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MEANING APPROACHED VIA PROOFS

ABSTRACT. According to a main idea of Gentzen the meanings of the logical constants are reflected by the introduction rules in his system of natural deduction. This idea is here understood as saying roughly that a closed argument ending with an introduction is valid provided that its immediate subarguments are valid and that other closed arguments are justified to the extent that they can be brought to introduction form. One main part of the paper is devoted to the exact development of this notion. Another main part of the paper is concerned with a modification of this notion as it occurs in Michael Dummett's book *The Logical Basis of Metaphysics*. The two notions are compared and there is a discussion of how they fare as a foundation for a theory of meaning. It is noted that Dummett's notion has a simpler structure, but it is argued that it is less appropriate for the foundation of a theory of meaning, because the possession of a valid argument for a sentence in Dummett's sense is not enough to be warranted to assert the sentence.

1. INTRODUCTION

The term proof-theoretic semantics would have sounded like a *contradictio in adjecto* to most logicians and philosophers half a century ago, when proof theory was looked upon as a part of syntax, and model theory was seen as the adequate tool for semantics. Michael Dummett is one of the earliest and strongest critics of the idea that meaning could fruitfully be approached via model theory, the objection being that the concept of meaning arrived at by model theory is not easily connected with our speech behaviour so as to elucidate the phenomenon of language. Dummett pointed out at an early stage that Tarski's T-sentences, i.e., the various clauses in Tarski's definition of truth, cannot simultaneously serve to determine both the concept of truth and the meaning of the sentences involved. Either one must take the meaning as already given, which is what Tarski did, or one has to take truth as already understood, which is the classical approach from Frege onwards.

This latter alternative amounts to an account of meaning in terms of truth conditions depending on a tacit understanding of truth. In the

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case of a construed formal language, the T-sentences become postulated semantic rules that are supposed to give the formulas a meaning (a representative presentation of this view is in *Introduction to Mathematical Logic* by Alonzo Church (1956)). If the T-sentences are to succeed in conferring meaning to sentences, this must be because of some properties of the notion of truth. A person not familiar with the notion of truth would obviously not learn the meaning of a sentence by being told what its truth condition is. It therefore remains to state what it is about truth that makes the semantic rules function as genuine meaning explanations – the semantics has to be embedded in a meaning theory as Dummett puts it.

In the case of an already given natural language, the T-sentences become instead hypotheses, which must somehow be connected with speech behaviour. Here one may follow Donald Davidson's suggestion which may roughly be put: if "A is true iff C" is a correct T-sentence for the sentence A in a language L, then a speaker of L who asserts A normally believes that the truth condition C is satisfied; cases when a speaker is noticed both to observe that C is satisfied and to assert A therefore constitute data supporting the T-sentence.

By making this connection between T-sentences and speech behaviour for at least observation sentences, one begins spelling out the concept of truth, which is needed to support the claim that the T-sentences give the meaning of the sentences of a language. However, as argued by Dummett (e.g., in Dummett 1983), it is only a beginning, because the assertion of sentences is only one aspect of their use. If the T-sentences are really to be credited with ascribing meaning to sentences, they must be connected with all aspects of the use of sentences that do depend on meaning. In other words, there are further ingredients in the concept of truth that must be made explicit, if the truth condition of a sentence is to become connected with all features of the use of the sentence that do depend on meaning. One such feature is the use of sentences as premisses of inferences. When asserting a sentence we are not only expected to have grounds for the assertion, we also become committed to certain conclusions that can be drawn from the assertion taken as a premiss.

I shall leave the prospects of rightly connecting the meanings of expressions with our use of them within a theory of meaning developed along these lines, and shall instead review some approaches to meaning that are based on how we use sentences in proofs. One advantage of such an approach is that from the beginning meaning is connected with aspects of linguistic use.

One very simple version of an approach of this kind is to take meaning to be determined by all the rules for a language. Restricting oneself to deductive uses of language and thinking of proofs as determined by a set of inference rules, meaning simply becomes determined by all the inference rules of the language. This way of literally following the slogan "meaning is use" – the inference rules that determine the use of sentences also determine their meaning – fell in some disrepute, when Prior (1960) introduced a sentential operator tonk governed by rules similar to the introduction rule for disjunction and the elimination rule for conjunction. Since the effect of adding tonk to a language is to make all sentences derivable, a person who adheres to the idea that an arbitrary set of inference rules determines meaning must be prepared to allow that even inconsistent languages are entirely meaningful.

An interesting defence of such a standpoint is given by Cozzo (1994). He develops a theory in which the meaning of a sentence is given by arbitrary argumentation rules concerning the terms that occur in the sentence. The theory is interesting because, in spite of the fact that it gives a meaning to tonk and thus to inconsistent languages, it makes meaning compositional, it rejects semantic holism but respects epistemological holism, and it allows criticism of a language; a meaningful language may not be a good language (or in Cozzo's terminology: a "correct" language). However, this is not a line of thought that I shall follow here.

