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Delay-dependent Robust Stabilization and H_∞ Control for Uncertain Stochastic T-S Fuzzy Systems with Discrete Interval and Distributed Time-varying Delays

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Abstract: In this paper, delay-dependent robust stabilization and H_∞ control for uncertain stochastic Takagi-Sugeno (T-S) fuzzy systems with discrete interval and distributed time-varying delays are discussed. The purpose of the robust stochastic stabilization problem is to design a memoryless state feedback controller such that the closed-loop system is mean-square asymptotically stable for all admissible uncertainties. In the robust H_∞ control problem, in addition to the mean-square asymptotic stability requirement, a prescribed H_∞ performance is required to be achieved. Sufficient conditions for the solvability of these problems are proposed in terms of a set of linear matrix inequalities (LMIs) and solving these LMIs, a desired controller can be obtained. Finally, two numerical examples are given to illustrate the effectiveness and less conservativeness of our results over the existing ones.

Keywords: Robust stability, robust H_∞ control, stochastic fuzzy systems, distributed delay, linear matrix inequality (LMI).

1 Introduction

In modeling of dynamical systems, Takagi-Sugeno (T-S) fuzzy systems[1] provide an alternative approach to the control of plants that are complex, uncertain, and ill-defined. In the last two decades, with wide applications from consumer products to industrial processes, T-S fuzzy model[1−5] is proven to be effective universal approximations over differential geometric and differentiable algebraic methods. By making use of simple fuzzy reasoning rules and fuzzy inference methods, it provides a basis for development of systematic approaches to stability, stabilization, H_∞ control and filtering problems[6−13].

Time delays are often encountered in many industrial and engineering systems such as chemical processes, rolling mill systems, networked control systems, etc. It is well known that time delays can cause poor performance or instability. Therefore, the problem of delay-dependent stability analysis and controller synthesis for T-S fuzzy systems with time delays have received great efforts by many researchers in recent years. Moreover, delay-dependent approaches[6, 9, 14] are generally less conservative than delay-independent[2] ones when the sizes of time delays are small. Recently, the delay-dependent stabilization and H_∞ control of T-S fuzzy systems with interval time-varying delay are discussed[15, 16]. Robust stability, stabilization and H_∞ controller design of discrete and distributed time delays with or without fuzzy systems are considered[17−19].

In the past few years, stochastic nonlinear systems have received much attention since stochastic modeling has come to play an important role in many branches of science and engineering applications. For instance, stabilization, H_∞ control, and H_∞ filtering problems for linear and nonlinear stochastic systems have been considered[20−26]. The control technique based on the so-called T-S fuzzy model has attracted lots of attention. Recently, some attempts have been made to use T-S fuzzy model based control technique for stochastic nonlinear systems[27−30]. Very recently, the delay-dependent robust H_∞ control for uncertain stochastic T-S fuzzy systems with time delays have been discussed in [31, 32]. However, to the best of our knowledge, the delay-dependent robust stabilization and H_∞ control for uncertain stochastic T-S fuzzy systems with discrete interval and distributed time-varying delays have not yet been fully investigated and this will be the goal of this paper.

In this paper, we investigate the problem of the delay-dependent robust stabilization and H_∞ control for uncertain stochastic T-S fuzzy systems with discrete interval and distributed time-varying delays. The uncertainties are assumed to be norm bounded and time-varying. For the robust stabilization problem, a state feedback fuzzy controller is designed such that the closed-loop system is mean-square asymptotically stable for all admissible uncertainties, while for the robust H_∞ control problem, a state feedback fuzzy controller is designed such that the closed-loop system is not only mean-square asymptotically stable but also guarantees a prescribed H_∞ performance level. Sufficient conditions for the solvability of these problems are obtained, and desired state feedback controllers can be constructed by solving certain LMIs. Further, two numerical examples are given to illustrate the effectiveness of the proposed approach.

Throughout this paper, notation $X \geqslant Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). I denotes the identity matrix of appropriate dimension. $L_2[0, \infty)$ is the space of square integrable vector. Moreover, let $(\Omega,\ \mathcal{F},\ \{\mathcal{F}_t\}_{t\geqslant 0},\ \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions (i.e., the filtration contains all $\mathcal{P}$-null sets and is right continuous). The symmetric elements of the symmetric matrix will be denoted by $*$. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible di-

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mensions for algebraic operations.

2 Problem formulation

Consider the following uncertain stochastic T-S fuzzy model with discrete and distributed time-varying delays described by

Plant rule i: If $\theta_1(t)$ is η_{i1} and $\theta_2(t)$ is η_{i2} and $\cdots$ and $\theta_p(t)$ is η_{ip},

then

$$(\Sigma):\ \mathrm{d}x(t) = \Big[(A_i+\Delta A_i(t))x(t)+(A_{di}+\Delta A_{di}(t))\times x(t-\tau(t))+ (B_{1i}+\Delta B_{1i}(t))u(t)+B_{v_{1i}}v(t)+B_{d_{1i}}\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big]\mathrm{d}t+ \Big[(C_i+\Delta C_i(t))x(t)+(C_{di}+\Delta C_{di}(t))\times x(t-\tau(t))+ (B_{2i}+\Delta B_{2i}(t))u(t)+B_{v_{2i}}v(t)+B_{d_{2i}}\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big]\mathrm{d}w(t) \tag{1}$$

$$z(t)=D_ix(t)+D_{di}x(t-\tau(t))+B_{3i}u(t) \tag{2}$$

$$x(t)=\phi(t),\quad \forall t\in[-\tau,0],\quad i=1,2,\cdots,r \tag{3}$$

where η_{ij} is the fuzzy set, $\theta_1(t),\theta_2(t),\cdots,\theta_p(t)$ are the premise variables, r is the number of IF-THEN rules of T-S fuzzy model, $x(t)\in\mathbf{R}^n$ is the state, $u(t)\in\mathbf{R}^m$ is control input, $v(t)\in\mathbf{R}^p$ is a disturbance input which belongs to $L_2[0,\ \infty)$, $z(t)\in\mathbf{R}^q$ is controlled output vector, and $\omega(t)\in\mathbf{R}^n$ is a one-dimensional Brownian motion defined on the probability space $(\Omega,\ \mathcal{F},\ \{\mathcal{F}_t\}_{t\geqslant 0},\ \mathcal{P})$ satisfying $\mathcal{E}\{\mathrm{d}w(t)\}=0$, $\mathcal{E}\{\mathrm{d}w(t)^2\}=\mathrm{d}t$. In the above system (Σ), A_i, A_{di}, B_{1i}, $B_{v_{1i}}$, $B_{d_{1i}}$, C_i, C_{di}, B_{2i}, $B_{v_{2i}}$, $B_{d_{2i}}$, D_i, D_{di} and B_{3i} are known real constant matrices with appropriate dimensions. $\Delta A_i(t)$, $\Delta A_{di}(t)$, $\Delta B_{1i}(t)$, $\Delta C_i(t)$, $\Delta C_{di}(t)$ and $\Delta B_{2i}(t)$ are unknown matrices representing time-varying parameter uncertainties, $\tau(t)$ and $d(t)$ are bounded continuous time-varying delays satisfying

$$0\leqslant\tau_m\leqslant\tau(t)\leqslant\tau_M,\quad \dot{\tau}(t)\leqslant\mu<\infty,\ 0\leqslant d(t)\leqslant d_M \tag{4}$$

where τ_m, τ_M, μ and d_M are real constant scalars. Let $\tau=\max\{\tau_M,d_M\}$. $\phi(t)$ is real valued continuous initial function on $[-\tau,0]$. In this paper, the parameter uncertainties are assumed to be of the form

$$\Big[\Delta A_i(t)\ \Delta A_{di}(t)\ \Delta B_{1i}(t)\ \Delta C_i(t)\ \Delta C_{di}(t)\ \Delta B_{2i}(t)\Big]= E_iF_i(t)\Big[H_{1i}\ H_{2i}\ H_{3i}\ H_{4i}\ H_{5i}\ H_{6i}\Big] \tag{5}$$

where E_i, H_{1i}, H_{2i}, H_{3i}, H_{4i}, H_{5i} and H_{6i} are known real constant matrices with appropriate dimensions, and $F_i(t)$ is an unknown real time-varying matrix function satisfying

$$F_i^{\mathrm{T}}(t)F_i(t)\leqslant I. \tag{6}$$

It is assumed that all elements of $F_i(t)$ are Lebesgue measurable. $\Delta A_i(t)$, $\Delta A_{di}(t)$, $\Delta B_{1i}(t)$, $\Delta C_i(t)$, $\Delta C_{di}(t)$ and $\Delta B_{2i}(t)$ are said to be admissible if both (5) and (6) hold.

By using center average defuzzifier, product inference and singleton fuzzifier, the global dynamics of the T-S fuzzy system (Σ) can be inferred as

$$(\Sigma_1):\ \mathrm{d}x(t)= \sum_{i=1}^{r}h_i(\theta(t))\Big\{\Big[(A_i+\Delta A_i(t))x(t)+ (A_{di}+\Delta A_{di}(t))x(t-\tau(t))+(B_{1i}+\Delta B_{1i}(t))u(t)+ B_{v_{1i}}v(t)+B_{d_{1i}}\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big]\mathrm{d}t+ \Big[(C_i+\Delta C_i(t))x(t)+(C_{di}+\Delta C_{di}(t))x(t-\tau(t))+ (B_{2i}+\Delta B_{2i}(t))u(t)+B_{v_{2i}}v(t)+ B_{d_{2i}}\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big]\mathrm{d}w(t)\Big\} \tag{7}$$

$$z(t)=\sum_{i=1}^{r}h_i(\theta(t))\Big\{D_ix(t)+D_{di}x(t-\tau(t))+B_{3i}u(t)\Big\} \tag{8}$$

$$x(t)=\phi(t),\quad \forall t\in[-\tau,0] \tag{9}$$

where $h_i(\theta(t))=\frac{\nu_i(\theta(t))}{\sum_{i=1}^{r}\nu_i(\theta(t))}$, $\nu_i(\theta(t))=\Pi_{j=1}^{p}\eta_{ij}(\theta_j(t))$, and $\eta_{ij}(\theta_j(t))$ is the grade of membership value of $\theta_j(t)$ in η_{ij}. In this paper, we assume that $\nu_i(\theta(t))\geqslant 0$ for $i=1,2,\cdots,r$ and $\sum_{i=1}^{r}\nu_i(\theta(t))>0$ for all t. Therefore, $h_i(\theta(t))\geqslant 0$ (for $i=1,2,\cdots,r$), and $\sum_{i=1}^{r}h_i(\theta(t))=1$ for all t. In the sequel, for simplicity, we use h_i to represent $h_i(\theta(t))$.

