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ORIGINAL RESEARCH

Putnam's model-theoretic argument (meta)reconstructed

In the mirror of Carpintero's and van Douven's interpretations

Krystian Jobczyk[1,2]

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Abstract

In "Models and Reality", H. Putnam formulated his model-theoretic argument against "metaphysical realism". The article proposes a meta-reconstruction of Putnam's model-theoretic argument in the light of two mutually compatible interpretations of it–elaborated by Manuel Garcia-Carpintero and Igor van Douven. A critical reflection on these interpretations and their adequacy for Putnam's argument allows us to expose new theses coherent with Putnam's reasoning and indicate new paths to improve this argument for our reconstruction task. In particular, we show that Putnam's position may be coherent with van Douven's versions of Global Descriptivism under some conditions, but Putnam cannot reject realism as quickly as Carpintero suggests. We show that Suszko's canonic axiomatic system and Sneed's concept of theory may provide valuable support for Putnam's argument. Finally, we critically evaluate Carpintero's theses about the genesis of unintended interpretations of our languages, adopting the machinery of the Upward Skolem–Loewenheim Theorem and Knight's Theorem.

Keywords The model-theoretic argument · Putnam's anti-realism · Carpintero's interpretation of the model-theoretic argument · van Douven's interpretation of the model-theoretic argument · Global descriptivism · The Skolem-Loewenheim theorem

1 Introduction

In the early '80s, Hilary Putnam formulated his model-theoretic argument against metaphysical realism-shortly specified by him as an "externalist perspective" whose "favourite point of view is a God's Eye point of view" (Putnam, 1981, p. 49). Putnam

✉ Krystian Jobczyk
kajobczyk@gmail.com

1 Department of Applied Computer Science, AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Kraków, Poland

2 The Saul Kripke Center, CUNY, 365 Fifth Avenue, New York, NY 10016-4309, USA

develops his characteristics of this position[1] through a conjunction of the following theses:

- **P1**: The world consists of mind-independent objects. (See: Putnam, 1981, p. 49; cf. Putnam, 1978, p. 125; Anderson, 1993, p. 312.)
- **P2**: Our statements about the world express realist claims about mind independent reality.(See: Putnam, 1981, p. 49; cf. 1989, p. 214, Anderson, 1993, p. 312.)

Putnam constructs his model-theoretic argument—as a unique reductio argumentation by adopting the theses **P1, P2** as premises of his reasoning, and he completes them with the following new premises–referring to the so-called operational and theoretic constraints imposed on our theories.

- **P3**: Operational and theoretical constraints alone are insufficient to fix a determinate reference relation between the terms of our language and mind-independent reality. (See Anderson, 1993, p. 313; cf. Putnam, 1980. pp. 475–476, 482; Putnam, 1983, p. 494.)

Putnam then argues that

- **P4**: There exists nothing else in the universe that (in addition to theoretical and operational constraints) could fix a determinate referential relation to a mind-independent reality. (See Anderson, 1993, p. 313; cf. Putnam, 1980, pp. 475–476, 482; Putnam, 1983, p. 494.)

He finally concludes that, therefore,

Conc : Our statements are semantically indeterminate. There is no fact of the matter concerning the reference of our terms. (See Anderson, 1993, p. 313; cf. Putnam, 1980, pp. 475–476,482.)

Because Putnam finds *Conc* absurd, he postulates abandoning the premises **P1** and **P2**. Whereas realists pay special attention to disconfirming **P4**, Putnam spends a lot of time justifying **P3**. He does it in terms of his theorem[2] in Putnam (1980, p. 468).

The model-theoretic argument, in contrast to this opportunity to depict it in such a concise way, is deceptively simple—as Anderson claims in Anderson (1993)—and evokes mixed feelings in the philosophical world. Some commentators of Putnam's work-as Haukioja in Haukioja (2001)—seem to be delighted by its hard-hitting nature. Some of them-as Anderson in Anderson (1993), Bellotti in (2005), or Button in (2011), formulate some hints on how to improve and defend Putnam's argument. Finally, many others would insist that Putnam's argument is "completely wrongheaded"(Cleve, 1992, p. 349), "fatally flawed" (Lewis, 1984, p. 236), or "question-begging." (See Devitt, 1991, p. 227; cf. Bays, 2001, pp. 344–346.)[3]

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[1] A similar juxtaposition may be found in Button (2011, p. 321).

[2] This theorem is called the Putnam Theorem in the body of the paper. It will be presented in detail in Sect. 2.

[3] The history of philosophical reaction to Putnam's argument is long. Its part regarding the formal side of the model-theoretic argument might be thematized as follows. D. Lewis-author of the chronologically first and most complex reconstruction of Putnam's argument–argued in Lewis (1984) that the model-theoretic argument reflects the trivial truth that each theory can be "mistaken" about its models, and the conclusions of Putnam's model-theoretic argument may be inferred without any model-theoretic machinery. Some gaps in the formal tissue of the argument were identified in Shapiro (1985), Levin (1997), Velleman (1998). In particular, M. Levin and D. Velleman argued that Putnam's model has non-constructible objects inside-

1.1 The motivation of the article

The diversity of remarks and comments on the model-theoretic argument necessitates its retrospective (meta)reconstruction[4], especially since some questions remain unanswered: "Which theses and tools may Putnam accept to support his argument?"(Q1), "When is the proper moment of Putnam's rejection of realism?"(Q2), "What is responsible for the existence of unintended interpretations of our conceptual apparatus?"(Q3).

It is, therefore, methodologically reasonable to consider a philosophical interpretation (or even two) as a methodological "mirror" for such a reconstruction. In this subjective author's opinion, the interpretation of Manuel Garcia-Carpintero from Garcia-Carpintero (1996) and the interpretation of Igor van Douven from Douven (1999a, b) seem to be the most suitable for this role. They thematize the first Lewis' difficulties with Putnam's argument from Lewis (1984) in an interesting way; they are mutually coherent[5] and have a lot of explanatory power for Putnam's argument. This new reconstruction is elaborated by a retrospective return to Putnam's argument-provoked by Carpintero's and van Douven's interpretations-and by pointing out discrepancies between Putnam's theses and those of Carpinero and van Douven. These differences allow us to find new answers and solutions coherent with the original Putnam theses. They will collectively determine the content of this new reconstruction. They will, by the way, expose some inadequacies of these interpretations of Putnam's argument in specific points.

In the reconstruction task, we refer to the problem when Putnam rejects realism in his argument (question Q2). We understand the moment of *Putnam's rejection of realism* used in Q2 as a moment in which Putnam is already willing (not necessary the moment when he can) to deny metaphysical realism-due to his theses put forward in Putnam (1980), and there is some reference to Putnam (1983, 1978). Indicating this moment is crucial to understanding the role of the model-theoretic side of Putnam's argument and its reconstruction limitations. On a polemic level, it allows us to slightly disprove Carpintero's central thesis that models have nothing to do with the classic debate realism/anti-realism and to demonstrate how model-theoretic analysis can (and should) enrich this debate.

1.2 The goals and structure of the article

The main goal of this article is to propose a meta-reconstruction of Putnam's semantic anti-realism and his model-theoretic argument-due to its exposition in Putnam (1980,

Footnote 3 continued
against Putnam's intentions. This critique met a reaction in Bellotti (2005). Many authors, such as Benacerraf and Wright (1985) and Bays (2001, 2007), found Putnam's "Just-More-Theory" postulate-as a remedy for these difficulties-entirely question-begging. Its complete recapitulation of Putnam's model-theoretic arguments and in different reconstruction variants—is put forward and defended by Button in (2011). This work thematizes almost all previous controversies around the formal tissue of Putnam's argument-mainly Putnam's Theorem and its proof from Putnam (1980).

[4] We define the "meta-reconstruction of the model-theoretic argument" as an attempt to reframe this argument in light of existing interpretations.

[5] Van Douven's interpretation might be seen as an interesting extension of Carpintero's one for the problem of Global Descriptivism and its reference to Putnam's argument.

1983) and visible in the light of Caprintero's and van Douven's interpretation of Putnam's argument from Garcia-Carpintero (1996), Douven (1999a, b)[6]. We achieve this objective by some evaluating insight into Carpintero's and van Douven's interpretations; more specifically, by justifying the following theses.

T_1: Carpintero's interpretation (mainly represented by the VF premise)[7] does not entirely capture Putnam's model-theoretic argument, and Putnam's rejection of realism is executable much earlier than in *Models and Reality* (Sect. 3).

T_2: Putnam's anti-realism may be coherent with Carpintero's interpretation and van Douven's versions of Global Descriptivism (GD) under some conditions. Admittedly, there is no remedy—acceptable by Putnam—for neutralizing the pro-realism consequences of GD, but confronting Putnam's ideas with GD indicates an interesting path to improve Putnam's pragmatics by adopting some ideas from the non-sentential concept theory of Sneed. Unfortunately, these ideas require a semantic "connector" to be adopted by Putnam (Sect. 4).

T_3: Carpintero fails in his belief that Putnam finds the genesis of unintended interpretations of formal theories in the existence of non-interpreted terms. This point of view is indefensible; it is visible in the light of the proofs of the upward Skolem–Loewenheim Theorem (USL) and Knight's Theorem—as its far generalization (Sect. 5.)

2 Terminological frame, Putnam theorem and Carpintero's iiinterpretation of Putnam's argument

2.1 Terminological frame of the analysis

Before presenting Putnam's Theorem and Carpintero's interpretation, we clarify some concepts used in the paper analysis. We understand the *model- theoretic argument* (also called "Putnam's argument") following Anderson's style-depiction (cf. Anderson, 1993, pp. 312–313) in Sect. 1 and based on Putnam (1980) and also Putnam (1978, 1981, 1983). Having recourse to its original Putnam's depiction in Putnam (1978, p. 125) and Putnam (1981, p. 49), *metaphysical realism* is defined as a conjunction of premises **P1** and **P2**. *Putnam's anti-realism* is used here as a synonym for Putnam's "internal realism."[8] Whenever we use the expression *Putnam's pragmatics* (alternatively: "the pragmatics of the model-theoretic argument"), we mean all Putnam's theses from Putnam (1978, 1980, 1983) about relations between theories in our (formal and natural) languages, their semantic models (broader: interpretations) and their users, mainly oriented to the problem of unintended interpretations and methods of their determination. Thus, the semantic field of "Putnam's pragmatics" is included in the semantic field of "Putnam's anti-realism."We shall later elucidate its meaning in a confrontation with Sneed's ideas of pragmatics. Nevertheless, this concept does not refer to Putnam's pragmatism-a philosophical (meta)position elaborated after years as in Putnam (1994).

[6] This (meta)-reconstruction should be understood as extracting new theses from Putnam (1980) that are compatible with Putnam's original ideas, visible in the light of Caprintero's and van Douven's interpretation of Putnam's ideas, and thematically organized by questions Q1–Q3.

[7] We introduce and describe this premise in Sect. 2. As a result, we only mention it in this paragraph.

[8] The author of the paper believes that this description describes Putnam's debate with his realism-oriented opponents more accurately than the original Putnam's auto-description.

It is noteworthy that, while neither Putnam's nor Carpintero's work provides an explicit explanation of the concept of pragmatics, both authors focus on this issue. For instance, Carpintero makes it when he paraphrases Putnam's objection to "causal explanatory talk (...) so ridden with *pragmatic* or otherwise anthropomorphic aspects to fulfil its intended role properly."(cf. Garcia-Carpintero, 1996, p. 313.) By *pro-realistic interpretations*—we mean any position in the realism-anti-realism debate incoherent with Putnam's theses **P3**, **P4**, and *Conc* or demonstrated to be inconsistent with the logical consequences of **P3**, **P4**, and *Conc*. Similarly, we categorize all meta-theses about the model-theoretic argument (its nature, utility, role, etc.) and Putnam's argumentation formulated from various realistic positions. Through the paper, we refer to the position of *Global Descriptivism*. Unfortunately, D. Lawis, its father and inventor, does not leave any concise explanation of it. In the author's opinion, the most concise depiction of GD is McGowan's: "Global descriptivism (...) is tantamount to saying that any term of the theory it relates to everything it must relate to for this theory to be true." (McGowen, 2002, p. 32; cf. Douven, 1999a, p. 343.). We adopted it after I. van Douven.

2.2 Carpintero's interpretation of the model-theoretic argument

In the opinion of M. Garcia-Garpintero, the initial Putnam's definition of realism (in terms of any given theory T in a given language L) is expressible by a premise:

(R): T might be false.

As Carpintero argues in Garcia-Carpintero (1996), Putnam tries to bear this premise on (or at most, falsify it). To accomplish this, he assumes the following:

(LR): L is logically regimentable (where L is at least first order).

It follows from (LR) that T will have models or interpretations (as an ideal and—as a result—also a consistent theory) in a sense logicians give to these words (Garcia-Carpintero, 1996, pp. 306–307). Because of all the terms of a language L of T—other than logical ones –our theory T will have many possible models, satisfying many possible constraints. It could support Putnam's new hypothesis:

(MT): T - viewed as a partially uninterpreted theory - will have many different models.

Carpintero now poses a provocative question about the most reasonable (in the spirit of Putnam's reasoning) strategy for exposing some logical conflict between (MT) and (IM)-presuppositions for (R):

(IM): L has a determinate intended model.

He describes this strategy as having an "emotional resemblance to verificationist contentions" (cf. Garcia-Carpintero, 1996, p. 307.), expressing it in the following way:

(VF): Except for the logical vocabulary (and perhaps also for the observational vocabulary), T provides the only intelligible way to specify the intended model for L: The intended model is "the" model which satisfies T, viewing T as a formal theory. Any proposed constraint could intelligibly bear on the specification of the intended model only in this way, by belonging to T. (Garcia-Carpintero, 1996, pp. 308).

In the opinion of the Catalonian philosopher, VF is already a direct rejection of realism. As Carpintero concludes, it contradicts realism much earlier than Putnam's model-theoretic argument against realism (Garcia-Carpintero, 1996, p. 313).

