



Paradoxes of truth-in-context-X

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Abstract We may suppose that the truth predicate that we utilize in our semantic metalanguage is a two-place predicate relating sentences to contexts, the *truth-in-context-X* predicate. Seeming paradoxes pertaining to the truth-in-context-X predicate can be blocked by placing restrictions on the structure of contexts. While contexts must specify a domain of contexts, and what a context constant denotes relative to a context must be a context in the context domain of that context, no context may belong to its own context domain. A generalization of that restriction appears to block all of the paradoxes of truth-in-context-X. This restriction entails that, in a certain sense, we cannot talk about the context we are in. This result will be defended, up to a point, on broadly ontological grounds. It will also be conjectured that our semantic metalanguage can be regarded as semantically closed.

Keywords Semantic paradox · Context-relativity · Two-place truth predicate · Semantic closure

1 The relativity of truth to context

When we are writing a semantic theory for natural languages, we need to acknowledge that sentences are neither true nor false simpliciter but only true or false relative to a context. The sentence “Barack Obama is tall” is neither true nor false simpliciter. It is true in a context if and only if Barack Obama is taller than the contextually determined standard of size. The sentence “Everyone is present” is neither true nor false simpliciter. It is true in a context if and only if every member of the contextually determined domain of discourse is at the contextually

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determined location in time and space. These examples suggest that the truth predicate we need for our semantic metalanguage is the relational predicate “is true in”, which may be written between an expression denoting a sentence and an expression denoting a context. I will call this the *truth-in-context-X predicate*.

If the truth predicate that we need for our semantic metalanguage is only the two-place truth-in-context-X predicate, then the semantic paradoxes present a problem for our semantic metalanguage only if there are paradoxes involving the truth-in-context-X predicate. Let us try to construct one. Let us assume that λ names the sentence $[\lambda \text{ is not true in } \Gamma]$, where $[\Gamma]$ denotes a context. (I will use square brackets as quotation marks, sometimes as selective quotation marks.) Then we seem to be able to reason as follows:

1	$\lambda = [\lambda \text{ is not true in } \Gamma]$	premise
2	$\lambda \text{ is true in } \Gamma$	hypothesis
3	$[\lambda \text{ is not true in } \Gamma] \text{ is true in } \Gamma$	1,2
4	$\lambda \text{ is not true in the context that } [\Gamma] \text{ denotes in } \Gamma$	3
5	$\lambda \text{ is not true in } \Gamma$	4
6	\perp	2, 5
7	$\lambda \text{ is not true in } \Gamma$	2-6
8	$\lambda \text{ is not true in } \Gamma$	hypothesis
9	$[\lambda \text{ is not true in } \Gamma] \text{ is not true in } \Gamma$	1, 8
10	$\lambda \text{ is true in the context that } [\Gamma] \text{ denotes in } \Gamma$	9
11	$\lambda \text{ is true in } \Gamma$	10
12	\perp	8, 11
13	$\lambda \text{ is true in } \Gamma$	8-12
14	$\lambda \text{ is true and not true in } \Gamma$	6, 13

The steps from 3 to 4 and from 9 to 10 just apply the presumable account of the truth conditions of the sentences quoted in 3 and 9. (I assume that the quotation name $[\lambda]$ denotes the sentence λ .) But notice that at two steps in this reasoning, at the step from 4 to 5 and the step from 10 to 11, we assume that $[\Gamma]$ denotes Γ relative to Γ . That step seems questionable. The reasoning could be blocked if we could deny that.

The denotation of a context by a context constant is relative, I will suppose, to a context. When a context constant δ denotes a context Δ relative to a context Γ , I will write: $Den_{\Gamma}(\delta) = \Delta$. What we must affirm in order to block the above reasoning is that for each context Γ and each context constant δ , the context that δ denotes in Γ is not Γ , i.e., $Den_{\Gamma}(\delta) \neq \Gamma$. In English words, we can express the restriction this

way: *Reflexive reference to contexts is forbidden*. Pictorially, the prohibition we require is shown in Fig. 1.

The context domain for a context is the set of contexts that a context constant may denote relative to the context. What Fig. 1 shows is that, while the context that a context constant denotes relative to a context Γ belongs to the context domain for context Γ , the context denoted must not be the same context Γ relative to which the context constant denotes it.

My plan in this paper is to inflate this observation into a solution to a broad swath of semantic paradoxes. (I will not try to say what might lie beyond the swath that can be handled in this way.) This will involve placing a considerably more general constraint on the structure of contexts than the restriction against reflexive reference to contexts.

First, I will propose that the semantic predicate that should interest us in seeking a solution to semantic paradoxes is really the two-place truth-in-context-X predicate, not a one-place truth predicate. Then by examining several other types of semantic paradox, I will identify the more general restriction on the structure of contexts that we need. Following that, I will show on general, ontological grounds, independent from consideration of the semantic paradoxes, that we should expect at least part of the necessary restriction to be respected. However, I will not be able to provide an independent rationale for the full restriction. In the course of this, I will address a puzzling consequence of the restriction, namely, that, in a sense, we cannot talk about the context we are in.

