use of it. The first constraint is that phonetic expressions are less official. The scientific community, especially in China, usually avoids using phonetic expressions for scientific concepts. This is often the last option in the Chinese case and usually comes with a footnote explanation. The second constraint is the length of the term. The Japanese phonetic expression is often longer than the usual term: for example, the term *document* could be written with *Katanaka* as, ドキュメント (dokyumento), with six letters, while its translation, 文書, uses only two letters. Another constraint, especially in Japan, is that it may already have other commonly used meanings, if it already exists in the Japanese language. The issue of the *translation shift* appears here again at the semantic level.

4.1.4 Rules of derived words

One of the biggest challenges shared among us was the translation of a word derived from changing its category, an issue that usually resulted from the differences in the grammatical rules of creating the derived words between the East and West. For example, *instrument*, which is a key concept in DAD, is a noun that can be verbalized (*instrumentate* or *instrument*) or nominalized (*instrumentation* or *instrumentalization*) differently in English. There are also some rules for such word building in Chinese and Japanese. For instance, we may add a suffix 化 (huà) after the original term, which denotes an act of transforming something into an object with a target property. In the case of *instrumentalization* in Chinese, by adding the suffix 化 (huà) after 工具 (gōng-jù, *instrument*), the new term 工具化 (gōng-jù huà) obtained a meaning of the act of making something (e.g., artifact) instrumental.

The term *instrumentation*, however, has a specific meaning in DAD (Trouche et al., 2020). *Instrumentation* and *instrumentalization* denote the opposite processes when dealing with an instrument: the former is a process where “the affordances of the resource/s influence teachers’ practice” (p. 239); while the latter is a process where “the teachers’ dispositions and knowledge guide the choices and transformation processes between different resources” (p. 239). We also understand that *instrumentation* is a process of transforming a given object into an instrument with its intrinsic usage, whereas *instrumentalization* is a process of transforming anything general into an instrument. In addition to the linguistic rules and relying on these meanings, we chose two different terms implying an instrument and nominalized them with a suffix for the Japanese translation: 用具化 (yōguka) and 道具化 (dōguka) for *instrumentation*

and *instrumentalization* respectively. In general, 用具 (yōgu) denotes a more specific instrument than 道具 (dōgu) in Japan.

4.2 Issues of cultural origin

Despite the finding of an equivalent word (at literal or contextual level) of a given English (or French) word, other translation issues were faced owing to cultural differences in teachers' and researchers' terminologies in mathematics education, as well as researchers' perspectives on translation.

4.2.1 Teachers' terminologies

Some technical terms by DAD are something new for teachers in China and Japan. For instance, most teachers in both countries are not very familiar with the term *resource*, 资源 (zī-yuán) in Chinese and リソース (risōsu) in Japanese, in their professional communities. One reason is that there are already other terminologies that are used in the communities of teachers or researchers. The Lexicon project investigates such terminologies used by the teachers (e.g., Cao, et al., 2021; Shimizu et al., 2021). An example in Japanese, which is also found in China, is the terms 教材 (kyōzai; teaching material) and 教具 (kyōgu; teaching instrument), which are similar but not identical to the term *resource* in DAD. These terms differ from *resources* in two aspects. First, kyōzai or kyōgu is mainly used when referring to students and not teachers. For example, textbooks are seen as primary kyōzai and are also considered *resources* for students to learn something. However, curricular documents, such as the national curriculum and its teacher's guide, are not considered kyōzai but *resources* for teachers to prepare for teaching. Second, the use of these terms is practice-oriented rather than research-oriented, unlike the term *resource* in DAD. For these reasons, the meaning of the translated terms is not necessarily evident to Chinese and Japanese teachers, educators, or even researchers.

Furthermore, some Chinese and Japanese terms also exist that denote the teachers' work with resources from a different perspective. For example, the expression, 教材研究 (kyōzai kenkyū), in Japan refers to studying teaching materials, including textbooks, mainly for classroom teaching (see also Sect. 3.3.2). This is also the case for the Chinese term, 教材研究 (jiào-cái yán-jiū). These can be regarded as teachers' documentation work with different resources (e.g., Shinno & Mizoguchi, 2021), while they are customarily used in teachers' communities without any theoretical underpinning as terminologies that support their practices.