A critical reflection on Carpintero's interpretation—as the leading interpretation of the model-theoretic argument in the paper – delivers material for further reconstruction of this argument. Before we proceed with this reconstruction, $\boldsymbol{T}_1, \boldsymbol{T}_2, \boldsymbol{T}_3$ (as depicted in Sect. 1.2) must be justified.

2.3 Putnam theorem

Putnam requires this theorem to formalize his premise **P3** that even operational and theoretical constraints alone are insufficient to establish a fixed reference relation between the terms of our language and mind-independent reality. Putnam accomplishes this by developing a novel model-theoretic theorem. It asserts the existence of a denumerable (ω) model of a far extension of set-theory $\boldsymbol{ZF}+\boldsymbol{V}=\boldsymbol{L}$, i.e., set theory $\boldsymbol{ZF}$ with the *axiom of constructibility*[9]. As Putnam postulates, the so-far extension of $\boldsymbol{ZF}$ formally represents our entire conceptual apparatus, all that could be constructed by *total science*. (Cf. Putnam, 1980, pp. 466, 468, 473.) This theory and its ω-model internalize the operational and theoretical constraints (imposed on the theory). As a result, they were unable to identify this model as *intended*. Putnam appeals to the downward Skolem–Loewnheim Theorem and the so-called Shoenfield's absoluteness to prove his theorem and obtain his ω-model. The exact formulation of the Putnam Theorem and its proof are given below.

Theorem 1 *(Putnam)* $\boldsymbol{ZF} + \boldsymbol{V} = \boldsymbol{L}$ *has* ω*- model which contains any given countable subset of real numbers.*

Outline of the proof: (Putnam, 1980, p. 468.): Thesis of the theorem is equivalent to the statement ($\star$): if X is a countable set of reals, then there exists ω-model M, $M \models \boldsymbol{ZF} + \boldsymbol{V} = \boldsymbol{L}$ and M contains an abstract "copy" of X. If X is countable, one can encode both X and M by a single real, say s, by standard techniques. Thus, the initial statement ($\star$) has a logical form of two-place arithmetical Π_2-formula: (for each real s)(there is such M) that $(\ldots, M, s\ldots)$. Consider this Π_2-sentence in some inner model $\boldsymbol{V}=\boldsymbol{L}$. For each s, there is some model which satisfies "$\boldsymbol{V}=\boldsymbol{L}$" and contains s, for example: $\boldsymbol{L}$ itself. By downward Skolem–Loewenheim Theorem, there is a denumerable submodel of $\boldsymbol{L}$ which contains s, and Π_2 is satisfied in it. □

[9] We adopt $\boldsymbol{ZF}$ as a standard abbreviation for set theory in the axiom system proposed by Zermelo and Fraenkel here. This system is described in every set theory handbook. For example, the position (Dzamonja, 2021) contains a relatively concise and gentle introduction to this system and its models with the constructibility axiom (symbolically: $\boldsymbol{V}=\boldsymbol{L}$). This unique axiom is a postulate to treat all sets as constructible. Consider Goedel's constructible universe $\boldsymbol{L}$ as a class of sets that can be described entirely in terms of simpler sets, for a more in-depth understanding of the model-theoretic sense of the constructibility axiom. More precisely, $\boldsymbol{L}$ is defined by the so-called transfinite recursion as follows: $L_0 := \varnothing$, $L_{\alpha+1} := \mathrm{Def}(L_\alpha)$, where $\mathrm{Def}(L_\alpha) := \{A : A$ is definable over L_α by some first-order formula ϕ and parameters from $L_\alpha\}$. If λ is a limit ordinal, then $L_\lambda := \bigcup_{\alpha<\lambda} L_\alpha$, and $L := \bigcup_{\alpha\in\mathbf{Ord}} L_\alpha$.'**Ord**' denotes the class of all ordinals. Meanwhile, $\boldsymbol{V}$ is the cumulative hierarchy of *all* sets. Then, the 'axiom of constructibility' ("all sets are constructible") is formally represented by $\boldsymbol{V}=\boldsymbol{L}$. This equation formally asserts that the constructible universe ($\boldsymbol{L}$) is identical to all sets' classes (cumulative hierarchy). (Cf. Dzamonja, 2021, pp. 32–39.)

Being equipped with knowledge about the formal tissue of Putnam's argument, we can face the question Q1: "Which theses and tools may be accepted by Putnam?". Section 3 provides an answer to this question.

3 What can be properly accepted by Putnam?

We shall face the title problem in light of Carpintero's and van Douven's interpretations, with a particular emphasis on Carpintero's VF premise. Because of the nature of the following analysis, it is convenient to represent VF as the conjunction of the two premises:

VF_1 : The intended model is the model which satisfies T, viewing T as a formal theory.
VF_2 : Any proposed constraint could intelligibly bear on the specification of the the intended model only in this way, by belonging to T.

Let us first explore Putnam's dictionary with the concepts of operational and theoretical constraints in a unique role.

3.1 The operational and theoretic constraints in Putnam's dictionary

Despite numerous references to so-called operational (putnam, 1978, pp. 485,486; Putnam, 1980, pp. 471–477, 481) and theoretic constraints (Putnam, 1978, pp. 481; Putnam, 1980, pp. 466–469, 471-477, 482) constraints, Putnam's depiction of these concepts leaves a slight deficiency. In fact, Putnam tends to mention them together and approximates them through his favourite reference to theories and their models which *satisfy* (cf. Putnam, 1978, p. 494) or preserve (cf. Putnam, 1980, p. 482) all theoretic and operational constraints. Despite it, only *theoretic constraints* (OT) deserved to be specified more clearly by him as coming "from the set theory itself or total science." (See, for example, Putnam, 1980, p. 466.) Furthermore, Putnam makes an effort to enumerate some of the constraints: AC (cf. Putnam, 1980, pp. 471, 478), a determinacy axiom[10](cf. putnam, 1980, pp. 471), and his global axiom of constructibility $\boldsymbol{V=L}$ (for example, cf. Putnam, 1980, pp. 469–471), treating this class of constraints as an *open reservoir* of axioms or properties rather than a closed collection. Are certain axioms and properties forbidden in this collection? Admittedly, L. Bellotti has reservations about the non-well-foundedness property[11](cf. Bellotti, 2005, p. 405.) in this role, but Putnam's collection appears to be arbitrarily extensible due to his "just more theory" postulate.

Meanwhile, Putnam's approach to the *operational* constraints is distinct. They are not immediately apparent but are specified by 'three things,' as Putnam clarifies: a) a sufficiently large "observational vocabulary"—the set of 0-terms such as "red", "touches", "hard", "push" (cf. Putnam, 1980, p. 472), b) an assumption on the exis-

[10] This axiom states that infinite games with perfect information are determined, i.e., there is a winning strategy for either the first or second player (cf. Putnam, 1980, p. 471.)

[11] The non-well-foundedness appears to be a formal property of Putnam's ω-model if it codes a non-constructible ("in reality") set inside. For more information, see: (Shapiro, 1985, p. 724; Bellotti, 2005, p. 405). We return to this issue briefly in Sect. 3.2.

tence of a set, say S, of macroscopically observable things and events, and c)—a valuation (OP). Its role is to assign "the correct truth value to each n-place 0-term (for $n = 1, 2, 3, ...$) on each n-tuple of elements of S on which it is defined." (See, for example, Putnam, 1980, p. 472)[12]. Meanwhile, OP—the correct assignment of values to a countable set of physical magnitudes MAG—is a countable set of real numbers which suitably encodes all possible measurements over MAG at each rational space-time point. Because of the Putnam Theorem, these reals can encode physical magnitude measurements from MAG inside the ω-model. In this sense, OP and operational constraints "lose" their external perspective and potential ability to indicate this ω-model as the intended one. This role is similar to that of OTs—as they remain internal with respect to ***ZF*** + **V** = **L**.

This property of OP explains why OP and OT play together in Putnam's game against the "metaphysical realism" supporters. Even more, it appears that Putnam's use of OP makes his game more provoking. In fact, after allowing his opponents to state that there are "genuinely non-constructible" sets, he later shows them in his theorem that these sets are encoded by some reals in his ω-model. In the meantime, this model for ***ZF*** is also a model for ***V*** = ***L***. As a result, a non-constructible set "in real" has its code in theory, asserting that "all is constructible" (cf. Button, 2011, p. 424). Because realists find this dichotomy unacceptable, their confusion appears to be a source of Putnam's malicious satisfaction. Does Putnam win his game this way? We leave this intriguing question unanswered to return to it in Sect. 3.2, where we connect it to the central question of this section (Q1).

3.2 Problem with acceptance of VF—the perspective of Carpintero's clarification

We now intend to confront the issue of Putnam's acceptance of VF in light of Carpintero's clarification from Garcia-Carpintero (1996) before facing the same problem in light of van Douven's interpretation and his modifications of Global Descriptivism. Carpintero's interpretation provides the most natural conceptual environment for VF, and as we will show, it reveals Putnam's ability to accept this premise, but only conditionally.

Let us begin with some thoughts on the VF premise. As a combination of its subpremises VF_1 and VF_2, VF conveys two messages: there is a special relationship between formal theories and their intended models (part VF_1), and each constraint imposed on a theory T has something to do with the specification of their intended models if only it belongs to T ((part VF_2)).

Is Putnam willing to accept VF_1? First, we must comprehend the VF_1 message. How does the desired model (of a given theory T) "see" it as a formal theory? Carpintero's analysis reveals two interpretation lines for this VF metaphor. The first is literally borrowed from Carpintero's comments: the intended model "views" its theory "as a formal theory" in the sense that it is "defined relative to the language"(cf. Garcia-

[12] Since these "three things" constitute three aspects of the operational constraints rather than their components—the operational constraints themselves remain elusive. Fortunately, it does not negatively influence the explanatory power of Putnam's clarifications-as he needs a reference to OP in his argument. Meanwhile, operational constraints seem to be viewed as all the restrictions, rules, or principles imposed on the measurements of physical magnitudes from MAG and their truth assignments.

Carpintero, 1996, p. 307) and "is left unspecified to the same extent that the language is vague" (cf. Garcia-Carpintero, 1996, p. 307). Thus, "viewing a theory as a formal by its model," according to this "relativization and similarity" interpretation, means: "the model is relativized to its language and shares the same vagueness." In the alternative interpretation of VF_1, we can understand the phrase "an intended model's views" as "it has some reference to our pragmatic external intentions" (regarding, for instance, the formal theory).

Is Putnam willing to accept VF_1 in the first interpretation? It appears so, but only partially. Obviously, the thesis about the relativization of intended models to a language seems to be consistent with Putnam's attitude, provided we interpret the thesis as a paraphrase of his conclusion from "Models and Reality": "Models are not lost noumenal waifs looking for someone to name them; they are constructions within our theory itself, and they have names from birth." (See Putnam 1980, p. 482). In other words, the model cannot be conceived apart from the theory. Unfortunately, Putnam's intellectual requirements do not appear to be met by the thesis about the same vagueness of languages and their intended models, as he is not a follower of this thinking about possible sources of unintended interpretations of our languages[13]. Furthermore, this thesis may smuggle (under certain conditions) the idea that syntax and semantics (of a given theory T) have the same "expressive power," which contradicts Tarski's indefinability theorem. It isn't easy to believe Putnam doesn't respect it.

Does Putnam agree with VF_1 in the second interpretation? As previously stated, the situation will be much more promising if we maintain the spirit of Putnam's conclusions from Putnam (1980, p. 482). What is the semantic content of "viewing theory through an intended model," defined as "accessing our pragmatic intentions"? For example, it could imply that the intended model materializes some beneficial metalogical properties due to our preferences or pragmatic intentions (e.g., the intended model of PA arithmetic as the standard model for this theory, etc.).

Fortunately, the situation with VF_2 appears to be less complicated. Indeed, Putnam appears to formulate a similar manifesto with his special reverence for theoretic and operational constraints: if something can specify intended models of a theory T, then it must be either a theoretical or operational constraint imposed on T. Thus, his manifesto closely resembles the spirit of VF_2 but is irreducibly broader than VF_2, and could be expressed as follows.

$VF_2^\star$: Any proposed constraints could bear on a specification of the intended model only in this way—belonging to T or being encoded in ω-model for T.

Thus, Putnam's $VF_2^\star$ and VF_2 can be identified if (1) OP is removed from the Putnam Theorem and the entire model-theoretic argument, or (2) the phrase "belonging to the theory" is interpreted as "belonging to the theory and its models." Obviously, postulate (1) contradicts Putnam's empirical attitude, as evidenced by his efforts to build his ω-model which satisfies *everything is constructible* and assigns the correct values to "all physical magnitudes in MAG at all rational space-time points". (See Putnam, 1980, p. 468.) Putnam certainly agrees with (2) because it simply reflects his intentions regarding the theoretical and operational constraints. (Another question is whether Carpintero will agree to such a "model" extension of VF_2.)

13 This problem is the subject of deeper analysis in Section 5.

Meanwhile, we can also consider the problem of Putnam's approval for VF less casuistically, "from the bird's eye view". It is sufficient to return to the following clarification of VF by the Catalonian philosopher:

> (VF) asserts exactly what the realist denies; namely, that the extension of 'causes', 'explain', and the theoretical vocabulary is fixed (to the extent that they are fixed at all) by facts about the epistemically relevant aspects of the psychological endowment of human beings (Garcia-Carpintero, 1996, p. 312).

Carpintero defines (and proposes to Putnam) VF in the anti-realistic spirit of a pragmatics-oriented approach to determining intended models (Garcia-Carpintero, 1996, p. 312). If Putnamists take this slightly enigmatic reference to pragmatics at face value, they should accept VF and incorporate it into their facilities.

Nevertheless, Putnam's intellectual acceptance of VF–as coherent with his model-theoretic argumentation-should be distinguished from his *willingness* to accept VF as its *proper and convenient* representation. One might question whether the second situation holds. On the one hand, Putnam's model-theoretic argument's general philosophical motto is similar to Carpintero's clarification of VF: our conceptual apparatus cannot indicate its intended interpretation without our pragmatic intentions[14].