The approaches that I shall discuss are inspired by the main idea behind Gentzen's systems of natural deduction, and take a quite different view with respect to the kind of inference rules that are considered as a possible basis for a theory of meaning. I shall mainly restrict myself to an approach that I first proposed in Prawitz (1973) and to a somewhat modified approach suggested by Dummett (1991). These two approaches will be compared and problems concerning the possibility of embedding them into a full theory of meaning will be discussed.

2. GENTZEN'S IDEA OF INFERENCE RULES DETERMINING MEANING

There is a remark by Gentzen (1934) which he made after having constructed his system of natural deduction and which I have quoted before both as a key to the normalization theorem for natural deduction (or the *Hauptsatz*) and as a basis for a proof-theoretic semantics. It reads:

The introductions constitute, as it were, the 'definitions' of the symbols concerned, and the eliminations are, in the final analysis, only consequences of this, which may be expressed something like this: At the elimination of a symbol, the formula with whose outermost symbol we are dealing may be used only 'in respect of what it means according to the introduction of that symbol'.

In contrast to meaning theories inspired by model theory, meaning is now given not by truth conditions but by certain ways in which truth is established, what Gentzen calls *introductions*. The truth of a sentence may however be established also in other ways, which is to say that a sentence may also occur in other connections than introductions. Gentzen is now careful to stress something which was noted to be absent but needed in the approach to meaning based on truth conditions, viz., that the other uses of a sentence are accounted for in terms of (or as Gentzen expresses it: are 'consequences of') the meaning ascribed.

To develop Gentzen's idea we have thus firstly to state more exactly how the introductions determine the meaning of the logical constants; the phrase saying that the introductions represent definitions is clearly not meant to be taken literally. The view that I am taking is that the introductions represent what we may call the canonical ways of inferring a sentence. Other ways of inferring a sentence have to be justified by reducing them to the canonical ways.

Gentzen considers besides introductions certain specific inferences that he calls *eliminations*. We cannot expect these eliminations to be derivable from the introductions in the ordinary sense of being derived inference rules in the system given by the introduction rules. Instead, we have to show that they can be justified in some semantic way, which is to say that they can be shown to be valid in view of the meaning of the sentences involved.

The task is thus to develop an appropriate notion of validity and to show that certain legitimate forms of reasoning are valid in the sense defined. We should then not restrict ourselves to the eliminations given by Gentzen but consider what it is for any non-canonical inference to be valid.

3. DEFINING THE VALIDITY OF ARGUMENTS

3.1. *Argument Skeletons*

Validity has thus to be defined not only for derivations in some given system but for more arbitrary ways of reasoning.[1] I shall therefore define validity for what I call arguments. Furthermore, it seems strange to speak of the validity of proofs. A false sentence is still a sentence, but an invalid proof is not really a proof. In contrast, an argument may be valid or invalid – if it is valid, it represents a proof.

By an *argument skeleton* I shall understand a tree arrangement of formulas; if all the formulas are sentences (i.e., closed formulas), the arrangement is to be understood as claiming for each sentence in the tree except the ones at the top that it follows from the sentences (premisses) standing immediately above. For each top sentence of the tree there is to be indicated whether it is claimed outright as holding (to follow from zero premisses) or if it is entered as an assumption made for the sake of the argument, in which case there may also be an indication at which step in the argument the assumption is discharged or *bound* as I shall say. An example of a step that is allowed to bind assumptions is implication introduction, i.e., an inference of the form

$$\frac{\begin{array}{c}[A]\\ \mathcal{D}\\ B\end{array}}{A \supset B} \tag{1}$$

There may also be indications that a variable that is free in the formulas in which it occurs is *bound by some step in the argument*. An example of a step that binds variables is universal introduction, i.e., an inference of the form

$$\frac{\begin{array}{c}\mathcal{D}\\ A(x)\end{array}}{\forall x A(x)} \tag{2}$$

An inference binds only occurrences of an assumption or a variable that appear in the part of the tree that is above the conclusion of the inference. When a variable is bound by a step it must not occur in the conclusion of the step or in assumptions that are not bound by the step or by some step higher up in the tree (cf. the conditions on so called *Eigenvariablen*). An occurrence that is not bound is said to be *free*.

An argument skeleton is *closed*, if all occurrences of assumptions are bound and likewise all occurrences of variables that are free in the formulas are bound in the argument skeleton; it is *open* otherwise. An open argument skeleton is to be understood as a schema, from which closed argument skeletons can be generated by first substituting closed terms for the free variables and then closed argument skeletons for the free assumptions (in the argument skeleton resulting from the first substitution); the result is said to be an *instance* of the open argument skeleton.