4.2.2 Educational researchers' terminologies

In addition to the community of teachers, there are educational researchers who use their own terminologies independently. In this community, as mentioned earlier, the scientific work published in the West has at times translated into our own Eastern language. Accordingly, some technical terms of Western origin already have their translations. For example, the term *didactics* already had its translation in Chinese and Japanese.

The presence of such terms does not, however, imply that the translation is easy, because the meaning of the pre-existing translation may not correspond to the meaning of the source texts. The DAD has emerged within the European tradition of didactics (of mathematics), in particular French didactics (Trouche et al., 2020), which shares common features such as a strong connection with mathematics, mathematicians, and the key role of theory (Blum et al., 2019). The term *didactics* in Europe denotes the art and science of teaching and learning, specific to the teaching content. The Japanese translation of didactics, 教授学 (kyōjugaku), is not necessarily used in this sense. It often refers to general pedagogy, without a particular focus on any subject matter content. Furthermore, the Chinese translation of didactics, 教学论 (jiào-xué lùn), is also misused between pedagogy and didactics, because pedagogy is also translated into this same term.

We also faced this issue with the term *scheme*, which had already been translated into Chinese and Japanese in the context of Piaget's theory. In Japan, however, there is confusion regarding this technical term as a result of its English translation and the two similar French terms *schème* and *schéma* (Nakagaki, 2007). In an English translation (Piaget, 1936), the English term *schema* (and *schemata* in plural form) is attributed to the French term *schème* (Piaget, 1952). In addition, since the term *schéma* means a diagram in everyday French language, the term *schème* of Piaget's work has been often translated into 図式 (zushiki), which means a diagram, or into シェマ (shema), which is a phonetic expression of the French term *schéma*. There are Japanese translators that have adopted other phonetic expressions such as スキーマ (sukīma) (English term *schema*) and シェーム (shēmu) (French term *schème*). We accordingly chose the English phonetic expression スキーム (sukīmu) for the term *scheme* in DAD, since this term became common today in Japan more than the French phonetic expression シェーム (shēmu).

4.2.3 Researchers' perspectives

When making use of a new term for translation, we usually face issues regarding how to preserve the original meaning.

In this process, the researchers' perspectives strongly affect their choices and decisions. The translation is under the effect of what the translator thinks the purpose should be: the perspective called *purpose paradigm* in translation studies (Pym, 2014). For example, one may see this issue in the translation of the theory's name, *Documentational Approach to Didactics*.

In relation to the Chinese translation, we decided to use 文献纪录教学论 (*wén-xiàn jì-lù jiào-xué lùn*). The term *didactics* was translated into 教学论 (*jiào-xué lùn*) as mentioned above, *documentational* into 文献纪录 (*wén-xiàn jì-lù*), while *approach* was omitted in the translation. The issue was the last character 论 (*lùn*), which means *theory*. Another option to preserve the meaning of *approach* was to use 法 (*fǎ*) instead of 论 (*lùn*), which means the method or approach. However, 教学法 (*jiào-xué-fǎ*) refers to the method of teaching or approach, while 教学论 (*jiào-xué lùn*) is the theory of teaching and learning. We chose the latter because, from the researcher's perspective, we considered DAD as a theory even though there is an "approach" in the name.

For the Japanese side, we chose, after a lengthy discussion, the long translation 教授文書活動研究法 (*kyōju bunsho katsudō kenkyūhō*). The term *didactics* is translated into 教授 (*kyōju*), *documentational* into 文書活動 (*bunsho katsudō*), and *approach* into 研究法 (*kenkyūhō*). Our translation of the term *approach* is a common term in scientific areas and is often used to denote an approach or a research method. In general, one may understand the translation 教授文書活動 (*kyōju bunsho katsudō*) as the activities related to the documents used for teaching, and thus the DAD as an approach to studying these activities. Regarding the term *didactics*, we decided to use the term 教授 (*kyōju*) instead of 教授学 (*kyōjugaku*). 教授 (*kyōju*) without the character 学 (*gaku*) means teaching or instruction. While the translation is not exactly what the source text literally denotes, we consider that the target text fits well with the scientific work done with DAD, based on the understanding obtained of DAD as a researcher. In other words, the choice is made in accordance with the researchers' interpretations of the theoretical framework and expectations (or *purposes*) of how the theoretical term could be received in their local contexts.

