

Moderate structural realism about space-time

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Abstract This paper sets out a moderate version of metaphysical structural realism that stands in contrast to both the epistemic structural realism of Worrall and the—radical—ontic structural realism of French and Ladyman. According to moderate structural realism, objects and relations (structure) are on the same ontological footing, with the objects being characterized only by the relations in which they stand. We show how this position fares well as regards philosophical arguments, avoiding the objections against the other two versions of structural realism. In particular, we set out how this position can be applied to space-time, providing for a convincing understanding of space-time points in the standard tensor formulation of general relativity as well as in the fibre bundle formulation.

Keywords Fibre bundles · Hole argument · Intrinsic properties · Quantum entanglement · Relations · Space-time points · Structural realism

1 Moderate in contrast to radical structural realism

Structural realism is a position in the philosophy of science that has been much debated recently. It is the view that only structure in the sense of the relations that are instantiated in the world is real or at least is all that we can know. The latter position is known as epistemic structural realism, the former one as ontic structural realism (this distinction goes back to [Ladyman, 1998](#)). The main motivation of epistemic structural realism is to steer a middle course between the no miracle argument for scientific realism and the argument from pessimistic induction for instrumentalism. The main motivation—and application—of the form of ontic structural realism that has been

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developed hitherto is the interpretation of quantum physics. The aim of this paper is to set out (1) a moderate ontic or metaphysical structural realism that puts objects on the same footing as structure and (2) to apply this position to space-time, arguing in particular that it leads to a convincing view about space-time points.

The current discussion on structural realism goes back to Worrall (1989, in particular pp. 117–123). Worrall's aim is to pay heed to both the argument from pessimistic induction—that is, the claim that since most of our past scientific theories have turned out to be false, it is likely that our present and future scientific theories will endure the same fate—and the no miracle argument, that is, the claim that the predictive success of our scientific theories would be a miracle if they were not tracking truth. Worrall's middle way consists in three theses:

- (1) Structure in the sense of relations among physical objects and as captured by the mathematical equations of a scientific theory is all that we can know.
- (2) There is continuity in our views about structure despite theory change: the views about structure of a predecessor theory can be construed as an approximation of the views about structure of the successor theory.
- (3) We cannot know the intrinsic properties of the physical objects that underlie structure.

Thus, in a nutshell, the no miracle argument applies to structure, and the argument from pessimistic induction applies to our—futile—attempts to gain knowledge of intrinsic properties.

According to the standard view, intrinsic are all and only those properties that an object has irrespective of whether or not there are other contingent objects; in brief, having or lacking an intrinsic property is independent of accompaniment or loneliness (see Langton & Lewis, 1998 and for a refinement Lewis, 2001). All other properties are extrinsic or relational, consisting in the object bearing certain relations to other objects. By properties, we understand in this paper all and only those properties whose instantiation does not depend on the existence of any particular individual; properties such as being that individual (e.g. being John Smith, or being identical with John Smith—if these are properties at all) will hence not be considered in this paper. We shall employ in the following the terminology of relations instead of the terminology of extrinsic or relational properties. Our position is to regard structure as the concrete relations that obtain between physical objects.

The intrinsic properties that are at issue in the debate about structural realism are fundamental intrinsic properties of basic physical objects, sometimes referred to as intrinsic essences—that is, intrinsic properties that cannot be traced back to more fundamental properties. It goes without saying that there are intrinsic properties of complex systems as a whole—for instance, even the property of being composed of two sub-systems is an intrinsic property of a whole. Such intrinsic properties of complex systems can, of course, be known. Recognizing them poses no problem for metaphysics. In the last resort, one can regard the whole world as one complex system. All the properties attributed to that whole trivially are intrinsic ones (leaving aside relations to other possible worlds). The point at issue is intrinsic properties of basic physical objects that somehow ground the relations that these objects bear to one another.

Worrall's first claim is supported by a widespread view in the philosophy of science. According to this view, scientific theories reveal the relations between physical objects, but not their fundamental intrinsic properties. The argument is, in brief, that we gain

knowledge of physical objects owing to the causal relations that obtain between the objects and our senses or our measuring instruments. It is not excluded that the relational descriptions that we set up on this basis refer to intrinsic properties on which the relations are grounded. However, we cannot know what the properties of physical objects are insofar as they are intrinsic. In other words, the fundamental intrinsic properties of the physical objects are beyond the scope of our knowledge, because we have access to these objects only in a relational way. In sum, our knowledge is not limited to the relations that hold between physical objects and our measuring instruments—these relations do not have any special ontological status—but it is limited to the relations in which the physical objects stand. The statements of laws of nature that our scientific theories contain describe such relations (see Jackson, 1998, pp. 23–24 for a clear statement of that type of argument).

For the sake of this paper, we shall assume without further discussion that this argument is correct. If, however, this argument, which supports Worrall's first claim, is sound, then it is evident that there is a tension between Worrall's first and his second claim. According to this argument, structure in the sense of relations is all that our scientific theories describe. Thus, it is not possible to draw a line within a scientific theory between what is the description of structure and what is a purported description of intrinsic properties. Hence, if there is a theory change, there is a change in our views about structure. But in that case, all the well-known arguments from the history of science against cumulative progress on the conceptual level—including the argument from pessimistic induction—apply to our views about structure as well. Consequently, structural realism as such does not rescue scientific realism from the standard objections mounted against this position. In the following, therefore, we presuppose scientific realism, but we do not intend to use structural realism in support of scientific realism.

If epistemic structural realism is separated from the ambition to make a case for scientific realism, its main claims can be put in this way:

- (1) Physical objects have intrinsic properties over and above the relations (structure) in which they stand.
- (2) No claim is made about the link between the intrinsic properties and the relations (that is, notably, whether or not the relations supervene on the intrinsic properties of the related objects).
- (3) We cannot know the intrinsic properties.

This position hence implies the claim that there is something beyond structure that we cannot know. That is why this position may with reason be described as *epistemic structural realism*.¹

If it is claimed that there is something that exists but that we cannot know, we need an argument why we should accept that there is any such thing. The master argument for intrinsic properties can be summed up in this way:

- (1) Relations require relata, that is, objects that stand in the relations.
- (2) These objects have to be something in themselves, that is, they necessarily have some intrinsic properties over and above the relations that they bear to one another—even if the relations do not supervene on the intrinsic properties and even if we cannot know the intrinsic properties.²

¹ For another epistemic version of structural realism, see van Fraassen (1997).