I have been speaking about argument skeletons because what I shall take to be arguments will contain something in addition to the trees of formulas (with indications of how assumptions and variables are bound) which we have been considering so far. The notion of validity will be defined for arguments, i.e., argument skeletons supplemented in a way that remains to be specified. The need for this supplementation does not arise in connection with the introduction inferences, which will now be considered in more detail.

3.2. *Canonical Forms*

An argument skeleton whose last step is an introduction will be said to be in *canonical form*. For each sentence there are given forms of arguments for the sentence which count as canonical. The idea is that these forms determine the meaning of the sentence. The sentence is to be understood as standing for something whose canonical proof, if there is a proof at all, is of the form specified. An argument step that has the form of an introduction is therefore valid by the very meaning of the sentence occurring as conclusion. We shall take care of this idea by saying that an argument whose skeleton is closed and is in canonical form is valid provided its immediate subarguments (i.e., the arguments for the premisses of the last inference step) are valid.

Closed arguments whose skeleton has the form exhibited in (1), (2) or

$$\frac{\begin{array}{cc}\mathcal{D}_1 & \mathcal{D}_2\\ A_1 & A_2\end{array}}{A_1 \& A_2} \qquad \frac{\begin{array}{c}\mathcal{D}\\ A_i\end{array}}{A_1 \vee A_2} \qquad \frac{\begin{array}{c}\mathcal{D}\\ A(t)\end{array}}{\exists x A(x)} \tag{3}$$

are thus valid provided the immediate subarguments resulting from leaving out the last step are valid.

When a skeleton has one of the forms shown in (3), one could impose a more stringent requirement on the canonical forms, namely that the skeletons of the immediate subarguments are themselves in canonical form. However, when the last inference step is an implication introduction or a universal introduction as in (1) or (2), then, as we have seen, it binds occurrences of an assumption or of a variable, respectively. Therefore, the argument skeleton obtained by leaving out the last step may not be closed, and it would be too stringent to impose on the canonical forms that also such an open part of the skeleton is canonical.

3.3. *Open Argument Skeletons*

In line with the understanding of open argument skeletons as schemata, we shall adopt the principle that an open argument is valid provided those instances are valid that are obtained by substituting closed terms for the free variables (supposed to denote objects that belong to the range of the variable) and valid closed arguments for the free assumptions. Let us call such an instance an *appropriate instance*. We have thus the following

Principle of validity for open arguments: An open argument is valid if and only if all its appropriate instances are valid.

3.4. *Justifications of Non-Canonical Arguments*

How is then an inference step that is not an introduction to be justified with reference to the meaning of the sentences involved? Consider a closed argument whose skeleton ends with modus ponens:

$$\frac{\begin{matrix}\mathcal{D}_1 \\ A\end{matrix} \quad \begin{matrix}\mathcal{D}_2 \\ A \supset B\end{matrix}}{B} \tag{4}$$

Suppose that the immediate subarguments are valid. Their skeletons $\mathcal{D}_1$ and $\mathcal{D}_2$ that end with A and $A \supset B$ are closed, and by the meaning of $A \supset B$, it should be possible to bring the valid argument $\mathcal{D}_2$ for $A \supset B$ into canonical form with a skeleton as exhibited in (1) above. It should remain valid, and its immediate subargument with skeleton $\mathcal{D}$ should then also be valid. $\mathcal{D}$ is open, but by substituting $\mathcal{D}_1$ for the open occurrences of the assumption A in $\mathcal{D}$ we obtain

$$\begin{matrix}\mathcal{D}_1 \\ [A] \\ \mathcal{D} \\ B\end{matrix} \tag{5}$$

i.e., a closed argument for B, which should also be valid, being an appropriate instance of a valid closed argument schema.

This is a rough outline of how modus ponens is justified in terms of a notion of validity that is not yet defined. The main idea is that there is an operation that transforms an argument skeleton of the form (4) where the part $\mathcal{D}_2$ is in canonical form into another argument skeleton (5) still ending with B but from which the exhibited application of modus ponens is eliminated. An operation of this kind I shall call a *justification* (strictly speaking one should say an alleged justification) in this case of modus ponens. A justification of modus ponens should show that a closed argument for B whose skeleton has the form exhibited in (4) and whose immediate subarguments are valid could be brought into a valid closed canonical argument for B.

My approach is now to let the arguments for which validity is defined consist of argument skeletons together with proposed justifications of all the inferences that are non-canonical. A bare argument skeleton is not regarded in itself as a valid argument. In other words, it is not enough that there exist effective means for finding another argument skeleton for A in canonical form for counting a given argument skeleton for a sentence A as a valid argument. It is the skeleton together with such effective means, operating on the given skeleton, that constitute an argument for A, as I see it.