5 Discussion

In this section, we return to the two research questions (see Sect. 2) and discuss their answers and contributions to the DAD.

5.1 Conditions and constraints in the translation work

The RQ1 addresses the conditions and constraints that produce translation issues from a comparative perspective. In terms of ATD, the conditions and constraints that emerged during the dialogue between the Chinese and the Japanese scholars mostly belong to relatively higher levels of didactic co-determinacy (such as pedagogy, school, society, and civilization), rather than lower levels within a didactic system (such as the theme, sector, domain, or discipline). This can be attributed to the methodology of our work, which deals with the translation of the theoretical framework from the Western to the Eastern languages, and not with the empirical data of mathematics teaching, such as teachers' practices or interviews.

Some of the conditions and constraints entailed in our translation work have already been described in Sect. 3 as our local contexts regarding the 'language', 'education', and 'research'. Among them, the language was one of the biggest constraints that caused difficulties in the translation and shaped the theoretical concepts of DAD. The Chinese and Japanese languages share many characteristics that do not exist in Western languages. One may see here a constraint at the level of "civilization" beyond the "society".

We have observed that there are different cultural and institutional elements that affect the development of terminologies. In particular, the local contexts of the teachers' practices constitute a constraint when choosing the translation terms, which led to the consideration of how the translated terms would be received by the teachers in each country. This is a constraint at the "school" or "pedagogy" level.

It is also noted that some difficulties and our translation work can be explained in terms of the concepts introduced in translation studies. For example, the concepts of natural or directional equivalence between the source and target texts, formal or dynamic translations, purpose paradigm, and translation shift shed light on the ideas implicitly used during the translation work, which could be modeled as a logos block of *translation* or *transposition praxeology*. Further investigation of this praxeology would reveal some conditions and constraints specific to the translation work.

5.2 Research praxeology in the East

In relation to the RQ2, a critical aspect highlighted in the exploration of the transposition of research praxeology, is the nature of research praxeology (C-RP and J-RP in Fig. 1) with respect to didactic/paradidactic praxeology (C-DP and J-DP) and the conditions and constraints that shape them in the East.

Translation work is a practice that most Chinese and Japanese researchers are engaged in when conducting research or writing research papers. For example, Sato-Rossberg and Wakabayashi (2012) mentioned that the translation work in general has led to a considerable body of research and thinking in the Japanese culture. This is also the case in China (Yang, 2015). As far as mathematics education is concerned, the transposition of research knowledge is one of the important tasks that constitute research praxeology in the East. As we have seen, this practice is strongly influenced by the linguistic and cultural specificities of the target institutions. Mathematics education researchers generally play multiple roles in China and Japan. In addition to scientific research, they are very often engaged in practice or development such as teacher education, working with teachers, and playing the role of a “knowledgeable other” in the Lesson Study or TRG (Huang & Shimizu, 2016; Miyakawa & Xu, 2019); as well as writing and editing official school textbooks, writing books and articles for teachers, and so forth. This is one of the reasons why we often considered terminologies used by teachers (such as *kyōzai kenkyū*) in relation to theoretical terms (such as *documentation work*) during the translation work (see Sect. 4.2.1).

In contrast, the evolution of scientific discipline requires the development of new theoretical concepts and terminologies, other than those spontaneously used in practice, to better understand the object of study (Bourdieu et al., 1968/2005). The use of such terminology is more or less shared within the European tradition of didactics (Blum et al., 2019). However, in mathematics education in the East, teachers and researchers often share terminology for collaborative work. In other words, research praxeology and didactic/paradidactic praxeology often share elements of a logos block. Accordingly, the existence of the terminology used by teachers affects the researchers’ translation work. One may identify specificity in the East, that is, the close relationship between research praxeology and didactic/paradidactic praxeology.