² See, for instance, Langton (1998, Chap. 2, in particular p. 22).

Structural realism as a *metaphysical* thesis—in contrast to *epistemic* structural realism as a thesis about the limits of our knowledge—rejects this argument. The ontic structural realism set out notably by Steven French and James Ladyman in recent papers is a radical structural realism—or a radical metaphysics of relations—because this position puts aside both parts of the master argument for intrinsic properties.³ According to French and Ladyman (2003), in brief, there is only structure in the sense of relations, and there are no objects standing in the relations.

One can put forward three types of arguments for this position:

- *The argument from coherence*: Our metaphysics should be coherent with our epistemology. Postulating intrinsic properties that are unknowable leads to a gap between metaphysics and epistemology. Metaphysics has it that there are, at the basic level of the world, objects whose fundamental properties are intrinsic ones. On epistemological reflection, however, we have to concede that we do not have access to these properties insofar as they are intrinsic.
- *The argument from parsimony*: We have to recognize relations (structure) in our metaphysics anyway. It is not possible to reduce all relations to intrinsic properties. Even if the world consists of objects whose fundamental properties are intrinsic ones, there have to be some relations: at least spatio-temporal relations are not supervenient on—and consequently not reducible to—intrinsic properties (that much is conceded even by David Lewis in his famous thesis of Humean supervenience; see Lewis (1986, IX–X)). On the other hand, it is questionable whether we have to recognize both relations and intrinsic properties in our metaphysics. Parsimony (Occam’s razor) tells us that we shall not admit entities beyond necessity. Thus, the claim is that the metaphysics of structural realism is parsimonious, because it does not recognize more than is necessary anyway, namely relations (structure).
- *The empirical argument from quantum physics*: French and Ladyman present two examples from quantum physics of what they consider as cases of underdetermination that challenge an ontology of objects:
 - (1) They claim that the issue of whether or not quantum particles are individuals is underdetermined by the empirical fact that quantum particles of the same kind are indistinguishable (the physical state of a system composed of quantum particles of the same kind is invariant under permutation). Despite being indistinguishable, one can maintain that quantum particles possess a primitive thisness (haecceity) each.⁴ French and Ladyman use this issue as an argument in favour of shifting from an ontology of objects—which is plagued according to them by this problem of underdetermination—to an ontology of relations, whereby, according to them, questions of individuality or non-individuality do simply not arise.⁵ However, one can retort that primitive thisness is a purely metaphysical position that one can take anyway, quantum physics be as it may, so that there is no *empirical* underdetermination. As regards all empirical evidence, quantum particles are not individuals, because there are no empirical

³ See, in particular, Ladyman (1998) and French and Ladyman (2003). For another version of a radical metaphysics of structural realism, see Dipert (1997).

⁴ As regards primitive thisness, see Adams (1979). As regards the issue of whether or not quantum particles are individuals, see French and Redhead (1988).

⁵ See Ladyman (1998, pp. 419–420); French and Ladyman (2003, pp. 36–37, 41–42).

- properties whatsoever that distinguish one quantum particle from other quantum particles of the same kind in the case of entanglement (see Cao, 2003b, p. 62).
- (2) Furthermore, according to French and Ladyman (2003, Sect. 6), the issue of whether quantum particles or quantum fields are fundamental entities is a similar case of underdetermination due to the attempt to consider objects (either particles or fields) as forming a genuine ontological category. However, there are in fact strong reasons in favour of a field ontology on the basis of quantum field theory: the notion of a quantum particle, with all its physical content, can be derived from the primary notion of a quantum field and is actually not always well-defined (see Cao, 2003a, pp. 17–19). Therefore, the particle interpretation is physically not equivalent to and explanatory less successful than the field interpretation.

Whereas the empirical argument from quantum physics as set out by French and Ladyman thus is doubtful, the argument from coherence is based on a methodological principle that is well respected. The same goes for the argument from parsimony. The main objection against the ontic structural realism of French and Ladyman (2003), however, is that it is too parsimonious: relations presuppose relata, that is, objects between which the relations obtain and of which they are predicated.

More precisely, a Platonist may maintain that relations as such exist as abstract structures, that is, abstract entities that are universals. However, when it comes to the physical world, the point at issue are concrete relations that are instantiated in the physical world and that hence are particulars in contrast to universals. For the relations to be instantiated, there has to be something that instantiates them, that is, something that stands in the relations. In brief, the objection is that in eliminating objects, the position of French and Ladyman is not intelligible as a theory of the physical world: it is not able to distinguish between structure as an abstract entity and the structure of the physical world.⁶

This objection is cogent to our mind. However, it does not refute structural realism as a metaphysical thesis about the physical world. In order to accommodate this objection, we put forward a position that may be called *moderate structural realism*. This is a version of ontic or metaphysical structural realism, but in contrast to radical structural realism, this position accepts the first part of the master argument for intrinsic properties; it only rejects its second part:

- (1) Relations require relata, that is, objects that stand in the relations.
- (2) *It is not the case* that these objects necessarily have intrinsic properties over and above the relations that they bear to one another.

Moderate structural realism proposes that there are objects, but instead of being characterized by intrinsic properties, all there is to the basic physical objects are the relations in which they stand. Admitting objects provides for an empirical anchorage of the relations. Consequently, this position is not touched by the standard objection against the radical structural realism of French and Ladyman (2003).

According to this position, neither objects nor relations (structure) have an ontological priority with respect to the physical world: they are both on the same footing, belonging both to the ontological ground floor. It makes no sense to assign an ontological priority to objects, because instead of having fundamental intrinsic properties,

⁶ See Cao (2003b); Chakravartty (2003, pp. 871–872); Busch (2003); Psillos (2004, Sect. 2).

there are only the relations in which they stand. In other words, an object as such is nothing but that what bears the relations. As regards the relations, it makes no sense to attribute an ontological priority to them, for at least insofar as they exist in the physical world, they exist as relations between objects. In sum, as far as the physical world is concerned, there is a mutual ontological as well as conceptual dependence between objects and structure (relations): objects can neither exist nor be conceived without relations in which they stand, and relations can neither exist in the physical world nor be conceived as the structure of the physical world without objects that stand in the relations.

