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Of Marriage and Mathematics: Inferentialism and Social Ontology

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Abstract
The semantic inferentialist account of the social institution of semantic meaning can be naturally extended to account for social ontology. I argue here that semantic inferentialism provides a framework within which mathematical ontology can be understood as social ontology, and mathematical facts as socially instituted facts. I argue further that the semantic inferentialist framework provides resources to underpin at least some aspects of the objectivity of mathematics, even when the truth of mathematical claims is understood as socially instituted.

Keywords Abstraction principles · Deontic scorekeeping · Philosophy of mathematics · Semantic inferentialism · Social ontology

1 Introduction and the Plan of the Paper

Semantic inferentialism is now a well established, if not exactly an *establishment*, research programme in the philosophy of language.[1] To this date, relatively little has been said about its import for mathematical vocabularies in particular.[2] But the framework of semantic inferentialism provides a rich set of conceptual resources to probe philosophical questions about mathematical vocabularies and mathematical practices. Here, I press it into this task. In particular, I argue, firstly that semantic inferentialism provides a framework within which mathematical ontology can be understood as social ontology, and mathematical facts as socially instituted facts. Secondly, I argue that the same framework provides resources to underpin at least some aspects of the objectivity of mathematics, even when the truth of mathematical claims is understood as socially instituted.[3]

This will involve some scene-setting regarding inferentialism itself. The inferentialist takes semantic meaning to be instituted by norms regarding proprieties of inference—the network of inferential connections a claim has to other claims. These norms are themselves socially instituted. Both grasping semantic meanings and communicating with others requires being able to navigate this network of inferential proprieties; it requires acknowledging entitlements and commitments to claims, and keeping track of the further entitlements and commitments that arise as a result: a practice Brandom (1994, ch. 3) calls 'deontic scorekeeping'. Though Brandom's development of this core idea is multifaceted and complex, I will set out just enough of his account to make intelligible how it can be naturally extended to make sense of the institution of social entities. And with this scene-setting in place, I will sketch how the semantic inferentialist account of meaning is naturally extended to apply to the social construction of other social facts, facts concerning things like monetary value and

[1] See Brandom (1994, 2000a, b) and Peregrin (2014) for extended defences of semantic inferentialism.

[2] Dummett (1978, 1993, 2000), Prawitz (1974, 1977), Tennant (most recently (2022)), and Warren (2020) have all developed broadly inferentialist views, with an eye to mathematics. They differ, however, in fundamental respects from the semantic inferentialism of Brandom and Peregrin—Dummett, Prawitz, and Tennant all take inferentialism to underwrite intuitionistic logic, and Warren's view is more austerely naturalistic, aiming to parse inferential rule following in dispositionalist terms. Semantic inferentialism, moreover, brings a very different set of explanatory resources to the table, some of which are indispensable to the view sketched here.

[3] The social constructivist account of mathematics that is made possible with a semantic inferentialist framework has some affinities with Cole's social constructivist account of mathematics (Cole 2013, 2015). But the semantic inferentialist framework provides distinct (though perhaps complementary) resources to explain aspects of the objectivity of mathematics, understood in social constructivist terms.

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connubiality. We will then be in a position to see why the semantic inferentialist framework makes possible an understanding of mathematical facts as both socially instituted and, in some important senses, objective.

2 Inference, Substitution, Anaphora

Brandom's inferentialist account of meaning-instituting practices, I have said, is complex and multifaceted. But here, three nested layers of practices will concern us.

2.1 Layer One: Inference

The first layer consists of the practices that institute sentential meaning. The animating thought behind semantic inferentialism is that semantic meaning is instituted by inferential relationships–relationships of *incompatibility*, *commitment-preservation*, and *entitlement-preservation*—between claims, expressible in sentences. Say that on the basis of perception in good conditions, one undertakes, and becomes epistemically entitled to undertake, a commitment to the claim *The animal over there is a lion*. In doing so, one thereby *precludes* oneself from undertaking a commitment or gaining entitlement to *The animal over there is a reptile*, undertakes a *further* commitment and entitlement to *The animal over there is a mammal*, and becomes committed and entitled to *The animal over there is dangerous*. (This latter commitment and entitlement is defeasible; it can be lost by undertaking certain other collateral commitments such as *The animal over there has been tranquillised.*)

Understanding the meaning of a claim requires a practical mastery of these relationships of incompatibility, commitment-preservation, and entitlement-preservation; it involves being able to navigate, in practice, this network of inferential relationships. This practical mastery will involve things like *not* undertaking a commitment to *The animal over there is a lizard* when one has undertaken a commitment to *The animal over there is a lion*, answering in the affirmative (in appropriate circumstances) when asked whether the animal is a mammal, and so on. One's practical mastery need not be perfect, but one must be able to keep track of one's commitments and entitlements to at least some significant extent—to competently engage in *deontic scorekeeping*—to count as grasping the meanings of the commitments one acknowledges to at least some significant extent. (For the inferentialist, grasping meaning is something that comes in degrees.)[4]

2.2 Layer Two: Substitution

At the second layer are subsentential expressions. Inferential relationships hold between claims, expressible as sentences. Subsentential expressions are meaningful insofar as they make *contributions* to the inferential roles of claims, and grasping subsentential expressions requires being able to navigate, in practice, the way they alter the inferential proprieties of sentences when substituted into those sentences. Take, for instance, the inferential contribution of singular terms: expressions such as names and definite descriptions, whose role is to pick out exactly one thing. Briefly, and roughly, grasping the inferential contribution of singular terms such as 'Margaret Thatcher', 'The first woman UK Prime Minister', and 'The inventor of Mr Whippy Ice Cream' involves grasping that in undertaking a commitment to, for example, *Margaret Thatcher introduced an unpopular poll tax* one thereby undertakes a commitment to, for example, *The first woman UK Prime Minister introduced an unpopular poll tax*, and *The inventor of Mr Whippy ice cream introduced an unpopular poll tax*. It involves grasping, furthermore, that these relationships of commitment-preservation are *de jure* symmetrical (as well as transitive and reflexive), so that in taking it that a commitment to *Margaret Thatcher introduced an unpopular poll tax* licenses a commitment to *The inventor of Mr Whippy ice cream introduced an unpopular poll tax*—and that it does so in virtue of the inferential contribution of the singular terms 'Margaret Thatcher' and 'the inventor of Mr Whippy ice cream'—one is thereby committed to the goodness of the reverse inference from *The inventor of Mr Whippy ice cream introduced an unpopular poll*

