

## The need for introduction of non-probabilistic interval conceptions into structural analysis and design

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Received August 31, 2016; accepted September 13, 2016; published online September 23, 2016

**Citation:** Z. P. Qiu, and L. Wang, The need for introduction of non-probabilistic interval conceptions into structural analysis and design, *Sci. China-Phys. Mech. Astron.* **59**, 114632 (2016), doi: [10.1007/s11433-016-0329-3](https://doi.org/10.1007/s11433-016-0329-3)

With the growing complexity of engineering structures and the severity of their service environments, the uncertainties related to material properties, external loads, boundary conditions, measurement noises, etc., have increased. Uncertainty greatly influences structural safety, and hence, it is necessary to completely understand all the uncertainties involved when tackling analysis and design issues [1].

Traditional approaches to uncertainty analysis and design derived from probability models have been widely applied to various industries in the past decades. Indeed, precise probability distributions of the uncertain parameters based on a large amount of experimental samples may be required in these industries. In many engineering applications, however, the experimental data are limited. In view of this problem, several non-probabilistic interval methods for evaluating reliability and guiding structural design were developed; these methods have gained attention because of insufficient parametric data [2]. The recent research by scholars and engineers worldwide are summarized as follows:

### (1) Interval propagation analysis

Interval propagation analysis is essential for obtaining more reasonable descriptions of uncertain structural responses in cases of missing information. In 1994, Wang et al. [3] first proposed the interval perturbation analysis in mechanics. Recently, they presented the novel stochastic collocation method and the dimension-wise method (DWM) to predict static and dynamic mechanical responses accurately [4,5]; the collocation

method is based on the equivalent weak form of a multivariate function integral, and the DWM is based on the Legendre orthogonal polynomial approximation (as shown in Figure 1). Muhanna et al. [6] presented an element-by-element technique for interval finite element analysis in mechanics to overcome the drawbacks of the traditional interval analysis due to the dependency phenomenon. By means of a novel approximate relationship based on an interval-valued Sherman-Morrison-Woodbury formula, Nicola et al. [7] developed a numerical method for evaluating the response of structures with interval axial stiffness; this method takes into account the high-order perturbation terms.