Throughout most of the paper I will avoid giving any detailed specification of a sample language, or any detailed account of the contents of contexts, or any detailed account of truth-in-a-context conditions, because I do not want to bury the main idea under distracting details. I will say about these things only what I need to say in order to explain the basic strategy for blocking paradoxes of truth-in-context-X. But then in the last main section, I will provide a precise account of truth (and falsehood) conditions for a simple language that contains a truth-in-context-X predicate in order to convey my hope that along these lines we can aspire to a language that is, in a sense, semantically closed.

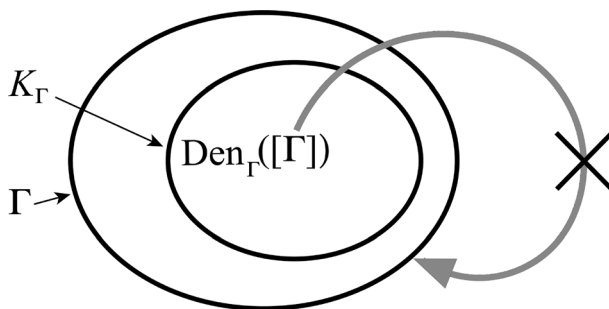


Fig. 1 Γ is a context. K_Γ is the context domain in Γ . The diagram shows that the denotation of $[\Gamma]$ in Γ , that is, $Den_\Gamma([\Gamma])$, which is in K_Γ , cannot be Γ itself

Occasionally other authors besides me have considered paradoxes involving a two-place truth predicate like the truth-in-context- X predicate. Parson (1974), Burge (1979), Koons (1992), Simmons (1993, 2018) and Williamson (1998) all make the relativity to context more or less explicit in their treatments of the semantic paradoxes. I will not here attempt to evaluate the things they have had to say about those paradoxes (but see my comments in Gauker 2006). Suffice it to say that none of them proposes a diagnosis comparable to the one I will propose here, which consists in attributing some structure to contexts and then placing restrictions on the structure of contexts. In particular, none of them says that a context includes a context domain that includes all of the contexts that context constants might denote relative to a context.

Since I propose to avoid semantic paradoxes by means of restricting the structure of contexts, my approach could be called a *contextualist* approach to paradox. Unfortunately, the name is already taken and refers to those approaches that propose to undercut the reasoning that leads to a contradiction by positing a shift in context that renders the apparent contradiction not really a contradiction. Examples of this approach include Barwise and Etchemendy (1987), Simmons (1993, 2018), Glanzberg (2001, 2004) and Murzi and Rossi (2018). Although I will also have occasion to speak of a shift in context (in Sect. 4), the context shift that I will posit plays no rule in undercutting the reasoning to a contradiction. On my account, there is no need to posit a shift in context to avoid deriving a contradiction. The attempt to derive a contradiction simply transgresses the restriction on the structure of contexts.

2 A one-place truth predicate?

So first, I want to locate the problem that interests us in the two-place truth-in-context- X predicate. I propose that the semantic predicate for our semantic metalanguage that should interest us is the two-place truth-in-context- X predicate, not a one-place truth predicate. An initial reason for this is provided by examples, such as those I began with in the previous section. However, this is perhaps not an entirely persuasive reason, because one might suppose, following Kaplan (1989), that the two-place predicate expressing the truth of a sentence in a context can be reduced to the one-place predicate for the truth of a proposition via the following scheme:

A sentence s is true in a context Γ if and only if the proposition that s expresses in Γ is true.

Here we imagine that a domain of propositions having properties such as truth is given and that the task of semantics is to explain how a sentence in a context reaches out and grabs one of these. In that case, the context-relativity of the truth of sentences can be accommodated while treating truth as fundamentally what is denoted by a one-place predicate predicable of expressions for propositions.

So there is a question whether we should think of truth as fundamentally a relation between a sentence and a context or should think of truth as fundamentally a property of propositions and then explain the truth of sentences in a context in accordance with the above scheme. There would be other options as well, such as treating truth as fundamentally a property of utterances of sentences. (Presently I will explicate utterance-truth in terms of the truth of sentences in a context.) This is an issue that can be debated quite apart from consideration of the paradoxes. However, for present purposes I will offer just the following as a reason not to reduce truth-in-a-context for sentences to truth for propositions. We *can* formulate our semantic theories in terms of the two-place truth-in-context-X predicate and doing so opens up the possibility of avoiding semantic paradoxes by placing restrictions on the structure of contexts. Our semantics may take the form of an account of the truth of sentences in a context if we can provide a recursive definition of truth-in-a-context for the languages that interest us. We will avoid semantic paradox by restricting the structure of contexts in the ways to be identified in what follows. If, on the contrary, we treated truth as fundamentally what is denoted by a one-place predicate predicable of expressions for propositions, we would be faced with potential paradoxes involving the truth of propositions. We could ask, for instance, whether there is a proposition that says of itself that it is not true, and, if so, whether it is true or not.