Based on the parallel distributed compensation schemes, a fuzzy model of a state feedback controller for the system (Σ_1) is formulated as follows:

Control rule i: If $\theta_1(t)$ is η_{i1} and $\theta_2(t)$ is η_{i2} and $\cdots$ and $\theta_p(t)$ is η_{ip}, then

$$u(t)=K_ix(t),\quad i=1,2,\cdots,r. \tag{10}$$

The overall state feedback fuzzy control law is represented by

$$u(t)=\sum_{i=1}^{r}h_iK_ix(t) \tag{11}$$

where K_i $(i=1,2,\cdots,r)$ are the local control gains. Under control law (11), the overall closed-loop system is obtained as

$$(\Sigma_2):\ \mathrm{d}x(t)= \Big[A_Kx(t)+A_dx(t-\tau(t))+B_{v_1}v(t)+B_{d_1}\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big]\mathrm{d}t+ \Big[C_Kx(t)+C_dx(t-\tau(t))+B_{v_2}v(t)+B_{d_2}\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big]\mathrm{d}w(t) \tag{12}$$

$$z(t)=D_Kx(t)+D_dx(t-\tau(t)) \tag{13}$$

$$x(t) = \phi(t), \quad \forall t \in [-\tau, 0] \tag{14}$$

where

$$A_K = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (A_i + B_{1i}K_j + \Delta A_i(t) + \Delta B_{1i}(t)K_j)$$

$$A_d = \sum_{i=1}^{r} h_i (A_{di} + \Delta A_{di}(t))$$

$$B_{v_1} = \sum_{i=1}^{r} h_i B_{v_{1i}}$$

$$B_{d_1} = \sum_{i=1}^{r} h_i B_{d_{1i}}$$

$$C_K = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (C_i + B_{2i}K_j + \Delta C_i(t) + \Delta B_{2i}(t)K_j)$$

$$C_d = \sum_{i=1}^{r} h_i (C_{di} + \Delta C_{di}(t))$$

$$B_{v_2} = \sum_{i=1}^{r} h_i B_{v_{2i}}$$

$$B_{d_2} = \sum_{i=1}^{r} h_i B_{d_{2i}}$$

$$D_K = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j (D_i + B_{3i}K_j)$$

$$D_d = \sum_{i=1}^{r} h_i D_{di}.$$

Let us introduce the following definition and lemmas that are useful for the development of our results.

Definition 1[25]**.** The nominal system (7) and (9) with $u(t) = 0$ and $v(t) = 0$ is said to be mean-square stable if for any $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that $\mathcal{E}\{|x(t)|^2\} < \varepsilon$ when

$$\sup_{-\tau \leqslant s \leqslant 0} \mathcal{E}\{|\phi(s)|^2\} < \delta(\varepsilon).$$

In addition,

$$\lim_{t\to\infty} \mathcal{E}\{|x(t)|^2\} = 0$$

for any initial conditions, then the nominal system (7) and (9) with $u(t) = 0$ and $v(t) = 0$ is said to be mean-square asymptotically stable. The uncertain stochastic system (7) and (9) is said to be robustly stochastically stable if the system associated to (7) and (9) with $u(t) = 0$ and $v(t) = 0$ is mean-square asymptotically stable for all admissible uncertainties $\Delta A_i(t)$, $\Delta A_{di}(t)$, $\Delta B_{1i}(t)$, $\Delta C_i(t)$, $\Delta C_{di}(t)$ and $\Delta B_{2i}(t)$.

In this paper, our aim is to develop techniques of robust stochastic stabilization and robust H_∞ control for the stochastic fuzzy system (Σ_2). More specifically, we are concerned with the following two problems:

1) Robust stabilization problem: Design a state feedback controller (11) for the system (7) and (9) with $v(t) = 0$ such that the resulting closed-loop system (12) and (14) with $v(t) = 0$ is mean-square asymptotically stable for all admissible uncertainties. In this case, the system (12) and (14) with $v(t) = 0$ is robustly stochastically stablilizable.

2) Robust H_∞ control problem: Given a scalar $\gamma > 0$, design a state feedback controller in the form of (11) for system (Σ_1) such that, for all admissible uncertainties, the resulting closed-loop system (Σ_2) is mean-square asymptotically stable, and for any non-zero $v(t) \in L_2[0, \infty)$, $\|z(t)\|_{\mathcal{E}_2} < \gamma\|v(t)\|_2$ is satisfied under zero initial condition. In this case, the system (Σ_2) is robustly stochastically stabilizable with disturbance attenuation level γ.

Lemma 1[33]**.** For any vectors $x, y \in \mathbf{R}^n$, matrices $P \in \mathbf{R}^{n\times n}$, $D \in \mathbf{R}^{n\times n_f}$, $E \in \mathbf{R}^{n_f\times n}$ and $F \in \mathbf{R}^{n_f\times n_f}$ with $P > 0$, $\| F \| \leqslant 1$, and scalar $\varepsilon > 0$, we have

1) $2x^{\rm T}y \leqslant x^{\rm T}P^{-1}x + y^{\rm T}Py$,
2) $DFE + E^{\rm T}F^{\rm T}D^{\rm T} \leqslant \varepsilon^{-1}DD^{\rm T} + \varepsilon E^{\rm T}E$.

Lemma 2[21]**.** For any constant matrix $M > 0$, any scalars a and b with $a < b$, and a vector function $x(t) : [a, b] \to \mathbf{R}^n$ such that the integrals concerned are well defined, the following holds:

$$\Big[\int_a^b x(s){\rm d}s\Big]^{\rm T} M \Big[\int_a^b x(s){\rm d}s\Big] \leqslant (b-a)\int_a^b x^{\rm T}(s)Mx(s){\rm d}s.$$

Lemma 3[27]**.** For any real matrices X_{ij} for $i, j = 1, 2, \cdots, r$ and $\Lambda > 0$ with appropriate dimensions, we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i h_j h_k h_l X_{ij}^{\rm T}\Lambda X_{kl} \leqslant \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j X_{ij}^{\rm T}\Lambda X_{ij}$$

where h_i $(1 \leqslant i \leqslant r)$ are defined as $h_i(\theta(t)) \geqslant 0$, $\sum_{i=1}^{r} h_i(\theta(t)) = 1$.

3 Robust stochastic stabilization

In this section, we shall present a sufficient condition for the uncertain stochastic fuzzy system (12) and (14) with $v(t) = 0$ to be robustly stochastically stabilizable in terms of LMIs. The design of the fuzzy controller is to determine the local feedback gains $K_i (i = 1, 2, \cdots, r)$ such that the system (12) and (14) with $v(t) = 0$ is robustly stochastically stabilizable. When there are no parameter uncertainties in the system (12) and (14) with $v(t) = 0$, Theorem 1 is specialized as follows.

Theorem 1. For given scalars τ_m, τ_M, d_M and μ, the time-varying delays satisfying (4), the closed-loop stochastic fuzzy system (12) and (14) with $v(t) = 0$ and $\Delta A_i(t) = \Delta A_{di}(t) = \Delta B_{1i}(t) = \Delta C_i(t) = \Delta C_{di}(t) = \Delta B_{2i}(t) = 0$ is stochastically stabilizable if there exist matrices $X > 0$, $\bar{Q}_s > 0$ $(s = 1, 2, 3)$, $\tilde{R}_l > 0$ $(l = 1, 2, 3, 4)$, $\bar{Z} > 0$ and real matrices $\bar{N}_{lij}$, $\bar{M}_{lij}$, $\bar{S}_{lij}$, Y_j $(l = 1, 2,\ 1 \leqslant i \leqslant j \leqslant r)$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} \Xi_{11}^{ii} & \Xi_{12}^{ii} & \Xi_{13}^{ii} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} < 0, \quad 1 \leqslant i \leqslant r \tag{15}$$

$$\begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & \Xi_{13}^{ij} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} + \begin{bmatrix} \Xi_{11}^{ji} & \Xi_{12}^{ji} & \Xi_{13}^{ji} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33} \end{bmatrix} < 0,$$

$$1 \leqslant i < j \leqslant r \tag{16}$$

where

$$\Xi_{11}^{ij}=\begin{bmatrix}\phi_{11}^{ij} & \phi_{12}^{ij} & \bar{M}_{1ij} & -\bar{S}_{1ij} & B_{d_{1i}}X\\ * & \phi_{22}^{ij} & \bar{M}_{2ij} & -\bar{S}_{2ij} & 0\\ * & * & -\bar{Q}_2 & 0 & 0\\ * & * & * & -\bar{Q}_3 & 0\\ * & * & * & * & -\dfrac{1}{d_M}\bar{Z}\end{bmatrix}$$

$$\Xi_{12}^{ij}=\begin{bmatrix}\hat{C}_{ij}^{\rm T} & \tau_M\hat{A}_{ij}^{\rm T} & \bar{\tau}\hat{A}_{ij}^{\rm T} & \tau_M\hat{C}_{ij}^{\rm T} & \bar{\tau}\hat{C}_{ij}^{\rm T}\end{bmatrix}$$

$$\Xi_{13}^{ij}=\begin{bmatrix}\tau_M\bar{N}_{ij} & \bar{\tau}\bar{M}_{ij} & \bar{\tau}\bar{S}_{ij} & \bar{N}_{ij} & \bar{M}_{ij} & \bar{S}_{ij}\end{bmatrix}$$

$$\Xi_{22}=\text{diag}\Big\{-X,\ -\tau_M\tilde{R}_1,\ -\bar{\tau}\tilde{R}_2,\ -\tau_M\tilde{R}_3,\ -\bar{\tau}\tilde{R}_4\Big\}$$

$$\begin{aligned}\Xi_{33}=\text{diag}\Big\{&-2\tau_M X+\tau_M\tilde{R}_1,\ -2\bar{\tau}X+\bar{\tau}\tilde{R}_2,\\ &-4\bar{\tau}X+\bar{\tau}\tilde{R}_1+\bar{\tau}\tilde{R}_2,\ -2X+\tilde{R}_3,\ -2X+\tilde{R}_4,\\ &-4X+\tilde{R}_3+\tilde{R}_4\Big\}\end{aligned}$$

with

$$\begin{aligned}\phi_{11}^{ij}=&\bar{Q}_1+\bar{Q}_2+\bar{Q}_3+\bar{N}_{1ij}+\bar{N}_{1ij}^{\rm T}+(A_iX+B_{1i}Y_j)+\\ &(A_iX+B_{1i}Y_j)^{\rm T}+d_M\bar{Z}\\ \phi_{12}^{ij}=&\bar{S}_{1ij}-\bar{N}_{1ij}+\bar{N}_{2ij}^{\rm T}-\bar{M}_{1ij}+A_{di}X\\ \phi_{22}^{ij}=&-(1-\mu)\bar{Q}_1-\bar{N}_{2ij}-\bar{N}_{2ij}^{\rm T}+\bar{S}_{2ij}+\bar{S}_{2ij}^{\rm T}-\bar{M}_{2ij}-\bar{M}_{2ij}^{\rm T}\\ \hat{A}_{ij}^{\rm T}=&\Big[A_iX+B_{1i}Y_j\quad A_{di}X\quad 0\quad 0\quad B_{d_{1i}}X\Big]^{\rm T}\\ \hat{C}_{ij}^{\rm T}=&\Big[C_iX+B_{2i}Y_j\quad C_{di}X\quad 0\quad 0\quad B_{d_{2i}}X\Big]^{\rm T}\\ \bar{N}_{ij}=&\Big[\bar{N}_{1ij}^{\rm T}\quad \bar{N}_{2ij}^{\rm T}\quad 0\quad 0\quad 0\Big]^{\rm T}\\ \bar{M}_{ij}=&\Big[\bar{M}_{1ij}^{\rm T}\quad \bar{M}_{2ij}^{\rm T}\quad 0\quad 0\quad 0\Big]^{\rm T}\\ \bar{S}_{ij}=&\Big[\bar{S}_{1ij}^{\rm T}\quad \bar{S}_{2ij}^{\rm T}\quad 0\quad 0\quad 0\Big]^{\rm T}\\ \bar{\tau}=&\tau_M-\tau_m.\end{aligned}$$

Moreover, the state feedback gain can be constructed as $K_j=Y_jX^{-1}$ $(j=1,2,\cdots,r)$.

