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EDITORIAL

# Editorial introduction to the special issue on 'Strategic Queueing: Game-Theoretic Models in Queueing Theory'—part 2

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We are pleased to present the second part of the special issue of *Queueing Systems* on Strategic Queueing. The first part consisted of five papers and appeared in Volume 96. In its editorial introduction (see [1]), there is a brief description of Strategic Queueing as a subfield of Queueing Theory with some key references. This second part comprises five papers that we summarize below.

*Mark Fackrell, Peter Taylor and Jiesen Wang* consider an M/M/1 feedback queue, where each customer, after being served, departs with probability $q$ or joins the end of the queue to wait to be served again. The system is observable, and the strategic customers face the dilemma of whether to join or balk. The authors use a matrix-analytic approach to determine the equilibrium strategies of the customers in two cases, according to whether reneging is permitted or not. Several counterintuitive findings show that the feedback feature induces unexpected peculiarities in strategic customer behavior.

*Moshe Haviv and Binyamin Oz* introduce a new framework for the study of strategic customer behavior in queueing models that regenerate every time that a new arrival finds an empty system. Whenever this happens, not only renewed arrival and service processes begin, but also a renewed set of customers–players interact with each other. This approach enables the isolated analysis of each such "cycle" of the process as a genuine non-cooperative game with a random, yet finite and well-defined, set of players. Then, the study of the corresponding queueing games can be carried out within the realm of classical Game Theory with the appropriate use of its relevant tools.

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*Corine Laan, Judith Timmer and Richard J. Boucherie* study a game on a network of single-server queues with fixed routes. There are a finite number of players, and each one of them controls a Poisson arrival process. There is a set of available routes, and each player must decide which routes to use for his arrival stream with the objective of minimizing the expected sojourn time of his customers. Two cases are considered, according to whether a player can divide his customers over multiple routes or can select only a single route. For the multiple-route case, it is shown that a unique pure equilibrium strategy profile exists. For the single-route case, the existence of pure equilibria is proved for several special cases of this framework.

*Liron Ravner and Yutaka Sakuma* consider a Poisson population of customers who strategically choose their arrival times to a single-server queue with opening and closing times, with the objective of minimizing their expected waiting times. The distinctive feature of the model is that there are two types of customers that differ in their beliefs regarding the service time distribution and who are aware of the different beliefs. The authors characterize the Nash equilibria for exponentially distributed service times and provide an explicit solution for a fluid version of the game. Moreover, they propose an algorithm for computing the equilibrium in a discrete-time setting and an associated agent-based model.

*Zhongbin Wang, Luyi Yang, Shiliang Cui and Jinting Wang* study a model where customers may pay and upgrade to priority at any time during their stay in the queue. The novelty of the contribution lies in the departure from the standard restriction in the extant literature that allows customers to purchase priority only upon arrival. The authors consider two cases, according to whether the customers make their decisions simultaneously or sequentially. In the simultaneous case, the authors show that pure-strategy equilibria do not exist under some intuitive criteria. However, in the sequential case, threshold-type equilibria may exist. More explicit results are proved under additional assumptions, for example in the case of light traffic.

## Reference

1. Economou, A., Kulkarni, V.: Editorial introduction to the special issue on 'Strategic queueing: game-theoretic models in queueing theory'- part 1. Queueing Syst. **96**, 201–203 (2020). https://doi.org/10.1007/s11134-020-09680-w