On the other hand, this clarification, together with VF itself, "tastes" like Carpintero's anti-model-theoretical appeal to Putnam to "better forget about models."(cf. Garcia-Carpintero, 1996, p. 314.) Perhaps—there is a reason why Carpintero avoids mentioning models in his formulation VF–against an explanatory optimism that he only borrows Putnam's *facon de parler* to refer to the models within our theory as constructions. (See, for example, Putnam, 1980, p. 482.) Meanwhile, expecting Putnam to make a similar anti-model-theoretic declaration is misguided, given his formally-oriented Putnam Theorem and his efforts throughout his work to expose the mutual relationships between models and their theories. (See Putnam, 1980, pp. 467–469, 471–473, pp. 480–482.) Finally, the model-theoretic tissue of Putnam's argument appears to be a convenient testing ground for grasping and elucidating some of realism's difficulties and critical moments in the debate between realism and anti-realism.

Therefore, if Carpintero, with his manifesto, plays in the same Lewis-Van Cleve's anti-model-theoretic league (cf. Lewis, 1984, Cleve, 1992), then Putnam should think twice before he takes Carpintero's gift.

3.3 Problem with acceptance of VF—the perspective of van Douven's interpretation

Van Douven's analyses from his paper (Douven, 1999a) provide another perspective on Putnam's acceptance of VF. We employ them to gain a more profound understanding of Putnam's intellectual distance from Carpintero's VF. This new viewpoint is especially attractive because it reveals what was hidden in Carpintero's clarification.

In fact, there are two solid motivations for incorporating van Douven's notes from Douven (1999a, b) into the paper analysis and our reconstruction attempt. The first is delivered by I. van Douven himself, who observes (cf. Douven, 1999a, p. 343)

[14] These pragmatic intentions refer to, e.g., the required and preferred properties of intended models.

that Carpintero's VF forms a manifesto similar to the following version of Global Descriptivism[15].

> GD: The intended interpretation(s) of $\mathcal{L}$(T) (insofar as it is/they are not determined by operational constraints) is/are the one(s) that assign(s) things, classes of things (and classes of such classes, etc.) in the world as extensions to the terms of $\mathcal{L}$(T) in such a way that T comes out true. (See Douven, 1999a, p. 343.; cf. Lewis, 1984, pp. 222–224.)

D. Lewis' remarks on GD's profound relationship with Putnam's manifesto (that ideal epistemic theory cannot be false) provide another reason to focus on van Douven's analysis and modification of GD. (See Lewis, 1984, pp. 224, 232.) Meanwhile, as repeated by van Douven, Lewis's diagnosis, as recounted by van Douven, is clear: GD may support Putnam's manifesto if *GD* "is the whole truth about reference."(See: Lewis, 1984, p. 224; Douven, 1999a, p. 342.)

It just sets up the whole debate between Putnam and his adversaries. They deny GD, leaving Putnam with the philosophical task of assimilation of a digestible (for him) version of GD. Although van Douven discloses some pessimism here, we intend to explore the problem again-trusting that some advanced van Douven's modifications of GD not only explain a discrepancy between Carpintero's VF and Putnam's manifesto (goal 1) but also deliver some hints to improve the pragmatics of Putnam's model-theoretic argument (goal 2).

Keeping in mind these two goals, let us start with an interesting modification of GD, denoted in Douven (1999a) by GDQ⋆:

> GDQ⋆: The intended interpretation(s) of a language is/are the one(s) on which a maximal number of the beliefs of the speakers of that language comes out true. (See Douven, 1999a, p. 345.)

The perfidy of GDQ⋆ manifests itself in what Putnam may regard as its attractiveness: GDQ⋆ associates "being an intended interpretation" (of a given language) with the beliefs of language speakers rather than just theories. As a result, it appears to develop the notion of the intended interpretation precisely in the pragmatic direction Putnam desires. Meanwhile, as van Douven points out, GDQ⋆ puts Putnam into some impasse - as GDQ⋆ blocks Putnam's strategy to refute realism or, even more, helps to expose the realistic thesis that "Even epistemic ideal theory might be false."(See Douven, 1999a, pp. 345–346.) In order to illustrate how Putnam might fall into the trap of accepting GDQ⋆, let us recall the following fragment of van Douven's reasoning (denoted later as *Dou*) from Douven (1999a, pp. 346–347).

Dou : Let M_O be as before, and let TO be the set of all observation-sentences (in $\mathcal{L}$(T) or some other language) which are true in this interpretation. Let us now consider the set of all consistent and complete theories in the full language which contain TO as a subset. Call this set T. Every element of T will satisfy all operational constraints (since these have only to do with T_O). Will every element of T also satisfy all theoretical constraints? Probably not. However, Putnam would be the last to claim that these constraints are guaranteed to pick out exactly one member of T, i.e., there will be a unique member of T that can unambiguously be said to score best in this respect. Suppose, in our case; they do not. Then let $T^\star$ denote the set of theories that do, overall, equally well concerning simplicity, mathematical elegance, etc. Suppose further that T_{14} is an element of $T^\star$, and that it happens to be the one we believe (for whatever reason). Presuming something like GDQ, the interpretation M_{14} (extending MO) which makes T_{14} come out true, is (among) the

15 Following van Douven, we adopt this conjecture without further justification.

intended interpretation(s) of our language. Since M_{14} is an extension of MO, the other members of $T^{\star}$ will on M_{14} still satisfy all operational constraints. However, it is clear that, on the intended interpretation, they all come out false. Hence, an epistemically ideal theory needs not be true.

Because an epistemically ideal theory does not have to be true-due to the conclusion of *Dou* and against Putnam's wishes-this constitutes an obvious trap for Putnam. In any case, this confusing (at least for Lewis' or Carpintero's Putnam) thesis of van Douven's reasoning is no longer surprising in light of the GDQ⋆ identification of "being an intended interpretation" with the "beliefs of a majority (only!) of users of a given language". It is not difficult to figure out such an epistemically ideal theory (in a sense dictated by the current needs of a discourse) on which some language speakers' beliefs turn out to be false. It's also no surprise that such a theory could be false even if it wins a competition under the banner "maximize the number of true beliefs of language speakers." Although the pragmatics of *Dou* is treacherous from Putnam's position, one might ask: 'Maybe it is only incomplete and might be developed due to Putnam's expectations?'.

We now address this issue by considering two potential remedies to Putnam's unacceptance of *Dou*(GDQ⋆). The first possible remedy is an idea to refute *Dou*'s conclusions by rigidly adhering to a postulate of "true theories." The second remedy under the banner of "objectivization of pragmatics" is a postulate to make an adopted portion of pragmatics less subjective and more coherent with model-theoretic semantics of formal theories (mainly through formalization)[16]. (We find this remedy more promising than the first one[17].) We accept the postulate of pragmatic objectivization through formalization, believing that mathematical clarification satisfies a postulate of "intersubjective communicability"-a constituent of objectivity.

A. Diagnosis. Before examining the first remedy, we shall identify the real cause of Putnam's rejection of GDQ⋆. Therefore, let us think about two pairs of concepts. The first pair of concepts, P_1, includes the concepts "a theory being true" and "a theory recognized by the greatest number of speakers," whereas P_2 consists of the concepts "being a model" (of a theory) and "being an intended model" (of a theory). The first notions in each pair -"theory being true" and "intended model" (resp.) - clearly have a semantic character, whereas the second concepts in P_1 and P_2 have a pragmatic nature. As a result, the first pair of concepts, P_1, exposes the relationship between semantics and pragmatics in GDQ⋆(*Dou*). Meanwhile, the second pair, P_2, may represent the relation between semantics and pragmatics in Putnam's anti-realism.

If the second pair, P_2, plays such a role, then the source of the difficulty with his acceptance of GDQ⋆(*Dou*) is relatively easy to explain. It is based on the fact that a "gap" between semantics and pragmatics in GDQ⋆(*Dou*)[18] is larger than the corresponding "distance" between semantics and pragmatics in Putnam's anti-realism[19]. It follows from the fact that GDQ⋆ pragmatics (as determined by "beliefs of language

16 This taxonomy corresponds to van Douven's distinction between two types of Global Descriptivism. However, this taxonomy is not motivated by the distinction.

17 The reasons will be explained later.

18 That is, a "distance" between the concepts: a theory "being true" and a theory "being recognized by a maximal number of speakers".

19 That is, between the concepts of "being a model (in general)" and "being an intended model(of a given theory)".

speakers") has almost nothing to do with GDQ⋆ semantics (as determined by the falsehood/truth of theories in their models). Meanwhile, Putnam's pragmatics stems from his semantics; in some ways, it is a complement to it. The mutual relationships between the concepts of the model and the intended model perfectly reflect this relationship. (Each intended model must be a model.) Meanwhile, the pragmatics of GDQ⋆(*Dou*) may be "opposite" to its semantics. It is reflected in the fact that being "an intended model" has only a subjective anchoring in human beliefs.

Firstly, before delving into the postulate of "objectivisation of pragmatics," it is reasonable to assess (seemingly) the most natural postulate—as a potential remedy for the problems with GDQ⋆: strengthen Putnam's pragmatics-oriented anti-realism by rejecting all *clearly* realistic theses (such as the thesis R or IM in Garcia-Carpintero (1996) + "even ideal theory may be false") and incorporating Putnam's opposite thesis and its logical consequences. Unfortunately, even if this solution reflects some sensitivity to Putnam's postulate of "more pragmatics," simply considering ideal theories as "true theories" does not solve the problem. It does not create a real space for dialogue—as it immediately introduces Putnam's acceptance problem to the centre of the emotional debate between realists and his opponents[20]. One of the leading causes of this state is realists' inability to provide their opponent with a satisfactory (for Putnamists) clarification on how to distinguish the truth *simplicier*[21] from the truth in all models (as anti-realists expect)-or, as compared in Douven (1999a, p. 342)—a theory T as true *simplicier* from T as being true-on-SAT, where SAT is a given interpretation of a language of T by a bijective mapping onto the world (cf. Kroon, 2001; Chambers, 2000; 2001; Haukioja, 2001; Douven, 1999a). Concurrently, the anti-realist postulate of identifying "true in a metaphysical sense" with "true in *all models*" is inacceptably maximalistic due to the general quantification involved in this identification. Furthermore, a realist may see this identification as her/his unfavorable a modal "multi-world" game.

B. A remedy —the idea of objectivization of pragmatics. The proper remedy that we offer to make GDQ⋆ potentially acceptable by Putnam appears to be a reinforcement of Putnam's model-theoretic semantics by the appropriate portion of objectivized pragmatics[22]. We shall examine this idea by having recourse to the non-sentential concept theory of Joseph Sneed, elaborated in his groundbreaking monograph (Sneed, 1971). Before we move to Sneed's ideas, let us note that the idea of enriching the semantics of formal theories with pieces of formalized pragmatics is not new. A common consensus to identify the notion of the intended model of Peano arithmetic (PA) with the notion of the standard model for this theory delivers a spectacular argument for it.

Meanwhile, the situation of the concept of the intended model for ***ZF*** is drastically different. In contrast to Gaifman's optimistic suggestion (cf. Geifman, 2004) to iden-

20 It is a bit of a challenge to define even the main lines and levels of the discussion. However, it appears that the issues raised here are somehow compatible with those raised in the debates between Kroon (2001), Chambers (2000, 2001), and Haukioja (2001).

21 That is true if the terms of the language of T are interpreted as we intend them to be interpreted.

22 This concept has already been approximated in the initial part of this subsection. We shall explore it more in the context of Sneed's ideas. Whereas the first remedy refers more to Putnam's anti-realism, the second remedy is slightly differently oriented. It refers more to Putnam's models (the semantics of Putnam's argument) than to Putnam's position as a whole.

tify such a model with the model built on linearly ordered domains–the problem with the metalogical (read: objectively communicable) depiction of the "intended models" remains unresolved. A discussion of possible methods of improving Putnam's Theorem (cf. Shapiro, 1985, p. 724; Levin, 1997, pp. 61–66; Velleman, 1998, p. 1364; Bellotti, 2005, pp. 398–400; Button, 2011, pp. 324–326.) reveals more: there is no common consensus regarding even the minimal conditions which should be satisfied by such models. Putnam's model is also non-well founded because it contains non-constructible sets (cf. Shapiro, 1985, p. 724). In Bellotti's opinion, this property discredits this model as the intended one (Bellotti, 2005), but it is unproblematic for Putnam (cf. Putnam, 1980, p. 469.).

Fortunately, this situation is only seemingly inconvenient for Putnam. Indeed, the classical models of set theory are static and non-reactive—due to Gabbay's idea of reactivity from Gabbay (2013)[23])—and, therefore, usually unsuitable for incorporating a significant portion of formal pragmatics. Meanwhile, Putnam's speech is about theories with an ever-expanding set of physical magnitudes (MAG), principles, rules, operational and theoretical constraints imposed on these theories. He adopts this open paradigm of reasoning to capture the entire dynamism of empirically interpreted theories in their historical development[24].

In this context, Sneed's non-sentential concept of theory (as elaborated in Sneed 1971) provides another premise for believing in the successful materialization of the idea of objectivized pragmatics. His groundbreaking work exemplifies how the pragmatics (here: people's beliefs, observations, etc.)[25] of empirically interpreted theories in the historical development of natural sciences can be formalized and deeply coincide with the semantics of the theories. He tends to treat theories in terms of their models, say N, M, etc., and their expansions, N_ξ, M_ξ, etc. (associated with a new scientific statement, ξ), in his so-called "non-sentential" discourse[26]. He also tends to consider the model expansions of his theories at a concrete time point, t, of their historical development[27]. Finally, each theory has its own development 'core' as a pair (H, I), where H is understood as a triple $\langle M_0, N_0, r, C\rangle$, where M_0 is an abstract model of a given theory, N_0—a set of possible applications, C—a set of constraints imposed on elements of M_0, and $I \subseteq N_0$[28].