An (alleged) justification is any operation that is defined for argument skeletons of some form and transforms them to other argument skeletons for the same formulas without introducing additional free variables or free assumptions. In addition we only need to impose a few formal requirements on the operations such as commuting with substitutions. A set of such operations with mutually disjoint domains of definitions will be said to be a *consistent* justification set. By an *argument* I shall understand an argument skeleton together with a consistent justification set.

An argument consisting of an argument skeleton $\mathcal{D}$ and a justification set $\mathcal{J}$ will be said to reduce to another argument consisting of the skeleton $\mathcal{D}'$ and the justification set $\mathcal{J}$, if $\mathcal{D}$ reduces to $\mathcal{D}'$ in the same way as natural deductions are said to reduce to each other in connection with normalizations, but now using the justification set $\mathcal{J}$ instead of the reductions defined for natural deductions. Notions introduced for argument skeletons may be carried over to arguments in the obvious way. In particular, an argument consisting of an argument skeleton $\mathcal{D}$ and a justification set $\mathcal{J}$, written $\langle \mathcal{D}, \mathcal{J} \rangle$, will be said to be open or closed, if $\mathcal{D}$ is open or closed, respectively. Similarly, an immediate subargument of $\langle \mathcal{D}, \mathcal{J} \rangle$ is an argument $\langle \mathcal{D}', \mathcal{J}' \rangle$ where $\mathcal{D}'$ is an initial part of $\mathcal{D}$ ending with a premiss of the last inference step in $\mathcal{D}$ and $\mathcal{J}'$ is the subset of $\mathcal{J}$ obtained by leaving out justifications of steps not occurring in $\mathcal{D}'$.

3.5. *Principles of Validity*

We may now sum up the ideas outlined above by stating three principles that the notion of validity for arguments should satisfy.

Principle 1. A closed argument in canonical form is valid iff its immediate subarguments are valid.

Principle 2. A closed argument not in canonical form is valid iff it reduces to a valid argument in canonical form, i.e., to an argument that is valid by principle 1.

Principle 3. An open argument $\langle \mathcal{D}, \mathcal{J} \rangle$ is valid iff all those instances $\langle \mathcal{D}', \mathcal{J}' \rangle$ of $\langle \mathcal{D}, \mathcal{J} \rangle$ are valid where $\mathcal{J}'$ is a consistent extension of $\mathcal{J}$ and $\mathcal{D}'$ is an appropriate instance of $\mathcal{D}$, i.e., the argument is to be appropriate in the sense that for any argument $\mathcal{E}$ that is substituted for a free assumption in $\mathcal{D}$ in order to form $\mathcal{D}'$ it should hold that $\langle \mathcal{E}, \mathcal{J}' \rangle$ is valid.

Principles 1 and 2 articulate the idea that the meaning of a sentence is given by what counts as a canonical proof of the sentence: the use of an introduction in an argument preserves validity by the very meaning of the inferred sentence, and non-canonical arguments are valid if and only if they reduce to valid canonical ones. Principle 3 articulates the idea that an open argument is seen as an argument schema.

Together the three principles also constitute an inductive definition of the notion of validity, provided that a set of valid canonical arguments for atomic sentences is given as an induction base. Principle 3 refers the validity of open arguments to the validity of closed arguments as determined by principles 1 and 2. Principle 2 refers to validity as determined by principle 1. Principle 1 finally refers the validity of an argument for a given sentence to the validity of other arguments as determined by all the principles but with respect to formulas of lower complexity than the given one. Clearly we can extend the present approach to other sentence forming operations, if we can formulate introduction rules for the operations in such a way that each inference that proceeds according to the rule satisfies the requirement that the premisses of the inference and the assumptions that are bound by the inference are of lower complexity than the conclusion of the inference.

When the induction base B is made explicit by a set of inference rules for atomic formulas (both conclusion and premisses are to be atomic), I shall speak about validity relative to an *atomic base* B. A logically valid argument must be valid relative to an arbitrary base B.

But we should require more. Otherwise the atomic formulas get a special status, not congruent with the idea that when speaking of logical validity the atomic formulas are thought to stand for arbitrary propositions. Of logical validity one should therefore require that the validity is invariant also for substitutions for atomic formulas.

3.6. *The validity of inference rules*

An inference rule may be defined as valid relative to a justification j if it preserves validity. More precisely it is to hold for each argument skeleton $\mathcal{D}$ whose last inference step is an application of the rule and for each consistent extension $\mathcal{J}$ of $\{j\}$ that the argument $\langle \mathcal{D}, \mathcal{J} \rangle$ is valid if its immediate subarguments are. A rule whose validity is invariant for variations of atomic base and substitutions for the atomic formulas may be said to be logically valid.

All arguments where the skeleton is formed according to the intuitionistic rules of natural deduction for predicate logic and where the justifications assigned to the elimination steps consist of the ordinary reductions defining normalizations for natural deductions are (logically) valid. This is most easily proved by first showing that each intuitionistic elimination rule is (logically) valid with respect to its reduction operation.