Nothing would prevent us from using the word “proposition” in writing the truth-in-context-X predicate. Instead of saying, “Sentence *s* is true in context Γ ”, we could say “Sentence *s* expresses a true proposition in context Γ ”. But so long as this is just a long-form way of expressing the same thing, and we do not suppose that propositions have truth values quite apart from their being expressed by sentences in contexts, then we have offered no reduction of the two-place relation of the truth of a sentence in a context to the one-place property of truth as a property of propositions. By the same token, nothing is gained by this way of writing the truth-in-context-X predicate, and so I will not use it.

Still, in terms of the two-place truth-in-context-X predicate we can define two one-place truth predicates, one for utterances and one for propositions. As a consequence of specifying the conditions under which sentences are true in contexts, we will also be able to describe *utterances* of sentences as true or false (or neither) *simpliciter*. By an *utterance* here I mean a concrete event in a place at a time. Utterance truth in this sense is absolute, not relative to a context, and whether an utterance is true is entirely determined by whether the sentence uttered is true in the context that *pertains* to that utterance. (I will have more to say in Sect. 4 about the way in which utterance truth depends on the truth of the sentence uttered in a context that pertains to the utterance.) However, no new paradoxes should arise from the use of this one-place truth predicate if there are none involving the truth-in-context-X predicate.

We might also have reason to countenance a one-place truth predicate denoting a property of propositions that we can define in terms of truth-in-context-X thus: A proposition *p* is *true* if and only if there is a sentence *s* and a context Γ such that *s* expresses *p* in Γ and *s* is true in context Γ . (Steps will have to be taken if we want to ensure that no proposition can be both truth and false. Otherwise, we might find

that a proposition p is true when expressed by sentence s in context Γ and false when expressed by sentence r in context Δ .) But if we want to think of propositions as corresponding to sentences in contexts in this way, then we should suppose that their semantic properties, such as truth, are likewise entirely determined by the semantic properties of sentences in contexts. In that case, if there are no paradoxes of truth-in-context- X , there will be none for this kind of truth of propositions.

We may also use a one-place truth predicate that applies to sentences in speaking elliptically about the truth of sentences in a context. “That sentence is true” might be elliptical for “That sentence is true in context Γ ”. I assume that this elliptical one-place truth predicate also creates no additional paradoxes.

Perhaps there is reason to countenance a non-elliptical one-place truth predicate that is not tied to the truth-in-context- X predicate in either of the ways I have indicated, neither as a property of utterances definable in terms of truth-in-context- X nor as a property of propositions definable in terms of truth-in-context- X . Call this an *independent* one-place truth predicate. For example, one might suppose that this is the truth predicate we use in endorsing what someone has said, as when we say, “That’s true!”. Or one might suppose that this is the truth predicate we use in quantification over propositions, as when we say, “Everything written in this book is true”.

If there is an independent one-place truth predicate, then we will be able to write apparently paradoxical sentences in terms of it. And in that case, the tasks for the semantic theory that we formulate in terms of the two-place truth-in-a-context- X predicate will include that of providing a semantics for sentences formed with the independent one-place truth predicate and then showing that, according to this semantics, the apparently paradoxical sentences are not so paradoxical after all. That is, from empirical facts about the identity of these sentences or the propositions they express (such as that $\lambda = “\lambda$ is not true”), we cannot derive contradictions.

How best to formulate the semantics for such an independent one-place truth predicate, if there is one, is not my concern in this paper. Since we do not expect to formulate our semantics in terms of the independent one-place truth predicate, some of the desiderata that people have wished to impose can perhaps be suspended. For instance, it is not obvious that our semantics has to secure the truth in all contexts of all instances of the Tarski T-schema, “ s is true if and only if p ”, where “ s ” is quotation name of a sentence of the object language and “ p ” is its translation into the metalanguage (as Field 2008 thinks it must). So we might find that a treatment of the independent one-place truth predicate that blocks semantic paradoxes for that predicate does not require anything very fancy. Perhaps the prosentential theory of truth offers the right account of such a truth predicate (Grover 1992). Or it might be enough to say that all non-grounded sentences (in the sense of Kripke 1975) are neither true nor false in any context and to evaluate sentences in accordance with the strong Kleene scheme. (See Gauker 2005 for one approach along these lines.) We can then deny the validity (or the validity preservingness) of the instances of the inference rules, such as *reductio ad absurdum*, that are used to derive contradictions from empirical facts about paradoxical sentences. Beyond that, we might expect the logic of a language containing the independent one-place truth predicate but not the truth-in-context- X predicate to be similar to the logic of a language containing both.

In any case, since we formulate our semantics in terms of truth in a context and not in terms of truth *simpliciter*, there is no risk that our treatment of the paradoxes involving an independent one-place truth predicate, if there is one, will result in revenge paradoxes for the one-place truth predicate. For example, we will not find ourselves saying, “So the liar sentence is not true; but that’s what it says; so it must be true after all”, ...and so on. We will not fall into that trap because our conclusion will definitely not be that the liar sentence *is not true*, but perhaps that it *is not true in any context*, which is not what the liar sentence involving the independent one-place truth predicate says.