Proof. Let

$$\begin{aligned}A_K&=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j(A_i+B_{1i}K_j)\\ A_d&=\sum_{i=1}^{r}h_iA_{di}\\ B_{d_1}&=\sum_{i=1}^{r}h_iB_{d_{1i}}\\ C_K&=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j(C_i+B_{2i}K_j)\\ C_d&=\sum_{i=1}^{r}h_iC_{di}\\ B_{d_2}&=\sum_{i=1}^{r}h_iB_{d_{2i}}\end{aligned}$$

then the closed-loop nominal system (12) with $v(t)=0$ can be represented as

$$\mathrm{d}x(t)=f(t)\mathrm{d}t+g(t)\mathrm{d}w(t) \tag{17}$$

where

$$\begin{aligned}f(t)&=A_Kx(t)+A_dx(t-\tau(t))+B_{d_1}\textstyle\int_{t-d(t)}^{t}x(s)\mathrm{d}s\\ g(t)&=C_Kx(t)+C_dx(t-\tau(t))+B_{d_2}\textstyle\int_{t-d(t)}^{t}x(s)\mathrm{d}s.\end{aligned}$$

Choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V(x_t,t)=&V_1(x_t,t)+V_2(x_t,t)+V_3(x_t,t)+\\ &V_4(x_t,t)+V_5(x_t,t)\end{aligned} \tag{18}$$

where

$$\begin{aligned}V_1(x_t,t)=&x^{\rm T}(t)Px(t)\\ V_2(x_t,t)=&\int_{t-\tau(t)}^{t}x^{\rm T}(s)Q_1x(s)\mathrm{d}s+\int_{t-\tau_m}^{t}x^{\rm T}(s)Q_2x(s)\mathrm{d}s+\\ &\int_{t-\tau_M}^{t}x^{\rm T}(s)Q_3x(s)\mathrm{d}s\\ V_3(x_t,t)=&\int_{-\tau_M}^{0}\int_{t+\theta}^{t}f^{\rm T}(s)R_1f(s)\mathrm{d}s\mathrm{d}\theta+\\ &\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t}f^{\rm T}(s)R_2f(s)\mathrm{d}s\mathrm{d}\theta\\ V_4(x_t,t)=&\int_{-\tau_M}^{0}\int_{t+\theta}^{t}g^{\rm T}(s)R_3g(s)\mathrm{d}s\mathrm{d}\theta+\\ &\int_{-\tau_M}^{-\tau_m}\int_{t+\theta}^{t}g^{\rm T}(s)R_4g(s)\mathrm{d}s\mathrm{d}\theta\\ V_5(x_t,t)=&\int_{-d(t)}^{0}\int_{t+\theta}^{t}x^{\rm T}(s)Zx(s)\mathrm{d}s\mathrm{d}\theta\end{aligned}$$

where P, Q_s $(s=1,2,3)$, R_l $(l=1,2,3,4)$ and Z are symmetric positive definite matrices with appropriate dimensions.

By using Itô's formula[34], we have

$$\mathrm{d}V(x_t,t)=\mathcal{L}V(x_t,t)\mathrm{d}t+2x^{\rm T}(t)Pg(t)\mathrm{d}w(t) \tag{19}$$

where

$$\mathcal{L}V(x_t,t)=\sum_{i=1}^{5}\mathcal{L}V_i(x_t,t). \tag{20}$$

It is easy to know

$$\begin{aligned}\mathcal{L}V_1(x_t,t) =&2x^{\mathrm{T}}(t)Pf(t)+g^{\mathrm{T}}(t)Pg(t)=\\&2x^{\mathrm{T}}(t)P\Big(A_Kx(t)+A_dx(t-\tau(t))+\\&B_{d_1}\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big)+g^{\mathrm{T}}(t)Pg(t)\end{aligned}$$

$$\begin{aligned}\mathcal{L}V_2(x_t,t)\leqslant&x^{\mathrm{T}}(t)Q_1x(t)-(1-\mu)x^{\mathrm{T}}(t-\tau(t))Q_1x(t-\tau(t))+\\&x^{\mathrm{T}}(t)Q_2x(t)-x^{\mathrm{T}}(t-\tau_m)Q_2x(t-\tau_m)+\\&x^{\mathrm{T}}(t)Q_3x(t)-x^{\mathrm{T}}(t-\tau_M)Q_3x(t-\tau_M)\end{aligned}$$

$$\begin{aligned}\mathcal{L}V_3(x_t,t)=&\tau_Mf^{\mathrm{T}}(t)R_1f(t)-\int_{t-\tau_M}^{t}f^{\mathrm{T}}(s)R_1f(s)\mathrm{d}s+\\&(\tau_M-\tau_m)f^{\mathrm{T}}(t)R_2f(t)-\int_{t-\tau_M}^{t-\tau_m}f^{\mathrm{T}}(s)R_2f(s)\mathrm{d}s\end{aligned}$$

$$\begin{aligned}\mathcal{L}V_4(x_t,t)=&\tau_Mg^{\mathrm{T}}(t)R_3g(t)-\int_{t-\tau_M}^{t}g^{\mathrm{T}}(s)R_3g(s)\mathrm{d}s+\\&(\tau_M-\tau_m)g^{\mathrm{T}}(t)R_4g(t)-\int_{t-\tau_M}^{t-\tau_m}g^{\mathrm{T}}(s)R_4g(s)\mathrm{d}s\end{aligned}$$

$$\mathcal{L}V_5(x_t,t)\leqslant d_Mx^{\mathrm{T}}(t)Zx(t)-\int_{t-d(t)}^{t}x^{\mathrm{T}}(s)Zx(s)\mathrm{d}s.$$

From the Newton-Leibnitz formula, the following equalities are true for matrices $N_{lij},\ M_{lij},\ S_{lij}\ (l=1,2)$ with appropriate dimensions:

$$\begin{aligned}0=&2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big[x^{\mathrm{T}}(t)N_{1ij}+x^{\mathrm{T}}(t-\tau(t))N_{2ij}\Big]\times\\&\Big[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}f(s)\mathrm{d}s-\int_{t-\tau(t)}^{t}g(s)\mathrm{d}w(s)\Big]\end{aligned}\tag{21}$$

$$\begin{aligned}0=&2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big[x^{\mathrm{T}}(t)M_{1ij}+x^{\mathrm{T}}(t-\tau(t))M_{2ij}\Big]\times\\&\Big[x(t-\tau_m)-x(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_m}f(s)\mathrm{d}s-\int_{t-\tau(t)}^{t-\tau_m}g(s)\mathrm{d}w(s)\Big]\end{aligned}\tag{22}$$

$$\begin{aligned}0=&2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big[x^{\mathrm{T}}(t)S_{1ij}+x^{\mathrm{T}}(t-\tau(t))S_{2ij}\Big]\times\\&\Big[x(t-\tau(t))-x(t-\tau_M)-\int_{t-\tau_M}^{t-\tau(t)}f(s)\mathrm{d}s-\int_{t-\tau_M}^{t-\tau(t)}g(s)\mathrm{d}w(s)\Big].\end{aligned}\tag{23}$$

By Lemma 1 1), for matrices $R_l\geqslant 0\quad (l=1,2,3,4)$, the following inequalities hold:

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)N_{ij}\int_{t-\tau(t)}^{t}f(s)\mathrm{d}s\leqslant\\&\tau_M\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)N_{ij}R_1^{-1}N_{ij}^{\mathrm{T}}\xi(t)+\int_{t-\tau(t)}^{t}f^{\mathrm{T}}(s)R_1f(s)\mathrm{d}s\end{aligned}\tag{24}$$

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)M_{ij}\int_{t-\tau(t)}^{t-\tau_m}f(s)\mathrm{d}s\leqslant\\&\bar{\tau}\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)M_{ij}R_2^{-1}M_{ij}^{\mathrm{T}}\xi(t)+\int_{t-\tau(t)}^{t-\tau_m}f^{\mathrm{T}}(s)R_2f(s)\mathrm{d}s\end{aligned}\tag{25}$$

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)S_{ij}\int_{t-\tau_M}^{t-\tau(t)}f(s)\mathrm{d}s\leqslant\\&\bar{\tau}\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)S_{ij}\Big(R_1+R_2\Big)^{-1}S_{ij}^{\mathrm{T}}\xi(t)+\\&\int_{t-\tau_M}^{t-\tau(t)}f^{\mathrm{T}}(s)\Big(R_1+R_2\Big)f(s)\mathrm{d}s\end{aligned}\tag{26}$$

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)N_{ij}\int_{t-\tau(t)}^{t}g(s)\mathrm{d}w(s)\leqslant\\&\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)N_{ij}R_3^{-1}N_{ij}^{\mathrm{T}}\xi(t)+\\&\Big(\int_{t-\tau(t)}^{t}g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}}R_3\Big(\int_{t-\tau(t)}^{t}g(s)\mathrm{d}w(s)\Big)\end{aligned}\tag{27}$$

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)M_{ij}\int_{t-\tau(t)}^{t-\tau_m}g(s)\mathrm{d}w(s)\leqslant\\&\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)M_{ij}R_4^{-1}M_{ij}^{\mathrm{T}}\xi(t)+\\&\Big(\int_{t-\tau(t)}^{t-\tau_m}g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}}R_4\Big(\int_{t-\tau(t)}^{t-\tau_m}g(s)\mathrm{d}w(s)\Big)\end{aligned}\tag{28}$$

$$\begin{aligned}&-2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)S_{ij}\int_{t-\tau_M}^{t-\tau(t)}g(s)\mathrm{d}w(s)\leqslant\\&\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\xi^{\mathrm{T}}(t)S_{ij}\Big(R_3+R_4\Big)^{-1}S_{ij}^{\mathrm{T}}\xi(t)+\\&\Big(\int_{t-\tau_M}^{t-\tau(t)}g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}}\Big(R_3+R_4\Big)\Big(\int_{t-\tau_M}^{t-\tau(t)}g(s)\mathrm{d}w(s)\Big)\end{aligned}\tag{29}$$

where

$$\begin{aligned}\xi^{\mathrm{T}}(t) =& \Big[x^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t-\tau(t)) \quad x^{\mathrm{T}}(t-\tau_m) \quad x^{\mathrm{T}}(t-\tau_M) \\ & \Big(\int_{t-d(t)}^{t} x(s)\mathrm{d}s\Big)^{\mathrm{T}}\Big] \\ N_{ij} =& \; [N_{1ij}^{\mathrm{T}} \quad N_{2ij}^{\mathrm{T}} \quad 0 \quad 0 \quad 0]^{\mathrm{T}} \\ M_{ij} =& \; [M_{1ij}^{\mathrm{T}} \quad M_{2ij}^{\mathrm{T}} \quad 0 \quad 0 \quad 0]^{\mathrm{T}} \\ S_{ij} =& \; [S_{1ij}^{\mathrm{T}} \quad S_{2ij}^{\mathrm{T}} \quad 0 \quad 0 \quad 0]^{\mathrm{T}} \\ \bar{\tau} =& \; \tau_M - \tau_m.\end{aligned}$$

Using Lemma 3, one can derive that

$$\begin{aligned}& f^{\mathrm{T}}(t)\Big(\tau_M R_1 + \bar{\tau} R_2\Big) f(t) = \Big[A_K x(t) + A_d x(t-\tau(t)) + \\ & B_{d_1}\int_{t-d(t)}^{t} x(s)\mathrm{d}s\Big]^{\mathrm{T}}\Big(\tau_M R_1 + \bar{\tau} R_2\Big)\times \\ & \Big[A_K x(t) + A_d x(t-\tau(t)) + B_{d_1}\int_{t-d(t)}^{t} x(s)\mathrm{d}s\Big] = \\ & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r} h_i h_j h_k h_l \xi^{\mathrm{T}}(t)\tilde{A}_{ij}^{\mathrm{T}}\Big(\tau_M R_1 + \bar{\tau} R_2\Big)\tilde{A}_{kl}\xi(t) \leqslant \\ & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \xi^{\mathrm{T}}(t)\tilde{A}_{ij}^{\mathrm{T}}\Big(\tau_M R_1 + \bar{\tau} R_2\Big)\tilde{A}_{ij}\xi(t)\end{aligned} \tag{30}$$

where

$$\tilde{A}_{ij}^{\mathrm{T}} = \Big[A_i + B_{1i}K_j \quad A_{di} \quad 0 \quad 0 \quad B_{d_{1i}}\Big]^{\mathrm{T}}.$$