To exemplify we may again look at modus ponens to which we assign an operation j that transforms a skeleton of form

$$\dfrac{\begin{array}{c}\mathcal{D}_1\\ A\end{array} \quad \dfrac{\begin{array}{c}[A]\\ \mathcal{D}\\ B\end{array}}{A \supset B}}{B} \tag{6}$$

to a skeleton of the form exhibited in (5) above. We want to show that an argument is valid when its skeleton has the form exhibited in (4) and its set $\mathcal{J}$ of justifications is a consistent extension of $\{j\}$ given the assumption that its immediate subarguments $\langle \mathcal{D}_1, \mathcal{J} \rangle$ and $\langle \mathcal{D}_2, \mathcal{J} \rangle$ are valid. We first note that the validity of $\langle \mathcal{D}_2, \mathcal{J} \rangle$ implies that it reduces to a valid argument in canonical form. Remembering that the justifying operations commute with substitutions, it follows that the given argument reduces to an argument whose skeleton has the form (6), which in turn reduces to (5) by applying j. That the argument $\langle (5), \mathcal{J} \rangle$ is valid, then follows from principle 3 by the fact that it is an instance of a valid open argument $\mathcal{D}$ obtained by substituting the valid argument $\langle \mathcal{D}_1, \mathcal{J} \rangle$ for the free assumption A.

4. DUMMETT'S PROOF THEORETIC JUSTIFICATIONS

In his book *The Logical Basis of Metaphysics*, Dummett (1991) describes and discusses what he calls proof-theoretic justifications of logical laws, which in many respects follow the approach presented in the previous section. There are a couple of noteworthy differences however. The major difference is that Dummett defines validity for what I call argument skeletons. Another minor one is that the canonical forms are defined slightly differently. To facilitate a comparison of Dummett's treatment with mine I shall keep the terminology of the previous section when I state Dummett's definitions. (I shall ignore some other small differences such as one concerning the definition of what I call an instance of an argument skeleton. In Dummett's definitions of corresponding notions, he happens to pay no attention to atomic sentences that occur as free assumptions or to free variables that do not occur in the conclusion or in some free assumption. The difference may be due to the fact that Dummett does not operate explicitly with the notion of open and closed argument (skeleton).)

4.1. *Hereditary Canonical Form*

Dummett makes the more stringent requirement on the canonical forms that was noted above (in Section 3.2) to be possible to make in some cases. He then obtains something that we may call *hereditary canonical forms* defined inductively as follows: an argument skeleton is in hereditary canonical form iff (a) its last step is an introduction, and (b) in case the introduction does not bind any assumption or variable, its immediate subarguments are also in hereditary canonical form. (It is to be assumed that the atomic base specifies introduction rules for atomic formulas.)

It is easy to see (by induction over the definition of validity) that if we replace "canonical form" by "hereditary canonical form" in the definition of validity (i.e., in principles 1 and 2), the extension of the notion of validity stays the same.[2] We may therefore disregard this difference between Dummett's definition and mine.

4.2. *Leaving out the Justifications*

Returning to the major difference between the two definitions, we may try to phrase in my terminology Dummett's definition of what it is for an argument skeleton to be valid by stating three principles similar to the ones in Section 3.5:

*Principle 1**. A closed argument skeleton in canonical form is valid if and only if its immediate parts are valid.

*Principle 2**. A closed argument skeleton for a sentence A that is not in canonical form is valid if and only if a closed valid argument skeleton for A in canonical form can be found effectively.

*Principle 3**. An open argument skeleton $\mathcal{D}$ is valid if and only if all those instances of $\mathcal{D}$ are valid that are obtained by substituting closed valid canonical argument skeletons for free assumptions.

These three principles constitute as before an inductive definition. Leaving out the justifications, as I called them, the notion of validity now defined is much simpler. It may be simpler than the notion intended by Dummett, however. If we apply principles 2* and 3* together to an open non-canonical argument skeleton $\mathcal{D}$ for the formula A, we find that $\mathcal{D}$ is now defined to be valid if and only if for any closed instance $\mathcal{D}^\sigma$ of $\mathcal{D}$ obtained by a substitution σ that substitutes closed valid canonical arguments for free assumptions in $\mathcal{D}$ and terms for free variables in $\mathcal{D}$, we can find effectively a closed argument skeleton $\mathcal{D}'$ for A^σ. Here it is not required that the argument skeleton $\mathcal{D}'$ to be found for A^σ is in any way related to $\mathcal{D}^\sigma$. It is only required that for any σ there is an effective method to find a valid closed canonical argument skeleton $\mathcal{D}'$ for A^σ, not that there is an effective uniform method which applied to any $\mathcal{D}^\sigma$ finds such a $\mathcal{D}'$.