This position makes metaphysics coherent with epistemology, since it does not assume that there are intrinsic properties that are unknowable. Furthermore, it is parsimonious, for it does not postulate fundamental intrinsic properties of objects over and above the relations in which they stand. However, one may raise the following question: if there are objects, don't they require intrinsic properties as identity condition (master argument for intrinsic properties)? Hence, if there are only relations and no intrinsic properties, doesn't this imply that there are no objects either (radical structural realism)?⁷ These concerns are motivated by a metaphysical view that is widespread, but that is not mandatory: there is no need to assume that there first are objects and that these objects are then put into relations; in that case, of course, the objects would first of all need to have intrinsic properties on the basis of which they would then enter into relations ("first", "then" in the sense of a metaphysical, not a temporal order).

Recall that, as mentioned above, our position is that structure always consists in certain specific, concrete relations, these relations being as determinate as intrinsic properties are supposed to be. Our position is therefore not subject to what is known as the Newman objection against structuralism.⁸ Consequently, since relations are as specific and determinate as intrinsic properties were (if they existed), they are exactly on the same footing as intrinsic properties as far as identity conditions are concerned: insofar as intrinsic properties account for identity conditions, relations can perform that task as well. For instance, if *A* is bigger than *B*, heavier than *C*, etc., these relations individuate *A* and distinguish *A* from *B* and *C*. It goes without saying that there is in structural realism no question of identity conditions for an object independently of other objects. But this does not mean that relations cannot provide identity conditions. Which relations make up for identity conditions for which types of objects depends obviously on the case under consideration.

Consider an analogy: since Quine's seminal paper on "Two dogmas of empiricism" (Quine, 1951) and the subsequent development of semantic holism (inferential role semantics), we are familiar with the notion of a web of beliefs. We are used to thinking of beliefs as points in a web that are individuated by their position in the web, that is, their relations to other beliefs. Content (meaning) is not an intrinsic property of a belief, but consists in inferential relations to other beliefs (the same goes for other properties of beliefs such as confirmation or justification). Semantic holism has no problem in individuating beliefs on that basis: each belief is defined by its position in the web, being distinguished from all the other beliefs in the web, for no two beliefs stand in exactly the same relations to all the other beliefs in the web. The problem is

⁷ We are grateful to an anonymous referee and to Christian Wüthrich for insisting on that point.

⁸ See Demopoulos and Friedman (1985). As regards the point that concrete relations are not subject to this objection, see Chakravartty (2004, Sect. 3).

that we do not want any old change of relations in the system to amount to a change in the content of *all* the beliefs in the system. Some inferential relations thus have to be distinguished as being more important than others. But this problem does not touch the central issue that it is relations that provide the identity conditions for the members of the system. Moderate structural realism can be received as proposing to transfer this idea from semantics to metaphysics, the objects being now physical entities instead of beliefs. If this idea is intelligible in semantics, then so it is in metaphysics.

Hence, insofar as intrinsic properties can provide for identity conditions, so can relations. However, there are cases in physics where neither relations nor intrinsic properties are able to provide for identity conditions. Quantum systems of the same kind whose states are entangled are indistinguishable, although in the common cases there is a definite number of them that is greater than one. These systems do not have an identity in time. An analogous consideration applies to space-time points on certain symmetry assumptions about space-time: space-time points can stand in exactly the same spatio-temporal relations and, yet, be numerically distinct. One may receive these cases as speaking against a bundle theory of objects: quantum systems and space-time points can neither be bundles of intrinsic properties nor can they be bundles of relations; for the intrinsic properties or the relations may be as concrete as is physically possible and, nevertheless, fail to establish a distinction between quantum systems or space-time points. A bundle theory of objects accords ontological priority to intrinsic properties or relations over objects: objects are constituted by intrinsic properties or relations on that theory.

One may therefore be inclined to accept a primitive thisness (haecceity) in these cases. However, at least as far as space-time points are concerned, there is a strong argument against primitive thisness on which we shall elaborate in the next section, namely the hole argument. The view of each object having a primitive thisness accords ontological priority to objects over intrinsic properties or relations: objects are first constituted by a primitive thisness that provides for their identity and then equipped with intrinsic properties or put into relations. The view of objects being constituted by a primitive thisness stands in opposition to the spirit of structural realism.

The bundle theory and the view of objects as bare particulars are not the only options in the metaphysics of objects. In the cases where neither intrinsic properties nor relations provide for identity conditions one can simply accept a numerical distinction (diversity)—among quantum systems or space-time points—as primitive (a similar view is held by Pooley, 2005, Sect. 4). A numerical distinction tells us that there is a number of objects that is greater than one—in many cases of quantum entanglement even a definite, finite natural number of objects—and that is all that it tells us. A numerical distinction is not a primitive thisness, for it does not establish an identity in time—or any other sort of an identity—that is not empirically accessible. Accepting a numerical distinction as primitive is motivated by the physical cases—quantum entanglement, space-time points—in which there is a plurality of objects without these objects being distinguished from one another by any intrinsic properties or relations in which they stand and without primitive thisness being an open way out, since there are strong physical arguments against primitive thisness. This empirical situation—and thus the motivation for acknowledging numerical distinction as a primitive—is independent of structural realism. Any position in the metaphysics of science has to come to terms with this empirical situation.

Nonetheless, having to acknowledge a numerical distinction as primitive is the reason why we are committed to the view that objects and relations are interdependent, being on the same ontological footing: we get the relata and the relations at once as the internal structure of a whole, neither of them being eliminable or reducible to the other one. We cannot dispense with objects on pain of running into absurdity; we cannot accord priority to relations or intrinsic properties over objects, because we cannot conceive objects as bundles of either relations or intrinsic properties, for these fail to provide for a distinction in the case of quantum entanglement as well as in the case of space-time points; and we cannot grant priority to objects, for this would commit us to primitive thisness.