3 The necessary restriction on contexts

From the example of λ in the first section, I drew the conclusion that $Den_{\Gamma}(\delta) \neq \Gamma$. One way to look at this would be as a restriction on the denotation of contexts. But this is not the route I want to take. I do not think I could provide a good motivation for a restriction on denotation. If a context could be a member of its own context domain, then nothing would prevent a context constant from denoting that member of the context domain relative to that same context. Rather, we need to place the restriction on the structure of contexts themselves. I will assume for simplicity that contexts are denoted only by context constants (and not, for instance, by definite descriptions), but since the sought-for restrictions are restrictions on the structure of contexts and not restrictions on denotation, this assumption will not affect the nature of the restrictions to be proposed.

Before I can identify the necessary restriction, I need to make the following observation. We should expect an argument of the form,

Existential Generalization over Context Constants

s is true in Δ .

Therefore, s is true in some context.

to be logically valid. To secure that result, we can make two assumptions:

First, each context Γ contains a set of contexts, K_{Γ} , called the *context domain*, relative to which quantifications over contexts are evaluated relative to Γ . (It may be the empty set.)

Second, for each context Γ and each context constant δ , such as $[\Delta]$ in the above argument, the denotation of δ relative to Γ is either undefined or a member of the context domain K_{Γ} of Γ .

Then we can say that a sentence [s is true in Δ] is true in Γ if and only if the denotation of $[\Delta]$ in Γ is defined and the denotation of $[s]$ is true in the context that $[\Delta]$ denotes in Γ . And we can say that a sentence [s is true in some context] is true in Γ if and only if the denotation of $[s]$ is true in some context in K_{Γ} . Under these assumptions, Existential Generalization over Context Constants will be logically valid in the sense that for each context, if the premise is true in it, then so is the conclusion.

In view of the second of these assumptions, we can strengthen our restriction on the structure of contexts as follows: *No context can belong to its own context domain.* As a consequence of the assumption that for each context Γ and each context constant δ , the denotation of δ relative to Γ is either undefined or a member of the context domain K_Γ of Γ , this restriction implies the restriction that no context constant can denote a context relative to that same context ($Den_\Gamma(\delta) \neq \Gamma$). This is the restriction that I have expressed by saying that reflexive reference to contexts is forbidden.

The restriction against contexts belonging to their own context domains suffices to block another apparent paradox of truth-in-context-X. Let μ denote the sentence [μ is not true in any context]. Then it seems we might reason as follows:

1	$\mu = [\mu \text{ is not true in any context}]$	premise
2	Γ is arbitrary	hyp
3	μ is true in Γ	hyp
4	[μ is not true in any context] is true in Γ	1, 3
5	For all δ in K_Γ , μ is not true in δ	4
6	μ is not true in Γ	5
7	\perp	3, 6
8	μ is not true in Γ	3-7
9	μ is not true in any context	2-8
10	μ is not true in any context	hyp
11	[μ is not true in any context] is not true in any context	1, 10
12	For each γ , there exists δ in K_γ , such that μ is true in δ	11
13	μ is true in some context	12
14	\perp	10, 13
15	μ is true in some context	10-14
16	\perp	9, 15

The steps from 4 to 5 and from 11 to 12 are just supposed to be the application of the presumable truth conditions to the sentences in 4 and 11, respectively, where K_Γ is the context domain for Γ . To block this reasoning, we do not need to introduce any further restrictions. Our restriction against contexts' being members of their own context domains already suffices. In this reasoning we violate this restriction when we pass from line 5 to line 6. Here we are assuming that Γ belongs to K_Γ , which cannot be. We violate it again when we pass from line 12 to line 13. Here we assume that the context domain relative to which the universal quantification in line

12 is evaluated ranges over the context of our own reasoning, which, by our restriction, it cannot do.

However, other apparent paradoxes show that we need to strengthen the restriction on the structure of contexts even further.¹ Consider a context-relative version of the notecard paradox. Imagine a notecard, on one side of which is written the sentence $[\alpha \text{ is true in } \Gamma]$. Let us say that the context pertinent to this sentence-token is Δ . On the other side of the notecard is written the sentence $[\beta \text{ is not true in } \Delta]$. Let us say that the context pertinent to this sentence-token is Γ . Further, let us suppose that relative to Δ , the context constant $[\Gamma]$ denotes Γ and that relative to Γ , the context constant $[\Delta]$ denotes Δ . Finally, relative to Δ , $[\alpha]$ denotes $[\beta \text{ is not true in } \Delta]$, and relative to Γ , $[\beta]$ denotes $[\alpha \text{ is true in } \Gamma]$. Then we seem to be able to reason as follows:

1	$Den_{\Delta}([\alpha]) = [\beta \text{ is not true in } \Delta]$	premise
2	$Den_{\Gamma}([\beta]) = [\alpha \text{ is true in } \Gamma]$	premise
3	$Den_{\Delta}([\Gamma]) = \Gamma$	premise
4	$Den_{\Gamma}([\Delta]) = \Delta$	premise
5	$[\beta \text{ is not true in } \Delta] \text{ is true in } \Gamma$	hyp
6	$Den_{\Gamma}([\beta]) \text{ is not true in } Den_{\Gamma}([\Delta])$	5
7	$[\alpha \text{ is true in } \Gamma] \text{ is not true in } \Delta$	2, 4, 6
8	$Den_{\Delta}([\alpha]) \text{ is not true in } Den_{\Delta}([\Gamma])$	7
9	$[\beta \text{ is not true in } \Delta] \text{ is not true in } \Gamma$	1, 3, 8
10	\perp	5, 9
11	$[\beta \text{ is not true in } \Delta] \text{ is not true in } \Gamma$	5-10
12	$[\beta \text{ is not true in } \Delta] \text{ is not true in } \Gamma$	hyp
13	\perp	Similarly, using 1, 2, 3, 4, 12
14	$[\beta \text{ is not true in } \Delta] \text{ is true in } \Gamma$	12-13
15	\perp	11, 14

The trouble here lies in our assuming both $Den_{\Delta}([\Gamma]) = \Gamma$ and $Den_{\Gamma}([\Delta]) = \Delta$ (premises 3 and 4). This is not a case of a context's being in its own context domain. It is, rather, a matter of a context Γ 's being in the context domain of a context Δ , which in turn is in the context domain of Γ . This is something we need to prevent. In pictures, what we *cannot* have is depicted in Fig. 2.

¹ This was pointed out to me by Anil Gupta many years ago.

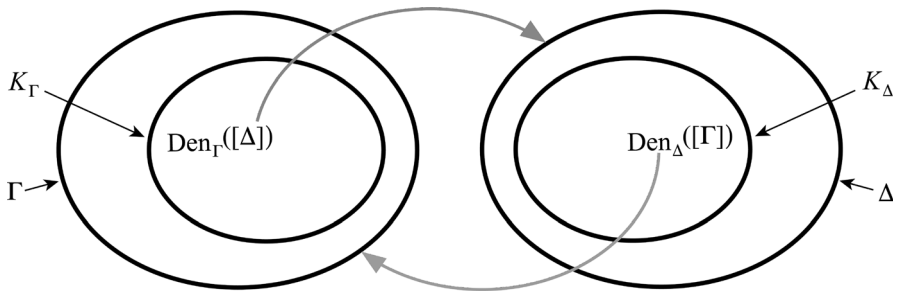


Fig. 2 This must not happen. A context constant $[\Delta]$ denotes in Γ a context Δ , and a context constant $[\Gamma]$ denotes in Δ context Γ

Likewise, we will want to prevent larger circles of this kind, such as a case in which $Den_{\Gamma}([\Delta]) = \Omega$, $Den_{\Omega}([\Gamma]) = \Delta$ and $Den_{\Delta}([\Omega]) = \Gamma$. To rule out such circles, let us define *pointing* thus:

Context γ *points* to context δ if and only if either (a) δ is in the context domain for γ or (b) for some context ω , γ points to ω and ω points to δ .

The necessary restriction can now be formulated thus: *No context can point to itself*. This of course implies our previous restriction, that no context can belong to its own context domain.

The restriction on the structure of contexts must be tightened even further in light of a context-relative Yablo-paradox.² (For the original Yablo paradox, see Yablo 1993.) Suppose there is an infinite sequence of contexts, $\Gamma_0, \Gamma_1, \Gamma_2, \dots$ having context domains as follows:

$$K_{\Gamma_0} = \{\Gamma_1, \Gamma_2, \Gamma_3, \dots\}.$$

$$K_{\Gamma_1} = \{\Gamma_2, \Gamma_3, \Gamma_4, \dots\}.$$

$$K_{\Gamma_2} = \{\Gamma_3, \Gamma_4, \Gamma_5, \dots\}.$$

\vdots

Suppose also that the denotations of context constants and sentence names obey the following laws, respectively:

For every $i \geq 0$, for every $j > i$, $Den_{\Gamma_i}([\Gamma_j]) = \Gamma_j$.

For every $i \geq 0$, for every $j \leq i$, $Den_{\Gamma_i}([\Gamma_j])$ is undefined.

For every $i, j \geq 0$, $Den_{\Gamma_i}([S_j]) = S_j$.

(None of these stipulations violates any restriction.) Finally, suppose that there is an infinite sequence of sentences as follows (with the name of the sentence to the left of the colon, the sentence it names to the right):

S_0 : For all $n > 0$, S_n is not true in Γ_n .

S_1 : For all $n > 1$, S_n is not true in Γ_n .

² I thank Gabriel Uzquiano and James Studd for pressing me on this point.

S_2 : For all $n > 2$, S_n is not true in Γ_n .