This may not be intended, and perhaps Dummett's notion of validity is instead rendered by principle 1* and a modified combination of Principles 2* and 3* as follows:

*Principles (2–3)**. An argument skeleton $\mathcal{D}$ for a formula A that is not in canonical form is valid if and only if there is an effective method M such that for any closed instance $\mathcal{D}^\sigma$ of $\mathcal{D}$ obtained by a substitution σ that substitutes terms for free variables in $\mathcal{D}$ and closed valid canonical arguments for free assumptions in $\mathcal{D}$, M applied to $\mathcal{D}^\sigma$ yields a valid canonical argument skeleton for A^σ.

How are the two notions of validity related to each other? Could it be that an argument skeleton $\mathcal{D}$ is valid as defined by principles 1* and (2–3)* if and only if there are justifying operations $\mathcal{J}$ to assign to the non-introductory steps of $\mathcal{D}$ so that the argument $\langle \mathcal{D}, \mathcal{J} \rangle$ is valid as defined by principles 1, 2 and 3? If a closed argument $\langle \mathcal{D}, \mathcal{J} \rangle$ for a sentence A reduces to a valid canonical argument $\langle \mathcal{D}', \mathcal{J} \rangle$ (as required for $\langle \mathcal{D}, \mathcal{J} \rangle$ to be valid by principle 2), then $\mathcal{D}'$ can of course be found effectively and $\mathcal{D}'$ is an argument skeleton for A in canonical form (as required for $\mathcal{D}$ to be valid by principle (2–3)*). But conversely, it is not obvious that given an effective method for finding a canonical argument skeleton $\mathcal{D}'$ for a sentence A, the existence of which makes any closed argument skeleton $\mathcal{D}$ for A valid provided that $\mathcal{D}'$ is valid, we can find justifying operations $\mathcal{J}$ to assign to the non-introductory steps of such a $\mathcal{D}$ so that $\langle \mathcal{D}, \mathcal{J} \rangle$ reduces to $\langle \mathcal{D}', \mathcal{J} \rangle$. The two notions are therefore not easily compared to each other.

For Dummett's notion of validity it holds, as he himself remarks, that an argument skeleton $\mathcal{D}$ for a sentence A from open premisses $A_1, A_2, \ldots, A_n$ is valid if and only if the one step argument

$$\frac{A_1, A_2, \ldots, A_n}{A}$$

is valid. Thus, regardless of how irrelevant the steps of $\mathcal{D}$ are for inferring A from $A_1, A_2, \ldots, A_n$, $\mathcal{D}$ is valid if the corresponding one step argument is valid. In other words, it is the existence of effective means for finding a closed valid canonical argument skeleton for A, given closed valid canonical argument skeletons for $A_1, A_2, \ldots, A_n$, that makes $\mathcal{D}$ valid, not what goes on in the skeleton $\mathcal{D}$. It is these means and not the skeleton alone that carries epistemic force, and this was my motivation for including them, i.e., what I have called justifying operations, in the arguments.

However, it remains to discuss whether the notions developed so far form a reasonable basis for a theory of meaning which is supposed to represent the idea that the meaning of a sentence is determined by how it is established as true.

5. RELATIONS TO VERIFICATIONISM

The verificationism of the logical positivists was an early attempt to relate the meaning of a sentence to how we establish its truth, i.e., how we verify it. The slogan "the meaning of a sentence is its method of verification" is not very apt however. It seemed both from this slogan and from what some of the early verificationists said that knowing the meaning of a sentence involved knowing how to decide the truth of the sentence in principle.

That a viable verificationism cannot require that a meaningful sentence is decidable but should relate the understanding of a sentence with the ability to recognize a verification of the sentence when presented by one was pointed out long ago by Michael Dummett.[3]

What determines the meaning of a sentence is thus not its method of verification but rather what it is to verify it, or what counts as a verification of it, as Per Martin-Löf (1985) formulates it. Furthermore, there is now a need to single out a class of direct or canonical verifications – in the first place, they are what is related to meaning.[4]

The basic idea of verificationism as construed here is thus that the meaning of a sentence is given by what counts as a direct verification of it. Gentzen's suggestion that the meanings of the logical constants are determined by their introduction rules can be seen as a special case of this verificationist idea. So as to conform better to this way of expressing the general verificationist idea, the suggestion may be slightly reformulated as saying, firstly, that the meaning of a compound sentence in the language of first order predicate logic is given by what counts as a direct verification of it, and, secondly, that the forms of these direct verifications are given by the introduction rules, i.e., a direct verification has the form of an argument whose last step is an introduction.

