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DISCUSSION

Rejoinder on: Remarkable polyhedra related to set functions, games and capacities

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I address all my thanks to the three discussants, who carefully read the paper, appreciated it and made many interesting and stimulating comments.

Marina Nuñéz made many suggestions and completed the references about the problem of finding the vertices of the core in general and for some classes of games, a topic to which she has greatly contributed, and I am very thankful to her for that. In fact, Peter Sudhölter and I also started to study the question of finding the vertices of the core, in the more general context of games with precedence constraints, using reduced games as in Nuñéz and Rafels (1998). We discovered that the idea of obtaining vertices of the core by choosing an order on the players and then successively for each player maximizing or minimizing his payoff within the core was mentioned in Derks and Kuipers (2002), after Chapter 6 of the Ph.D. thesis of Kuipers (1994). As mentioned by Marina Nuñéz, this procedure is known to work for several classes of games (assignment games, minimum cost spanning tree games, cyclic permutation games, etc.) but not in general. Anyhow, the suggestion of using the exact game being the lower envelope of the core of the considered game as in Izquierdo et al. (2007) seems to be an idea to be exploited.

Hans Peters raised the interesting question whether there exist interpretations in the decision theory of several concepts introduced in game theory, like multichoice games, and the Möbius transform. The context and aims of these theories being radically different, I am afraid that it is difficult to find such interpretations, at least in decision under risk an uncertainty. The main concern of decision theory is to provide

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a numerical representation of preferences, and most of the time, this is done through a kind of integral with respect to a capacity, or a lower envelope of integrals with respect to probability measures, a device which is absent in game theory. Another fundamental difference is that the set function which is considered (capacity) is representing uncertainty, and not power as in voting games or benefit due to cooperation in general TU games. Then, the idea of k-ary capacity or multichoice game (more generally, fuzzy game) would mean that an event A realizes to some degree, which in most cases is hard to imagine. However, the two theories become much closer if we consider decision under multiple criteria instead. There, the set N of players becomes the set of criteria, and the capacity models the importance (power) of a set of criteria, more precisely, $v(A)$ gives the overall evaluation of an alternative being fully satisfactory on all criteria in A, and nonsatisfactory otherwise. The generalization to k-ary capacities is then immediate: it consists in distinguishing several levels of satisfaction. In addition, the Möbius transform can be interpreted in this context as an interaction or synergy between the criteria [see a survey of decision under multiple criteria using capacities in Grabisch and Labreuche (2010)].

Peter Sudhölter raises the question that, although it has interesting mathematical properties, the k-additive core does not seem to have a clear interpretation in game theory. It seems that despite my efforts in the past, the concept of k-additive core is not well accepted in the community of game theory, and the remark of Peter Sudhölter is not isolated. I admit that, when Pedro Miranda and I proposed this idea, we were more attracted by the mathematical side, where it appeared as a very natural generalization: in some sense, it is an approximation problem, and passing from the core to the k-additive core is like passing from polynomials of degree 1 to polynomials of degree k. We then discovered two unpleasant features: the k-additive core is a very large set, and it is not a set of payoff vectors, like any game theorist would expect to obtain from a solution concept. I am, however, still convinced that this could open the door to a new kind of solution concept, done in two steps: in the first step, while keeping the fundamental idea of coalitional rationality, payoffs are given to individuals, but also to some coalitions (in which case, these payoffs may be negative). In a second step, these coalitions should find an agreement between their members by any means (using, e.g., bargaining theory or bankruptcy theory) to come up with a sharing among the players. Using results in Gonzalez and Grabisch (2015), one can select in the k-additive core elements, such that there is a minimum number of non-individual payoff vectors, so as to minimize the size of bargaining problems to solve. This subset of the k-additive core is called the minimum negotiation set. Let me finally remark that the extended core of Bejan and Gomez (2009) is in a similar vein, but provides a solution which is at the other extremity: in the extended core, the whole society N of players has to pay some debt when the classical core is empty, while in the minimum negotiation set, only some coalitions as small as possible have to pay some debt.

As remarked by Peter Sudhölter, the paper cannot provide a complete account of all the mentioned topics, and some important references are missing, among which Derks and Kuipers (2002), which I added in the meantime, together with another paper of Kuipers et al. (2010). I apologize for any other important reference I may have overlooked.

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