Similarly

$$\begin{aligned}& g^{\mathrm{T}}(t)\Big(P + \tau_M R_3 + \bar{\tau} R_4\Big) g(t) \leqslant \\ & \quad \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \xi^{\mathrm{T}}(t)\tilde{C}_{ij}^{\mathrm{T}}\Big(P + \tau_M R_3 + \bar{\tau} R_4\Big)\tilde{C}_{ij}\xi(t)\end{aligned} \tag{31}$$

where

$$\tilde{C}_{ij}^{\mathrm{T}} = \Big[C_i + B_{2i}K_j \quad C_{di} \quad 0 \quad 0 \quad B_{d_{2i}}\Big]^{\mathrm{T}}.$$

Then, it follows from Lemma 2, that

$$\begin{aligned}& -\int_{t-d(t)}^{t} x^{\mathrm{T}}(s) Z x(s)\mathrm{d}s \leqslant \\ & \quad -\frac{1}{d_M}\Big(\int_{t-d(t)}^{t} x(s)\mathrm{d}s\Big)^{\mathrm{T}} Z \Big(\int_{t-d(t)}^{t} x(s)\mathrm{d}s\Big).\end{aligned} \tag{32}$$

Combining (20) to (32), we get

$$\begin{aligned}\mathcal{L}V(x_t,t) \leqslant & \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \xi^{\mathrm{T}}(t)\Xi^{ij}\xi(t) + \\ & \Big(\int_{t-\tau(t)}^{t} g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}} R_3 \Big(\int_{t-\tau(t)}^{t} g(s)\mathrm{d}w(s)\Big) + \\ & \Big(\int_{t-\tau(t)}^{t-\tau_m} g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}} R_4 \Big(\int_{t-\tau(t)}^{t-\tau_m} g(s)\mathrm{d}w(s)\Big) + \\ & \Big(\int_{t-\tau_M}^{t-\tau(t)} g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}} \Big(R_3 + R_4\Big)\times \\ & \Big(\int_{t-\tau_M}^{t-\tau(t)} g(s)\mathrm{d}w(s)\Big) - \int_{t-\tau(t)}^{t} g^{\mathrm{T}}(s) R_3 g(s)\mathrm{d}s - \\ & \int_{t-\tau(t)}^{t-\tau_m} g^{\mathrm{T}}(s) R_4 g(s)\mathrm{d}s - \\ & \int_{t-\tau_M}^{t-\tau(t)} g^{\mathrm{T}}(s)\Big(R_3 + R_4\Big) g(s)\mathrm{d}s\end{aligned} \tag{33}$$

where

$$\begin{aligned}\Xi^{ij} =& \Psi_{11}^{ij} + \tilde{C}_{ij}^{\mathrm{T}} P \tilde{C}_{ij} + \tau_M \tilde{A}_{ij}^{\mathrm{T}} R_1 \tilde{A}_{ij} + \bar{\tau}\tilde{A}_{ij}^{\mathrm{T}} R_2 \tilde{A}_{ij} + \\ & \tau_M \tilde{C}_{ij}^{\mathrm{T}} R_3 \tilde{C}_{ij} + \bar{\tau}\tilde{C}_{ij}^{\mathrm{T}} R_4 \tilde{C}_{ij} + \tau_M N_{ij} R_1^{-1} N_{ij}^{\mathrm{T}} + \\ & \bar{\tau} M_{ij} R_2^{-1} M_{ij}^{\mathrm{T}} + \bar{\tau} S_{ij}(R_1 + R_2)^{-1} S_{ij}^{\mathrm{T}} + \\ & N_{ij} R_3^{-1} N_{ij}^{\mathrm{T}} + M_{ij} R_4^{-1} M_{ij}^{\mathrm{T}} + S_{ij}(R_3 + R_4)^{-1} S_{ij}^{\mathrm{T}}\end{aligned}$$

$$\Psi_{11}^{ij} = \begin{bmatrix} \psi_{11}^{ij} & \psi_{12}^{ij} & M_{1ij} & -S_{1ij} & PB_{d_{1i}} \\ * & \psi_{22}^{ij} & M_{2ij} & -S_{2ij} & 0 \\ * & * & -Q_2 & 0 & 0 \\ * & * & * & -Q_3 & 0 \\ * & * & * & * & -\dfrac{1}{d_M}Z \end{bmatrix}$$

with

$$\begin{aligned}\psi_{11}^{ij} =& Q_1 + Q_2 + Q_3 + N_{1ij} + N_{1ij}^{\mathrm{T}} + P(A_i + B_{1i}K_j) + \\ & (A_i + B_{1i}K_j)^{\mathrm{T}} P + d_M Z \\ \psi_{12}^{ij} =& S_{1ij} - N_{1ij} + N_{2ij}^{\mathrm{T}} - M_{1ij} + PA_{di} \\ \psi_{22}^{ij} =& -(1-\mu)Q_1 - N_{2ij} - N_{2ij}^{\mathrm{T}} + S_{2ij} + \\ & S_{2ij}^{\mathrm{T}} - M_{2ij} - M_{2ij}^{\mathrm{T}}.\end{aligned}$$

It can be known that

$$\begin{aligned}& \mathcal{E}\Big\{\Big(\int_{t-\tau(t)}^{t} g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}} R_3 \Big(\int_{t-\tau(t)}^{t} g(s)\mathrm{d}w(s)\Big)\Big\} = \\ & \mathcal{E}\Big\{\int_{t-\tau(t)}^{t} g^{\mathrm{T}}(s) R_3 g(s)\mathrm{d}s\Big\}\end{aligned} \tag{34}$$

$$\mathcal{E}\Big\{\Big(\int_{t-\tau(t)}^{t-\tau_m} g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}} R_4\Big(\int_{t-\tau(t)}^{t-\tau_m} g(s)\mathrm{d}w(s)\Big)\Big\} =$$

$$\mathcal{E}\Big\{\int_{t-\tau(t)}^{t-\tau_m} g^{\mathrm{T}}(s)R_4 g(s)\mathrm{d}s\Big\} \tag{35}$$

and

$$\mathcal{E}\Big\{\Big(\int_{t-\tau_M}^{t-\tau(t)} g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}}\Big(R_3+R_4\Big)\Big(\int_{t-\tau_M}^{t-\tau(t)} g(s)\mathrm{d}w(s)\Big)\Big\} =$$

$$\mathcal{E}\Big\{\int_{t-\tau_M}^{t-\tau(t)} g^{\mathrm{T}}(s)\Big(R_3+R_4\Big)g(s)\mathrm{d}s\Big\}. \tag{36}$$

Taking the mathematical expectation on both sides of (33) and using (34)−(36), we get

$$\mathcal{E}\Big\{\mathcal{L}V(x_t,t)\Big\} \leqslant \mathcal{E}\Big\{\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \xi^{\mathrm{T}}(t)\Xi^{ij}\xi(t)\Big\}. \tag{37}$$

If $\Xi^{ii} < 0$ for $1 \leqslant i \leqslant r$ and $\Xi^{ij} + \Xi^{ji} < 0$ for any $1 \leqslant i < j \leqslant r$, it yields $\mathcal{E}\Big\{\mathcal{L}V(x_t,t)\Big\} < 0$. Employing the Schur complement, $\Xi^{ii} < 0$ and $\Xi^{ij} + \Xi^{ji} < 0$ are equivalent to

$$\tilde{\Xi}^{ij} + \tilde{\Xi}^{ji} < 0 \tag{38}$$

for any $1 \leqslant i \leqslant j \leqslant r$, where

$$\tilde{\Xi}^{ij} = \begin{bmatrix} \Psi_{11}^{ij} & \Psi_{12}^{ij} & \Psi_{13}^{ij} \\ * & \Psi_{22} & 0 \\ * & * & \Psi_{33} \end{bmatrix}$$

with

$$\Psi_{12}^{ij} = \Big[\tilde{C}_{ij}^{\mathrm{T}} P \quad \tau_M \tilde{A}_{ij}^{\mathrm{T}} \quad \bar{\tau}\tilde{A}_{ij}^{\mathrm{T}} \quad \tau_M \tilde{C}_{ij}^{\mathrm{T}} \quad \bar{\tau}\tilde{C}_{ij}^{\mathrm{T}}\Big]$$

$$\Psi_{13}^{ij} = \Big[\tau_M N_{ij} \ \bar{\tau} M_{ij} \ \bar{\tau} S_{ij} \ N_{ij} \ M_{ij} \ S_{ij}\ \Big]$$

$$\Psi_{22} = -\operatorname{diag}\Big\{P,\ \tau_M R_1^{-1},\ \bar{\tau} R_2^{-1},\ \tau_M R_3^{-1},\ \bar{\tau} R_4^{-1}\Big\}$$

$$\Psi_{33} = -\operatorname{diag}\Big\{\tau_M R_1,\ \bar{\tau}R_2,\ \bar{\tau}(R_1+R_2),\ R_3, R_4,\ (R_3+R_4)\Big\}$$

and Ψ_{11}^{ij} is defined previously.

Pre- and post-multiply (38) by $\operatorname{diag}\{X, X, X, X, X, X, I, I, I, I, X, X, X, X, X, X\}$ and its transpose, respectively, and applying the change of variables such that $P = X^{-1}$, $XQ_sX = \bar{Q}_s$ $(s = 1,2,3)$, $XZX = \bar{Z}$, $XN_{lij}X = \bar{N}_{lij}$, $XM_{lij}X = \bar{M}_{lij}$, $XS_{lij}X = \bar{S}_{lij}$ $(l = 1,2)$, then it gives

$$\hat{\Xi}^{ij} + \hat{\Xi}^{ji} < 0 \tag{39}$$

for $1 \leqslant i \leqslant j \leqslant r$, where

$$\hat{\Xi}^{ij} = \begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & \Xi_{13}^{ij} \\ * & \hat{\Xi}_{22} & 0 \\ * & * & \hat{\Xi}_{33} \end{bmatrix}$$

$$\hat{\Xi}_{22} = \operatorname{diag}\Big\{-X,\ -\tau_M R_1^{-1},\ -\bar{\tau}R_2^{-1},\ -\tau_M R_3^{-1},\ -\bar{\tau}R_4^{-1}\Big\}$$

$$\hat{\Xi}_{33} = \operatorname{diag}\Big\{-\tau_M X R_1 X,\ -\bar{\tau}XR_2X,\ -\bar{\tau}X(R_1+R_2)X,$$
$$-XR_3X,\ -XR_4X,\ -X(R_3+R_4)X\Big\}$$

and Ξ_{11}^{ij}, Ξ_{12}^{ij}, Ξ_{13}^{ij} are defined in statement of Theorem 1.

It follows from inequalities

$$XR_lX - 2X + R_l^{-1} = (X - R_l^{-1})R_l(X - R_l^{-1}) \geqslant 0$$

that

$$-2X + R_l^{-1} \geqslant -XR_lX,\ l = 1,2,3,4.$$

Let us assume that $R_l^{-1} = \tilde{R}_l$ $(l = 1,2,3,4)$. Then, LMI (39) is equivalent to the LMIs defined in (15) and (16). Therefore, by Definition 1 and [35], the closed-loop nominal stochastic fuzzy system (12) and (14) is stochastically stable with $v(t) = 0$. □

In the following part, using Lemma 1 2), we extend the above result to the uncertain stochastic fuzzy system (12) and (14) with $v(t) = 0$ to obtain a delay-dependent criterion as stated in the following theorem by means of the feasibility of LMIs.