\vdots

It can be shown that both the assumption that S_0 is true in context Γ_0 and the assumption that S_0 is not true in context Γ_0 lead to a contradiction. (The proofs involve only identity-substitutions and applications of presumable truth conditions analogous to those employed in the previous examples.) To prevent this it will suffice to stipulate that no such series of contexts is possible.

Toward a precise formulation, let us say that $\Gamma \succ \Gamma^*$ if and only if $\Gamma^* \in K_\Gamma$. A sufficient restriction can be formulated as follows: *Every maximal chain of contexts ordered by \succ must have a least member that has an empty context domain*. Let us formulate the restriction this way: *Every maximal chain of contexts must bottom out*. The series of contexts $\Gamma_0, \Gamma_1, \Gamma_2, \dots$ defined above for the context-relative Yablo paradox does not bottom out.

If we alter the series so that it bottoms out, then paradox is thereby blocked. Suppose that Γ_0, Γ_1 and Γ_2 have context domains as defined above, but $\Gamma_3, \Gamma_4, \Gamma_5 \dots$ all have empty context domains. So for every $2 \geq i \geq 0$, for every $j > i$, $Den_{\Gamma_i}([\Gamma_j]) = \Gamma_j$, but if $j \leq i$ or $i > 2$, then $Den_{\Gamma_i}([\Gamma_j])$ is undefined. This set of contexts satisfies the restriction. For instance, while $\Gamma_0 \succ \Gamma_1 \succ \Gamma_2 \succ \Gamma_3$, there is no context Γ_i such that $\Gamma_3 \succ \Gamma_i$, because Γ_3 has an empty context domain. Then we will find that for every sentence S_i in the sequence of sentences defined above and every context Γ_j in the sequence defined in this paragraph, we can prove without contradiction either that S_i is true in Γ_j or that S_i is false in Γ_j or that S_i is neither true nor false in Γ_j . For example, S_3 is neither true nor false in Γ_3 , S_2 is true in Γ_2 , S_1 is false in Γ_1 and S_0 is false in Γ_0 . (These evaluations rest on the account of truth conditions that will be presented in Sect. 5, but augmented with an account of quantification over sentences.)

The requirement that maximal chains bottom out implies our earlier restriction, according to which no context can point to itself. I do not know whether this is the weakest requirement that would be sufficient to rule out all Yablo-type paradoxes, in which we define a sequence of contexts and a sequence of sentences such that for some context in the sequence of contexts and some sentence in the sequence of sentences, we find that both the assumption that the sentence is true in the context and the assumption that it is not true in the context lead to contradictions.

However, we will want to rule out all bottomless chains in any case, for the following reason. Suppose that $\Gamma_0, \Gamma_1, \Gamma_2, \dots$ is an arbitrary infinite sequence of contexts such that $\Gamma_0 \succ \Gamma_1 \succ \Gamma_2 \dots$ without end. Then we can write a sequence of sentences thus:

S_0 : S_1 is not true in Γ_1 .

S_1 : S_2 is not true in Γ_2 .

S_2 : S_3 is not true in Γ_3 .

\vdots

We may assume also that for every i , $Den_{\Gamma_i}([\Gamma_{i+1}]) = \Gamma_{i+1}$, since we place no restrictions on the context-relative denotation of contexts in the context domain. In

this case, we find that if we try to evaluate S_0 in Γ_0 , we are taken to the question of the value of S_1 in Γ_1 , which takes us to the question of the value of S_2 in Γ_2 , and so on, which means that we get no answer to any of these questions. In order to avoid finding ourselves with a semantics that for some sentences and some contexts yields simply no answer to the question whether the sentence is true, false, or neither in the context, we need to forbid chains of contexts that fail to bottom out.

It remains to define in a general way the total set of contexts in such a way that our restrictions are satisfied. To do this, we can start by defining a base layer of contexts as the set of all contexts whose context domains are empty. Then we define the next layer of contexts as the set of all contexts that are either in the base layer or whose context domains are subsets of the base layer. Then we define the next layer as the set of all contexts that are either in the previous layer or whose context domains are subsets of the previous layer. And so on. We define the set of all contexts as the union of all of these layers. In symbols:

1. $M_0 = \{\Gamma \mid K_\Gamma = \emptyset\}$.
2. For each $i > 0$, $M_i = \{\Gamma \mid \Gamma \in M_{i-1} \text{ or } K_\Gamma \subseteq M_{i-1}\}$.
3. The set of contexts is $M = \bigcup_{i=0}^{\infty} M_i$.

Call the set of contexts so defined the *well-foundedness conception of contexts*.

The well-foundedness conception of contexts ensures that every maximal chain of contexts bottoms out, which in turn implies that no context points to itself, which implies that no context belongs to its own context domain, which implies that reflexive reference to contexts is forbidden. I conjecture, but cannot prove, that the well-foundedness conception is the most permissive definition of the set of contexts that will allow us, without contradiction, to evaluate each sentence in each context as true, false or neither.