[4] Note that, for Brandom, inferential relationships between claims are cashed out in *normative* terms. We have talked of *deontic* scorekeeping and *proprieties* of inference (which inferences would be proper

Footnote 4 (Continued)

or improper) rather than in terms of *dispositions* to infer. Some of the advantages of this approach are discussed by Peregrin (2014, ch. 1), and by myself (Collin 2013, 2017). These proprieties of inference are themselves 'brought into play by social linguistic practices of giving and asking for reasons, of assessing the propriety of claims and inferences.' (Brandom 2000a, b, p. 26) It is of course very plausible *that* linguistic norms are instituted by linguistic practices. (One is tempted to ask: *how else could they be instituted?*) Making out exactly *how* linguistic norms are instituted by linguistic practices is however a large task, requiring making sense of how one can treat another in practice as a normative authority (Brandom 2019, ch. 8), making sense of reciprocal recognition of the other as both subject to one's own normative authority and as holding normative authority over oneself (Brandom 2019, chs. 9 & 10), and making sense of how our treatments of things as having a given normative status (our normative attitudes) can themselves *institute* norms that we can be both responsible to and mistaken about (Brandom 1994, chs. 1 & 8; 2019, ch.15; 2022, ch. 2). The details of this richly developed account will not concern us here. However, going forward I will take it as plausible that linguistic norms are instituted by social linguistic practices.

tax to *Margaret Thatcher introduced an unpopular poll tax*.[5] Again, this practical mastery need not be perfect, but one must be able to keep track of the inferential contribution of subsentential expressions to at least some significant extent to qualify as using them *as* subsentential expressions of a particular kind, and to grasp their meaning to at least some significant extent.

2.3 Layer Three: Anaphora

But these abilities are themselves only intelligible in the context of another kind of bookkeeping. In order to keep track of the inferential contribution of subsentential expressions (and of the sentences they compose), one must, and most fundamentally, be able to keep track of when one is using the *same* subsentential expression in *distinct* inscriptions, utterances, or gestures. That is, one must be able to keep track of when a given *token* or *tokening* is a token of a particular *type*. This is the third layer in Brandom's account of meaning-instituting practices—as we will see, it plays a pivotal role in explaining linguistic objectivity. Consider the argument:

> Artemis is a woman.
> All women are mortal.
> ∴ Artemis is mortal.

Is the argument classically valid? Not necessarily. The validity of the argument depends on the status of the first and second tokenings of the orthographic type <Artemis>. For if the first tokening of <Artemis> denotes a human and the second a (nonidentical) member of the Greek pantheon, then the conclusion is false, and the argument invalid. So the validity of the argument depends on its being the case that—and, in fact, one would typically be entitled to assume that it *is* the case that—we are treating the second tokening of <Artemis> as a *recurrence* of the first. As Brandom (1994, p. 451) notes, one cannot make this presupposition explicit using the identity claim 'Artemis = Artemis', on pain of regress. One can however introduce a notation to do so. Let terms of the form /a/$_i$ pick out distinct tokenings of the type <a>, so that we can form expressions such as '/a/$_2$ is a recurrence of /a/$_1$', to make explicit anaphoric recurrences of this sort. We can then say that the argument above is valid, just in case /Artemis/$_2$ is a recurrence of /Artemis/$_1$ (and sound just in case both /Artemis/$_1$ and /Artemis/$_2$ refer to a woman). For the inferentialist, then, the ability to book-keep with respect to anaphoric recurrence is necessary if one is to have the ability to book-keep with respect to the inferential contribution of subsentential expressions. And the ability to book-keep with respect to the inferential contribution of subsentential expressions is necessary if one is to have the ability to book-keep with respect to the commitments and entitlements one undertakes in assertorically deploying sentences. The sentential deontic scorekeeping practices constituting the first layer are intelligible only if supported by the subsentenial bookkeeping practices constituting the second layer. These in turn are only intelligible if supported by the anaphoric bookkeeping practices constituting the third layer.

3 The Role of Anaphora in the Objectivity of Discourse

*Intra*personal reasoning requires this third kind of bookkeeping. One cannot infer *Artemis is mortal* from *Artemis is a woman* and *All women are mortal* unless one is able to deploy the second tokening of <Artemis> as a *recurrence* of the first. And the point generalises to any inference one may make. If one is to reason at all, one must keep track of anaphoric recurrence. But, for the inferentialist, anaphora are also essential for *inter*personal communication. Recall that for the inferentialist, the meaning of a sentence is given by its inferential significance, i.e. what, in undertaking a commitment to that sentence, one thereby also undertakes a commitment to and one thereby precludes oneself from undertaking a commitment to, and what, in gaining entitlement to a sentence, one thereby also gains entitlement to. Note firstly that what commitments and entitlements follow from commitment or entitlement to a given sentence differs depending on which collateral commitments one has. I acknowledge a commitment to *The animal over there is a lion* and thereby undertake a commitment to *The animal over there is dangerous*. You similarly acknowledge a commitment to *The animal over there is a lion*, but because you are also committed to *The animal over there has been tranquillised* you do not undertake the further commitment to *The animal over there is dangerous*. So, because of our differing collateral commitments, the acknowledged inferential proprieties of *The animal over there is a lion* differ between you and I. If we did not have different sets of commitments, communication would be superfluous. But because we do, if I am to communicate with you, I must keep track, at least to some significant extent, of both my own commitments and your commitments. (Brandom 1994, p. 475).