[23] Gabbay's concept of reactivity is based on the ability of some models to change in response to interactions with operating agents. Some arcs/accessibility relations between states, for example, may be cancelled; others may be re-cancelled.

[24] In fact, one could accuse Putnam of using inappropriate "static" models in his model-theoretic argument.

[25] We can see that Sneed's pragmatics is oriented differently. Whereas Putnam is focused on the user's intentions regarding the intended interpretations of formal theories, Sneed is interested in a person's beliefs and abilities to observe empirical phenomena and objects.

[26] The common sense-based meaning of the concept of an expansion may be naturally substituted by its algebraic depiction in Sneed's approach. Sneed's expansions are formally expressed by the appropriate n-tuples, i.e., in relational calculus, as they represent empirical theories in their historical development. The algebraic foundations of expansions were characterized in Lindstrom's groundbreaking papers (Lindstrom, 1966, 1969) by the occasion of his first-order characterization of elementary logic.

[27] In that case, an expansion N_ξ will be denoted as N_{ξ_t}, etc.

[28] Each singled-out element has its own technical definition in Sneed's work. Their recall, however, essentially exceeds the scope of the analysis of the article; thus, we omit it. For details, see (Sneed, 1971), pp. 170–184.

Sneed proposes a couple of pragmatic definitions in such a terminological environment. His definition D62-1 provides an illustrative example. It formally depicts a person p having a specific mathematical physics theory at a given time point. Let us recall Sneed's definition (D62-1, see: Sneed, 1971, p. 266) to see how he integrates formal pragmatics with semantic considerations.

Definition(D62-1). If p is a person and $\langle H, I\rangle$ is a theory of mathematical physics, then p has $\langle H, I\rangle$ at time t if and only if:

1. There is an expansion of H, ξ, such that p believes at t that N_ξ;
2. If ξ is an expansion of H such that, for all expansions ξ_t of H such that p believes at t that $I \in N_\xi$, $N_{\xi_t} \subseteq N_\xi$ and p believes at t that $I \in N_{\xi_t}$ then:
   - p has observational evidence at t that N_ξ;
   - p believes at t that there exists an ξ such that $I \in N_\xi$ and $N_\xi \subset N_{\xi_t}$.

As Sneed himself suggestively explains later:

> (D62-1) just requires a person who has the theory $\langle H, I\rangle$ to believe at least one of the statements associated with this theory. (D62-2) characterizes the strongest statement associated with $\langle H, I\rangle$ that the person believes at time t-the statement that $I \in N_{\xi_t}$'. It further requires that he have observational evidence for this statement and that he believes that he can, in some sense, make an even stronger statement of the theory than this.(See: Sneed, 1971, p. 267.)

Can Sneed's approach help Putnam with his potential project of objectivization of pragmatics? At the most fundamental conceptual level, Sneed's approach meets all Putnam's requirements. Indeed, it demonstrates how to expose the historical development of theories and formally introduce a piece of pragmatics (human agents with their beliefs at a given time) to them. Hence, Sneed delivers to Putnam a couple of hints on constructing a pragmatics-aware model theory for empirically interpreted theories. In this way, Sneed's approach meets Putnam's empirically-oriented requirement of encoding "all possible measurements of a countable set of physical magnitudes MAG at each rational space-time point." (See, for example, Putnam, 1980, pp. 466; Bellotti, 2005, p. 397.)

Unfortunately, some issues arise as a result of this adaptation task. (Fortunately, some of them are only of a technical nature.) In fact, Putnam would prefer objectivized pragmatics for models of his favorite $\boldsymbol{ZF}$ set theory over other systems, such as Suszko's $\overline{M^{\star}}$[29]. One could also expect that Putnam's philosophical manifesto is likely to be defended by Putnam using his preferred technical instrumentation with OP and OT which may be 'enclosed' in $\boldsymbol{ZF} + \boldsymbol{V} = \mathbf{L}$ and its models-even in his 'small' ω-model. Meanwhile, the model-theory for $\boldsymbol{ZF}$, with its rigid concept of the model and non-consensual idea of the intended model, is insufficiently prepared to incorporate Sneed's solutions directly. Thus, Putnam's and Sneed's approaches require a semantic "connector."

It seems that the Kripke frame-based model theory for non-classical modal logic systems, such as Fagin's *behavioural semantics* from Fagin et al. (1995) (cf. also Lomuscio and Michaliszyn, 2013) might be a convenient conceptual bridge between Putnam's standard model theory and Sneed's pragmatics-sensitive model theory[30].

29 We shall discuss this system and its role later.

30 In Fagin's behavioural semantics, we're dealing with a system of agents who have some epistemic abilities and access to some sources. For example, in Lomuscio and Michaliszyn (2013), the model agents can recognize interval equalities.

Putnam's model theory, equipped with a piece of Fagin's behavioural semantics, would become very similar to Sneed's model theory. To bridge the gap, one could exploit a condition of time-dependent agent capabilities or a series of such conditions. Due to Gabbay (2013), this time sensitivity could be borrowed from Gabbay's reactive semantics. (Let us call this approach the "KreFab-model".) Secondly, for symmetry, Sneed's approach should be further specified. Fortunately, it lends itself to further formalization; for example, the person's beliefs can be represented in an epistemic logic system using box- and diamond-type operators ("knows that..." and "believes that..." *resp.*). Thus, the meeting point of Putnam's and Sneed's formal pragmatics is located in a pragmatics-oriented model theory for non-classical logic systems.

Putnam's fair that integrating pragmatics and semantics (e.g., due to Sneed's ideas) can potentially weaken his Putnam program is the only serious reason to keep a distance from Sneed's approach. Indeed, consider the KreFab-model as formally encoding our pragmatic intentions (e.g., regarding the intended models of our theories). Could theories interpreted using dynamic semantics (as in our KreFab-model) indicate their intended models without our intervention? This point of view is obvious when we consider some similar preference change models based on Convolutional Petri nets (CP-nets) from Spohn (2009)—a significant improvement over the so-called "AGM-models"[31] for change revision. (Call them CP-AGM-models.) They are appropriate for dealing with agents discovering new possible states and actions during their activities and updating them. The models are conceived of as collections of partial models that are then updated in response to the activities of the agents and changes in their preferences. Perhaps surprisingly, both the KreFab and the CP-AGM models offer a kind of a compromise between Putnam and realists: although a piece of (formal) pragmatics is added here—as Putnam wishes—it becomes *internal* pragmatics of the CP-based agent system. Meanwhile, it appears that Putnam is content to keep our various pragmatic intentions (for example, regarding the concept of an intended model) as purely *external*—even at the cost of some nonchalance elusiveness of them, even without "full manifestability in use" (cf. Garcia-Carpintero, 1996, p. 313). External intentions significantly sharpen his philosophical manifesto.

Despite these challenges, Sneed's approach should be methodologically beneficial to Putnam. Its universal significance stems from its ability to demonstrate that an objectivization of pragmatics is possible and materializable in objective terms delivered by the appropriate formalization.

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What is the takeaway from the Section analysis? The Putnam Theorem and the model-theoretic argument appear to say much more than VF_2, which cannot capture the moral of the Putnam Theorem. Although Putnam's position is consistent with VF, this premise does not constitute a proper or convenient representation of his model-theoretic argument. It appears that reinterpreting VF toward Global Descriptivism and van Douven's modifications can lead to discovering its new philosophical content. Unfortunately, there is no completely successful remedy to their potentially realistic (and, according to Putnam, unacceptable) connotations. The "objectivization of pragmatics" postulate—in the spirit of Sneed's ideas—provides Putnam with a noteworthy

31 These models' names are derived from the names of their creators. See (Alchourron et al, 1985).

pattern to reconcile semantics with the pragmatics of formal theories, but it cannot be immediately adopted in Putnam's models and requires a semantic communicator, such as reactive-behavioural Kripke-Fagin semantics (KreFab-models). Adopting Sneed's ideas contrasts with Putnam's vision of pragmatics as an external mediator between our languages and their semantic interpretations.

After delving into the issue of what is properly accepted by Putnam, we can turn our attention to the closely related issue of rejecting realism in Putnam's anti-realism. This is the subject of Section 4's investigation.

4 The problem of rejecting of realism

As previously stated, instead of the VF-based strategy of attacking realism from Garcia-Carpintero (1996), Putnam employs his non-standard reductio strategy to demonstrate that "mind-independent objects are not the type of object to which our words could conceivably bear a determinate referential relation"-as Anderson aptly clarifies (cf. Anderson, 1993, p. 312)[32]. Putnam focuses on justifying his **P4** premise that there is nothing else in the universe that (along with OP and OT) could fix a determinate referential relation to a mind-independent reality. Meanwhile, Carpintero reveals an overly optimistic attitude when he encourages Putnam to employ the VF-based rejection strategy which he believes is more natural for the realism/anti-realism debate (cf. Garcia-Carpintero, 1996, p. 314). In fact, the previous analysis of T_1 focused on the question $Q1$: 'What could Putnam accept?' (cf. Sect. 3), demonstrated that several obstacles prevent this strategy from being accepted uncritically.

It is still necessary to address the issue raised by question Q2: "When is the proper time for Putnam's rejection of realism?" We should investigate whether this is the "early" moment of rejecting realism via (R), its (IM) presupposition, and VF, as Carpintero suggests. We now examine the problem from two complementary perspectives: that of the Putnam Theorem itself and the entire model-theoretic argument.

4.1 Moment of rejecting realism: a perspective of the Putnam theorem

As previously stated, the core of the Putnam Theorem is to justify the existence of Putnam's ω-model encoding OP within and satisfying all theoretic constraints of $\boldsymbol{ZF}$. Meanwhile, proving the existence of Putnam's ω model necessitates using several model-and set-theoretic tools. It includes Shoenfield's absoluteness, the Gödel' constructible universe, and the Skolem–Lowenheim Theorem (DSL). While incorporating these two initial technical tools is not difficult, DSL itself necessitates some special attention or even an earlier declaration on the logical foundation of Putnam's favorite $\boldsymbol{ZF} + \boldsymbol{V} = \mathbf{L}$. Indeed, DSL may be—unexpectedly and paradoxically—problematic from the standpoint of Putnam's model-theoretic argument, his pro-intuitionistic attitude, and his "Just More Theory" approach for several reasons (called later "difficulties"):

[32] Anderson interprets Putnam's approach as a travesty of the Kantian approach to the existence and role of mind-independent objects. (See Anderson, 1993, p. 312.) The author of this paper shares this viewpoint with Anderson.

D1: DSL, as formulated for first-order theories, appears to be non-reconstructible in the intuitionistic set theory ***IZF*** (as a metatheory for these theories)[33].

D2: Even in some variants of classical logic-based systems of set theory (class theory), such as Suszko's canonic axiomatic system, DSL disappears. Simultaneously, the role and presence of Shoenfield's absoluteness for the potential Putnam Theorem reconstructions outside $\boldsymbol{ZF}^1$ are unclear[34].

D3: There is no general consensus on whether Putnam can directly apply DSL to his model because it is a class model, whereas DSL only refers to set models. (See Bays, 2001)[35].

How can these circumstances influence Putnam's moment of rejection of realism? Let us examine the issue starting from the difficulty **D1**[36].

Difficulty D1. If DSL is non-reconstructible in intuitionistic ***IZF*** for first-order theories, we can try to avoid the difficulty by substituting DSL for the Skolem Hull strategy (as recommended in other contexts, for example, in Bellotti, 2005, p. 376). This approach's optimism appears to be justified in light of some "translation" results between ***ZF*** and ***IZF***. Their general pattern is as follows: If $\boldsymbol{ZF} \vdash \phi$, then $\boldsymbol{IZF} \vdash tr(\phi)$, for a property ϕ and a carefully chosen translation function tr[37]. If a given property is also expressible as a Pi_2-formula and provable in PA, it is also provable in Hayting algebra (HA), its intuitionistic analogue. (See: Friedman, 1973; 1978.) So, do Putnamists have a chance of reconstructing the DSL in intuitionistic set theory (even in a reduced Skolem Hull form)?

Unfortunately, using the Skolem Hull instead of full DSL only mildly alleviates Putnam's difficulty and does not bring Putnam closer to his rejection of realism via the Putnam Theorem. In order to demonstrate this, consider McCarty and Tennant's proof (cf. McCarthy and Tennant, 1987, pp. 186–87.) They use neither DSL nor the Skolem Hull strategy to show that no desired denumerable model exists for each theory. In order to show this, they start from a theory, say T, that describes the relation "=" and build unique models for it from immune sets[38], taking advantage of the fact that such

33 Here, we refer to the set theory in Zermelo-Fraenkel's axiom system in an intuitionistic logic language. If DSL^T is the downward Skolem–Loewnehein Theorem for a theory T, then non-reconstructibility means $\boldsymbol{IZF} \vdash DSL^T$. See: (McCarthy & Tennant, 1987, pp. 186–192).

34 $\boldsymbol{ZF}^1$ denotes first-order ***ZF***. Another thing is that DSL holds for theories outside first-order logic in a restricted or a significantly modified, weak form. For instance, we have only partial results on denumerable models and quantifiers for $\boldsymbol{ZF}^{L(Q)}$, i.e., for ***ZF*** in logic with Mostowski's quantifiers—due to Dubiel (1977). The so-called Loewenheim numbers indicate the existence of these weak forms of DSL for extensions of first-order logic. They are the smallest cardinals such that a weak DSL holds for a given abstract logic by this cardinal. We omit a detailed definition of them-as it would require a concept of abstract logic. Notably, they constitute a twin concept to the so-called Hanf numbers. They will be a subject of our interest (from another perspective) in Sect. 5.

35 In general, a majority of researchers agree that this problem is solvable by moving to a class theory (of Kelley–Morse)—due to (Bellotti, 2005; Button, 2011).

36 Since some conclusions elaborated for difficulty D1 appear to be useful by difficulty 3, we consider it achronologically before difficulty 2.