In sum, structural realism as a metaphysical thesis certainly is a radical position, because it does away with fundamental intrinsic properties. Nonetheless, one can distinguish two versions of this position—a radical and a moderate one. The moderate version fares well as regards philosophical arguments (coherence, parsimony), and it is not plagued with the intelligibility objection against the more radical version of structural realism. Over and above that, moderate structural realism is supported by empirical arguments as well.

As regards quantum physics, the point is again that insofar as quantum theory provides a description of the physical world, it cannot dispense with objects. As far as quantum mechanics is concerned, when it comes to the application of the formalism of quantum theory, it always refers to cases in which there is a definite number of quantum systems taken for granted (although these systems are not individuals). In a nutshell, what is challenging about quantum physics is not that there are no objects, but that the properties of objects are remarkably different from the properties that classical physics considers (notably entanglement and its consequence that there are no properties that distinguish quantum objects of the same kind from one another). Unless one goes for hidden variables, there are no intrinsic properties underlying the entanglement, and the way in which entangled states are conceived in the formalism of quantum theory shows that there is no need to postulate unknowable intrinsic properties of quantum systems underlying the relations of entanglement. It can therefore be argued that moderate structural realism is an appropriate metaphysics for quantum physics (see [Esfeld, 2004](#)).

We shall now apply moderate structural realism to space-time, claiming that this position is a convincing metaphysics of space-time as well: there undoubtedly are space-time points that fulfil the function of objects.⁹ But instead of these objects having intrinsic properties, all there is to them are the relations in which they stand.

2 Moderate structural realism applied to general relativity in the standard tensor formulation

In this section, we consider space-time as described by general relativity (GR) in the standard tensor formulation. We shall briefly go into the fibre bundle formulation of GR in the next section. We do not consider the possible purely algebraic formulation of GR in this paper. In this section and the next one, we mainly limit ourselves to “empty” solutions of the Einstein field equations or pure gravitational cases, that is,

⁹ We limit our considerations to classical general relativity; for a brief comment on the possible generalizations of the notion of a space-time point in the framework of a still to be developed theory of quantum gravity, see the last paragraph of the paper.

models of the theory without non-gravitational (energy-)matter fields (this is essentially a methodological move for the sake of simplicity and clarity; it does not alter the argumentation).

Structural realism about space-time is a realist position about the space-time structure as described by our best current physical theory, namely GR. This position subscribes to the ontological commitment of there being space-time as a mind-independent physical structure. The standard tensor formulation of GR makes use of the concepts of a four-dimensional differentiable manifold as a mathematical background structure and of tensor fields defined on this manifold—in particular the Lorentz metric tensor field. Basically, a four-dimensional differentiable manifold, sometimes simply denoted “manifold” in the following, is a set of points with a topology and a differentiable structure—such that subsets of the manifold locally “look like” subsets of R^4 : roughly, we have sets of local smooth assignments of four real numbers for the manifold points together with some conditions. A Lorentz metric tensor field, sometimes simply denoted “metric” in the following, is a geometric object providing geometric relations like distances between manifold points. The metric plays a crucial role in the standard tensor formulation of GR, because it encodes the fundamental space-time relations. These are the chronogeometrical relations (like space-time intervals), the inertio-gravitational or affine relations (they tell us, through the potential of the gravitational field, how freely falling test particles behave in a gravitational field) and causal relations (they enable us to define a light cone structure at each space-time point and to make a distinction between spatial and temporal directions). Therefore, the Lorentz metric tensor field can be regarded as incorporating the structure of space-time—indeed together with the (metric-preserving linear or Lorentz) connection, which can be considered as an independent variable (as in the Palatini formulation of the theory, see Rovelli, 2004, Chap. 2). However, for sake of simplicity, we restrict our considerations to the Lorentz metric tensor field.

As explained in the first section, there are three versions of structural realism, one epistemological and two metaphysical ones. If epistemic structural realism is applied to space-time, this position commits us to fundamental intrinsic properties of space-time points. Only the structure in the sense of the relations between these points can be known. Applied to the framework of the standard tensor representation of space-time, epistemic structural realism implies that the identity of the space-time points is constituted by their fundamental intrinsic properties, independently of the space-time structure—that is, independently of the metric.¹⁰

Such an intrinsic individuation of space-time points is not supported by physical theory. It leads to conceptual difficulties as is evident from the recent versions of the famous hole argument (see in particular Earman & Norton, 1987; Stachel, 1993). In the general case of a space-time with non-gravitational (energy-)matter fields, we consider a hole in the space-time manifold, that is, an open subset of the manifold where all non-gravitational fields vanish. We furthermore consider a non-trivial active diffeomorphism on the hole that smoothly reduces to the trivial diffeomorphism, that is, the identity, on the boundary and outside the hole. The GR-principle of active general covariance tells us that if we have a space-time model, that is, a solution of the Einstein field equations, then any diffeomorphism applied on this model will generate a space-time model.

¹⁰ Slowik (2005), however, proposes a version of epistemic structural realism applied to space-time that deviates from Worrall’s epistemic structural realism in that Slowik does not commit himself to intrinsic properties; his proposal simply remains agnostic about the ontology of space-time.

Any primary and metric-independent intrinsic individuation of the manifold points forces us to consider any two mathematical models of the theory related by this diffeomorphism as distinct physical solutions. For instance, a given manifold point in the hole will be “coloured” by distinct metric tensors in these two diffeomorphic models. A complete physical model outside the hole—or a set of Cauchy data in the initial value formulation of the theory—then is insufficient to provide a unique physical solution inside the hole. No unique evolution can be determined from the set of Cauchy data. This lack of common determinism is quite problematic for a physical theory—in the sense that, from any complete set of initial data, it is not possible to make any physical prediction of a unique solution: two physically possible space-times, in which we consider the same foliation, may agree till a time t and then disagree for any time $t' > t$ in the foliation.

A wide range of philosophers of physics and physicists agree on the fact that this dramatic feature is a consequence of the non-physical primary individuation of space-time points independently of the metric.¹¹ This unpalatable feature can hence be avoided by claiming that *there is no physical individuation within GR of space-time points independently of the metric*. Indeed, this can be seen as the moral of the fundamental GR principle of active general covariance—or, in more mathematical terms, of the invariance under active diffeomorphisms, that is, invariance under certain manifold point transformations taking one point into another together with the induced drag-along maps acting on the tensor fields (see Stachel, 1993). Therefore, and in agreement with the common practice among working relativists with respect to the invariance of the theory under active diffeomorphisms, two diffeomorphic space-time models of the theory should be considered as representing the same physical situation (solution) and the same physically possible space-time structure. This is often referred to as Leibniz equivalence in the philosophical literature about space-time.