4 A rationale for restriction

In this section I have several objectives. First, I need to clarify some of my assumptions about contexts and context-relativity. Second, I want to acknowledge a puzzling consequence of the proposed restriction on contexts, namely, that, in a sense, we cannot talk about the context we are in. I will derive that consequence from some general features of contexts and context-relativity. But then, third, I wish to defend the proposed restrictions on contexts in two ways. One way will be just to point out that, even given these restrictions, no context must remain forever unmentionable. The other way will be to justify some of the restrictions on broadly ontological grounds, grounds that do not appeal to semantic paradox. What I will be able to justify in this way is only the restriction against contexts pointing to themselves. I will not be able to justify the complete restriction to the well-foundedness conception.

Contexts, I am assuming, are formal structures of some kind. Again, I am postponing any further specification in order to avoid burying the lead. Furthermore, I assume that contexts have a lot of content. They have so much content, in fact, that

for each sentence, and each context, it is formally determinate whether the sentence is true, false or neither in the context. In this respect, contexts resemble the models of traditional model-theoretic semantics. So truth in a context is a purely formal relation between a sentence and a context. It is not the case that after we have settled the structure of a context, we still have to look at what the actual world is like in order to know whether a sentence is true relative to the context.

Contexts, so defined, have to be distinguished from *situations*, by which I mean the concrete arrangements of objects and events in which *utterances* of sentences occur. One is free to use the term “context” to refer to what I call *situation*, as many people do (even Kaplan 1989 sometimes), but that is just not how I will use the term. An utterance of a sentence, considered as a component of a situation, may be true or false *simpliciter*. (In saying this, I employ one of the one-place truth predicates that I introduced in Sect. 2.)

The relation between the truth (*simpliciter*) of an utterance and the truth-in-a-context of a sentence can be explained in terms of a relation of *pertaining* that holds between a context and an utterance. Roughly, the context that pertains to an utterance is the context that is relevant to the utterance in the sense that it is the context we have to look to in evaluating the utterance as true or not. I will thus assume that at most one context pertains to an utterance, but of course we could consider reasons to relax that assumption. In general, the truth of an utterance can be defined thus:

An utterance is *true (simpliciter)* if and only if the sentence uttered is true relative to the context that pertains to the utterance (Gauker 2010, 2012).

For example, suppose that the context that pertains to an utterance of the sentence “Everyone is present” is a context in which the domain of discourse is the set of people on the committee and the present time and place is the meeting room at 9 o’clock in the morning. Then that utterance of the sentence “Everyone is present” will be true (*simpliciter*) if and only if everyone on the committee is in the meeting room at 9 o’clock in the morning. Although the truth predicate applicable in this way to utterances is a one-place truth predicate, its applicability to an utterance is entirely governed by the applicability of the two-place truth predicate to the sentence uttered and the context that pertains to the utterance. So if the two-place predicate is not a source of paradoxes, then this one-place truth predicate will not be one either.

We can say something similar about the reference of a demonstrative expression, such as “that”. The reference of the *word* “that” is relative to a context, but the reference of an *utterance* of the word is absolute and dependent on the pertaining relation. The reference of an utterance of “that” is the object that the context that pertains to the utterance assigns to the word. More generally,

The Principle of Demonstrative Reference: An utterance of a demonstrative δ denotes object o if and only if, where Γ is the context that pertains to that utterance, $Den_{\Gamma}(\delta) = o$.³

As a special case of this general principle, we may assert the following principle:

The Principle of Referring to Contexts: An utterance of a context constant δ denotes the context Ω if and only if, where Γ is the context that pertains to that utterance, $Den_{\Gamma}(\delta) = \Omega$.

The truth of this biconditional does not mean that the left-hand side and the right-hand side, when both true, express the same fact. But it does mean that any explanation of why the left-hand side is true must equally be an explanation of why the right-hand side is true and vice versa.

As we have seen at the beginning of Sect. 3, the prohibition against contexts belonging to their own context domains implies that reflexive reference to contexts is forbidden, that is, that for each context constant δ and each context Γ , $Den_{\Gamma}(\delta) \neq \Gamma$. By this assumption and the Principle of Referring to Contexts, we may conclude that (i) no utterance of a context constant can denote the context that pertains to that utterance. Consider now an utterance of a sentence of the form, [s is true in every context] (s being any sentence). The context that pertains to that utterance is not in its own context domain. So the domain of contexts over which the utterance quantifies does not include the context that pertains to that utterance. Generalizing, we may conclude that (ii) no utterance can refer indirectly, via quantification, to the context that pertains to that utterance. A sloppy formulation of (i) and (ii) together would be: *We cannot talk about the context we are in!*

Off hand, this conclusion can seem quite paradoxical. It seems to mean that I cannot talk about the context in which I am asserting my semantic theory. So if I make a general claim about the conditions under which a sentence of a certain form is true in a context, then the context that pertains to my own utterance is not one of those I am talking about. Even if I say, as I have just done, “We cannot talk about the context we are in”, I have not said anything about the context I am in and have consequently not ruled out that I can talk about the context I am in in the context I am in.