As a result, inferentialists, of the Brandomian stripe, reject broadly Lockean transmission accounts of communication. Communication does not take place because

[5] The inferential contributions of singular terms are made explicit using claims of identity of the form $\iota x(\phi x) = \mathbf{a}$, such as *The inventor of Mr Whippy ice cream is Margaret Thatcher*. The inferential contribution of predicates, another kind of subsentential expression, is not uniformly symmetrical. Even though predicates typically license some symmetrical relationships of commitment-preservation—e.g. from *The shape is a triangle* to *The shape is a trilateral with three straight sides* and back again—they will always license some asymmetric relationships of commitment preservation—e.g. from *The shape is a triangle* to *The shape is a convex polygon*, but not back the other way.

some semantic content is transmitted, through inscriptions, utterances, gestures, and the like, from one interlocutor to another. Instead, communication involves deontic scorekeeping. In communication, one keeps *two* sets of books: the first of one's *own* commitments and entitlements, and the second of the commitments and entitlements of one's *interlocutor*. But since the inferential significance of any given sentence—and, in fact, any given subsentential expression, including singular terms—can differ from interlocutor to interlocutor, what kind of practice would allow us to treat the talk of two distinct interlocutors as being *about* the same things? Here, the third layer of practices—anaphoric bookkeeping—is key. Consider again <Artemis>, and the pair of claims:

Artemis is a human.
Artemis is a god.

Are the claims incompatible? Again, it depends on whether /Artemis/$_{2}$ is a recurrence of /Artemis/$_{1}$. If so then the claims are incompatible, if not then the claims are not incompatible. More generally, *treating* two tokenings as anaphoric recurrences of each other involves treating them as having a particular kind of inferential significance: as making claims *incompatible* when they involve contrary predicates.

This, it turns out, is a central part of what it is to treat a term as having singular referential purport, to treat a term as referring to a particular object. This is perhaps easiest to see by first considering the issue at the *objectual* level—in terms of *objects* and *properties*—before considering what kinds of inferential practices would allow us to treat singular terms as picking out objects and predicates as picking out properties. We understand some *properties* to be incompatible with each other. The property of having positive charge is incompatible with the property of having negative charge, the property of being a god is incompatible with the property of being a human. But properties *themselves* are not incompatible. Rather, there are properties that cannot be had by one and the same *object*. No one *thing* can be simultaneously positively charged and negatively charged. No one *thing* can be both a god and a human. No incompatibility arises however when contrary properties are had by *distinct* objects.[6] It is possible for *A* to be negatively charged while, at the same time, *B* is positively charged, and for one member of the human race, 'Artemis', to be a human while, at the same time, for a nonidentical member of the pantheon, 'Artemis', to be a god. Objects are loci of incompatibility. (The incompatibility here is alethic; it is *impossible* for a single object to be simultaneously both positively and negatively charged.)

What sorts of deontic scorekeeping practices then would allow us to talk *as of* particular objects with particular properties? We have already noted that there are also incompatibilities between *claims*. If both tokenings of <Artemis> are anaphoric recurrences of each other, then *Artemis is a human* and *Artemis is a god* are incompatible. (Here the incompatibility is *deontic*; it is not *impossible* for someone to undertake commitments both to *Artemis is a human* and *Artemis is a god*. It is however *improper*. One *ought* not to be committed both to *Artemis is a human* and to *Artemis is a god*. This is why the scorekeeping at issue is *deontic* scorekeeping.) Treating tokenings of singular terms as loci of incompatibility is part of what it is to treat singular terms as referring to *objects*:

> The judgement that *A* is a dog is *not* incompatible with the judgement that *B* is a fox. The judgement that *A* is a dog *is* incompatible with the judgement that *A* is a fox. That means that taking a dog-judgement to be materially incompatible with a fox-judgement *is* taking them to refer to or represent an object: the *same* object. And the same thing holds for relations of material inferential consequence. Taking it that *A* is a dog does *not* entail that *B* is a mammal. But taking it that *A* is a dog *does* entail that *A* is a mammal. So drawing the inference is taking it that the two judgements refer to one and the same object. (Brandom 2009, pp. 43–44)[7]

It is our ability to use tokenings as anaphoric recurrences of one another—and so as playing the inferential role of making atomic claims containing contrary predicates incompatible with one another when they contain singular terms that are anaphoric recurrences of one another—that allows for the *intra*personal treatment of claims as being co-referential. And it is the same practice that allows for the *inter*personal treatment of claims as being co-referential. This is because we can treat the tokenings of *others* as anaphoric recurrences of our own tokenings (and vice versa)—as forming parts of the same anaphoric chain. It is possible to do this even when one takes it that the commitments and entitlements of the other *at the sentential level* are very different from one's own. By doing so, one treats one's own commitments and the *different* commitments of the other, as being *about* the *same* objects, as occupying two different *perspectives* on the same *things*.

[6] Here I'm restricting attention to properties picked out by 1-place predicates. A very similar point applies to predicates of higher arity however (Brandom 2009, p. 44).

[7] Brandom continues: 'Represented objects show up as something like *units of account* for the inferential and incompatibility relations judgeable contents stand in to one another. If two properties are incompatible, then it is impossible for *one and the same* object to exhibit both, but not impossible for *two different* objects to do so. And if possession of one property entails possession of another, then any object that exhibits the first will necessarily exhibit the second. But it is not necessary that some other object do so.' (Brandom 2009, pp. 44–45).