Radical structural realism about space-time avoids the difficulties that the hole argument poses for epistemic structural realism by eliminating the constituents of the space-time structure: there are simply no space-time points that function as objects on which the metric tensor field is defined. Thus, there is no question of there being fundamental intrinsic properties that constitute the identity of space-time points independently of the space-time structure. However, the radical version of structural realism about space-time faces the problem of explaining how the space-time structure can be conceived without any reference to constituents. This is the intelligibility objection mentioned in the first section. In concrete terms, what does a space-time relation (of the space-time structure) relate? How can the space-time structure be represented by the metric alone without any reference to space-time points, given that the metric tensor field itself is defined as an assignment of a metric tensor at each manifold point?

The ambiguous status of the bare manifold point is evident. On the one hand, it has no physical meaning because of the active diffeomorphism invariance of the theory. There is no bijective correspondence between physical space-time points and mathematical bare manifold points; indeed, an equivalence class of diffeomorphic manifold points corresponds to a space-time point. On the other hand, the bare manifold point is conceptually indispensable when conceiving space-time relations or the

¹¹ See notably Stachel (1993); Brighouse (1994); Hoefer (1996); Dorato (2000). Indeed, it is usually claimed, within the debate about the ontological status of space-time (see Sect. 4), that it is the consequence of a literal realist interpretation of the differential manifold (within the standard tensor formulation of GR), which is usually called “manifold substantivalism”.

metric tensor field itself. Hence, neither can the bare manifold point be abandoned, nor can it be physically considered independently of the space-time structure that it supports and to which it belongs.

The moderate version of structural realism as set out in the first section, being based on the mutual ontological and conceptual dependence between the structure and its constituents, reconciles these two aspects. Applied to space-time, this version does not face the difficulties that plague the other two versions. It can therefore be regarded as an appropriate conceptual framework for the physical understanding of the representation of space-time in the standard tensor formulation of GR. Indeed, following the physical understanding, as we have seen above, a pure gravitational space-time model representing a physically possible space-time with no non-gravitational energy-matter fields is given by an equivalence class of diffeomorphic four-dimensional Lorentz manifolds, which are differentiable manifolds equipped with a Lorentz metric tensor field.

The moderate structural realist interpretation accounts for this physical understanding in considering space-time as a network of space-time relations among space-time points that do not possess any intrinsic properties over and above bearing the relations. The space-time structure is represented by the Lorentz metric tensor field together with the four-dimensional, differentiable manifold on which it is defined. The very notion of an equivalence class (of Lorentz manifolds) generated by diffeomorphisms is understood in structural terms. Insofar as the metric tensor field grounds all the chronogeometrical, inertio-gravitational (or affine) and causal relations (as described by GR)—that is, all the fundamental space-time relations—there is a natural way of interpreting it as a physical structure, that is, as a network of physical relations among physical objects. These latter are the indispensable constituents of the structure, the things that stand in the relations, the physical space-time points. It is obvious that a chronogeometrical relation—like a space-time distance—makes only sense as a relation between space-time points (at least in the pure gravitational cases).

Therefore, in contrast to radical structural realism, the moderate version admits space-time points (not bare manifold points!) as physical objects in its ontology. It accepts the first part of the master argument for intrinsic properties mentioned in the first section. However, as we said above, following moderate structural realism, the relations and the objects that stand in the relations are on the same ontological footing and are also conceptually interdependent. On the one hand, the metric tensor field defines space-time relations between space-time points, which are necessary for the definition of the field: the space-time relations cannot be defined without making reference to space-time points (at least in the pure gravitational cases). On the other hand, the metric tensor field completely determines the structural identity of the space-time points: these latter do not possess any physical intrinsic properties over and above the metric relations that the metric tensor field attributes to them (denial of the second part of the master argument for intrinsic properties).

In the same way as it makes no sense to consider a space-time relation without the space-time points—the relata standing in the relation—it is physically meaningless to consider a space-time point independently of the space-time structure and in particular independently of the space-time relations that define its position in the network. The very fact of designating a bare manifold point through coordinatization can be understood as a kind of intrinsic individuation. However, this individuation is purely mathematical. It has no physical meaning. For instance, coordinatization is subject

to passive diffeomorphism invariance, that is, invariance under coordinate transformations, and cannot be considered to provide any intrinsic characterization of bare manifold points. The bare manifold points (or rather the sets of manifold points) only get their—structural—physical identity and meaning through the specification of the metric tensor field (turning them into space-time points).

In the moderate structural realist perspective, active general covariance implies that it makes no sense to only perform either active diffeomorphisms on manifold points independently of the metric tensor field or the (active) induced drag-along map on the metric tensor field against fixed (independently individuated) manifold points (indeed, the term “active diffeomorphism” designates, by a common abuse of the language, both the active manifold diffeomorphisms and the active induced maps acting on tensors). The two types of active transformations are closely linked in the sense that when an active diffeomorphism is applied on the metric, it does not leave any identity to the manifold points behind (see [Dorato & Pauri, 2005](#), Sect. 3.1). We will see in the next section that this interpretation receives an accurate geometric expression in the framework of the fibre bundle formulation of GR. Therefore, two diffeomorphic Lorentz manifolds (M, g) and (M, g') represent the same physically possible space-time and the very notion of an equivalence class of Lorentz manifolds generated by active diffeomorphisms is naturally understood in this structural realist framework (it naturally leads to the acceptance of the so-called Leibniz equivalence).