Theorem 2. For given scalars τ_m, τ_M, d_M and μ, the time-varying delays satisfying (4), and the closed-loop uncertain stochastic fuzzy system (12) and (14) with $v(t) = 0$ is robustly stochastically stabilizable, if there exist matrices $X > 0$, $\bar{Q}_s > 0$ $(s = 1,2,3)$, $\tilde{R}_l > 0$ $(l = 1,2,3,4)$, $\bar{Z} > 0$, and real matrices $\bar{N}_{lij}$, $\bar{M}_{lij}$, $\bar{S}_{lij}$, Y_j $(l = 1,2)$ of appropriate dimensions and scalars $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$ $(1 \leqslant i \leqslant j \leqslant r)$ such that the following LMIs hold:

$$\begin{bmatrix} \Xi_{11}^{ii} & \Xi_{12}^{ii} & \Xi_{13}^{ii} & \Xi_{14}^{ii} \\ * & \Xi_{22} & 0 & \Xi_{24}^{ii} \\ * & * & \Xi_{33} & 0 \\ * & * & * & \Xi_{44}^{ii} \end{bmatrix} < 0, \quad 1 \leqslant i \leqslant r \tag{40}$$

$$\begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & \Xi_{13}^{ij} & \Xi_{14}^{ij} \\ * & \Xi_{22} & 0 & \Xi_{24}^{ij} \\ * & * & \Xi_{33} & 0 \\ * & * & * & \Xi_{44}^{ij} \end{bmatrix} + \begin{bmatrix} \Xi_{11}^{ji} & \Xi_{12}^{ji} & \Xi_{13}^{ji} & \Xi_{14}^{ji} \\ * & \Xi_{22} & 0 & \Xi_{24}^{ji} \\ * & * & \Xi_{33} & 0 \\ * & * & * & \Xi_{44}^{ji} \end{bmatrix} < 0,$$
$$1 \leqslant i < j \leqslant r \tag{41}$$

where

$$\Xi_{14}^{ij} = \begin{bmatrix} \varepsilon_{1ij}E_i & 0 & XH_{1i}^{\mathrm{T}} + Y_j^{\mathrm{T}}H_{3i}^{\mathrm{T}} & XH_{4i}^{\mathrm{T}} + Y_j^{\mathrm{T}}H_{6i}^{\mathrm{T}} \\ 0 & 0 & XH_{2i}^{\mathrm{T}} & XH_{5i}^{\mathrm{T}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{24}^{ij} = \begin{bmatrix} 0 & \varepsilon_{2ij}E_i & 0 & 0 \\ \varepsilon_{1ij}\tau_M E_i & 0 & 0 & 0 \\ \varepsilon_{1ij}\bar{\tau}E_i & 0 & 0 & 0 \\ 0 & \varepsilon_{2ij}\tau_M E_i & 0 & 0 \\ 0 & \varepsilon_{2ij}\bar{\tau}E_i & 0 & 0 \end{bmatrix}$$

$$\Xi_{44}^{ij} = \operatorname{diag}\Big\{-\varepsilon_{1ij}I,\ -\varepsilon_{2ij}I,\ -\varepsilon_{1ij}I,\ -\varepsilon_{2ij}I\Big\}$$

Ξ_{11}^{ij}, Ξ_{12}^{ij}, Ξ_{13}^{ij}, Ξ_{22} and Ξ_{33} are defined in Theorem 1. Moreover, the state feedback gain can be constructed as $K_j = Y_jX^{-1}$ $(j = 1,2,\cdots,r)$.

Proof. For the sake of presentation and simplicity, denote

$$\Omega_{1i} = \begin{bmatrix} E_i^{\mathrm T} & 0 & 0 & 0 & 0 & 0 & \tau_M E_i^{\mathrm T} & \bar\tau E_i^{\mathrm T} & 0_{1\times 8}\end{bmatrix}^{\mathrm T}$$

$$\Omega_{2i} = \begin{bmatrix} 0_{1\times 5} & E_i^{\mathrm T} & 0 & 0 & \tau_M E_i^{\mathrm T} & \bar\tau E_i^{\mathrm T} & 0_{1\times 6}\end{bmatrix}^{\mathrm T}$$

$$\Omega_{3ij} = \begin{bmatrix} H_{1i}X + H_{3i}Y_j & H_{2i}X & 0_{1\times 14}\end{bmatrix}$$

$$\Omega_{4ij} = \begin{bmatrix} H_{4i}X + H_{6i}Y_j & H_{5i}X & 0_{1\times 14}\end{bmatrix}.$$

Replacing A_i, A_{di}, B_{1i}, C_i, C_{di}, and B_{2i} in Theorem 1 with $A_i+\Delta A_i(t)$, $A_{di}+\Delta A_{di}(t)$, $B_{1i}+\Delta B_{1i}(t)$, $C_i+\Delta C_i(t)$, $C_{di}+\Delta C_{di}(t)$, and $B_{2i}+\Delta B_{2i}(t)$ respectively, we obtain the following corresponding uncertain stochastic fuzzy system (12) and (14) with $v(t)=0$

$$\begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & \Xi_{13}^{ij} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33}\end{bmatrix} + \Omega_{1i}F_i(t)\Omega_{3ij} + \Omega_{3ij}^{\mathrm T}F_i^{\mathrm T}(t)\Omega_{1i}^{\mathrm T} +$$

$$\Omega_{2i}F_i(t)\Omega_{4ij} + \Omega_{4ij}^{\mathrm T}F_i^{\mathrm T}(t)\Omega_{2i}^{\mathrm T} < 0. \tag{42}$$

By Lemma 1 2), we have

$$\begin{bmatrix} \Xi_{11}^{ij} & \Xi_{12}^{ij} & \Xi_{13}^{ij} \\ * & \Xi_{22} & 0 \\ * & * & \Xi_{33}\end{bmatrix} + \varepsilon_{1ij}\Omega_{1i}\Omega_{1i}^{\mathrm T} + \varepsilon_{1ij}^{-1}\Omega_{3ij}^{\mathrm T}\Omega_{3ij} +$$

$$\varepsilon_{2ij}\Omega_{2i}\Omega_{2i}^{\mathrm T} + \varepsilon_{2ij}^{-1}\Omega_{4ij}^{\mathrm T}\Omega_{4ij} < 0. \tag{43}$$

By Schur complement, we obtain (40) and (41). Then, by Theorem 1, the closed-loop uncertain stochastic fuzzy system (12) and (14) is robustly stochastically stable with $v(t)=0$. □

In the case of $u(t)=0$, $v(t)=0$, $B_{d_{1i}}=B_{d_{2i}}=0$, $\Delta C_i(t)=0$ and $\Delta C_{di}(t)=0$, the system (7) is reduced to the following model

$$\mathrm{d}x(t) = \sum_{i=1}^{r} h_i(\theta(t))\Big\{\Big[(A_i+\Delta A_i(t))x(t) + (A_{di}+\Delta A_{di}(t))\times x(t-\tau(t))\Big]\mathrm{d}t + \Big[C_i x(t) + C_{di}x(t-\tau(t))\Big]\mathrm{d}w(t)\Big\} \tag{44}$$

$$x(t)=\phi(t),\quad \forall t\in[-\tau_M,0] \tag{45}$$

where the time-varying delay $\tau(t)$ satisfies

$$0\leqslant \tau(t)\leqslant \tau_M<\infty,\quad \dot\tau(t)\leqslant \mu<\infty \tag{46}$$

with τ_M and μ are real constant scalars. In the system (44), the parameter uncertainties are assumed to be of the form

$$\begin{aligned}\Delta A_i(t) &= E_{1i}F_{1i}(t)H_{1i}\\ \Delta A_{di}(t) &= E_{2i}F_{2i}(t)H_{2i}\end{aligned} \tag{47}$$

where E_{1i}, E_{2i}, H_{1i} and H_{2i} are known real constant matrices with appropriate dimensions, $F_{1i}(t)$ and $F_{2i}(t)$ are unknown real time-varying matrix function satisfying

$$\begin{aligned}F_{1i}^{\mathrm T}(t)F_{1i}(t) &\leqslant I\\ F_{2i}^{\mathrm T}(t)F_{2i}(t) &\leqslant I.\end{aligned} \tag{48}$$

When there are no parameter uncertainties in the system (44), the following corollary can be obtained by using Theorem 1.

Corollary 1. For given scalars τ_M and μ, the time-varying delays satisfying (46), the nominal stochastic fuzzy system (44) is asymptotically stable in the mean square sense if there exist matrices $P>0$, $Q_1>0$, $Q_3>0$, $R_1>0$, $R_3>0$, and real matrices N_{li} and S_{li} $(l=1,2)$ of appropriate dimensions such that the following LMI holds:

$$\begin{bmatrix} \Phi_{11}^i & \Phi_{12}^i & \Phi_{13}^i \\ * & \Phi_{22} & 0 \\ * & * & \Phi_{33}\end{bmatrix} < 0,\quad i=1,2,\cdots,r \tag{49}$$

where

$$\Phi_{11}^i = \begin{bmatrix} \phi_{11}^i & \phi_{12}^i & -S_{1i} \\ * & \phi_{22}^i & -S_{2i} \\ * & * & -Q_3\end{bmatrix}$$

$$\Phi_{12}^i = \begin{bmatrix} C_i^{\mathrm T}P & \tau_M A_i^{\mathrm T}R_1 & \tau_M C_i^{\mathrm T}R_3 \\ C_{di}^{\mathrm T}P & \tau_M A_{di}^{\mathrm T}R_1 & \tau_M C_{di}^{\mathrm T}R_3 \\ 0 & 0 & 0\end{bmatrix}$$

$$\Phi_{13}^i = \begin{bmatrix} \tau_M N_{1i} & \tau_M S_{1i} & N_{1i} & S_{1i} \\ \tau_M N_{2i} & \tau_M S_{2i} & N_{1i} & S_{2i} \\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Phi_{22} = \mathrm{diag}\Big\{-P,\ -\tau_M R_1,\ -\tau_M R_3\Big\}$$

$$\Phi_{33} = \mathrm{diag}\Big\{-\tau_M R_1,\ -\tau_M R_1,\ -R_3,\ -R_3\Big\}$$

with

$$\phi_{11}^i = PA_i + A_i^{\mathrm T}P + Q_1 + Q_3 + N_{1i} + N_{1i}^{\mathrm T}$$

$$\phi_{12}^i = PA_{di} - N_{1i} + N_{2i}^{\mathrm T} + S_{1i}$$

$$\phi_{22}^i = -(1-\mu)Q_1 - N_{2i} - N_{2i}^{\mathrm T} + S_{2i} + S_{2i}^{\mathrm T}.$$

Remark 1. Choose the following Lyapunov-Krasovskii functional candidate as in (18) with $Q_2=0$, $R_2=0$, $R_4=0$, $Z=0$, replacing N_{lij} and S_{lij} $(l=1,2)$ with N_{li} and S_{li} $(l=1,2)$ in (21) and (23) respectively, and taking M_{lij} as zero in (22), the proof of Corollary 1 is easily obtained from Theorem 1.

For the system (44), the robust stability conditions can be obtained as stated in the following Corollary 2 by extending the proof of Corollary 1.