This co-ordination takes place when I treat some of your subsentential tokenings as belonging to the same anaphoric chain as some of my subsentential tokenings, *by* treating some of your sentential tokenings as incompatible with some of my own sentential tokenings. This is the scorekeeping practice by which we enact the idea that we are both purporting to talk *about* the same objects, and that our intended subject matter *transcends* either of our commitments and entitlements regarding it. For instance, I, in contrasting my commitments with your commitments, treat *your* commitments as being incorrect.[8] This is how our linguistic practices can embody the idea that the objects *of* our linguistic practices can transcend our conceptual grip on them, and how we can institute semantic norms according to which it is intelligible that our claims can be wrong in a way that transcends one's own entitlements and the entitlements of others.

4 Inferentialism and Social Ontology

We have seen in outline how the inferentialist understands semantic meaning, as well as the object-directedness of discourse. We are now in a position to see firstly how inferentialism provides a framework by which one can make sense of social ontology, and secondly how truths about mathematical objects themselves can be understood, by the inferentialist, as socially instituted, before, thirdly, seeing how this accounts for aspects of the objectivity of mathematics. Brandom does not himself develop a theory of social ontology.[9] The framework he develops to explain the institution of semantic meanings however is well adapted to be pressed into this task.[10]

Consider first the inferentialist account of semantic meaning as involving proprieties of inference, understood as relations of incompatibility, commitment-preservation, and entitlement-preservation between sentences. For the inferentialist, grasping the semantic meaning of a claim essentially involves the ability to place that claim within, to use Sellars' phrase, a *space of reasons*: grasping what would count as a reason for committing oneself to that claim, grasping what commitment and entitlement to that claim would provide reasons for, and grasping what commitment and entitlement to that claim would preclude entitlement to. This is a normative, deontic business, as the language of *commitment* and *entitlement* suggests.

Just as claims can be treated as having a particular deontic profile—a particular location in a space of reasons—so too can physical objects, aggregates, pluralities, persons and so on be treated as having a particular deontic profile—a particular location in a space of reasons. What makes the difference from a piece of paper (or polymer) being a *mere* piece of paper and being a banknote, is the position in a space of reasons that we assign to the piece of paper. To *treat* a piece of paper *as* a banknote is to treat it as being bound up with a constellation of commitments and entitlements. (At an earlier stage of its evolution, paper money was, explicitly, a promissory note, such that the person who possessed the appropriate piece of paper was *entitled* to request a given sum of gold from the institution that had issued the note, whereby the institution was *obligated* to pay that sum.) *Monetary value* can be understood as a socially instituted normative status in a way analogous to semantic meaning. Here, as with semantic meaning, our normative attitudes—our treating commitments as permissible, impermissible or obligatory—institutes normative statuses. Similarly, to treat a person as married is to regard that person as having a particular constellation of commitments and entitlements, as occupying a particular location in a space of reasons. Again, *connubiality* can be understood as a socially instituted normative status in a way analogous to semantic meaning.

Now there is a sense in which instituting social-normative statuses that are attributed to physical things already has the *potential* to institute a distinct social ontology, in that it has the potential to produce quantificational commitments over and above physical quantificational commitments. How so? In making explicit that certain physical objects have a given social-normative status—in introducing a vocabulary by which we can explicitly describe things as having a given social-normative status—we introduce a new predicates. We can, for instance, say that 'Thatcher is *ineligible* for membership of the Socialist Party of Great Britain'. Potential for expanding the domain of quantification comes into the picture when the predicates introduced are *sortal* predicates, for instance when we say that 'Thatcher was the *prime minister* of the UK'. The domain of quantification of a vocabulary is inextricably bound up with the sortal predicates of that vocabulary, as a result of the special relationship between sortals and identity judgements. Consider the predicate *ineligible*, in contrast to the sortal predicate *prime minister*; having learned that Thatcher was prime minister and that

[8] One is able also to treat one's *own* commitments as being potentially incorrect. One way this shows up is when one contrasts one's *current* commitments—which, by definition, one currently endorses—with one's *previous*, different commitments—which one now rejects. See (Brandom 2019, ch. 2). We will return to the idea that deontic scorekeeping makes space for one to treat one's own commitments as potentially incorrect below.

[9] Though social ontology as such is not one of his primary concerns—and he does not claim, as I do here, that the inferentialist can understand mathematical objects as socially instituted—Brandom does consider the institution of social statuses in detail. See, for instance, (Brandom 2019, ch. 10).

[10] Though a fuller development of the inferentialist account of social ontology sketched here would reveal many differences with these respective accounts, it shares with Ásta (2018) and Searle (1995, 2010) the plausible idea that social construction has to do with the conferral of *statuses*.

Johnson was prime minister, it makes sense to ask whether Thatcher and Johnson were the *same* prime minister, but having learned that Thatcher is ineligible and that Johnson is ineligible, it makes no sense to ask if Thatcher is the *same* ineligible as Johnson.[11]

When we introduce sortals to keep track of socially-instituted normative statuses, we talk of things that are the things they are—that have the criteria of identity they have—in virtue of their socially-instituted statuses. The potential for expanding the domain of quantification is due to the criteria of identity associated with some sortals (picking out socially-instituted roles) differing from the criteria of identity associated with other sortals (picking out physical objects). *University* is a sortal predicate, and we can intelligibly make identity judgements about universities, but the criteria of identity for a university may well not match those of any physical thing. If so, socially instituted facts require an expanded domain of quantification including universities not identical to any physical thing.

5 Mathematics and Social Ontology

There is however a clear sense in which something different is going on in if truths about mathematical objects are understood as socially instituted. In the case of the husband or banknote, we socially institute the existence of a social object *by* treating a physical physical object as having a particular normative status in a practice of giving and asking for reasons. Even universities, though perhaps not identical with any physical thing or plurality, at least depend for their existence on physical things being given normative statuses. There is no clear analogue of this in the mathematical case. If truths about mathematical objects are socially instituted, they appear to be so out of whole cloth, as it were. We cannot, at least cannot obviously, socially institute mathematical truths in a similar way to truths about banknotes, husbands, or universities, by treating physical objects as having particular normative statuses.