Research practices in the East have been strongly oriented toward the improvement of mathematics teaching and learning, in which teachers and researchers share the same types of tasks and goals. For example, Shimizu (2015) wrote: “Continuous working with, and learning from, teachers raises the issues and shapes the research questions originated in the efforts of improvement of teaching and learning mathematics in the classroom” (p. 1291). As a result of this orientation toward the practice, a “theoretical framework” is often received in China and Japan as a prescriptive or normative tool, which can be used for developing, designing, and improving educational practices. A ‘didactic theory’ is understood in such a case as a ‘pragmatic theory’ to be useful for the teachers’ practice (an element of Logos C-DP and

J-DP) rather than a scientific one to elucidate or understand the didactic phenomena (an element of Logos W-RP).

5.3 Contribution to DAD

One of the goals of the DAD Multilingual project was to deepen the concepts of DAD and diversify the theoretical framework by adapting it into different contexts. In the process of translation, we tried to better understand the concepts proposed in this theoretical framework, but did not necessarily deepen them. Instead, the specificity of the relationship between the research and didactic praxeologies implies that discussion on the viability of the theoretical framework as a result of the transposition is critical for the dissemination of DAD.

During the translation work, the focus was mainly on the logos block of the research praxeology, as the theoretical framework constitutes the logos block. However, the translation of a theoretical framework may not necessarily lead to the transposition of research praxeology as a whole, because it also includes the praxis block related to the research questions and method. In our case, while some descriptions of the methodology of DAD were given in the translated article (Trouche et al., 2020), this did not suffice for a theoretical framework to be viable as a research praxeology in the East. In the target institution, as discussed above, another cultural tradition of scientific research (or research praxeology) always exists. The types of tasks in the existing research praxeology in the East are not necessarily identical to those in DAD. In the case of Japan, the teachers’ work is a relatively new object of study in the community of mathematics education research, because it has been considered a practice to be engaged as a “knowledgeable other” rather than an object of study. To transpose the praxis block of DAD to the East, it would be necessary to have a further translation of the scientific papers or other activities (e.g., workshops, lectures, and supervision of PhD students), documenting and disseminating the research practices with DAD. In particular, the types of tasks or research questions, their rationales, and among others, the implications for the teaching practices or teacher education, are of relevance owing to the close relationship between research praxeology and didactic praxeology in the East.

6 Conclusion and perspectives

This study conceptualizes the translation work from the perspective of ATD as a transposition of research praxeologies from one institution to another. We initially expected such a study to identify several cultural specificities of teachers’ practices related to resources. Consequently, the

specificities found are more about the practices of researchers, particularly those related to the transposition of research praxeologies. This study allowed us to better understand the conditions and constraints entailed in the Chinese and Japanese researchers' work. Although the cultural specificities of mathematics educational practices have been discussed so far from a comparative perspective between the West and the East (e.g., Kaiser & Blömeke, 2013; Leung et al., 2006), such discussions are rarely extended to those in research practice. This is one of the contributions of this study.

There are several directions for future studies. Further investigations are necessary to elucidate the cultural and linguistic specificities of teachers' work in relation to the resources in China and Japan (or other countries). This aspect has not been fully deepened through the exploration of translation issues in this study. The investigation of the terminologies used by teachers and discourses related to the use or development of resources would allow us to better understand the teachers' practices and knowledge, as well as their expertise, as the Lexicon project does in the case of pedagogical terms associated with lessons (Mesiti et al. 2021). Concerning the linguistic aspect, linguistic theory would be helpful in more thoroughly analyzing how language acts in a context. It will also be interesting to further investigate the characteristics of translation work in mathematics education, as Ruthven (2022) did, from the perspective of translation studies (e.g., Pym, 2011, 2014), which have been developed in Western contexts (but it is worth investigating the case of non-Western languages in our research field). Furthermore, regarding research praxeology and its transposition, it is necessary to investigate the conditions for the transposition of the praxis block in an empirical study.

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