37 The methods used to select tr are unimportant in this case. They may be found, for example, in Friedman (1973).

38 Immune sets are those with *no* infinite recursively enumerable subsets. For more information, see (Hartly Rogers, 1967, pp. 107–109).

sets exist and the proof of this fact is constructive (cf. Theorem III in Hartly Rogers, 1967, p. 108).

How does the proof work? To use the analogy between infinite, recursively denumerable sets and infinite countable ones, consider an immune set, say I, and exchange it for its analogue, say $I^{\star}$, in a corresponding Kleene realizability model $V(Kl)$[39]. Because I does not have an infinite, recursively denumerable subset by definition, its analogue $I^{\star}$ does not have an *infinite countable* (= denumerable) subset. If $M =: (I^{\star}, =)$ is now a model for T[40], then M has no denumerable submodel (cf. McCarty and Tennant, 1987, pp. 186–187). This epistemically unexpected power of McCarthy and Tennant's work manifests itself in the fact that it might be extended to the whole series of similar variants of this theorem. (See Theorem 1, 2, 2.5, 3 and 4. in McCarty and Tennant, 1987, pp. 186–192.) Each of the theorems elucidates the phenomenon of non-provability of DSL in ***IZF*** in a slightly different way. Furthermore, the following Tennant and McCarthy's comment:

Int$_{McT}$: It is not the case that our proof shows no more than that an "effectivization' of the Loewenheim-Skolem theorem is independent of IZF +. Rather, it shows just what it purports to show-that the general Theorem itself is not constructively provable. (See McCarty and Tennant, 1987, p.192.)

enhances their effect. Can we, on the other hand, assume that McCarthy and Tennant's theorems with **Int**$_{McT}$ (when taken together) definitively establish this inconvenient polarization: 'intuitionistic set theory *versus* downward Skolem–Loewenheim Theorem'?

At first appearance, Putnamist may appeal to much stronger and potentially more robust results, such as the Markov Principle[41]. The Markov Principle establishes an acceptable inference rule in HA—due to Friedman's result from Friedman (1978). It brings traditional logic into HA by ensuring that if A is a Σ_1-formula and $\neg A$ is proveable in HA, then A is also proveable in HA. Unfortunately, if Putnamists still believe in a reconstruction of DSL in ***IZF*** thanks to the Markov Principle, they should accept Tennant and McCarty's strongest result (Theorem 4-cf. McCarty and Tennant, 1987, pp. 193–95)[42] which asserts that DSL remains independent of ***IZF***—even if equipped with the Markov Principle. Furthermore, even a cursory examination of the proof method in Theorem 4 reveals that ***IZF***+ Markov Principle cannot be reconciled with DSL in the sense that they all imply the law of excluded middle for arbitrary

39 Kleene realizability models are models (either for intuitionistic set theory or extensions of HA) that adopt the Kleene's idea of realizability. This idea constitutes an intuitionistic counterpart of the classical idea of satisfiability. Thus, it manifests itself in the definition of the realizability relation $\Vdash$, initially established for pairs (n, ϕ), where ϕ is a closed formula of a given arithmetic language, and $n \in \mathbb{N}$. These exemplary clauses illustrate how this definition works. (Cf. Kleene, 1945.)

1. $n \Vdash A \wedge B \equiv \exists n_1 \exists n_2 (n = \langle n_1, n_2 \rangle \wedge n_1 \Vdash A \wedge n_2 \Vdash B)$; $n \Vdash A \Rightarrow B \equiv \forall p(p \Vdash A \Rightarrow n \bullet p \Vdash B)$.
2. $n \Vdash \forall x A(x) \equiv \forall p(n \bullet p \Vdash A(p))$; $n \Vdash \exists x A(x) \equiv \exists p \exists m(n = \langle p, m \rangle \wedge m \Vdash A(p))$.

40 It means that M is a pair-model with $I^{\star}$ as the model universe and "=" as the relation defined on $I^{\star}$.

41 This principle has many different formulations. McCarthy and Tennant use the following formulation: "If $\neg\forall x \forall y(fx = fy)$," then $\exists x \exists y \neg fx = fy$." (See McCarty and Tennant, 1987, p. 193.) One could argue that Putnam used an informal version of this principle when discussing non-constructible sets in the ω-model universe with realists.

42 We're talking about the strongest'submodel' version of DSL here: "for every model M, there is a countable model M_1 that is elementarily equivalent to M."(See McCarty and Tennant, 1987, p. 193.)

sentences. (See McCarty and Tennant, 1987, pp. 193–94.) It constitutes another, more profound measure of the metamathematical "gap" between ***IZF*** and DSL.

This scenario has severe implications for Putnamist thinking: they are still forced to seek a new, more convenient methodological position between the 'poles' of intuitionism and DSL. As a result, it is difficult to attribute to Putnamists a willingness to reject realism without first carefully weighing the reasons for DSL and against intuitionism (and *vice versa*). However, the most ardent Putnamists are likely to be willing to investigate other formal tools, believing in the "just more theory" possibility of breaking the unfortunate dichotomy between DSL and intuitionism. As a result, there is little chance of prompt rejection of realism in this case.

Difficulty D3. Fortunately, the original Skolemization idea in the form of the Skolem Hull postulate provides a reasonable solution to difficulty D3 and to different objections to Putnam's using DSL for class-models[43]–Levin (1997), Bays (2001). Simultaneously, due to Levin (1997), Velleman (1998), Bellotti (2005), the idea of exploiting the formal apparatus of Kelley–Morse's class theory and the Skolem Hull might be replaced by an alternative method of assuming an inaccessible cardinal κ, which provides a corresponding model L_κ the desired set model (See: Bellotti, 2005, pp. 396).

Although each of the improvement methods is equally sound as a formal method, their philosophical weight differs from Putnam's point of view. Without a doubt, the improvement method based on Kelley-class Morse's theory's formal apparatus corresponds well with the philosophical tissue of the model-theoretic argument. Indeed, Putnam is particularly eager to employ class notion-based concepts and phrases like: *all* operational constraints (Putnam, 1980, pp. 466, 469, 474), *all* theoretic constraints (Putnam, 1980, p. 469), formalization of *all* our beliefs (Putnam, 1980, p. 466), etc. These concepts and phrases demonstrate the semantic "openness" property–typical for class-type entities. This vocabulary does not seem accidental, as Putnam's speech from Putnam (1978, 1980) is just about classes of different (conceptually accessible) entities. This mode of expression enables Putnam to do one more thing: to build a linguistic bridgehead to capture the "total science" in its historical development (as in Sneed's theory: Sneed (1971), which has already been discussed).

Meanwhile, Putnam's ambitions and the class notion-based language expose a methodological incoherence in Putnam's philosophical speech: he talks about class-type entities but formalizes his reasoning with set theory. Given this discrepancy, selecting a class theory apparatus is not only an admissible method of formalizing the model-theoretic argument but also a methodological *desideratum*. Is Kelley–Morse's class theory the only option? Von Neumann-Bernays-Gödel set theory (NBG) appears to be a convenient alternative for Putnam. In fact, NBG is a conservative extension of ***ZFC***; it introduces the key notion of class—definable by formulas with quantifiers ranging over sets only. Undoubtfully, the NBG-based class notion corresponds more closely to a pro-Skolemite methodological restraint of Putnam than to Kelley–Morse's maximalistic notion of class[44], but it is unclear whether the NBG-notion completely grasps the classes Putnam refers to.

[43] The class-models are the models with universes which constitute the proper classes. We adopt this idea from Bays. (Cf. Bays, 2001.)

[44] We mean the fact that the notion of class is definable by formulas with quantifiers which range over classes.

In contrast to the first improving method, the assumption of inaccessible cardinal *kappa* appears to be artificial in Putnam's argumentation-not only because it has little resonance with Putnam's pro-Skolemite sympathies and regardless of which Skolemite's face he prefers.[45]. There is another, more powerful argument against it. The inaccessible cardinal axiom is equivalent to the Tarski-Grothendieck axiom (TG): "Every set is contained in a Grothendieck universe" (assuming ***ZFC***). What, in Putnam's opinion, is undesirable about TG? To explain, consider this passage from his "Realism and Reason" in which he accuses his opponents of naive realism and the belief that "what she calls 'transitivity' really is transitivity." (See, for example, Putnam, 1978, p. 11.) Meanwhile, the TG axiom appears to support a realist in this set-theoretic discourse from Putnam (1978) against Putnam, as it ensures the existence of a Grothendieck universe[46] which is transitive from its definition; transitive in some "ontological reality" outside the language of discourse.

As a result, having recourse to Kelley–Morse's class theory to improve Putnam's Theorem is philosophically more sound than assuming an inaccessible cardinal. It appears to be even a kind of a methodological desideratum. Even if adopting this improvement is almost immediate, it cannot be identified with rejecting realism. Due to the chronology of Putnam's reasoning from Putnam (1980), it cannot even be recognised as concluding the model-theoretic argument.

Difficulty D2. In order to evaluate how dangerous for Putnam is difficulty D2 (lack of DSL and unclear role of Shoenfield's absoluteness), we shall consider Suszko's canonic axiomatic system[47] as paradigmatic for this situation. (This evaluation helps us to understand how D2 influences the moment of Putnam's rejection of realism.)

In Suszko (1951), Suszko constructs his *canonic axiomatic system* as an axiomatic theory of sets ($\overline{M^{\star}}$), starting with a system (M) close-related to Quine-Goedel's system from Quine (1946), Goedel (1940)[48]. Within the technical metasystem (μM) for (M), Suszko defines the notions of k-name and k-designation in (M): k-names are constant

[45] Bays appears to find Putnam's Skolemitism more in his affection for first-order logic or first-order theories. See, for example, Bays (2001), Sect. 4, pp. 341–345. He counters this viewpoint by claiming that first-order logic is insufficient to grasp such concepts as recursiveness and finitude. Although the properties of first-order logic listed by Bays are objectively true due to Lindstrom's Theorem, his allegation appears unsound. Putnam does not require the entire metalogical apparatus but rather a weak version of DSL. Indeed, he requires DSL in some form "at hand" to use it instead of a metalogical overview of first-order logic. T. Button expresses a similar conclusion: "Putnam's argument only requires the conditional: If a theory has any models, then it has an unintended model." See (Button, 2011, pp. 329). Meanwhile, this author seems to rather treat Putnam's affection for possibly weak theories as a manifestation of his Skolemitism. Some arguments for it might follow from the analysis of the author's argumentation against Bays' "Skolemization Dilemma." See (Button, 2011, pp. 327–329).

[46] Let $\in$ denotes the relation of 'being an element of'. The Grothendieck universe U meets the following conditions: a) if $x \in U$, and $y \in x$, then also $y \in U$(transitivity); if $x \in Y$, $y \in Y$, then $\{x, y\} \in U$; if $x \in U$, then $\mathcal{P}(x) \in U$(the powerset condition); if $x_{\alpha \in I} \in U$, for each α, and $I \in U$, then $\bigcup_{\alpha \in I} x_\alpha \in U$. An interesting but more general specification of this universe in categorial terms of toposes is given in Streicher (2006).

[47] The construction idea of canonic axiomatic systems of Suszko correlates to Myhill's similar idea of considering universes of namable objects. The construction itself of these universes supports the proof of their denumerability. See (Myhill, 1952).

[48] Suszko's also made references to some of Bernays's papers. The reader can find an exhaustive list of these references in Pogonowski's paper (Pogonowski, forthcoming). This paper forms an illustrative and erudite introduction to Suszko's system.

names (i.e., closed terms) built from atomic names (i.e., individual constants) by name-creating functors with nominal arguments, but k-designation is a designation by k-name. Suszko now defines a constructible object of (M) within the same metasystem (μM) by employing the notions of k-name and k-designation; an object is constructible in (M)if it is k-designated by a k-name. A system is said to be *canonic* if all the elements of its universe are only constructible objects. In the next construction stage, a new primitive notion of k-sets (introduced by a unary predicate $M^{\star}n()$), a new axiom, and a new rule of inference are added to the system (M), yielding a new system $(M^{\star})$. Suszko's next objective is to demonstrate that the concept of a k-set is equivalent to the concept of an object constructible in M (and also in $M^{\star}$). In order to prove this equivalence, he creates a new shared metasystem $\mu(M, M^{\star})$ for both M and $M^{\star}$[49]. Finally, in order to obtain the desired $\overline{M^{\star}}$, he adds to $M^{\star}$ the *canonicity axiom* "$\forall t M^{\star}n(t)$"("every t is k-set") which states that all objects are k-sets.

How is the denumerable universe of the canonic system model obtained here? Let us note that its denumerability (almost) immediately follows the definition properties of k-names and k-designation. It is a consequence of the fact that expressions in $\overline{M^{\star}}$ form finite sequences of symbols over a finite (or at most countable) alphabet, and that the k-designation of constructible objects by their k-names is bijective. This way, they decide that there is at most a denumerable diversity of elements in the universe of Suszko's system. (Let us note that the equivalence of constructible objects and k-sets allows us to consider k-sets as elements of the universe of $\overline{M^{\star}}$.) Thus, no form of DSL determines this process.

To examine the problem of Shoenfeld's absoluteness, let us delve more deeply into Suszko's construction. Is it completely absent in Suszko's reasoning if Suszko did not refer to an idea of absoluteness[50]? To answer this question, let us consider Suszko's construction as a "move" between a model, say V, with Suszko's constructible objects and another model, say L (of k-sets), with the canonicity axiom incorporated from V into L[51]. If we express this axiom in terms of k-names and Suszko's symbols, it takes the form "$\forall_t \exists_x k\text{-}Des^V(x,t)$"[52], thus it is expressible by a Π_2-type formula. We can "transfer" (in the sense of $\models$ relation) this formula from V to L and in the inverse direction using the same proof tricks as in Shoenfeld's Theorem (e.g., Basis Theorem for Σ_2 formulas—cf. Shoenfield, 1961; Moschovakis 1980, pp. 410–411)[53].