Indeed, it is mainly active general covariance (or invariance under active diffeomorphisms)—one of the main lessons of GR—that physically grounds the moderate structural realist interpretation of space-time proposed here. As a consequence, any attempt to identify and to individuate the space-time points independently of the space-time structure provided by the metric tensor field has no physical meaning. For instance, if we could identify a bare manifold point $p \in M$ as the genuine representation of a space-time point (independently of the metric), then the fact that, in general, a metric g and its induced drag-along image g' are such that $g'(p) \neq g(p)$, would lead to a breakdown of Leibniz equivalence. But the identity of the space-time points is completely determined by the space-time (chronogeometrical, inertio-gravitational, causal) relations they exhibit, that is, their “position” in the (generally covariant) network of space-time relations. Therefore considering $p \in M$ independently of g has no physical meaning. In other words, there is no possible definition or “location” (and therefore individuation) of a space-time point against a fixed background independently of the space-time structure. For instance, within GR, talk of the location (of a test particle for instance) at a space-time point is physically meaningful only if the location with respect to the gravitational field is intended (represented by the metric tensor field that individuates the considered space-time point).¹²

One can still ask how, in concrete terms and within the considered formalism of GR, the metric tensor field provides a structural physical identity for bare manifold points. An approach to this problem based on the original suggestion of [Synge \(1960\)](#) and on the work of [Bergmann and Komar \(1960\)](#) has been recently developed by [Lusanna and Pauri \(2005\)](#) and discussed by [Dorato and Pauri \(2005\)](#). The original Bergmann–Komar procedure aims at providing locally and in the absence of non-gravitational (energy-)matter fields a “pseudo-coordinatization” of space-time points from four suitable functions of the four functionally independent eigenvalues of the

¹² See [Rovelli \(2004, p. 75\)](#).

Weyl tensor (the “trace free part” of the Riemann tensor) (the coordinatization is “pseudo” because it does not define a chart in the atlas of the considered manifold). For space-times with no symmetries, this procedure seeks to provide distinct sets of four real numbers for distinct space-time points, which are therefore labelled by these non-coordinate sets of four real numbers. For a specific (although quite large) class of “empty” solutions of the Einstein field equations, Lusanna and Pauri (2005) develop the Bergmann–Komar procedure in the constrained Hamiltonian framework of GR: roughly speaking, an arbitrary foliation of space-time into three-dimensional hypersurfaces is considered, and the “dynamics” of the theory is given by a Hamiltonian function, provided that the canonical variables satisfy certain constraints.¹³

Within this framework, GR can be understood as a constrained Hamiltonian theory, that is, a theory whose physical content is invariant under gauge transformations generated by the first class primary constraints.¹⁴ The invariance under these gauge transformations corresponds in a well-defined sense to the diffeomorphisms invariance of GR (but only for the solutions of the Einstein field equations). The idea is to impose Bergmann–Komar’s non-coordinate labelling of space-time points as a gauge fixing for GR in the constrained Hamiltonian framework, that is, as further conditions that reduce the gauge freedom of the phase space. This non-coordinate labelling is physically meaningful in the sense that it depends uniquely on the dynamical degrees of freedom of the metric: it constitutes the structural identity of space-time points.

Although the structural identity of space-time points is obtained here through gauge-fixing and therefore through the explicit breaking of active general covariance (active diffeomorphisms invariance) and for a rather specific class of solutions, this procedure, which is a natural development of the standard tensor formalism of GR in the constrained Hamiltonian framework, fits well into the moderate structural realism proposed here. On the one hand, the physical identity of space-time points is not intrinsic but entirely provided by the metric in a well-defined and concrete sense. On the other hand, space-time points are not eliminable in the sense that they are parts of the physical structure we should be realist about in a scientific realist perspective. This moderate structural realist reading comes close to the position of Dorato and Pauri (2005) when they say that a space-time point “‘is’ the ‘values’ of the intrinsic degrees of freedom of the gravitational field”.¹⁵ Indeed, this procedure can be naturally understood as speaking in favour of a moderate version of structural realism about space-time in the sense that it provides a possible concrete realization within the standard tensor formalism of GR of the physical and structural individuation of space-time points by the metric. Moderate structural realism is however not tied to this gauge-fixing approach of GR. The peculiar gauge-theoretic aspects of GR can also be expressed in terms of a principal fibre bundle, the orthonormal frame bundle.

¹³ For a philosopher’s account of the Hamiltonian formulation of GR, see Belot and Earman (2001). For a more technical account, see Wald (1984), Chap. 10 and appendix E.

¹⁴ Earman (2006) argues indeed that the substantial content of (active) general covariance is best understood within the framework of a gauge symmetry of GR.

¹⁵ Dorato and Pauri (2005), p. 30 of the version on <http://philsci-archive.pitt.edu/archive/00001606/>

3 Moderate structural realism applied to general relativity in the fibre bundle formulation

The issue how space-time points receive their structural identity from the metric tensor (or gravitational) field can also be considered from the point of view of the fibre bundle formulation of GR. This formulation can be regarded as an extension (or a generalization) of the standard tensor formulation rather than being completely independent of it. The fibre bundle theory is worth being considered insofar as it constitutes a general geometric framework for (gauge) field theories describing all fundamental non-gravitational interactions. Insofar as GR can be considered as a field theory, the fibre bundle formulation may also provide a natural and conceptually fruitful formalism for GR.¹⁶ It constitutes "... a natural geometric setting for the gravitational field" (Rovelli, 2004, p. 61). Moreover, the fibre bundle formulation encodes gauge-theoretic aspects of GR allowing for a Palatini-type formulation of GR in terms of an orthonormal frame field and of a Lorentz connection leading to promising theoretical developments in (loop) quantum gravity (see Baez & Munian, 1994, part III).

The usual fibre bundle for GR is the orthonormal frame bundle over the space-time base space, the base space being the four-dimensional differentiable manifold of the standard tensor formulation. The fibres over base space points are the sets of all orthonormal frames (tetrads) at these points. An orthonormal frame at a base space point possesses all the relevant geometric information of the Lorentz metric tensor, which can be defined in terms of the orthonormal frame. It can therefore be considered as a derived notion, a function of the orthonormal frame. All the chronogeometrical and causal structure (spatio-temporal distances, local light cone structure) provided by the metric tensor field can be understood in terms of the orthonormal (co)frame field. The Einstein field equations can be written in terms of this latter field, which is then the physically relevant entity of the theory (possibly together with the Lorentz connection).