Corollary 2. For given scalars τ_M and μ, the time-varying delays satisfying (46), the uncertain stochastic fuzzy system (44) is robustly asymptotically stable in the mean square if there exist matrices $P>0$, $Q_1>0$, $Q_3>0$, $R_1>0$, $R_3>0$, real matrices N_{li} and S_{li} $(l=1,2)$ of appropriate dimensions, and scalars $\varepsilon_{1i}>0$ and $\varepsilon_{2i}>0$ such that the following LMI holds:

$$\begin{bmatrix} \Phi_{11}^i & \Phi_{12}^i & \Phi_{13}^i & \Phi_{14}^i \\ * & \Phi_{22} & 0 & \Phi_{24}^i \\ * & * & \Phi_{33} & 0 \\ * & * & * & \Phi_{44}^i\end{bmatrix} < 0,\quad i=1,2,\cdots,r \tag{50}$$

where

$$\Phi_{11}^i = \begin{bmatrix} \tilde{\phi}_{11}^i & \phi_{12}^i & -S_{1i} \\ * & \tilde{\phi}_{22}^i & -S_{2i} \\ * & * & -Q_3 \end{bmatrix}$$

$$\Phi_{14}^i = \begin{bmatrix} PE_{1i} & PE_{2i} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Phi_{24}^i = \begin{bmatrix} 0 & 0 \\ \tau_M R_1 E_{1i} & \tau_M R_1 E_{2i} \\ 0 & 0 \end{bmatrix}$$

$$\Phi_{44}^i = \begin{bmatrix} -\varepsilon_{1i} I & 0 \\ * & -\varepsilon_{2i} I \end{bmatrix}$$

with

$$\tilde{\phi}_{11}^i = PA_i + A_i^{\mathrm{T}} P + Q_1 + Q_3 + N_{1i} + N_{1i}^{\mathrm{T}} + \varepsilon_{1i} H_{1i}^{\mathrm{T}} H_{1i}$$

$$\tilde{\phi}_{22}^i = -(1-\mu)Q_1 - N_{2i} - N_{2i}^{\mathrm{T}} + S_{2i} + S_{2i}^{\mathrm{T}} + \varepsilon_{2i} H_{2i}^{\mathrm{T}} H_{2i}.$$

Further, Φ_{12}^i, Φ_{13}^i, Φ_{22}, Φ_{33} and ϕ_{12}^i are defined in Corollary 1.

4 Robust stochastic H_∞ control

In this section, a delay-dependent sufficient condition for the solvability of robust H_∞ control problem is proposed, and an LMI approach for designing a desired state feedback fuzzy controller is developed. The second main result is stated as follows.

Theorem 3. For a prescribed $\gamma > 0$, given scalars τ_m, τ_M, d_M and μ, the time-varying delays satisfying (4), there exists a fuzzy control law (11) such that the closed-loop uncertain stochastic fuzzy system (Σ_2) is robustly stochastically stabilizable with attenuation γ if there exist matrices $X > 0$, $\bar{Q}_s > 0$ $(s = 1, 2, 3)$, $\tilde{R}_l > 0$ $(l = 1, 2, 3, 4)$, $\bar{Z} > 0$, real matrices $\bar{N}_{lij}$, $\bar{M}_{lij}$, $\bar{S}_{lij}$, Y_j $(l = 1, 2)$ of appropriate dimensions, and scalars $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$ $(1 \leqslant i \leqslant j \leqslant r)$ such that the following LMIs hold:

$$\begin{bmatrix} \Upsilon_{11}^{ii} & \Upsilon_{12}^{ii} & \Upsilon_{13}^{ii} & \Upsilon_{14}^{ii} \\ * & \Upsilon_{22} & 0 & \Upsilon_{24}^{ii} \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & \Upsilon_{44}^{ii} \end{bmatrix} < 0, \quad 1 \leqslant i \leqslant r \tag{51}$$

$$\begin{bmatrix} \Upsilon_{11}^{ij} & \Upsilon_{12}^{ij} & \Upsilon_{13}^{ij} & \Upsilon_{14}^{ij} \\ * & \Upsilon_{22} & 0 & \Upsilon_{24}^{ij} \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & \Upsilon_{44}^{ij} \end{bmatrix} + \begin{bmatrix} \Upsilon_{11}^{ji} & \Upsilon_{12}^{ji} & \Upsilon_{13}^{ji} & \Upsilon_{14}^{ji} \\ * & \Upsilon_{22} & 0 & \Upsilon_{24}^{ji} \\ * & * & \Upsilon_{33} & 0 \\ * & * & * & \Upsilon_{44}^{ji} \end{bmatrix} < 0,$$
$$1 \leqslant i < j \leqslant r \tag{52}$$

where

$$\Upsilon_{11}^{ij} = \begin{bmatrix} \phi_{11}^{ij} & \phi_{12}^{ij} & \bar{M}_{1ij} & -\bar{S}_{1ij} & B_{v_{1i}} & B_{d_{1i}} X \\ * & \phi_{22}^{ij} & \bar{M}_{2ij} & -\bar{S}_{2ij} & 0 & 0 \\ * & * & -\bar{Q}_2 & 0 & 0 & 0 \\ * & * & * & -\bar{Q}_3 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -\dfrac{1}{d_M}\bar{Z} \end{bmatrix},$$

$$\Upsilon_{12}^{ij} = \begin{bmatrix} \check{C}_{ij}^{\mathrm{T}} & \tau_M \check{A}_{ij}^{\mathrm{T}} & \bar{\tau} \check{A}_{ij}^{\mathrm{T}} & \tau_M \check{C}_{ij}^{\mathrm{T}} & \bar{\tau} \check{C}_{ij}^{\mathrm{T}} \end{bmatrix},$$

$$\Upsilon_{13}^{ij} = \begin{bmatrix} \tau_M \overline{N}_{ij} & \bar{\tau} \overline{M}_{ij} & \bar{\tau} \overline{S}_{ij} & \overline{N}_{ij} & \overline{M}_{ij} & \overline{S}_{ij} & \check{D}_{ij} \end{bmatrix},$$

$$\Upsilon_{14}^{ij} = \begin{bmatrix} \Xi_{14}^{ij} \\ 0 \end{bmatrix}, \quad \Upsilon_{22} = \Xi_{22}, \quad \Upsilon_{24}^{ij} = \Xi_{24}^{ij},$$

$$\Upsilon_{33} = \begin{bmatrix} \Xi_{33} & 0 \\ 0 & -I \end{bmatrix}, \quad \Upsilon_{44}^{ij} = \Xi_{44}^{ij}$$

with

$$\check{A}_{ij}^{\mathrm{T}} = \begin{bmatrix} A_i X + B_{1i} Y_j & A_{di} X & 0 & 0 & B_{v_{1i}} & B_{d_{1i}} X \end{bmatrix}^{\mathrm{T}}$$

$$\check{C}_{ij}^{\mathrm{T}} = \begin{bmatrix} C_i X + B_{2i} Y_j & C_{di} X & 0 & 0 & B_{v_{2i}} & B_{d_{2i}} X \end{bmatrix}^{\mathrm{T}}$$

$$\check{D}_{ij}^{\mathrm{T}} = \begin{bmatrix} D_i X + B_{3i} Y_j & D_{di} X & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\overline{N}_{ij} = \begin{bmatrix} \bar{N}_{1ij}^{\mathrm{T}} & \bar{N}_{2ij}^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\overline{M}_{ij} = \begin{bmatrix} \bar{M}_{1ij}^{\mathrm{T}} & \bar{M}_{2ij}^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\overline{S}_{ij} = \begin{bmatrix} \bar{S}_{1ij}^{\mathrm{T}} & \bar{S}_{2ij}^{\mathrm{T}} & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}.$$

Further, Ξ_{14}^{ij}, Ξ_{22}, Ξ_{24}^{ij}, Ξ_{33}, Ξ_{44}^{ij}, ϕ_{11}^{ij}, ϕ_{12}^{ij}, ϕ_{22}^{ij} and $\bar{\tau}$ are defined as in Theorem 2. Moreover, the state feedback gain can be constructed as $K_j = Y_j X^{-1}$ $(j = 1, 2, \cdots, r)$.

Proof. For convenience, we set

$$f(t) = A_K x(t) + A_d x(t-\tau(t)) + B_{v_1} v(t) + B_{d_1} \int_{t-d(t)}^{t} x(s)\mathrm{d}s$$

$$g(t) = C_K x(t) + C_d x(t-\tau(t)) + B_{v_2} v(t) + B_{d_2} \int_{t-d(t)}^{t} x(s)\mathrm{d}s.$$

By (51) and (52), it is easy to see that the LMIs in (40) and (41) hold. Therefore, it follows from Theorem 2 that the closed-loop system (Σ_2) is robustly stochastically stable. Now, we show that under the zero initial condition, system (Σ_2) satisfies $\|z(t)\|_{\mathcal{E}_2} < \gamma \|v(t)\|_2$ for all non-zero $v(t) \in L_2[0, \infty)$. Choose a Lyapunov-Krasovskii functional candidate as defined in (18) and utilizing Itô's formula, we have

$$\begin{aligned}\mathcal{L}V(x_t,t)\leqslant&\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^{\mathrm{T}}(t)\Upsilon^{ij}\zeta(t)+\\&\Big(\int_{t-\tau(t)}^{t}g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}}R_3\Big(\int_{t-\tau(t)}^{t}g(s)\mathrm{d}w(s)\Big)+\\&\Big(\int_{t-\tau(t)}^{t-\tau_m}g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}}R_4\Big(\int_{t-\tau(t)}^{t-\tau_m}g(s)\mathrm{d}w(s)\Big)+\\&\Big(\int_{t-\tau_M}^{t-\tau(t)}g(s)\mathrm{d}w(s)\Big)^{\mathrm{T}}\Big(R_3+R_4\Big)\times\\&\Big(\int_{t-\tau_M}^{t-\tau(t)}g(s)\mathrm{d}w(s)\Big)-\int_{t-\tau(t)}^{t}g(s)^{\mathrm{T}}R_3g(s)\mathrm{d}s-\\&\int_{t-\tau(t)}^{t-\tau_m}g(s)^{\mathrm{T}}R_4g(s)\mathrm{d}s-\\&\int_{t-\tau_M}^{t-\tau(t)}g(s)^{\mathrm{T}}\Big(R_3+R_4\Big)g(s)\mathrm{d}s\end{aligned}\tag{53}$$

where

$$\begin{aligned}\Upsilon^{ij}=&\tilde{\Upsilon}_{11}^{ij}+\tilde{C}_{ij}^{\mathrm{T}}(t)P\tilde{C}_{ij}(t)+\tau_M\tilde{A}_{ij}^{\mathrm{T}}(t)R_1\tilde{A}_{ij}(t)+\\&\bar{\tau}\tilde{A}_{ij}^{\mathrm{T}}(t)R_2\tilde{A}_{ij}(t)+\tau_M\tilde{C}_{ij}^{\mathrm{T}}(t)R_3\tilde{C}_{ij}(t)+\\&\bar{\tau}\tilde{C}_{ij}^{\mathrm{T}}(t)R_4\tilde{C}_{ij}(t)+\tau_M\hat{N}_{ij}R_1^{-1}\hat{N}_{ij}^{\mathrm{T}}+\bar{\tau}\hat{M}_{ij}R_2^{-1}\hat{M}_{ij}^{\mathrm{T}}+\\&\bar{\tau}\hat{S}_{ij}(R_1+R_2)^{-1}\hat{S}_{ij}^{\mathrm{T}}+\hat{N}_{ij}R_3^{-1}\hat{N}_{ij}^{\mathrm{T}}+\\&\hat{M}_{ij}R_4^{-1}\hat{M}_{ij}^{\mathrm{T}}+\hat{S}_{ij}(R_3+R_4)^{-1}\hat{S}_{ij}^{\mathrm{T}}\end{aligned}$$