A kind of Shoenfeld's absoluteness holds (or may hold) in Suszko's system in a sense described above: even though Suszko makes no mention of it, his construction

49 He constructs the system in the same manner as $\mu(M)$ was built for M.

50 Obviously, he could not make *a direct* reference to Shoenfield's Theorem because it was formulated and proven several years after Suszko's article. Thus, we refer to the idea of absoluteness, currently associated with Sheonfield's result from Shoenfield (1961).

51 We omit here bolds to distinguish this pair from the Goedel's pair $\boldsymbol{V}$ and $\boldsymbol{L}$.

52 $k - Des^V(x, t)$ is a binary predicate read: "x k-designates t in V.

53 To be more precise, we intend to move from the condition $(\star)$ $V \models \forall_y \exists_t k\text{-}Des(x,t)^V$ to the condition $L \models \forall_y \exists_x k\text{-}Des^L(x,t)$ $(\star\star)$. In the proof, we use two results. The first is the so-called Basis Theorem for Σ_2-formulae which asserts that, for each $P(x, y) \in \Sigma_2$, it holds $\exists_y P(x, y) \iff \exists_{y \in \triangle_2} P(x, y)$. The second result (I. Shoenfield's Theorem) ensures that if $\alpha \in \triangle_2$, then $\alpha \in L$, allowing us to perform $(\star\star)$. (cf. Theorem 8F.9 in Moschovakis, 1980, p. 409). An analogy proof for Σ_2 may be found in Moschovakis (1980, pp. 411–412).

(specifically, this almost invisible move between V and L) appears to be naturally reconstructible in terms of Shoenfield's absoluteness. From a methodological standpoint, some proof methods and descriptive set theory results (such as The Basis Theorem for Σ_2) find applications in Suszko's constructions.[54]. Meanwhile, Suszko's construction demystifies denumerability, showing that it constitutes a natural property of canonic systems. The denumerability of the system universe—as follows from the definition of the system—makes DSL redundant or (at least) invisible[55]. Thus, DLS follows in the footsteps of the closely related Skolem's Paradox, which ceases to be mysterious from the perspective of Suszko's canonic system $\overline{M^{\star}}$.

What is the moral of this story? Suszko's canonic axiomatic system seems to create a much less dangerous environment for Putnam's model-theoretic argument than you might think. Admittedly, DSL disappears, but denumerable models may be obtained even more intuitively (from the definition of $\overline{M^{\star}}$, and a form of absoluteness might be easily recalled. Even more, it seems that there exists a mysterious relation between the goal of Suszko's construction and the goal of the Putnam Theorem. It is even more visible when we confront the main objective of the Putnam Theorem and the purpose of Myhill's approach from Myhill (1952) – similar to that of Suszko's. Myhill shows that his predicate $\mathcal{N}^{\star}(A)$ meaning "A is nameable" is definable in Goedel's set theory from Goedel (1940)[56]. He fights against the widespread conviction that "since there are non-denumerably many sets and only denumerably many names, therefore there must be nameless sets"(Myhill, 1952, p. 981). Putnam fights against the hypothesis that all measurements of physical magnitudes in the entire experimental science cannot be encoded in a denumerable model for $\boldsymbol{ZF}$ + **V = L**.

Let us return to our leading problem when Putnam rejects realism if we consider it in the described context. In light of Putnam's Theorem, he must do two things: apply DSL to his initial model for $\boldsymbol{ZF} + \boldsymbol{V} = \boldsymbol{L}$, and use Shoenfield's absoluteness to obtain the ω-model of constructible sets he desires. The situation appears to be—unexpectedly—more straightforward from the standpoint of Suszko's canonic system. Potential problems with sound using DSL are avoided, his ω-model can still be obtained, and the general idea of Shoenfield's absoluteness is defended. Hence, the potential philosophical costs of such a reconstruction are potentially low for Putnam-if only he finds an alternative way to encode measurements of physical magnitudes in his ω-model. (Perhaps, a modified Myhill's idea of naming by an arithmetization method might be helpful.) In any case, Putnam cannot deny realism before achieving his desired ω-model, regardless of the formal tools he decides to incorporate. Thus, not as quickly as Carpintero suggests.

[54] Suszko's equivalence of k-sets (V) and constructible sets (L)-as a profound analogue of the relation between Goedel's $\boldsymbol{V}$ and $\boldsymbol{L}$ universes-also appears to support the idea of a reconstructed Shoenfield absoluteness.

[55] It appears possible that a unique form of the Skolem Hull construction exists here. The author of the paper is, however, unsure about this hypothesis.

[56] More precisely, Myhill demonstrates that $\vdash \mathcal{N}^{\star}(A) = (\exists n)(A = \phi n)$, for some operator ϕ inductively definable in Goedel's set theory. This operator works like an inverse operator of arithmetization of syntax, i.e., it associates natural numbers to expressions of the language of Goedel's set theory (for example-to its axioms). For instance, $\phi 2 = E$, where E is some strong version of the axiom of choice adopted by Goedel (1940), $\phi(3^a 5^b) = \phi a \bullet \phi b$, etc. For more information, see Myhill (1952, pp. 979–980). The statements "A is nameable" means that A is a value of ϕ-operator for some $n \in \mathbb{N}$-according to the definition of $\mathcal{N}^{\star}(A)$.

4.2 Moment of rejecting realism: a perspective of the model-theoretic argument

Putnam's rejection of realism is somewhat elusive in light of Putnam's Theorem. However, we can assume that it will not happen until Putnam completes all of the formal improvements in his proof to obtain his ω-model and, potentially, to balance some philosophical problems "around" his theorem. The conflict between intuitionism and DSL (Difficulty D1–Sect. 4.1.) may be especially challenging for him here. Fortunately—and unexpectedly—some problematic DSL-related issues are eliminated in the case of Suszko's canonic system. Meanwhile, in Putnam (1980), Putnam performs philosophical "pre-processing" for his formal reasoning before proving his theorem. It explains why the overall model-theoretic argument appears to be more useful in reliable forecasting of the moment in which Putnam rejects realism. We now intend to indicate this moment in this new perspective and to demonstrate how new factors influence this moment in comparison to the Putnam Theorem's more narrow view.

To accomplish the goal, we must first understand-based on the chronology of the model-theoretic argument from Putnam (1980) - which tasks Putnam should complete before formulating his theorem. A timeline of his argumentation's "turning points"[57] essentially aids in identifying these tasks (cf. Putnam, 1980, p. 476 inn.). On the one hand, Putnam's task list for a sequential performance is relatively long: rejecting a causal theory of meaning (cf. Putnam, 1980, pp. 476–478); formulating the general frame of his theorem, proving it (cf. Putnam, 1980, pp. 468–69); defending the ideas of "liberalized intuitionism"(cf. Putnam, 1980, pp. 479–481); etc. On the other hand, a simple listing of Putnam's tasks and required actions could be predictively misleading. It can deceptively simplify the problem—not least because balancing philosophical reasons is rarely reduced to binary thinking. There are additional reasons. Firstly, it is difficult to establish an absolute importance hierarchy of Putnam's tasks from the standpoint of Putnam's argument's goal (for example, is the dilemma "DSL or intuitionism" more significant than the dilemma "$\boldsymbol{ZF}^1$ or Kelley–Morse's class theory"?). Secondly, even though the philosophical tasks in Putnam's argument are organised linearly, Putnam's decision tree has a branching structure. For example, Putnam's pro-Skolemite declaration in Putnam (1980) may necessitate further declarations on the type of Skolemitism: first-order-oriented (cf. Bays, 2001), weak-theory-oriented (cf. Button, 2011), or perhaps a more general one.

Although Putnam's declaration on this matter does not influence the moment of his rejection of realism from the standpoint of his theorem (see: 3.1), it is not excessive from the perspective of the whole model-theoretic argument: it may determine the direction of its further reconstruction, the choice of arguments, and admissible formal tools[58]. Finally, Putnam's "Models and Reality" and his "Realism and Reason" contain

[57] We define "turning points" as points in Putnam's reasoning from his model-theoretic argument in which Putnam rejects various philosophical positions (such as the causal theory of reference) as candidates for his philosophical manifesto.

[58] One can figure out that Putnam, as a first-order-oriented Skolemite, would be willing to accept only the first-order theories (and only these) as support of his model-theoretic argument. By contrast, Putnam, as a weak theory-oriented Skolemite, would prefer weak theories (in some sense) even if they are expressed in extensions of first-order logic. He might, for instance, accept some weak version of $\mathbf{ZF}^{\mathcal{L}(Q)}$

numerous explanatory and provocative comments about "hard-core metaphysical" realists, such as about their problems with "relativity of the truth value of '$V = \mathbf{L}$'"(cf. Putnam, 1980, p. 469), their attitude to Ramsay's sentence (cf. Putnam, 1980, p. 474), their postulate of "independence of the truth notion" (Putnam, 1980, p. 472), their beliefs in absolute coherence between reality and its description (cf. Putnam, 1978, p. 11), etc. Despite their highly impressive character, their real role in Putnam's model-theoretic argument is not completely clear. All these aspects elucidate the whole complexity of the situation, making a prediction of Putnam's rejection of realism difficult.

To make matters worse, Putnam goes above and beyond, performing not only "pre-processing" but also "post-processing" for his theorem. Putnam appears to be attempting to construct a "support system", or a collection of philosophical positions that correspond with his model-theoretic argument, such as "liberalized intuitionism" with its "non-truth-condition-based semantics" (cf. Putnam, 1980, pp. 479–80). Furthermore, some excerpts from Putnam's works (Putnam, 1980, 1983) show that Putnam *wishes to do much more than reject realism* only–even after accepting VF or (more realistically) after formulating his conclusion *Conc* from **P3** and **P4**. Let us note the following two examples.

1. He seems to highlight the troubles of his opponents with distinguishing between two "types of being true"–in Haukioja's depiction (cf. Haukioja, 2001, p. 700), i.e., truth *simplicier* from truth in *any model* (cf. Putnam, 1980, p. 474).
2. Putnam appears to be willing to extract some potential benefits of procedural and verification-based semantics for understanding mathematical theories[59]—even at the cost of some radicalization of his opinion on the role of models (cf. Putnam, 1980, p. 479).

What are the profits from this additional effort? One can argue that-in this way-Putnam has a chance to deliver "foundations without foundationalism"[60] to his semantic anti-realism instead of putting forward his worldview anti-realistic manifesto in a dogmatic way[61]. Moreover, if Haukioja is right in his diagnosis from (Haukioja 2001, p. 700), and Putnam is not dogmatically against the co-existence of these two types of truth, then the realist-Putnam debate has the more sophisticated nature of a *methodological* dispute instead of a purely *worldview one*.

To cut a long story short, Putnam's rejection of realism-viewed in the context of the entire model-theoretic argument-occurs chronologically much later than proving his theorem. It is also after he has completed his philosophical "pre-processing" for his

Footnote 58 continued
(as in Dubiel, 1977), i.e., **ZF** in a first-order language with Mostowski's quantifiers. The author of this paper believes that Putnam represents another, more conventional type of Skolemitizm, namely, a denumerability-oriented Skolemitizm.

59 Putnam writes here about "mastery of verification procedures" which enables us to understand mathematical theories such that this 'understanding does not presuppose the notion of a "model" at all, let alone an "intended model".' See (Putnam, 1980, p. 479).

60 It is a linguistic travesty of Shapiro's work "Foundation without Foundationalism." From the standpoint of his structuralism, the author of this erudite work proposes a significantly different program for defending second-order logic. See (Shapiro, 1991).

61 It is a matter of discussion how Putnam took advantage of the "non-dogmatic speech" opportunity.

theorem. Furthermore, Putnam appears unwilling to reject realism immediately after accepting VF or even concluding his model-theoretic argument against Carpintero's conviction.

5 Non-interpreted terms and Putnam's theorem

The previous chapter thoroughly verified the theses, T_1 and T_2. In this chapter, we shall investigate the soundness of thesis T_3. We shall consider the soundness of Carpintero's accusation against Putnam that he accepts the existence of unintended models for theories as a result of our languages' non-completely interpreted terms. Although Carpintero does not state it this way, he sees Putnam's model-theoretic argument as represented by the premise

MT) T-viewed as a partially uninterpreted theory-will have many different models. (See: Garcia-Carpintero, 1996, p. 307.)

In this context, the Catalonian philosopher formulated the following explanatory remark:

> Every language we know of that represents the material world is somehow vague. Models, however, (...) are defined relative to the language. It follows that the "intended" model for a given regimentable language is left unspecified to the very same extent that the language is vague. (See: Garcia-Carpintero, 1996, p. 308.)

Thus, Carpintero seems to explain the problem of unintended interpretations of our languages (lack of their determination) by the vagueness of these languages (being unfixed). It may also be noteworthy to expound this point by having recourse to the following evaluative remark of Carpintero:

> This is Putnam's contention that his opponents merely add "just more theory" or try "to determine the interpretation of an unfixed language with an equally unfixed metalanguage. (See: Garcia-Carpintero, 1996, p. 308.)

Hence, if determining the interpretations is an aftermath of a realist's operation-as Carpintero suggests in this interpretation of Putnam's point of view, then the natural matter of things is unfixed language and corresponding unintended interpretations of them[62]. Let us collect the separate pieces of Carpintero's convictions into the following integrated thesis, after its contextual relativization to first-order set theory and its models.

> **Carp:** The non-completely interpreted terms of the first-order logic of set theory are responsible for the unexpected existence of the unintended models of this theory.

The objective of the section is to decide—having recourse to the formal apparatus of USL and Knight's Theorem:

- whether the existence of the non-interpreted terms explains the existence of the unintended models—due to **Carp**;

[62] This fragment does not clearly indicate the unfixed languages as the sources of the unintended interpretations; it says more about some correlation between them. Nevertheless, it gives rise to interpreting the relations between languages and their interpretations in this manner. Indeed, we can think as follows: if determining the languages by realists establishes the intended interpretation, then leaving the languages unfixed establishes their unintended interpretation.

- whether (**Carp**) coincides with Putnam's vision of (un)intended interpretations (models) of our languages;
- which factors are really responsible for (un)intended interpretations of our natural language expressions.