To get a handle on the social institution of truths about mathematical objects we need to consider again the central role the inferentialist assigns anaphora. We saw earlier the role anaphora plays in explaining the objectivity of discourse. (And in fact we will soon see that, on the inferentialist account, they play the same role in explaining the objectivity of mathematical discourse.) In ordinary empirical discourse, the first link in an anaphoric chain may be picked out deictically, paradigmatically by combining a demonstrative with a sortal, for instance in '*That cat* is watching television'.[12] In ordinary cases where there is one salient cat, the demonstrative *that* along with the sortal *cat* succeeds in picking out the cat. As we noted before, the ability of *others* to treat *their* tokenings as anaphoric recurrences of *my* tokenings of <that cat> is what makes possible both interpersonal communication and interpersonal disagreement. It is also what is required, in practice, to treat our discourse as being about things, the statuses of which, in some sense, transcend our commitments regarding them.

In empirical vocabulary, anaphoric initiation is typically bound up with perceptions. Clearly mathematical objects cannot be picked out deictically in an analogous way. One cannot introduce talk of numbers by pointing at them and saying, for instance, 'That number is prime'.[13] But, at the same time, the practice of treating unique tokenings as links in an anaphoric chain is a necessary background for the intelligibility of practices of reasoning and communication. So, if we are capable of reasoning within and communicating using mathematical discourse, it is required that we treat unique mathematical tokenings as links in an anaphoric chain. How are anaphoric chains to be initiated in the mathematical case?

Consider the idealised but coherent example of a community with deontic scorekeeping practices sufficiently rich to institute things like semantic meanings, currencies, marriages and the like. The community however, and despite its relative norm-instituting sophistication in other domains, lacks mathematical vocabularies. There is nothing that would, in principle, prevent this community from introducing new vocabulary by setting out deontic scorekeeping norms for the use of that vocabulary. The community could introduce a cardinality operator Nx by stipulating an introduction rule:

$$\frac{F \approx G}{Nx : Fx = Nx : Gx}$$

and an elimination rule:

$$\frac{Nx : Fx = Nx : Gx}{F \approx G}$$

Here F and G pick out concepts, $\approx$ is an equivalence relation, and Nx is a term-forming operator: a function from concepts to cardinal numbers. We can read $F \approx G$ as 'The Fs are equinumerous with the Gs'[14] and $Nx : Fx = Nx : Gx$ as 'The number of Fs is equal to the

[11] We can perhaps intelligibly ask whether Thatcher and Johnson are *as* ineligible or even "equally" ineligible, but the relation expressed there would be one of *equivalence* rather than *identity*.

[12] The example is from (Brandom 1994, p. 468).

[13] One can, of course, point at, for instance, the *numeral* '7' and say 'That number is prime'. In taking this to succeed in referring however one must already presuppose that we are capable of referring to the number 7 by deploying the numeral '7'.

[14] Formally: $\exists R\big(\forall x\big[Fx \to \exists y(Gy \wedge Rxy \wedge \forall z(Gz \wedge Rxy \to z = y))\big]\big)$ $\big(\wedge \forall y\big[Gy \to \exists x(Fx \wedge Rxy \wedge \forall z(Fz \wedge Rzy \to z = x))\big]\big)$

number of Gs', so that the introduction and elimination rules tell us that we are permitted to infer 'The number of Fs is equal to the number of Gs' from 'The Fs are equinumerous with the Gs', and vice versa. This is of course a variant of Hume's Principle (HP).[15]

We should think then of HP as playing a dual role. In the first place, it acts as an *anaphoric initiator*. That is, it introduces reference to a class of entities—cardinal numbers—by introducing a new mathematical vocabulary in relation to a familiar, established vocabulary. In the second place, it lays down proprieties of inference regarding how we should *reason* about the things picked out by the new vocabulary. Grasping proprieties of inference regarding claims of the form $F \approx G$, via the introduction and elimination rules stipulated by HP, is sufficient for inferential mastery regarding claims of the form $Nx : Fx = Nx : Gx$.[16]

Why does this open up space to think of mathematical facts as, in some sense, socially instituted? Here, the inferentialist framework is crucial. In the mainstream representationalist paradigm, taking its cue from Davidson (1967, 1973), the meaning of an atomic sentence in a first-order language is given by its truth conditions, with the latter understood in terms of the n-tuples of worldly objects required to 'satisfy' the sentence (given an interpretation function). So, in an interpreted language, a sentence of the form $\Phi^n(t_1, \ldots t_n)$ consisting of an n-place predicate Φ^n and n terms $t_1, \ldots t_n$ is satisfied if an only if the interpretation function δ maps Φ^n to a set $\delta(\Phi^n)$ including as a member the n-tuple $\langle o_1, \ldots o_n \rangle$ and maps each consecutive term $t_1, \ldots t_n$ to each consecutive element of $\langle o_1, \ldots o_n \rangle$. Understood in these terms, what determines the truth of a sentence of this form is factored into two components: the meaning of the sentence given by the interpretation function (which fixes its truth conditions) and a structured domain of worldly objects (which resolves whether those truth conditions are satisfied). So even if semantic meanings are instituted by social conventions, the truth of sentences is not: the world must co-operate.