$$\tilde{\Upsilon}_{11}^{ij}=\begin{bmatrix}\gamma_{11}^{ij}&\gamma_{12}^{ij}&M_{1ij}&-S_{1ij}&PB_{v_{1i}}&PB_{d_{1i}}\\ *&\gamma_{22}^{ij}&M_{2ij}&-S_{2ij}&0&0\\ *&*&-Q_2&0&0&0\\ *&*&*&-Q_3&0&0\\ *&*&*&*&0&0\\ *&*&*&*&*&-\dfrac{1}{d_M}Z\end{bmatrix}$$

with

$$\begin{aligned}\gamma_{11}^{ij}=&\ Q_1+Q_2+Q_3+N_{1ij}+N_{1ij}^{\mathrm{T}}+P(A_i(t)+\\&B_{1i}(t)K_j)+(A_i(t)+B_{1i}(t)K_j)^{\mathrm{T}}P+d_MZ\\ \gamma_{12}^{ij}=&S_{1ij}-N_{1ij}+N_{2ij}^{\mathrm{T}}-M_{1ij}+PA_{di}(t)\\ \gamma_{22}^{ij}=&-(1-\mu)Q_1-N_{2ij}-N_{2ij}^{\mathrm{T}}+S_{2ij}+\\&S_{2ij}^{\mathrm{T}}-M_{2ij}-M_{2ij}^{\mathrm{T}}\end{aligned}$$

$$\tilde{A}_{ij}^{\mathrm{T}}(t)=\Big[A_i(t)+B_{1i}(t)K_j\quad A_{di}(t)\quad 0\quad 0\quad B_{v_{1i}}\quad B_{d_{1i}}\Big]^{\mathrm{T}}$$

$$\tilde{C}_{ij}^{\mathrm{T}}(t)=\Big[C_i(t)+B_{2i}(t)K_j\quad C_{di}(t)\quad 0\quad 0\quad B_{v_{2i}}\quad B_{d_{2i}}\Big]^{\mathrm{T}}$$

$$\begin{aligned}\hat{N}_{ij}=&\Big[N_{1ij}^{\mathrm{T}}\quad N_{2ij}^{\mathrm{T}}\quad 0\quad 0\quad 0\quad 0\Big]^{\mathrm{T}}\\ \hat{M}_{ij}=&\Big[M_{1ij}^{\mathrm{T}}\quad M_{2ij}^{\mathrm{T}}\quad 0\quad 0\quad 0\quad 0\Big]^{\mathrm{T}}\\ \hat{S}_{ij}=&\Big[S_{1ij}^{\mathrm{T}}\quad S_{2ij}^{\mathrm{T}}\quad 0\quad 0\quad 0\quad 0\Big]^{\mathrm{T}}\\ \bar{\tau}=&\tau_M-\tau_m\\ \zeta^{\mathrm{T}}(t)=&\Big[x^{\mathrm{T}}(t)\quad x^{\mathrm{T}}(t-\tau(t))\quad x^{\mathrm{T}}(t-\tau_m)\quad x^{\mathrm{T}}(t-\tau_M)\\&v^{\mathrm{T}}(t)\quad\Big(\int_{t-d(t)}^{t}x(s)\mathrm{d}s\Big)^{\mathrm{T}}\Big].\end{aligned}$$

It can be known that

$$\begin{aligned}z^{\mathrm{T}}(t)z(t)=&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}\sum_{l=1}^{r}h_ih_jh_kh_l\zeta^{\mathrm{T}}(t)\tilde{D}_{ij}^{\mathrm{T}}\tilde{D}_{kl}\zeta(t)\leqslant\\&\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^{\mathrm{T}}(t)\tilde{D}_{ij}^{\mathrm{T}}\tilde{D}_{ij}\zeta(t)\end{aligned}\tag{54}$$

where

$$\tilde{D}_{ij}^{\mathrm{T}}=\Big[D_i+B_{3i}K_j\quad D_{di}\quad 0\quad 0\quad 0\quad 0\Big]^{\mathrm{T}}.$$

Now, we set

$$J(t)=\mathcal{E}\Big\{\int_0^t\big[z^{\mathrm{T}}(s)z(s)-\gamma^2v^{\mathrm{T}}(s)v(s)\big]\mathrm{d}s\Big\}\tag{55}$$

where $t>0$. Because $V(\phi(t),0)=0$ under the zero initial condition, i.e., $\phi(t)=0$ for $t\in[-\tau,0]$, then by Itô′s formula, it follows that

$$\begin{aligned}J(t)=&\\&\mathcal{E}\Big\{\int_0^t\big[z^{\mathrm{T}}(s)z(s)-\gamma^2v^{\mathrm{T}}(s)v(s)+\mathcal{L}V(x_s,s)\big]\mathrm{d}s\Big\}-\\&\mathcal{E}\Big\{V(x_t,t)\Big\}\leqslant\\&\mathcal{E}\Big\{\int_0^t\big[z^{\mathrm{T}}(s)z(s)-\gamma^2v^{\mathrm{T}}(s)v(s)+\mathcal{L}V(x_s,s)\big]\mathrm{d}s\Big\}\leqslant\\&\mathcal{E}\Big\{\int_0^t\zeta^{\mathrm{T}}(s)\tilde{\Upsilon}^{ij}\zeta(s)\mathrm{d}s\Big\}\end{aligned}\tag{56}$$

where

$$\tilde{\Upsilon}^{ij}=\Upsilon^{ij}+\tilde{D}_{ij}^{\mathrm{T}}\tilde{D}_{ij}+\mathrm{diag}\Big\{0,\ 0,\ 0,\ 0,\ -\gamma^2I,\ 0\Big\}.$$

Then, considering LMIs (51) and (52), following similar line as in the proof of Theorem 2, we have $\tilde{\Upsilon}^{ii}<0$ and $\tilde{\Upsilon}^{ij}+\tilde{\Upsilon}^{ji}<0$, which imply that $J(t)<0$ for $t>0$. Therefore, we have $\|z(t)\|_{\mathcal{E}_2}<\gamma\|v(t)\|_2$. □

Remark 2. We mention that Theorem 3 provides a delay-dependent H_∞ control problem for a class of uncertain stochastic fuzzy systems with discrete interval and distributed time-varying delays. Note that, by Theorem 3, the problems of finding the maximum allowable upper bound of the delays are $\tau_M,\ d_M$, for given $\gamma,\ \mu$ and τ_m or the smallest γ for given $\tau_m,\ \tau_M,\ \mu$ and d_M can be easily solved. For instance, the smallest γ for given $\tau_m,\ \tau_M,\ \mu$ and d_M obtainable from Theorem 3 can be determined by solving the following convex optimization problem:

$$\begin{aligned}&\min\quad \chi\\ &\text{s.t.}\quad X>0,\ \bar{Q}_s>0\ (s=1,2,3),\\ &\tilde{R}_l>0\ (l=1,2,3,4),\ \bar{Z}>0,\\ &\varepsilon_{1ij}>0,\ \varepsilon_{2ij}>0\ (1\leqslant i\leqslant j\leqslant r)\\ &\text{and LMIs}\quad (51)-(52)\quad \text{with}\quad \chi=\gamma^2.\end{aligned}$$

Remark 3. By setting $B_{d_1}=0,\ B_{d_2}=0$ in Theorems 2 and 3, the delay-dependent robust stabilization and H_∞ control for uncertain stochastic fuzzy system with interval time-varying delay criteria can be obtained, corresponding proof is similar to Theorems 2 and 3 and hence omitted.

In the case when there is no parameter uncertainties in the system (Σ_2), Theorem 3 is specialized as follows.

Corollary 3. For a prescribed $\gamma>0$, given scalars $\tau_m,\ \tau_M,\ d_M$ and μ, the time-varying delays satisfying (4), there exists a fuzzy control law (11) such that the closed-loop stochastic fuzzy system (Σ_2) with $\Delta A_i(t)=\Delta A_{di}(t)=\Delta B_{1i}(t)=\Delta C_i(t)=\Delta C_{di}(t)=\Delta B_{2i}(t)=0$ is stochastically stabilizable with a disturbance attenuation γ, if there exist matrices $X>0,\ \bar{Q}_s>0\ (s=1,2,3),\ \tilde{R}_l>0\ (l=1,2,3,4),\ \bar{Z}>0$ and real matrices $\bar{N}_{lij},\ \bar{M}_{lij},\ \bar{S}_{lij},\ Y_j\ (l=1,2,\ 1\leqslant i\leqslant j\leqslant r)$ of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix}\Upsilon_{11}^{ii} & \Upsilon_{12}^{ii} & \Upsilon_{13}^{ii}\\ * & \Upsilon_{22} & 0\\ * & * & \Upsilon_{33}\end{bmatrix}<0,\quad 1\leqslant i\leqslant r \tag{57}$$

$$\begin{bmatrix}\Upsilon_{11}^{ij} & \Upsilon_{12}^{ij} & \Upsilon_{13}^{ij}\\ * & \Upsilon_{22} & 0\\ * & * & \Upsilon_{33}\end{bmatrix}+\begin{bmatrix}\Upsilon_{11}^{ji} & \Upsilon_{12}^{ji} & \Upsilon_{13}^{ji}\\ * & \Upsilon_{22} & 0\\ * & * & \Upsilon_{33}\end{bmatrix}<0,$$
$$1\leqslant i<j\leqslant r \tag{58}$$

where $\Upsilon_{11}^{ij},\ \Upsilon_{12}^{ij},\ \Upsilon_{13}^{ij},\ \Upsilon_{22}$ and Υ_{33} are defined in Theorem 3. Moreover, the state feedback gain can be constructed as $K_j=Y_jX^{-1}\ (j=1,2,\cdots,r)$.

5 Numerical examples

In this section, we provide illustrative examples to demonstrate the effectiveness of the method proposed in the previous section.

Example 1. Consider the uncertain stochastic T-S fuzzy system (44) with parameters as follows

$$A_1=\begin{bmatrix}-2.3 & 0\\ 0 & -5.7\end{bmatrix},\ A_2=\begin{bmatrix}-10 & 0.1\\ 0.1 & -12.9\end{bmatrix},$$

$$A_{d1}=\begin{bmatrix}0.5 & -0.1\\ 0.7 & -0.6\end{bmatrix},\ A_{d2}=\begin{bmatrix}0.2 & 0.5\\ 2 & 0.7\end{bmatrix},$$

$$C_1=\begin{bmatrix}-0.2 & 0\\ -0.1 & 0.1\end{bmatrix},\ C_2=\begin{bmatrix}-1 & 0.7\\ 0.3 & 0.5\end{bmatrix},$$

$$C_{d1}=\begin{bmatrix}0.5 & 0.3\\ 0.2 & 0.4\end{bmatrix},\ C_{d2}=\begin{bmatrix}2 & 0.2\\ -0.1 & 0.1\end{bmatrix},$$

$$\begin{aligned}E_{1i}=&0.1I,\ E_{2i}=0.2I,\ H_{li}=0.1I,\\ F_{li}(t)=&\mathrm{diag}\Big\{\sin(t),\ \cos(t)\Big\}\ (l=1,2,\ i=1,2).\end{aligned}$$

For this example, according to Corollary 2, system (44) is robustly asymptotically stable in the mean square. The maximal allowable upper bound of the time delay τ_M for various μ are shown in Table 1. Obviously, our result is less conservative than the method in [27]. Assuming $\tau_M=0.1328$ and $\mu=0.3$, solving LMI (50) in Corollary 2 by the Matlab LMI toolbox, we have the following feasible solutions:

$$P=\begin{bmatrix}4.5595 & 0.7706\\ 0.7706 & 6.3112\end{bmatrix}$$

$$Q_1=\begin{bmatrix}19.7757 & 4.0450\\ 4.0450 & 68.0995\end{bmatrix}$$

$$Q_3=\begin{bmatrix}2.4992 & -0.1721\\ -0.1721 & 0.3803\end{bmatrix}$$

$$R_1=\begin{bmatrix}4.7269 & -0.3315\\ -0.3315 & 0.0349\end{bmatrix}$$

$$R_3=\begin{bmatrix}19.9805 & -1.3832\\ -1.3832 & 0.4812\end{bmatrix}.$$

The time varying delay is assumed as $\tau(t)=0.13+0.0028\sin(t)$. For a membership function $h_1(x_1(t))=\frac{1}{1+\mathrm{e}^{(x_1(t)+0.5)}}$, $h_2(x_1(t))=1-h_1(x_1(t))$, and an initial function $\phi(t)=[-3,\ 3]^{\mathrm{T}}$, the simulation results of the state response of the system are plotted in Fig. 1.