5.1 The unintended models and the upward Skolem–Loewenheim theorem

It remains unclear how to comprehend the concept of unintended models in **Carp**. As V. Klenk clarifies, "unintended" often means "unexpected", and-further-unintended models "cannot be completely characterized by the structure of first-order logic since any first-order proposition can be constructed as a statement about a denumerable collection". (See: Klenk, 1976, p. 479.) This explanatory remark suggests a reference to the Upward Skolem–Loewenheim Theorem (USL) as a formal result suitable to elucidate the concept of unintended interpretations more profoundly-despite a lack of a common consensus about the comprehension of the intended models of set theory–against Gaifman's optimism from Geifman (2004). V. Klenk's statement that "there is absolutely no reason, according to the Platonist, to allow the countable models greater significance than the countable models" supports this thesis. (Klenk, 1976, p. 475.)

Although USL may be viewed as a classical bone of contention between Skolemites and their Platonist opponents (Klenk, 1976, p. 476)[63], this epistemic dichotomy between them must not concern us here. We are only interested in the model-theoretic face of this theorem, putting aside, for now, its potential philosophical connotations. We do it to extract a piece of knowledge about unintended models (broader: unintended interpretations) for a better understanding of their genesis. Meanwhile, one can expect that both Carpintero and Putnam are willing to accept USL as a formal tool suitable to elucidate the concept of unintended models in set theory[64]. This fact, the explanatory power of USL, and the anticipated benefits of analysis motivate us to conduct additional research concerning USL. We refer to its following formulation and its proof which runs as follows.

Theorem 2 *(USL) If a first-order theory T has an infinite model with a cardinality α, then it has models of cardinalities $> \alpha$.*

An outline of the proof Taking a language $\mathcal{L}$ and $T = \{F : A \models F\}$ in $\mathcal{L}$, we firstly define $\mathcal{L}^{\star} = \mathcal{L} \cup \{c_i, i \in I\}$ and $T^{\star} = T \cup \{\neg c_i \neq c_j, i \neq j\}$. Secondly, we consider finite subsets of $T^{\star\star}$. Let us note that each such finite S is of the form $S = F_1, \ldots, F_n \cup \{c_i \neq c_j, i, j \in I_{fin}\}$, where $I_{fin} \subset I$ is finite. The compactness theorem ensures the existence of a model for $T^{\star\star}$ if only each finite S has a model. To get a model for each S, it is sufficient to find a finite set of elements $a_i \neq a_j$, for $i \neq j$ and take $\langle A, a_i \rangle$ as the model for T. □

[63] By referencing the classical, let us say 'ontological' version of 'Skolemitism', Klenk explains that the USL constitutes a unique argument for the existence of uncountability relative to the formal system (Klenk, 1976, p. 475.) for Skolemites. It results from a unique weakness of the first-order theory: it is not 'powerful enough to generate the enumerating function' (Klenk, 1976, p.476.).

[64] Although Carpintero does not refer to USL in Garcia-Carpintero (1996), he does not underestimate the idea of formalization and the sense of Putnam's argument as radically as D. Lewis does in Lewis (1984). Putnam clearly does not find USL as interesting as DSL, but he treats all models as unintended, particularly those obtained through USL. From our perspective, this is sufficient.

It appears that the operation of *interpreting* the constant symbols $c_i, i \in I$, by corresponding elements $a_i, i \in I$ determines new unintended models (extensions of a given A structure-based one) more than the *existence itself* of the primary uninterpreted constant symbols of $\mathcal{L}^\star$. In order to establish the new extended model for T, each $\mathcal{L}^\star$-sentence of form $c_i \neq c_j$ (for $i, j \in I$) should first find its semantic materialization by the corresponding semantic inequality $a_i \neq a_j$ in some extension of A. In other words, each model extension of A is determined thanks to a precise identification of the a_i's elements that enrich the structure A as the initial model for T. These observations support the hypothesis that the A-*interpreted* terms of our language $\mathcal{L}^\star$ are just the required components to determine unintended models for our initial theory T—against Carpintero.

Nevertheless, it seems that an ardent supporter of Carpintero's thesis might still argue that the non-interpreted terms constitute the basis of the entire proof reasoning as a necessary condition for further complementing terms. Indeed, the initial syntactic construction and the invisible role of the compactness theorem in establishing the new extended model for T may support this type of argumentation. However, it is not easy to evaluate the soundness of **Carp** in light of USL alone. Therefore, we shall refer to some sophisticated model-theoretic results of Knight (1976) concerning the so-called Hanf numbers for omitting types over complete extensions of ***ZF* + *V* = *L***.

5.2 Knight's theorem and unintended models

Let Σ be a set of formulas of the type *t*. A theory T *locally realizes* Σ if and only if there is a formula $\phi(x_1, \ldots, x_n)$ of *t* such that (i) ϕ is consistent with T, (ii) for all $\rho \in \Sigma$, it holds $T \vdash \phi \rightarrow \rho$. T *locally omits* Σ if and only if T does not locally realize Σ. In other words, T locally omits Σ if and only if for every formula $\phi(x_1, \ldots x_n)$ consistent with T, there exists $\rho \in \Sigma$ such that the formula $\phi \wedge \neg\rho$ is consistent with theory T.

In order to formulate the theorem, we also need to introduce some unique types in a given language, say L. Let us assume that the language L is established and let Σ_1 be the type saying that variable $v_1 \in L$ is not definable. For each $n > 1$, let Σ_n be the type saying that the L-variables $v_1 \ldots v_n$ are such that no v_i is definable from the others, for $i = 1, \ldots, n$. Finally, let us repeat that, for two structures A and B, if $A \subseteq B$, and for every formula $\phi(x)$ and every a in A, we have

$$A \models \phi(a) \iff B \models \phi(a),$$

then we say that B is an *elementary extension* of A, denoted $A \preceq B$. It allows us to formulate the following theorem—a dual of (some version of) the Omitting Type Theorem[65].

65 Indeed, the Omitting Type Theorem is usually formulated in a downward variant. One of its possible formulations may be as follows. *Let T be a consistent theory in a countable language L, and let Σ be a type. If T locally omits Σ, then T has a countable model that omits Σ.*

Theorem 3 *Let A be a countable model of P, or of* $\mathbf{ZF} + \mathbf{V} = \mathbf{L}$. *If A omits* Σ_2, *then A has a proper elementary extension which omits* Σ_2.[66]

It is not difficult to find this theorem as a unique analogue of the classical USL with the corresponding idea of unintended models as models with arbitrary large cardinalities (obtained as proper extensions of the given model). In order to grasp a potentially more profound analogy between USL and Knight's theorem, it is reasonable to delve into its proof and trace its reasoning line. It allows us to approximate the answer to the question: is the existence of unintended models dependent more on uninterpreted terms than on interpreted ones?

Outline of the proof: In order to prove the theorem, it is enough to show that the theory $T(S) \cup \{e > a : a \in \mathcal{A}\}$, with the new constant symbol e, has a model which omits Σ_2. For any formula Ψ, $Q(x)\Psi$ is an abbreviation for the formula $\forall x_0 \exists x(x_0 < x \& \Psi)$. By the Omitting Types Theorem, the proof reduces to justifying that for any formula $\phi(u, x, v_1, v_2)$ in the language of $\mathcal{A}$, and any a in $\mathcal{A}$, if $\mathcal{A} \models Q(x)\exists v_1 \exists v_2 \phi(a, x, v_i, v_2)$, then $\mathcal{A} \models Q(x)\exists v_1 \exists v_2(\phi \wedge \neg\rho)$, for some ρ in Σ_2. For further proof, it is crucial to observe that $\neg\rho$ is equivalent to stating that v_1 and v_2 are mutually dependent, i.e., $v_1 = f(v_2)$ or $v_2 = f(v_1)$ for some definable function f.

Let $\phi(u, x, v_1, v_2)$ be a fixed formula. In the proof, we use the fact that the parameters a for which $\mathcal{A} \models Q(x)\exists v_1 v_2 \phi(a, x, v_1, v_2)$ must fall into one of three sets:

1. A—a set of all a's in $\mathcal{A}$ such that $\mathcal{A} \models \exists_{v_1} \exists_{v_2} Q(x)(\phi(a, x, v_1, v_2)$.
2. B $=\{a \in \mathcal{A} : \mathcal{A} \models \exists v_1 Q(x)\exists v_2 \phi(a, x, v_1, v_2) \& \neg\exists v_1 \exists v_2 Q(x)\phi(a, x, v_1, v_2)\}$.
3. C $= \{a \in \mathcal{A} : \mathcal{A} \models Q(x)\exists v_1 \exists v_2 \phi(a, x, v_1, v_2) \& \neg\exists v_1 Q(x)\exists v_2 \phi(a, x, v_1, v_2)\}$.

The set A is less problematic than B and C. Indeed, a choice of ρ for A is easy because $\mathcal{A}$ omits Σ_2. Meanwhile, a choice of the desired ρ for B and C refers to the existence of such new functions, say G and H (*resp.*), that

$\star$) $\exists v_1 Q(x)\exists v_2[\phi(a, x, v_1, v_2) \& G(v_1) = v_2]$
$\star\star$) $\exists v_1 Q(x)\exists v_2[\phi(a, x, v_1, v_2) \& H(v_1) = v_2]$.

Then, the desired ρ's may be found by negating the conditions that define G and H, i.e., the formula saying $G(v_2) \neq v_1$ will be the desired ρ in Σ_2 (for each a in the case B), and the formula $H(v_1) \neq v_2$ will be the desired ρ in Σ_2(for each a in the case C). It is enough to prove the existence of both G and H. The rest of the proof is focused on the method of obtaining G and H from a-parametrized functions g_a and h_a—definable in the structure $\mathcal{A}$[67], and such that $\star$) and $\star\star$) hold for them. Finally, it is enough to establish G and H as functions that agree with g_a and h_a on "enough values"[68]. □

Although this elementary method does not exhaust the whole spectrum of possible approaches to the justification of the proof[69], it delivers intriguing information about the role of completing terms of theories (in formal languages) in determining the intended models of the theories. The proof reasoning revolves around the problem of

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66 In many similar cases, a version of *the Hanf number existence theorem* for omitting types is formulated. Let T be a complete theory in a language L. *The Hanf number* for omitting types over T is the first infinite cardinal κ such that for all L-types, Σ, if T has models omitting Σ in all infinite powers less than κ, then T has models omitting Σ in all infinite powers. (See Knight, 1976.) The Hanf numbers for sentences of a given language may be viewed as 'pointwise' variants of the upward Skolem–Loewenheim theorem. Indeed, each Hanf number indicates the minimal cardinal, up to which USL holds for every single formula of a given language. Since σ forms collections of formulae (they may be seen as theories), the Hanf number for omitting types plays a role of a unique variant of a classical USL.

67 It is not important here which type of definability is adopted for the proof. It is enough to assume that a definition of definability in $\mathcal{A}$ is already established in the proof.

68 It means that $\mathcal{A} \models \forall a \in B \exists v_1 Q(x)\exists v_2[\phi(a, x, v_1, v_2) \& G(v_1) = g(a, v_1) = v_2]$. Similarly—for H and h_a: $\mathcal{A} \models \forall a \in C \exists v_1 Q(x)\exists v_2[\phi(a, x, v_1, v_2) \& H(v_1) = h(a, v_1) = v_2]$.

69 Indeed, the author proposes a concise but sophisticated forcing reasoning as an alternative approach, (see Knight, 1976, pp. 587–588).

some typology of sets of a's parameters from a given model structure $\mathcal{A}$ to ensure that each new extended model of $\mathcal{A}$ satisfies a unique condition (omits Σ_2-type) if only $\mathcal{A}$ does it. Therefore, the *Theatrum* is more located in semantic structure $\mathcal{A}$ and its extensions $\mathcal{A} \cup \{e > a : a \in \mathcal{A}\}$, even if we find a negation of the desired formula $\rho \in \Sigma_2$ to satisfy the Omitting Type Theorem in each case. Carpintero has a right on the matter of some indeterminacy of the terms of the language of ***ZF* + *V=L***. Indeed, in semantic terms, we only require the conditions $G(v_1) \neq v_2$ and $H(v_1) \neq v_2$, for the sets B and C of a's parameters, for v_1, v_2, G and H defined as above. In fact, these two semantic inequalities are *no more* determined than the semantic inequalities of the form $a_i \neq a_j$ from the proof of USL, for semantic constants a_i, a_j, and $i, j \in I$[70]. To some degree, the idea of indeterminacy wins here. In this sense, **Carp** is true. Nevertheless, the proof reasoning runs provided that the a-parametrized functions g_a and h_a, as some "surrogates" of G and H exist, and some semantic frames – in the form of sets A, B, or C— are determined for a's. In this sense, a's and all a-parametrized semantic entities (such as h_a, g_a, etc.) and their linguistic counterparts are determined against **Carp**. In this sense, **Carp** is false.

5.3 Unintended models as the intended one—a Case of Putnam's preferences

In this way, we've shown that unintended models (as defined by USL and Knight's Theorem) cannot be identified with **Carp** unintended models. However, it remains unclear how the concept of the unintended model (as defined by Putnam's model-theoretic argument in Putnam, 1980) applies to these two depictions. In our reconstruction task, we must complete this lack of reference.

Is Putnam's unintended model concept compatible with the concept of unintended models as exposed by USL and Knight's theorem? The answer does not seem to be straightforward, given Putnam's unambiguous attitude toward USL. On the one hand, he should be willing to accept USL as a convenient model-theoretic result elucidating the concept of unintended models. On the other hand, he makes no immediate reference to USL in his Putnam (1980). He does not use USL in the proof of his theorem—even though he begins his study with Skolem's Paradox and recalls the nondenumerable models numerous times in DSL contexts (See: Putnam, 1980, pp. 464–469.).

Meanwhile, this situation should not come as a surprise-perhaps unexpectedly. If Putnam celebrates his ω-model obtained via DSL to be the *intended* one, then the uncountable models obtained via USL appear to be *unintended* (and do not deserve to be celebrated by Putnam). Meanwhile, this type of naming must not be absolute. To grasp this phenomenon, recall why Putnam's ω-model is referred to as "the intended one." Putnam's ω-model is intended in this sense that it allows him to "close" the operational constraints imposed on ***ZF* + *V* = *L*** while satisfying the theoretic constraints imposed on this theory.