The orthonormal frame bundle encompasses mappings between the base space and the fibres, which are the crucial elements of the formalism: a surjection, called projection, from the total space into the base space, which maps each frame at some base space point into the considered base space point itself (it "projects" any orthonormal frame of a fibre into the base space point over which the fibre is defined). Sections are smooth assignments to each base space point within the domain of definition of the considered section of a unique orthonormal frame of the fibre over it. These mappings, together with the GR-principle of active general covariance, encode the structural identity of the space-time points, as Stachel points out: "For a theory with a general covariant structure, the map σ [a section] defines the points of space-time. ... Clearly, space-time points are not defined before a cross-section is given." (Stachel, 1986, p. 1861).

In fact, these mappings provide the bare differentiable manifold of the standard tensor formulation with the relevant geometric structures that turn it into a physically meaningful representation of space-time. Within this fibre-theoretic framework, it is often said that the fibres and the base space are "soldered": the geometry of the base space is determined by the geometry of the fibres over it (and vice versa). It makes

¹⁶ See Trautman (1980); Stachel (1986, 2002) and Stachel and Iftime (2005); Heller (1992, Chap. 4); Auyang (1995, pp. 59–60, Sect. 20 and appendix B).

no physical sense to consider the base space, the fibres or the orthonormal frame field (which is a section of the bundle) independently of the rest of the whole bundle structure.

In the same way, this formulation leaves no place for physically meaningless transformations occurring independently either only on the base space (only on the bare differentiable manifold) or only at the level of the fibres over the base space (only on the tensor fields). The principle of active general covariance can be expressed in an unambiguous way as the invariance under the group of all fibre-preserving horizontal and vertical automorphisms of the orthonormal frame bundle. The important point here is that the fibre bundle formulation of GR makes very clear the strong link between the automorphisms of the orthonormal frame bundle and the diffeomorphisms acting on the base space: these latter are uniquely determined by the former. On the other hand, a diffeomorphism acting on the base space uniquely determines an equivalence class of automorphisms of the orthonormal frame bundle (the equivalence relation is provided by the vertical ones, see [Stachel & Iftime, 2005](#)). According to this physically meaningful way of expressing the GR-principle of active general covariance, it is “impossible to *even formulate* the hole argument” ([Stachel, 2002](#), p. 235).

Our claim with respect to the fibre bundle formulation of GR is that it naturally receives a moderate structural realist interpretation and that it encodes in a precise geometric way some of the interpretative statements made in the framework of the tensor formulation:

- (1) The mappings between the base space and the fibres encode structural identity of the space-time points (this partially corresponds to the structural identity provided by the metric tensor field in the standard tensor formulation).
- (2) The base space cannot be considered independently of the fibres (and vice versa)—base space and fibres are “soldered”. The base space has to be considered within the whole bundle structure (this obviously corresponds to the denial of any independent ontological weight of the bare differentiable manifold within the standard tensor formulation).
- (3) Active general covariance is naturally conceived in a physically meaningful way as an invariance under fibre-preserving horizontal and vertical automorphisms of the orthonormal frame bundle, where the strong link with the diffeomorphisms acting on the base space is explicit and prevents any misleading (hole-type) considerations (this corresponds to the physically meaningful interpretation of active diffeomorphisms invariance in the standard tensor formulation).

Within this fibre-theoretic framework, space-time can be interpreted as a structure whose constituents are space-time points, their properties consisting in the spatio-temporal relations provided by the orthonormal frame field. Again, as in the standard tensor formulation, bare base space points do not represent space-time points (2). These latter get their structural identity from the fibres, more exactly from a section of the orthonormal frame bundle—from an orthonormal frame field (1). The fibres cannot be regarded as providing somehow intrinsic properties to space-time points because of the geometric structure of the whole bundle and because of the invariance under the group of automorphisms of the orthonormal frame bundle acting on the fibres (3). Again, it makes no sense to consider a fibre over a base space point independently of other fibres, and it cannot be considered independently of whether or not there are other fibres.

Thus, the fibre bundle formalism of GR does not contain any reasons that allow us to interpret space-time points as entities possessing an identity independently of the fibres (against the epistemic version of structural realism). However, fibres, and in particular the orthonormal frame field, are defined over (“soldered” with) base space points, turning them into space-time points (but not in a bijective way!): although a space-time point has a purely structural identity provided by the relevant section of the orthonormal frame bundle (1), it cannot be eliminated from the interpretation of space-time within the fibre-theoretic framework of GR, on pain of running into mathematical, physical and conceptual difficulties (against the radical version of structural realism). In particular, the (mathematical) possibility to define the base space as a quotient space does not imply the elimination of the space-time points (see [Stachel & Iftime, 2005](#), Sect. 3.2): it makes very clear mathematically the fact that any transformation on the base space cannot be carried out independently of the corresponding transformation at the level of the fibres (vertical or horizontal automorphisms). This fact underlines the fundamental and mutual interrelation between the space-time points and the space-time relations (structure) that the moderate structural realist interpretation of space-time proposes. Moreover, it shows that the fibre structure of space-time, which can be considered as an aspect of the metric structure of space-time, naturally encodes structural characteristics of space-time (in particular through the fibre-theoretic formulation of the principle of active general covariance), which, in a structural realist perspective, are best understood in the moderate version advocated here.

4 Moderate structural realism about space-time and the debate about the relationship between space-time and matter

Let us briefly consider the traditional debate between substantivalism and relationalism about space-time in the light of moderate structural realism. As a metaphysical claim about the nature of space-time, where does moderate structural realism about space-time stand in this debate? Substantivalism and relationalism cannot be considered independently of the issue of the relationship between space-time and matter. One can distinguish between three conceptions:

- (1) *Newtonian substantivalism*: space-time is considered as an independently existing entity that has its own properties, which are not reducible to the properties and relations of matter. Space-time and matter are two ontologically distinct beings.
- (2) *Leibnizean relationalism*: space-time is reduced to relations among matter or properties of matter. In order to avoid simply presupposing a network of spatio-temporal relations among physical objects, these relations and properties have to be conceived as non-spatio-temporal relations and non-spatio-temporal properties in the last resort (such as, for instance, the intrinsic properties of Leibnizean monads). Consequently, space-time and matter are not ontologically distinct, space-time being somehow derived from matter.
- (3) *Cartesian-Spinozean substantivalism*: space-time and matter are ontologically identical and form the same substantival entity.