Now, an argument cannot count as a direct verification just because its last step is an introduction, something more must be required. What must be added is something about the validity of the argument. The validity of the last step is of course not called in question – that is part of the essence of Gentzen's suggestion. What must be added is thus only that the rest of the argument is valid, i.e., that the immediate subarguments are valid. This is precisely how the validity of an argument in canonical form is defined both by me and by Dummett, except that an argument for Dummett is what I call an argument skeleton.

We arrive in this way at the following formulation of Gentzen's suggestion: A direct verification of a compound sentence A is the same as a valid argument in canonical form, i.e., an argument ending with an introduction whose immediate subarguments are valid, and this is what determines the meaning of A. In other words, it is the inductive definition of what it is to be a valid argument for A, which follows the inductive built up of A, that is proposed to be constitutive for the meaning of A.

I said in the introduction to this paper that the approach to meaning that I was going to review had the advantage that the meaning of a sentence is directly connected with aspects of its use. There is an obvious connection between assertions and verifications or valid arguments. Roughly speaking the assertion of a sentence is warranted iff a verification of the sentence is known. A fundamental requirement on the definition of validity of arguments is that it respects this equivalence: a person should be warranted in asserting a sentence iff she is in the possession of a valid argument for A and knows it to be a valid argument for A.

Does the definition of validity satisfy this fundamental requirement? Consider the case of a simple argument for a closed sentence $A \supset B$, whose skeleton is

$$\frac{\frac{A}{B}}{A \supset B}$$

Dummett counts this skeleton as a valid argument for $A \supset B$ if there is an effective method M for finding a closed valid argument for B given a closed valid argument for A. As already remarked in Section 4, to be in possession of such a skeleton does not amount to very much, certainly not to be entitled in asserting $A \supset B$. It is true that if we know that it is a valid argument in Dummett's sense, then we know that there exists such a method M. But what Dummett calls an argument, i.e., the skeleton shown above, plays virtually no role here.

This supports my more involved notion of argument, according to which a valid argument for $A \supset B$ whose skeleton is as shown above also contains as a second ingredient a method M that applied to any valid argument for A yields a valid argument for B. To be in possession of an argument is now to be in possession of such a method M. But again it can be said that it is not sufficient to be just in possession of M, we must also know that M is a method which applied to any valid argument for A yields a valid argument for B.

These considerations may be taken to speak in favour of counting a demonstration of the fact that M is such a method as an additional ingredient of a real argument for the truth of $A \supset B$, which was the approach of G. Kreisel (1962). A different response to these concerns is given by Per Martin-Löf (not in the paper by him quoted above but in later papers such as Martin-Löf 1995 and 1998). He separates what he calls proofs or proof objects from demonstrations. A proof (object) is an object in the type theory developed by Martin-Löf, while a demonstration is something which shows that an object is of a specific type. For instance, a canonical proof of $A \supset B$ is an object of the form $\lambda x b(x)$ such that $b(a)$ is a proof of B given that a is a proof of A. What in this way counts as a canonical proof of $A \supset B$ determines the meaning of $A \supset B$. But it is the act of demonstrating that something is a proof of $A \supset B$ that warrants the assertion of the truth of $A \supset B$. This approach differs from the verificationist idea that meaning is determined by how we establish truths. A more detailed comparison with the approach that I have outlined would take us outside the scope of this essay. We have therefore to leave it at that, although it must be admitted that the problematic feature of my approach noted above has not been resolved here.

The discussion so far has concerned the question whether knowledge of a valid argument for a sentence A is sufficient for the warranted assertion of A. But what about the necessity of such knowledge for being entitled to asserting A? Knowing a valid argument for A implies knowing how to find a valid argument for A in canonical form. But is it right that when we are entitled to assert a complex sentence A, we could in principle have arrived at that position by constructing a canonical argument for A? That the answer is yes is what Dummett (1991) calls the *fundamental assumption* of this approach to meaning. The answer is required to be yes, if the definition of validity is to respect the equivalence stated above between an assertion being warranted and a corresponding valid argument being known.

Dummett (1991) devotes a chapter to a discussion of this fundamental assumption, pointing out reasonable doubts that one can have about it. The doubts have the form of examples of sentences A with predicates that relate to ordinary empirical discourse and where it seems reasonable to say that the assertion of A may be warranted although the speaker knows no valid argument for A (or argument skeleton for A, the examples function equally well regardless which definition we choose).

Some of the examples are related to the fact that when we are concerned with tensed empirical sentences, the possibility of a having direct verification may be lost or may not yet be at hand. It is obvious that the notion of valid argument for empirical sentences has to be more lax than for mathematical ones. We cannot require that the argument is to give us a method for finding a valid canonical argument, but have to be satisfied if it demonstrates for sentences in the past time that a valid canonical argument could have been had at the time in question, and for sentences in future tense that a valid canonical argument will be possible to have at the future time in question.