But in the inferentialist paradigm it is possible, for some vocabularies, to understand both the semantic meaning of a sentence of the form $\Phi^n(t_1, \ldots t_n)$ *and* the truth of that sentence as being determined wholly by social conventions. Here is a sketch of why this is so. Note firstly that the inferentialist rejects the representationalist thought that the meaning of a sentence is instituted, at base, by interpretation functions from linguistic terms to worldly objects. Rather, the meaning of a sentence is instituted by its inferential relations to other sentences. So linking arithmetical sentences inferentially to other sentences, using the HP introduction and elimination rules, is sufficient to give them semantic meanings. Critically, there is nothing about deontic scorekeeping practices requiring that they be structured by rules in which worldly states of affairs are afforded authority over the correctness or incorrectness, truth or falsehood, of claims. If it were the case that, whenever we introduce some vocabulary, we *must* accord the world authority over the standards of correctness and incorrectness of atomic, declarative sentences of that vocabulary, then the truth of atomic, declarative sentences of that vocabulary would invariably make demands on the world. But we get to make the linguistic rules, and there is nothing incoherent about a deontic scorekeeping practice that does not accord the world this authority. (The harder task is showing how we can, for (e.g.) empirical vocabularies, institute linguistic standards that *do* accord the world authority over language: see (e.g.) Brandom (2008, ch. 6)). So we can introduce deontic scorekeeping practices whose standards of correctness and incorrectness, truth and falsehood, bottom out in, or are exhausted by, the deontic scorekeeping rules we bind ourselves to, and where none of the rules index the normative status of claims to the obtention of worldly states of affairs, perceptual inputs, or anything else outwith the vocabulary itself. In language games like these, the rules are all 'internal'; though they share the same surface syntax as games with some 'external' standards—both predicating properties of objects, both making use of sentences of the form $\Phi^n(t_1, \ldots t_n)$—claims made in the vocabulary are not made true by external states of affairs.[17] The inferentialist paradigm makes intelligible linguistic practices where there is no more fundamental arbiter of truth than the conventions we give to

[15] HP is usually parsed as the biconditional $F \approx G \leftrightarrow Nx : Fx = Nx : Gx$, rather than a pair of introduction and elimination rules. Wright (2016) discusses some epistemic advantages of treating HP in the latter way.

[16] And this inferential mastery takes us impressively far. Technical results (see Wright (1983), Boolos (1990), Heck (2011)) show that, from Hume's Principle, it is possible to derive (in second-order logic, with some additional definitions) theories that contain e.g. second-order arithmetic (PA_2). Note that the inferentialist need not—and should not, if she does not wish to trammel her mathematical practices—identify what is entailed by an axiom system in terms of the inferentialist notion of *commitment-preservation* with what follows from that axiom system with respect to its formal proof system. In the second-order case, the inferentialist is free to take it, for instance, that if some claim χ is semantically entailed (in the formal sense) by PA_2 then PA_2 is incompatible (in the inferentialist sense) with $\neg\chi$. (In fact, appeal to non-formal modal primitives may well be required to make sense our practices of deriving consequences from axioms in any case. See Leng (2007).) In the first order case, the inferentialist can appeal to omega rules, as discussed by Peregrin (2014, ch. 7). Note also that there is nothing preventing the linguistic community from introducing mathematical deontic scorekeeping norms axiomatically, with those axioms understood as introduction and elimination rules, as in Tennant (1990), Parsons (2007), and Warren (2020). What is important, from the inferentialist point of view, is that the rules both introduce a vocabulary and pin down rules of inference for that vocabulary.

[17] Brandom (2011, 2013) *does* argue that all autonomous vocabularies play a 'representational' role. But we should not be mislead on this point. What Brandom has in mind there is just the story sketched above, about how treating singular terms as part of the same anaphoric chain underwrites treating them as being *about the same objects*. This is something that can take place in deontic scorekeeping practices that bottom out in linguistic rules.

ourselves. There is no barrier then to thinking of the linguistic community's introduction of HP as *stipulating*[18] internal deontic scorekeeping norms for arithmetic, and so *determinative* of which arithmetical sentences are true or false.

Now, one might hold that there is nothing more required for a singular term to refer than that it occur in a true sentence, at least in extensional contexts. And here, inflationary theories of truth or reference—requiring that truth and reference involve substantive relations between linguistic terms and worldly objects—could prise open a gap between the *internal correctness conditions* and *external truth conditions* of arithmetical sentences, where the truth of arithmetical sentences depends on worldly co-operation. (This is, in a nutshell, the inflationary argument of (Benacerraf 1973, p. 677).) But inferentialists reject inflationary theories of truth and reference, instead understanding the expressive function of 'is true' and 'refers' in terms of 'horizontal' relationships between bits of language, explicable within the framework of deontic scorekeeping practices sketched above.[19] Similarly, the inferentialist understands quantifiers 'horizontally' in terms of their standard introduction and elimination rules. So when the truth or falsehood of some atomic sentence χ is fixed by linguistic conventions, so too is the truth or falsehood of sentences of the form $\exists v \chi[v/t_i]$ (where $\chi[v/t_i]$ is the result of substituting in the variable v to replace occurrences of a constant t_i in the sentence χ). In this way, inferentialism allows that the truth of arithematical 'existence' claims can be socially instituted, without having to do so via (as in the case of banknotes and husbands) the attribution of normative statuses to physical things.[20]

6 Mathematical Objectivity

An ostensible attraction of mathematical platonism is that it appears to ground mathematical objectivity in a straightforward way. If there is a pre-existent domain of mind- and language-independent mathematical objects, standing in mind- and language-independent relations to one another, then mathematical standards of correctness or incorrectness are not up to us, they are given by that pre-existent structured domain of mathematical objects. Our mathematical claims are correct just in case they accurately describe that domain. For a number of reasons the link between mathematical objects and mathematical objectivity is not as clear as one might initially suppose.[21] However it is incumbent on any account of mathematics to capture the ways in which mathematics is plausibly objective, and, ostensibly, social constructivist accounts may be thought ill-suited to this task.