Therefore, the situation does not exclude the unintended (= uncountable) models from the role of the *intended* ones. For example, we can build an uncountable model, say $\mathcal{M}$, of ***ZF* + *V* = *L*** with Putnam's ω-model as a submodel. Indeed, in such an uncountable $\mathcal{M}$, one can imagine OP and OT being "closed" in the same way (our

[70] I is a set of indices as in the proof of USL.

$\mathcal{M}$ contains only more components than the ω-model, but this is unimportant). In this sense, these unintended models perform the same task as Putnam's favoured ω-model. The only problem with them is the fact that these "greater" models slightly weaken the philosophical power of Putnam's solution. Nevertheless, they might be equally exploited in his experiment from a purely model-theoretic point of view. Thus, the *unintended* models (in the sense of USL) might serve as *intended* models (in the sense of satisfying Putnam's expectations). As a result, the perspectives of USL and Putnam's expectations are not entirely compatible.

It is still unclear what distinguishes Putnam's understanding of the 'unintended model' from the concept of the 'unintended model' as illuminated by **Carp**. Meanwhile, it seems that **Carp** stems from a principle like this:

> **Princ**: If a model M is the intended one, then another admissible intended model may be model isomorphic to M.

It may be easily inferred from Carpintero's statement: "Any model isomorphic to the allegedly intended one will do as well.[...]"(See: Garcia-Carpintero, 1996, p. 314.) Thus, **Princ** discloses non-denumerable models as the intended ones (in the sense of USL) if only the denumerable ones play this role. Meanwhile, Putnam's model-theoretic argument violates the principle: the non-denumerable models may play the role of the intended models, provided Putnam nominates them for this role. Furthermore, it is conceivable that Putnam's favourite ω-model and its non-denumerable rivals serve as *intended models*, meeting Putnam's expectations (if only Putnam decides for such egalitarianism). Obviously, these models are not isomorphic – against **Princ**. This principle also seems to reveal some commitment to second-order logic. Indeed, "being isomorphic" is a flag property of equinumerous models (of a given cardinality κ) of second-order theories—thanks to Morley's concept of κ-categoricity[71]. Meanwhile, Putnam's natural working environment is first-order logic, though the question of how orthodox he is in his attitude remains open.

To summarize, the broader perspective of "what Putnam could do" also indicates that **Carp** announces something different from Putnam's argument.

5.4 Unintended models as the intended ones—a case of an arbitrary user

A subjective perspective of Putnam's preferences concerning the intended model distances Putnam from this concept, as explained by the USL. To avoid this difficulty, let us adopt the perspective of an arbitrary natural language user. (Due to Putnam's declaration, the natural language users determine just the target group.) But even more importantly, we exploit previous model-theoretic considerations around USL and Knight's Theorem to build a bridge between the Putnam Theorem (broader: the model-theoretic part of Putnam's argument) and Putnam's references to natural languages and their intended interpretations. This way, we will reconstructively complete the original model-theoretic argument with new elements.

Because of this task, let us consider $\mathcal{L}$ (a part of our natural language) as a semantically interpreted language of our discourse and perform a not-so-serious thought

[71] We say that a theory T is κ-categorical for some cardinality κ if all of its models of the same κ cardinality are isomorphic. It means that T has a unique model (up to isomorphism).

experiment on it. Let us assume that the nature of our discourse necessitates adding two names $\bar{a_1}$ ="cat" and $\bar{a_2}$ = "causal theory of meaning" to our $\mathcal{L}$. As a result, we obtain a new language, say $\mathcal{L}^{\star\star}$. Having already interpreted $\mathcal{L}$ in a model, say M, we can build a model for $\mathcal{L}^{\star\star}$ as an enriched structured $M^{\star\star} = \langle M, \{a_1, a_2\}\rangle$, using the USL proof method, and such that the interpretation function $I^{\star\star}$ works as follows: $I^{\star\star}(\bar{a_1}) = a_1$, $I^{\star\star}(\bar{a_2}) = a_2$. The situation is clear as long as $\bar{a_1}, \bar{a_2}$ "live" as formal-logical terms. Let us suppose, however, that $I^{\star\star}$ behaves differently concerning $\bar{a_2}$ depending on the model in which it is defined, i.e., $I^{\star\star}(\bar{a_2})$ = causal theory of meaning, in $M_1^{\star\star}$, but $I^{\star\star}(\bar{a_2})$ = dog, in $M_2^{\star\star}$, as shown[72].

$$\begin{array}{lll}
\mathcal{L}^{\star\star}: & \ldots, \overbrace{\text{cat}}^{\bar{a_1}}, & \overbrace{\text{causal theory of meaning}}^{\bar{a_2}}, \ldots \\
M_1^{\star\star}: & \ldots, \overbrace{\text{cat}}^{a_1}, & \overbrace{\text{causal theory of meaning}}^{a_2}, \ldots \\
\mathcal{L}_{NL}^{\star\star}: & \ldots, \overbrace{\text{cat}}^{\bar{a_1}}, & \overbrace{\text{causal theory of meaning}}^{\bar{a_2}}, \ldots \\
M_2^{\star\star}: & \ldots, \overbrace{\text{cat}}^{a_1}, & \overbrace{\text{dog}}^{a_2}, \ldots \ .
\end{array}$$

How could we tell which interpretation is intended? On the one hand, the existence of non-interpreted terms (the **Carp** explanatory case) does not help. (To figure it out, suppose we adopt many more words in $\mathcal{L}$ but we don't know how $I^{\star\star}$ works on them.) Completing the non-interpreted terms (due to USL) is also ineffective. (This is the case when we adopt many more words in $\mathcal{L}$, and have knowledge about the "behaviour" of $I^{\star\star}$ with them.) In this second case, we can only identify the standard interpretation as consistent with a socio-linguistic *usus*—typical of the $\mathcal{L}$-user community. As a result, **Carp** and USL do not provide sufficient hints to determine the (un)intended interpretations of a natural language.

Meanwhile, this should be no surprise, given that USL and **Carp** are both blind to a taxonomy of non-logical natural language terms. Even if only referring to Putnam (1980), Sher (2016), the categories of *formal predicates* and *rigid designators* deserve special consideration among non-logical terms. Meanwhile, as G. Sher convincingly argues in Sher (2016, p. 215), the reference of formal predicates such as "is empty," "is a symmetric relation," or "has cardinality α" is preserved under permutations, i.e., they remain fixed if one of our terms' reference relations is replaced with another. It has an impact on their limited ability to identify the intended references. Does this fact aptly recapitulate a belief in the role of rigid designators as such indicators? The answer strictly depends on the way we use this concept. The original Kripke's meaning of the notion of rigid designator (as fixed in each modal world) from his Kripke (1980) makes it similar to the previous case of absolute formal terms preserved under permutation. Nevertheless, a pro-Putnam-oriented modification of this concept, say,

[72] We omit the redundant fact that $I^{\star\star}$ in the first and second model might be considered as different interpretation functions.

a $\star$-rigid designator as a linguistic bearer of our pragmatic intentions—and rigid only in these discourses in which we want it, might be helpful.

Let us return to the scheme mentioned above and investigate it. How do the newly modified rigid designators indicate the intended interpretation of $\mathcal{L}^{\star\star}$? Is it $M_1^{\star\star}$ or $M_2^{\star\star}$, or are both models equally suitable? The short answer is that it depends on our discourse's nature (need) and interpretation principles. In order to see that, suppose that we only need a language $\mathcal{L}+\bar{a_1}$, and our interpretation principle Pr_1 says: "Build your interpretation in your discourse based on a common socio-linguistic *usus* of $\mathcal{L}$". Let us also assume that our pragmatic intentions are such that $I(\bar{a_1}) = a_1$, i.e., $I(c\bar{a}t) = cat$ in all discourses where it is required. Thus, $c\bar{a}t$ is a $\star$-rigid designator. However, we can conclude that both $M_1^{\star\star}$, $M_2^{\star\star}$ are equally intended *relatively to the discourse* D_1. In a pessimistic version, $\bar{a_1}$ is *too weak* to specify the intended model. Meanwhile, if we engage in a new discourse D_2 in a language $\mathcal{L}^{\star\star}$ that also requires a term $\bar{a_2}$ =causal theory of meaning, then $M_1^{\star\star}$ should be the only intended interpretation. Why? Because only this interpretation works by our interpretation principle. In this case, we can regard the pair $(\bar{a_1}, \bar{a_2})$ as capable of recognizing $M_1^{\star\star}$ as the intended one. Furthermore, from the position of the extended discourse D_2, we can retrospectively indicate the same $M_1^{\star\star}$ as the intended model for $\mathcal{L}+\bar{a_1}$ in discourse D_1. Why? Because only $M_1^{\star\star}$ still follows our interpretation principle Pr_1 when we extend the discourse to D_2. However, $M_2^{\star\star}$ might deserve to be called "a much better interpretation" than a model $M_3^{\star\star}$ which does not respect Pr_1 much earlier—at the level of $\mathcal{L}$ itself.

What exactly does the story say? Firstly, it appears reasonable to speak about *hierarchy* of intended interpretations with varying degrees of adequacy rather than a single one. Secondly, even if we even prefer to indicate a single interpretation of a language $\mathcal{L}^{our}$ of our discourse D^{our}, we can do so precisely from the standpoint of a larger discourse and more expanded language. In other words, we require a broader set of $\star$-rigid designators than we have in our D^{our}. In any case, this collection cannot be closed and should be prone to expansion.

We have shown how the "gap" between the Putnam Theorem and the natural language-oriented conclusions of Putnam's "Models and Reality" can be filled in this way.

6 Summing up, closing remarks, and the further research perspectives

This article proposes a new reconstruction of Hilary Putnam's so-called model-theoretic argument from a meta-perspective determined by earlier Carpintero and van Douven interpretations of this argument. The reconstruction attempt was founded on three pillars determined by three questions: Q1-'What is properly accepted by Putnam?', Q2-'When should he reject realism?' and Q3-'What is the role of non-interpreted terms in the existence of unintended models of our languages?" Each of the questions determined the corresponding areas of the article analysis.

The first issue, "What is properly accepted by Putnam?", was addressed using Putnam's model-theoretic argument, Carpintero's clarifications, and van Douven's analysis of Global Descriptivism. The main point of reference is Carpintero's premise

VF which, in his opinion, allows for a direct rejection of realism. Furthermore, while Putnam's position may be partially consistent with VF, seeing VF as a proper and convenient representation of his model-theoretic argument is challenging. Van Douven's modifications to Global Descriptivism reveal VF's new philosophical content while having irreducible realistic connotations. As a result, Putnam cannot accept them without question. Fortunately, they point to two potential avenues for improving Putnam's pragmatics. Unfortunately, the first path quickly involves him in a contentious debate with his realistic opponents.

The second problem, "When should he reject realism?", was first described from the standpoint of Putnam's Theorem. In light of this, Putnam cannot deny realism before employing formal machinery such as DSL and Shoenfield's absoluteness to achieve his desired model. Surprisingly, Suszko's canonic axiomatic system suggests that the moment may have occurred earlier. Although DSL is not used in Suszko's system, denumerable models may be obtained even more intuitively due to system properties, and Schoenfield's absoluteness may be recalled. From the standpoint of the model-theoretic argument, Putnam's rejection of realism occurs much later than Putnam's Theorem suggests. From the perspective of the model-theoretic argument, Putnam's rejection of realism occurs much later than Putnam's Theorem suggests. It follows from Putnam's unique philosophical preprocessing and determining a general frame of further steps of his formal approach before proving his theorem. It also appears that, contrary to Carpintero's conviction, the moment of possible acceptance of the VF premise should not be identified with Putnam's rejection of realism. The chronology of the entire argument provides evidence for it. It also appears to result from Putnam's goal of rejecting realism and establishing a framework for methodological debate with his opponents.

The third issue, "What role do non-interpreted terms play in the existence of unintended models of our languages?" was examined in two ways. The first aspect is determined by Carpintero's thesis which asserts that non-interpreted terms are responsible for the existence of unintended models in Putnam's argumentation. The model-theoretic depiction of the idea of unintended models from the Upward Skolem–Loewenheim Theorem and Knight's Theorem is used to examine the thesis. Based on these theorems, we concluded that **Carp** fails in general, as unintended models are obtained through the specification of non-interpreted terms. Simultaneously, the unintended models—in a sense given by the Upward Skolem–Loewenheim Theorem–may be intended as beneficial for Putnam's philosophical manifesto. It also appears that neither **Carp** nor the Upward Skolem–Loewenheim Theorem (along with Knight's Theorem) is responsible for the existence of unintended interpretation of natural language expressions because they are not sensitive to the taxonomy of natural language non-logical terms, such as absolute concepts and rigid designators. Instead, we should talk about approximated intended interpretations of their terms in rich languages. However, stable sets of rigid designators cannot reach them.

Although Carpintero is partially right that models have nothing to do with the classical debate "realism/anti-realism", model-theoretic analysis can expose many new facts–as in Sect. 4 motivated by question Q2. Namely, Suszko's canonic axiomatic system indicates a new potential reconstruction path for the model-theoretic argument, and the non-reconstructibility of DSL in Kleene's models for intuitionistic set theory shows some limitations of Putnam's argument. These conclusions seem to be unachievable without model-theoretic analysis.

The entire collection of theses serves as a framework for the necessary meta-reconstruction of Putnam's semantic anti-realism. The word "frame" is not accidental here, as this meta-reconstruction does not aim to be exhaustive, necessitating complementation. The question "What is the proper logical foundation of Putnam's model-theoretic argument?" may provide an exemplary direction for a further extension of this argument. An expected benefit of such an extension could be knowledge about the limits of Putnam's model-theoretic argument's metalogical reconstruction and expertise about the weights of specific Putnam's theses.

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