There are counterexamples that cannot be dealt with in this way however. In my opinion, the most serious ones concern universal sentences in empirical discourse.[5] As may be expected, the discussions of these examples do not result in a suggestion that the canonical forms of arguments for the various kinds of sentences can be specified in some different way, which would be to replace Gentzen's introduction rules by some other introduction rules. The examples must rather be understood as casting doubts on the whole idea that it is possible to specify canonical forms of arguments (or verifications) such that the truth of a sentence can be identified with the existence of a valid canonical one. In other words, it is the whole verificationist project that is in danger when the fundamental assumption cannot be upheld. An essential prerequisite for this project is the distinction between direct and indirect verification as I have argued elsewhere (e.g., Prawitz 1995).

The discussion in this section indicates that the development of Gentzen's idea into a full theory of meaning along the lines considered here is not unproblematic. However, it should be recalled that here I have essentially confined myself to a review of two closely related lines of thought, and have only in passing considered alternative ways of developing Gentzen's idea or the general idea of approaching meaning via proofs.

NOTES

[1] When in Prawitz (1971) I started to use the term validity in this connection it was defined for derivations in given formal systems. To define it for arguments in general was one of the main ideas of Prawitz (1973).

[2] It is assumed in Dummett (1991) that the stronger notion of hereditary canonical form is needed when one is not confined to justify only given elimination inferences but is considering arbitrary inferences. As follows from the claim made above (easily proved by showing that a closed valid argument in canonical form reduces to one in hereditary canonical form), there is actually no such need.

[3] Most explicitely in for instance Dummett (1976).

[4] As pointed out in connection with proofs already by, e.g., Dummett (1973) and Prawitz (1974).

[5] I have briefly discussed them in for instance Prawitz (1987).

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REFERENCES

Church, A.: 1956, *Introduction to Mathematical Logic*, Princeton University Press, Princeton.

Cozzo, C.: 1994, *Meaning and Argument*, Almqvist & Wiksell, Stockholm.

Dummett, M.: 1973, *The Justification of Deduction*, The British Academy, London (republished in M. Dummett, *Truth and Other Enigmas*, Duckworth, London 1978).

Dummett, M.: 1976, 'What is a Theory of Meaning II', in G. Evans et al. (eds.), *Truth and Meaning*, Oxford, pp. 67–137 (republished in M. Dummett, *The Seas of Language*, Clarendon Press, Oxford 1993).

Dummett, M.: 1983, 'Language and Truth', in R. Harris (ed.), *Approaches to Language*, Oxford (republished in M. Dummett, *The Seas of Language*, Clarendon Press, Oxford 1993).

Dummett, M.: 1991, *The Logical Basis of Metaphysics*, Duckworth, London.

Gentzen, G.: 1934, 'Untersuchungen Über das Logische Schließen', *Mathematische Zeitschrift* **39**, 176–210 and 405–431.

Kreisel, G.: 1962, 'Foundations of Intuitionistic Logic', in E. Nagel et al. (eds.), *Logic, Methodology and Philosophy of Science*, Stanford University Press, Stanford, pp. 198–210.

Martin-Löf, P.: 1985, 'On the Meanings of the Logical Constants and the Justification of the Logical Laws', in *Atti degli Incontri di Logica Matematica*, Siena, **Vol. 2**, pp. 203–281 (reprinted in *Nordic Journal of Philosophical Logic* **1**, 11–60).

Martin-Löf, P.: 1995, 'Verificationism Then and Now', in E. Köhler et al. (eds.), *The Foundational Debate: Complexity and Constructivity in Mathematics and Physics*, Kluwer, Dordrecht, pp. 187–196.

Martin-Löf, P.: 1998, 'Truth and Knowability: on the Principles *C* and *K* of Michael Dummett', in H. G. Dales and G. Olivieri (eds.), *Truth in Mathematics*, Clarendon Press, Oxford, pp. 105–114.

Prawitz, D.: 1971, 'Ideas and Results in Proof Theory', in J. E. Fenstad (ed.), *Proceedings of the Second Scandinavian Logic Symposium*, North-Holland, Amsterdam, pp. 235–307.

Prawitz, D.: 1973, 'Towards a Foundation of a General Proof Theory', in P. Suppes et al. (eds.), *Logic, Methodology, and Philosophy of Science IV*, North-Holland, Amsterdam, pp. 225–250.

Prawitz, D.: 1974, 'On the Idea of a General Proof Theory', *Synthese* **27**, 63–77.

Prawitz, D.: 1987, 'Dummett on a Theory of Meaning', in B. Taylor (ed.), *Michael Dummett, Contributions to Philosophy*, Kluwer, Dordrecht, pp. 117–165.

Prawitz, D.: 1995, 'Quine and Verificationism', *Inquiry* **37**, 487–494.

Prior, A. N.: 1960, 'The Runabout Inference-Ticket', *Analysis* **24**.

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