In particular, the *continued recognition* of normative statuses by at least some people is, in many cases, required to sustain the existence of socially instituted objects. Our conceptions of money and of connubiality underwrite our acceptance of subjunctive conditionals such as 'If no-one recognised the entitlements and obligations associated with money, there would be no billionaires' and 'If no-one recognised the entitlements and obligations associated with marriage, there would be no husbands'. One tempting thought is that, if one regards arithmetical norms as instituted by our normative attitudes, then one must regard arithmetical claims as made true or false by consensus. Now I take it that for a philosophical account of mathematics to underwrite commitment to conditionals such as 'If no-one recognised that 2+2=4, then it would not be the case that 2+2=4' would be a Bad Result. At any rate, I will argue that within the inferentialist framework, one is not required to endorse problematic conditionals like these. For all arithmetical claims p and all persons S, the deontic scorekeeping practices initiated by HP do not require us to score either of the following principles as true:

$$\textit{No Communal Ignorance}(\text{NCI}):$$
$$\forall p(p \to \forall S(S \text{ acknowledges commitment to } p))$$

$$\textit{No Communal Error}(\text{NCE}):$$
$$\forall p \forall S(S \text{ acknowledges commitment to } p) \to p)$$

Brandom (1994, pp. 601–7, 2000a, b pp. 196–204) shows that this is the case for claims in general, and this carries over to claims expressible in arithmetical vocabularies. NCI and NCE are understood here not as material conditionals, but as conditionals expressing a modal force. So understood, NCI implies that it is not possible for an arithmetical fact to obtain without that fact being committed to by all persons. Recall the first layer of deontic scorekeeping that characterises semantic inferentialism, in which one keeps track of one's own commitments and entitlements, and attributes commitments and entitlements to others. If, for every arithmetical claim, undertaking a commitment to p requires one to undertake a commitment to *For all persons S, S acknowledges commitment to p*, then one would be required to score

[18] Compare Hale and Wright (2001, Introduction, ch. 5), who regard HP as an implicit *definition* of the concept *cardinal number*. See Stirton (2016) for critical discussion. Regarding arithmetical truths as stipulated in this way plausibly circumvents the kind of epistemic worries regarding mathematical platonism discussed by Benacerraf (1973), Field (1989a), and Collin (2018).

[19] Brandom (1994, ch. 5) understands 'is true' and 'refers' respectively as prosentence-forming and pronoun-forming operators.

[20] This sharply differentiates the inferentialist view from the fictionalism of Field (1989a, b, 2016) and Leng (2010), in which the content of the mathematical fiction might also be understood as fixed by social conventions, but where mathematical claims are false, or trivially true. Whether inferentialists should regard all quantifier commitments, and indeed 'existential' commitments, as full-fat *ontological* commitments is another issue: see (Collin forthcoming) for discussion.

[21] See, e.g., Field (2001a, b) and Clarke-Doane (2020, ch. 1).

NCI as true, that is, to endorse NCI oneself. But deontic scorekeeping, as explicated by Brandom, does not require in general that if one *undertakes* a commitment to some claim *p* then one must thereby *attribute* that commitment to everyone else. This is a structural feature of deontic scorekeeping understood as keeping *two* sets of books, one of one's own commitments and entitlements and one of the commitments and entitlements of one's interlocutor. As such, one can *compatibly* undertake a commitment both to *1729 is the smallest number expressible as the sum of two cubes in two different ways* and *Godfrey does not acknowledge that 1729 is the smallest number expressible as the sum of two cubes in two different ways*. The social constructivist inferentialist account does not commit one to NCI.

Consider also the case of first-personal ignorance, and the principle:

$$\textit{No First-Personal Ignorance}(\text{NFPI}):$$
$$\forall p(p \rightarrow \text{I acknowledge commitment to } p))$$

At first blush, this may be thought problematic for the inferentialist, since, plausibly, if one acknowledges commitment to *p* then one is thereby committed to *I acknowledge commitment to p*. There is no coherent way of keeping score such that one can endorse p without being committed to endorsing *I acknowledge commitment to p*; if one acknowledges commitment to *p*, one cannot be entitled to *I do not acknowledge commitment to p*. So (again, at first blush) it can look as though deontic scorekeeping practices are structured in such a way that one must treat *I acknowledge commitment to p* as a consequence of *p*. This would be a problem, because (e.g.) *I acknowledge 1729 is the smallest number expressible as the sum of two cubes in two different ways* is clearly not a consequence of *1729 is the smallest number expressible as the sum of two cubes in two different ways*. If we cannot use our deontic scorekeeping practices to treat the latter as true but the former as false, how can the inferentialist paradigm make sense of the fact that we can (and should) *not* regard the latter as a consequence of the former?

The problem however is chimearical; it *is* possible to use our deontic scorekeeping practices to treat *p* and *I do not acknowledge commitment to p* as *compatible*, and so to treat *I acknowledge commitment to p* as *not* being a consequence of *p*. This is something that becomes visible in the *social*, *intersubjective* account of deontic scorekeeping endorsed by Brandom, in which we keep two sets of books: one of our own commitments and one of the commitments of others. Consider a case in which the claim one acknowledges commitment to is itself an *attribution* of commitments to *another*. Say that, for some person S nonidentical to oneself, one endorses *S acknowledges commitment to p and S acknowledges commitment to: I do not acknowledge commitment to p*. Nothing in our deontic scorekeeping practices precludes us from making this attribution to S, and to do so without regarding S's commitments as incoherent. In doing so, one uses one's *own* deontic scorekeeping practice to treat *p* and *I do not acknowledge commitment to p* as compatible, and so one uses one's own deontic scorekeeping practice to treat *I acknowledge commitment to p* as failing to be a consequence of *p*.[22] Nor does inferentialism imply NCE. Consider the first-personal instance of NCE:

$$\textit{No First-Personal Error}(\text{NFPE}):$$
$$\forall p((\text{I acknowledge commitment to } p) \rightarrow p)$$

The situation is similar to before; since it is plausibly impossible to undertake a commitment to *I acknowledge commitment to p* without undertaking a commitment to *p*, it appears that, in one's deontic scorekeeping practices, one must always keep score in a way that would involve treating *p* as a consequence of *I acknowledge commitment to p*. But, as before, the appearance dissolves under closer inspection.