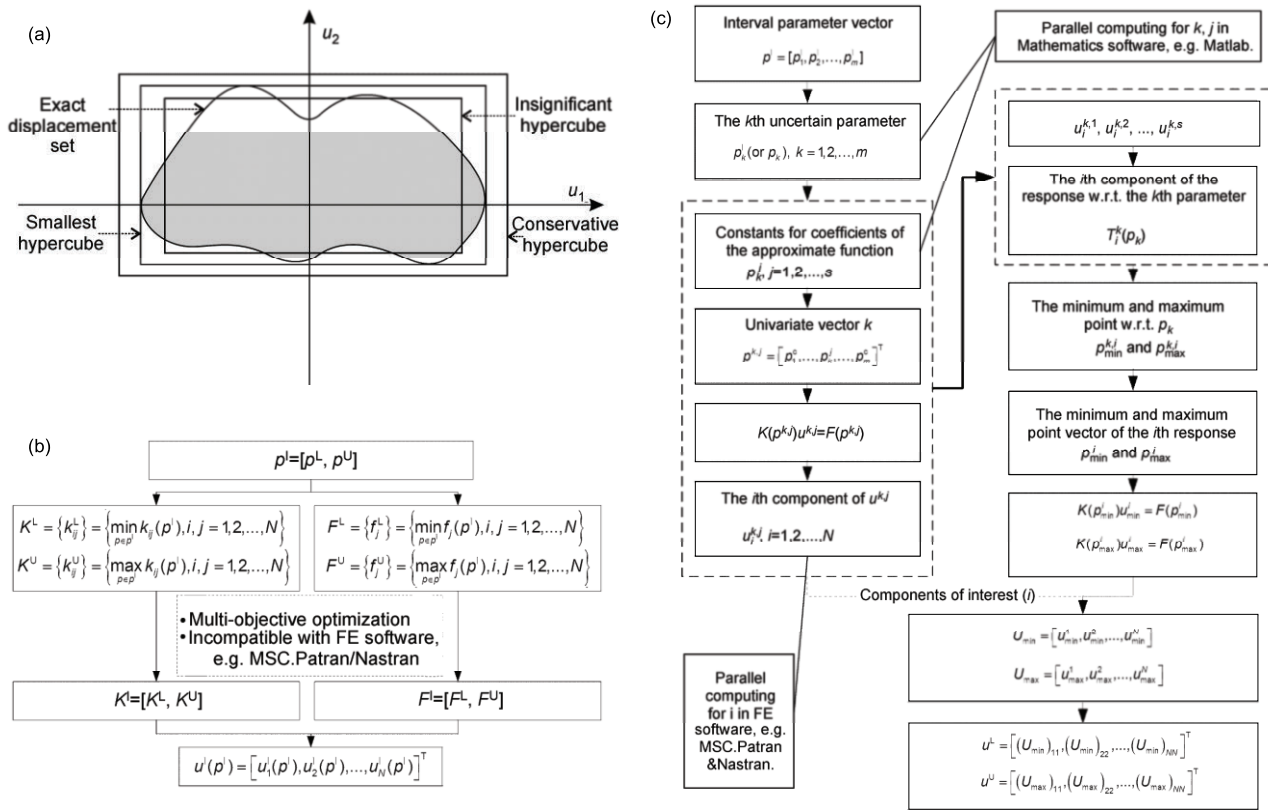
### (2) Interval reliability analysis

The idea of non-probabilistic reliability was first introduced by Ben-Haim and Elishakoff [8] in the early 1990s. In recent years, Vinot et al. [9] defined the performance criterion for structures by adopting a new non-probabilistic concept of reliability based on the info-gap robustness function. Jiang et al. [10] conducted a new structural reliability assessment method based on a multi-dimensional convex model and then explored the dependency between variables by using the proposed methodology. Wang et al. [11] defined a new time-dependent reliability index by combining the interval process model and the first-passage theory. It was successfully adopted to estimate the safety levels of time-varying truss and marine propeller structures.

(3) Reliability-based design optimization (RBDO) by interval mathematics

The ultimate aim of analysis is to guide the application of

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**Figure 1** Interval propagation analysis (from Xu et al. [5]). (a) Different hypercube approximations; (b) flowchart of the indirect method; (c) flowchart of DWM.

a design scheme in a better manner. In fact, economy and safety are the competing goals and major considerations in structural design, and thus, designing structural systems involves a tradeoff between safety and economy. Therefore, the reliability-based design optimization (RBDO) policy based on interval mathematics is gaining importance as the main research direction for future work. Chakraborty et al. [12] presented the RBDO of tuned mass damper (TMD) parameters in seismic vibration control, for which the interval model was employed to represent the uncertain-but-bounded parameters. Luo et al. [13] introduced interval analysis in mechanism synthesis, and explored a new robust methodology to accommodate truncated dimension variables and interval clearance variables.

(4) Interval inverse problems in mechanics

The above statements are generally classified as direct problems. In addition to these problems, another set of mechanical issues, namely, inverse problems exist; these problems are based on the two aspects of damage identification and load identification and have been systematically studied as well. Gabriele et al. [14] developed an interval method of model updating and global minimization for dynamic damage identification problems in framed structures. Song et al. [15] introduced the interval perturbation approach

for the load reconstruction of the statistical energy analysis (SEA) framework, and revealed the integral influences of the measurement errors in SEA parameters on identified excitations.

However, there still exist some problems unsolved and difficulties in the present structural interval analysis: (1) the quantification of multi-source uncertainties in a valid manner, (2) guaranteeing high precision in uncertainty propagation in order to balance the accuracy and applicability in a calculation, and (3) effective verification of the reasonability of non-probabilistic interval reliability analysis and design in complex systems.

In general, the non-probabilistic interval method has a significant influence when dealing with engineering problems, and certainly shows high potential for application in future research. The future work will focus on the generalization-oriented, elaboration-oriented, and engineering-oriented aspects.

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