Moderate structural realism about space-time fares well with the conceptions (1) and (3), but it does not seem to fit into (2).

Moderate structural realism claims that the space-time structure exists as a mind-independent physical network of spatio-temporal relations among spatio-temporal constituents (such as space-time points) that do not possess any intrinsic properties. Therefore, spatio-temporal relations as represented by the Lorentz metric tensor field or the orthonormal frame field are not reduced to something non-spatio-temporal such as non-spatio-temporal properties of matter.¹⁷ Our claim that space-time is purely relational in the sense that there are no fundamental intrinsic properties of the constituents of the (“empty”) space-time structure has nothing to do with any kind of relationalism about space-time understood in the reductive way of (2).

In this non-reductive sense, moderate structural realism about space-time can be understood as a kind of substantivalism in the sense of (1) or (3). In particular, a realist position towards space-time endorsing Leibniz equivalence—as the structural realist position does for instance (see Sect. 2)—is often called “sophisticated substantivalism” in the recent philosophical literature about space-time.¹⁸ But moderate structural realism about space-time remains open with respect to whether or not the space-time structure and (non-gravitational) energy-matter are distinct ontological beings. If we extend our moderate structural realist position to matter and interaction fields, these could be conceived as structures being ontologically independent of the space-time structure (but however physically interacting with it) (1) or as parts (substructures) of the total space-time or world structure (3). In particular, denying any possible physically relevant distinction between the gravitational field and other matter (and interaction) fields does not constitute an objection against moderate structural realism about space-time.¹⁹

In the approach of [Lusanna and Pauri \(2003\)](#) briefly discussed at the end of Sect. 2, the non-coordinate labelling of space-time points through the values of the degrees of freedom of the gravitational field is modified by the presence of matter fields. This modification is ruled by the Einstein field equations. In this framework, the question of whether matter fields, which also provide structural (but non-spatio-temporal) identity to space-time points through their universal coupling with the gravitational field, should be understood in the line of (1) or in the line of (3) cannot be settled by our moderate structural realist interpretation of space-time alone. [Dorato and Pauri \(2005\)](#) seem to think that this openness as regards the relationship between space-time and matter goes in the direction of (2) when they say that “the specific reality of spacetime depends (also) upon the (matter) fields it contains”.²⁰ But of course, non-spatio-temporal relations provided by matter fields only determine some part of the structural identity of space-time constituents.

5 Conclusion

We saw in Sect. 1 how moderate structural realism, as a metaphysical claim, is supported by the philosophical arguments of coherence and parsimony (in contrast to epistemic structural realism) and by empirical arguments from quantum theory,

¹⁷ Even if the Lorentz metric tensor field is merely relabelled as material or physical field like other physical fields (see [Rovelli, 2004](#), Chap. 2), this renaming has no reductive (or ontological) power.

¹⁸ See also [Belot and Earman \(2001, p. 228\)](#) and [Pooley \(2005, Sect. 5\)](#).

¹⁹ For such a denying see [Rovelli \(2004, Chap. 2\)](#) for instance.

²⁰ [Dorato and Pauri \(2005\)](#), p. 30 of the version on <http://philsci-archive.pitt.edu/archive/00001606/>

without facing any intelligibility problem as does the more radical version of ontic structural realism. In Sects. 2 and 3, we argued that moderate structural realism can be convincingly applied to space-time as described by GR in the standard tensor formalism and in the fibre bundle formalism respectively. We saw that it accounts for the fundamental GR-principle of active general covariance (or invariance under active diffeomorphisms) in a better way than the other two versions of structural realism, without facing hole-type problems (the hole argument) or intelligibility objections (by denying that there are space-time points underlying the space-time structure): the Lorentz metric tensor field, or the orthonormal frame field, is interpreted as representing the space-time structure and as providing structural identity to space-time points, which therefore cannot exist independently of the whole structure. The fibre bundle formalism, which yields a precise geometric framework for GR, suits well this moderate structural realist interpretation of space-time: within this framework, we saw that it is not possible to consider the base space independently of a section of the orthonormal frame bundle or to consider diffeomorphisms on the base space independently of the corresponding (vertical or horizontal) automorphisms on the total space.

To conclude, let us add a final thought about the link between moderate structural realism and the metaphysics of space-time points. We argued in this paper for a moderate structural realism about space-time (as described by *classical* GR in the standard geometric formalism) that recognizes space-time points as genuine physical entities that do not possess any intrinsic properties. However, moderate structural realism about space-time as a philosophical position is not tied to a set-theoretic representation of space-time in terms of a set of points. For instance, GR can be formulated in purely algebraic terms, without making use of the notion of a differentiable manifold; in this framework, the bare manifold points and the metric can be represented as derived algebraic structures (the manifold points correspond to maximal ideals in the relevant (abstract) algebra, see for instance Butterfield & Isham, 2001, Sect. 2.2.2). Moderate structural realism about space-time can account for such an algebraic representation of space-time: the identity of space-time points can be understood in purely structural (non-intrinsic) terms either at the geometric (tensor or fibre) level as we have seen in this paper or at the algebraic level, in terms of algebraic structures and properties (spelling out the details would obviously require a separate paper).

Moreover, we would like to stress that, in a broader sense and despite the ontological commitment to space-time points defended here, moderate structural realism as a metaphysical conception about space-time is *not* necessarily committed to the existence of space-time points. In this broader sense, it is the claim that space-time is a mind-independent physical structure whose basic constituents have no fundamental intrinsic properties independently of the structure they are part of. In particular and at a fundamental (possibly *quantum*) level, these latter do not have to be space-time points (such as described in this paper), but could also stem from a generalization of the notion of a space-time point. Indeed, with respect to the enormous empirical and explanatory success of quantum theory, it seems natural (but however not obvious) to look for possible quantum aspects of space-time which would be described by a still to be developed theory of quantum gravity (QG). In the framework of certain candidates for QG, such as loop quantum gravity or the algebraic generalization of GR, there may be no reference anymore to space-time points. Of course, if these approaches were to turn out to be plausible, they would require a rather radical modification of

the space-time ontology presented here. However, they would fit well into the main metaphysical claim of moderate structural realism.

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