For the inferentialist, the claims *I acknowledge commitment to p* and *p* are not in general treated as equivalent in deontic scorekeeping practices because they differ with respect to the circumstances under which one is entitled to them; one can typically gain entitlement to the former by introspectively surveying one's acknowledged commitments, while gaining entitlement to the latter typically involves more. One such case is where one finds oneself with incompatible commitments. In that kind of case, one loses entitlement to the commitments, but still retains them *as* acknowledged commitments and is entitled to acknowledge that they are commitments. Say that one is committed to *p*, *q*, and *r*, but discovers that *p*, *q* and *r* form an inconsistent triad. In Brandom's scorekeeping system, one then loses entitlement to *p*, *q*, and *r*. But one is still obliged to acknowledge them as commitments. So we have a case where one is entitled to *I acknowledge a commitment to p* but one is not entitled to *p*. Here one has a commitment (*q* & *r*) that is incompatible with *p* but not with *I acknowledge commitment to p*. Now, in Brandom's incompatibility semantics, *p* entails *q* iff_{df} every claim incompatible with *q* is incompatible with *p*. (Brandom 2008, p. 133) But we have just seen that it is possible to keep deontic score in a way that treats some commitment as incompatible with *p* but not with *I acknowledge commitment to p*. So deontic scorekeeping does not force us to

[22] Brandom summarises: 'Ascriptional locutions make explicit the possibility of taking up hypothetically a sort of third-person scorekeeping attitude toward my own present commitments and entitlements (much as I must do for my *past* commitments and entitlements in any case). Here such ascriptions show that what precludes entitlement both to the claim that *p* and to my denial of a self-ascription is a pragmatic matter concerning attitudes, not a semantic matter concerning the contents to which they are addressed.' (Brandom 1994, p. 605).

treat NFPE as obtaining. Since NFPE is an instance of NCE, the inferentialist is also not forced to treat NCE as obtaining.

Nothing then about deontic scorekeeping practices *as such* undermines arithmetical objectivity in this specific sense: they allow that arithmetical truth can be contrasted both with what arithmetical claims acknowledges oneself and with what arithmetical claims command corporate consensus. Additionally, the standards introduced by HP in particular do not underwrite conditionals such as 'If no-one recognised that $2+2=4$, then it would not be the case that $2+2=4$', for the simple reason that the standards introduced by HP do not treat the status of arithmetical claims as dependent on attitudes, or any physical states of affairs. This rightly differentiates arithmetical claims from claims about money, marriage, universities, and so on, the truth of which are more plausibly regarded as counterfactually depending on sustained corporate acknowledgement. Deontic scorekeeping practices in the latter instances *do* treat deontic statuses as indexed to certain attitudes or worldly states of affairs. One ceases to be a husband, according to the deontic scorekeeping rules governing marriage, if the right people regard you as divorced, or if you become a widower.

Secondly, arithmetic is objective in being about mathematical *objects*, and, thirdly, it is possible to make sense of *disagreement* about arithmetical claims. Both of these kinds of objectivity result from HP functioning as an anaphoric initiator, introducing singular terms. For the reasons sketched above, this makes possible a practice in which one treats one's own arithmetical commitments and the different arithmetical commitments of the other, as being about the same objects, as occupying two different perspectives on the same things. Against the backdrop of this kind of linguistic practice–one in which one treats the singular terms of others as anaphoric recurrences of one's own, and vice versa—mathematical *disagreement* is intelligible. One can understand the mathematical commitments of others as *incorrect* and not just *different*.

Fourthly, arithmetic is objective in the sense that what *follows* from HP is not up to us. It is logically necessary that PA_2 is implied by HP in second-order logic.[23] It is up to us then whether we bind ourselves to the linguistic norms of HP, but it is not then up to us what other commitments follow from binding ourselves to the norms of HP.

Finally, there is one kind of mathematical objectivity *not* underwritten by this account. Here is an illustrative example, though taken from set theory rather than arithmetic. Since the continuum hypothesis (CH)—that every set of reals is countable or equinumerous with $\mathbb{R}$—and its negation (¬CH), are both unprovable in ZFC, it is possible to introduce, axiomatically, coherent deontic scorekeeping practices governed by *either* ZFC + CH *or* by ZFC + ¬CH. In a context in which axioms are regarded as stipulating deontic scorekeeping rules for the new vocabulary, it appears to make little sense to regard one set of axioms or rules as uniquely correct. Here, the inferentialist view is most naturally paired with the 'plenitudinous' or pluralist view according to which, intuitively speaking, there is no one uniquely correct conception of set characterising a unique set-theoretic universe, but a plurality of conceptions of sets characterising different set-theoretic universes.[24] To change the inferential rules from ZFC + CH to ZFC + ¬CH is to grip onto a different anaphoric chain. For the inferentialist, social constructivist, there is then no objective fact of the matter about whether CH obtains *simpliciter*. In this regard the view is pluralistic and permissive.

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[23] It is true that one might hold, on broadly Wittgensteinian grounds, that what can be formally derived from what is not wholly objective (see Kripke (1982) and Wittgenstein et al. (1956)). But, even if one held this radical view, it would not put the social constructivist account on a different footing with regard to mathematical objectivity as any other account.

[24] See e.g. Linsky and Zalta (1995), Balaguer (1998) and Hamkins (2012). This does not imply that any *consistent* rules will do. At the very least, these rules must be *conservative* (Field 1989a, b; Wright 1997), because it ought not to be possible that they come into conflict with empirical commitments and, hence, deontically indexed to empirical states of affairs.

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