Table 1 Maximal allowable delay τ_M for various μ

μ	0	0.3	0.6	$\geqslant 0.9$
Theorem 2[27]	0.0813	0.0099	–	–
Corollary 2	0.2530	0.1328	0.1069	0.1017

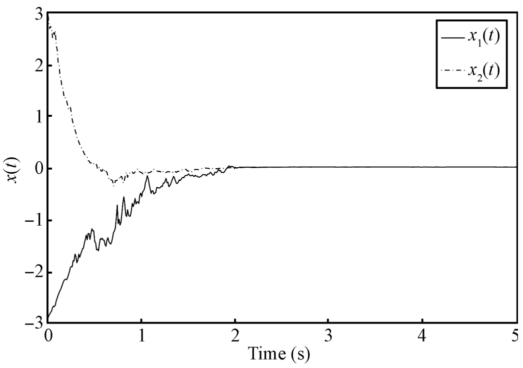

Fig. 1 State response of the system

Example 2. Consider the uncertain stochastic T-S fuzzy system (Σ_2) with parameters as

$$A_1 = \begin{bmatrix} -2 & 1 \\ 0.1 & -3 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.5 & 0 \\ 0 & -2 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.3 \end{bmatrix},\ A_{d2} = \begin{bmatrix} 0.5 & 0 \\ 0.4 & -0.3 \end{bmatrix},$$

$$B_{11} = \begin{bmatrix} -0.2 & 0 \\ -0.1 & 0.1 \end{bmatrix},\ B_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$B_{v_{11}} = \begin{bmatrix} -0.4 & 0.1 \\ 0 & -0.8 \end{bmatrix},\ B_{v_{12}} = \begin{bmatrix} -0.1 & 0 \\ -0.5 & 0.2 \end{bmatrix},$$

$$B_{d_{11}} = \begin{bmatrix} 0 & 0.2 \\ 0.1 & -0.2 \end{bmatrix},\ B_{d_{12}} = \begin{bmatrix} 0 & 0.5 \\ 0.2 & -0.3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.2 \end{bmatrix},\ C_2 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$C_{d1} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix},\ C_{d2} = \begin{bmatrix} -0.1 & 0.5 \\ 0.2 & -0.5 \end{bmatrix},$$

$$B_{21} = \begin{bmatrix} -0.2 & 0 \\ 0.1 & 0.1 \end{bmatrix},\ B_{22} = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.6 \end{bmatrix},$$

$$B_{v_{21}} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.2 \end{bmatrix},\ B_{v_{22}} = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix},$$

$$B_{d_{21}} = \begin{bmatrix} 0.3 & 0.2 \\ 0 & -0.3 \end{bmatrix},\ B_{d_{22}} = \begin{bmatrix} -0.4 & 0.3 \\ 0.2 & 0.3 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -0.03 & 0 \\ 0 & 0.03 \end{bmatrix},\ D_2 = \begin{bmatrix} -0.03 & 0 \\ 0 & 0.03 \end{bmatrix},$$

$$D_{d1} = \begin{bmatrix} -0.03 & 0 \\ 0 & 0.003 \end{bmatrix},\ D_{d2} = \begin{bmatrix} -0.13 & 0.2 \\ 0 & 0.4 \end{bmatrix},$$

$$B_{31} = \begin{bmatrix} 0.1 & -0.2 \\ -0.4 & 0.2 \end{bmatrix},\ B_{32} = \begin{bmatrix} -0.3 & 0.3 \\ 0.2 & -0.2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix},\ E_2 = \begin{bmatrix} 0.03 & 0 \\ 0 & -0.03 \end{bmatrix},$$

$$H_{11} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.3 \end{bmatrix},\ H_{12} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.3 \end{bmatrix},$$

$$H_{21} = \begin{bmatrix} 0.05 & -0.35 \\ 0.7 & 0.45 \end{bmatrix},\ H_{22} = \begin{bmatrix} 0.05 & -0.5 \\ 0.7 & 0.45 \end{bmatrix},$$

$$H_{31} = \begin{bmatrix} -0.11 & 0.2 \\ 0 & 0.01 \end{bmatrix},\ H_{32} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.15 \end{bmatrix},$$

$$H_{41} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.3 \end{bmatrix},\ H_{42} = \begin{bmatrix} -0.15 & 0.2 \\ 0 & 0.3 \end{bmatrix},$$

$$H_{51} = \begin{bmatrix} 0.05 & -0.35 \\ 0.7 & 0.45 \end{bmatrix},\ H_{52} = \begin{bmatrix} -0.1 & 0.2 \\ 0 & 0.01 \end{bmatrix}$$

$$H_{61} = \begin{bmatrix} -0.21 & 0.3 \\ 0 & 0.31 \end{bmatrix},\ H_{62} = \begin{bmatrix} -0.05 & 0.35 \\ 0.7 & 0.45 \end{bmatrix},$$

$$F_1(t) = F_2(t) = \mathrm{diag}\Big\{\sin(t),\ \cos(t)\Big\}.$$

In this example, our aim is to design a state feedback fuzzy controller such that, for all admissible uncertainties, the closed-loop system is robustly stochastically stable with disturbance attenuation $\gamma = 0.2$. The maximum allowable upper bounds of the time delay τ (for $\tau_M = d_M$) are obtained for different τ_m and various μ from Theorem 3 which are shown in the Table 2. For $\tau_m = 0.1$, $\mu = 0.2$, the time delay $\tau_M = 0.3432$, and $d_M = 0.3432$, solving the LMIs (51) and (52) through Matlab LMI control toolbox, the feasible solutions are given by:

$$X = \begin{bmatrix} 29.4343 & 1.7272 \\ 1.7272 & 7.8654 \end{bmatrix},\ \bar{Q}_1 = \begin{bmatrix} 22.4363 & -8.5634 \\ -8.5634 & 16.7466 \end{bmatrix},$$

$$\bar{Q}_2 = \begin{bmatrix} 0.0678 & 0.0114 \\ 0.0114 & 0.0027 \end{bmatrix},\ \bar{Q}_3 = \begin{bmatrix} 2.0366 & 0.0603 \\ 0.0603 & 0.5159 \end{bmatrix},$$

$$\tilde{R}_1 = \begin{bmatrix} 50.0733 & 4.1912 \\ 4.1912 & 12.9957 \end{bmatrix},\ \tilde{R}_2 = \begin{bmatrix} 58.7164 & 3.4330 \\ 3.4330 & 15.7265 \end{bmatrix},$$

$$\tilde{R}_3 = \begin{bmatrix} 29.2726 & 1.0348 \\ 1.0348 & 8.4426 \end{bmatrix},\ \tilde{R}_4 = \begin{bmatrix} 55.3893 & 2.6179 \\ 2.6179 & 15.5254 \end{bmatrix},$$

$$\bar{Z} = \begin{bmatrix} 80.7522 & 11.6452 \\ 11.6452 & 5.4140 \end{bmatrix}.$$

Table 2 Maximal allowable delay of τ with given τ_m and for various μ

μ	0	0.2	0.4	0.6	$\geqslant$ 0.8
$\tau_m = 0.1$	0.3960	0.3432	0.2696	0.2206	0.2134
$\tau_m = 0.3$	0.4726	0.4077	0.3405	0.3181	0.3174

By Theorem 3, we can obtain the desired state-feedback fuzzy controller as

$$K_1 = \begin{bmatrix} -0.3980 & -0.9984 \\ -0.0288 & -0.3490 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.4021 & 0.5118 \\ -0.2922 & -0.8733 \end{bmatrix}.$$

Define the membership functions as $h_1(x_1(t)) = \frac{1-\sin(x_1(t))}{2}$ and $h_2(x_1(t)) = \frac{1+\sin(x_1(t))}{2}$. The time-varying delays are assumed as $\tau(t) = 0.34 + 0.0032\sin(t)$ and $d(t) = 0.34 + 0.0032\sin(t)$, with an initial condition $\phi(t) = [-3,\ 2.5]^{\mathrm{T}}$. The disturbance input is assumed to be $v_1(t) = \frac{1}{0.2+t^2}$ and $v_2(t) = \frac{1}{1+t^2}$. Fig. 2 shows the state response of the closed-loop system. Figs. 3 and 4 show the graphical representation of the control input and controlled output respectively. From the above, it can be seen that the designed H_∞ controller satisfies the specified requirements.

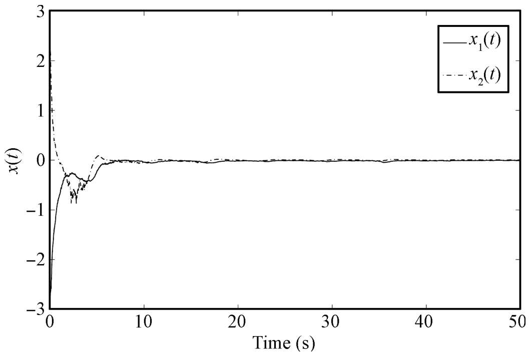

Fig. 2 State response of the closed-loop system

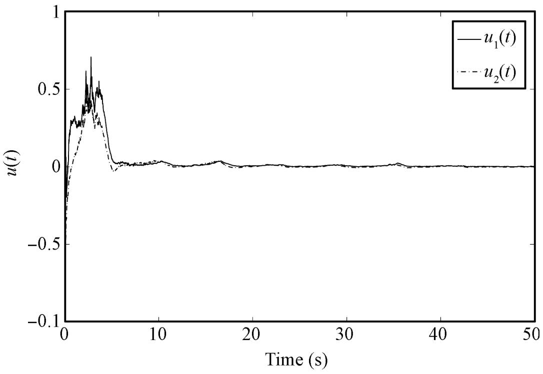

Fig. 3 Control input

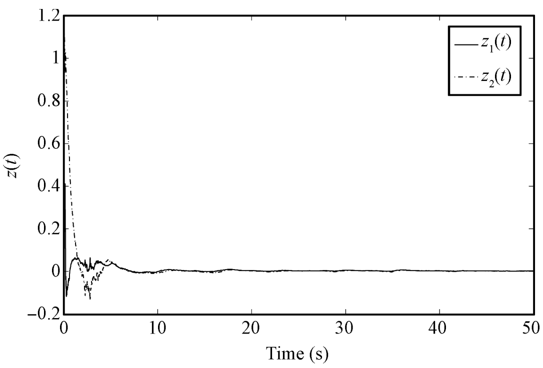

Fig. 4 Controlled output

6 Conclusions

In this paper, some sufficient conditions have been derived for the solvability of problems of delay-dependent robust stabilization and H_∞ controller design for uncertain stochastic T-S fuzzy systems with discrete interval and distributed time-varying delays. These conditions are expressed in terms of LMIs, which can be easily tested by using Matlab control toolbox. It has been shown that a desired state feedback controller can be constructed when the LMIs are feasible. Finally, two numerical examples have been given to illustrate the effectiveness of the developed techniques.

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