Without wishing to deny the unsettling nature of these conclusions, I will now try to defend them in two ways. The first way is to simply remove some of the sting by pointing out that they do not entail that there is any context that we cannot talk about. We cannot talk about the context that pertains to the very discourse we are engaged in. But the context that pertains to our discourse can become the object of a discourse to which a different context pertains. More generally, if we are conducting a discourse to which a context pertains whose context domain does not contain some other context that we are moved to talk about, then we can begin a discourse to which a context pertains whose context domain contains that context. The sense that

³ Here I ignore some questions one might have about how a context can assign different referents to two occurrences a single demonstrative, so that a sentence such as “That is not that” may be true in some context. For an answers to these questions, see Gauker (2014).

we must be able to talk about the context we are in might derive from mistaking the context we *were* in for the context we *are* in.

I emphasize that the shift to another context, i.e., to a discourse situation to which a different context pertains, is not a shift that is forced on us when we try to avoid the semantic paradoxes. The semantic paradoxes are blocked by the restriction on the structure of contexts. Nor need the context we shift to be in any sense an extension of the context from which we shifted. One possibility is that a sentence containing an indexical expression was true in the previous context but is false in the subsequent context. Or our semantics might allow that even nonsemantical sentences are neither true nor false in some contexts, in which case a sentence may pass from being true in a context to being neither true or false in the next context. Suppose, for example, that we were talking about birds. Some sentences about birds were true relative to this context and others were false. But then we take an interest in the context that pertains to our discourse about birds. The sentences about birds that were true or false relative to that context may be completely irrelevant, and neither true nor false, in the context that we shift to.

The second, more ambitious way to try to defend these conclusions is to argue without appeal to the paradoxes that the restrictions on the structure of contexts must be respected. That is what I will do, but I will not be able to justify in this way all of the restrictions we need. First, I will argue in some detail that reflexive reference to contexts must be forbidden for contexts that pertain to some utterance, which leads to the conclusion that no context that pertains to some utterance can belong to its own context domain. The basic idea will be that reflexive reference to contexts would create a kind of circularity in the grounding of contexts in the circumstances of utterance. Generalizing, I will conclude that no context whatsoever can belong to its own context domain, because we have no interest in contexts that could not pertain to any possible utterance. I will then take it as obvious that the style of argument could be generalized to show that contexts cannot point to themselves. I will not be able to justify the complete well-foundedness conception, including the assumption that chains of contexts must bottom out.

To get started, please set aside reference to contexts for a moment. Just think about the context-relativity of a sentence such as “Everyone is present”. For any given utterance of this sentence used to make a statement, there will some set of things and some place and time such that the utterance is true if and only if every person in that set is present at that place and time. There will be something about the structure of the situation in which the utterance takes place that makes *that* set and *that* place and time be the set and place and time relative to which the utterance must be evaluated. It might be that the utterance is being used to affirm that a precondition on the start of the meeting is fulfilled, namely, that all of the committee members are present. The fact that *that* set and *that* place and time are *the* set and *the* place and time relative to which the utterance must be evaluated *supervenes* somehow on the structure of the situation in which the utterance takes place. But as we have seen, the truth value of an utterance can be identified with the truth value of the sentence uttered relative to a context that pertains to the utterance. So the fact that a certain context pertains to the utterance of “Everyone is present” supervenes

on the structure of the situation in which the utterance takes place. Generalizing from this example, we may adopt the following principle:

The Principle of the Supervenience of Pertinent Contexts: A context's pertaining to an utterance supervenes on the structure of the situation in which the utterance takes place.

Still setting aside reference to contexts, consider demonstrative reference of other kinds. Consider what it takes for an utterance of "That" in an utterance of "That is dirty" to refer to a particular spoon (for eating). If the utterance of "that" refers to a certain spoon, then there has to be something about the structure of the situation in which the utterance takes place that makes that so. Some people say that the reference of a demonstrative can only be what the speaker intends it to be (King 2014). My own view is that that is wrong and the reference is determined by a competition between many factors, such as gestures, salience, charity in interpretation, rules of sequence and anaphora, and so on (Gauker 2008, 2019). In any case, the fact that an utterance of a demonstrative refers to a thing likewise supervenes on the structure of the situation in which it takes place. So by the Principle of Demonstrative Reference, stated above, the fact that a context Γ pertains to a given utterance of a demonstrative δ and is such that $Den_{\Gamma}(\delta) = o$ will likewise supervene on the structure of the situation in which the utterance takes place.

Returning now to reference to contexts, the fact that an utterance of a demonstrative refers to a given context will likewise supervene on the structure of the situation in which the utterance takes place. By the Principle of Referring to Contexts, stated above, from the fact that an utterance u of a demonstrative δ refers to a context Ω we may infer that, where Γ is the context that pertains to u , $Den_{\Gamma}(\delta) = \Omega$. So we may affirm the following as a special case of the Principle of the Supervenience of Pertinent Contexts:

The Principle of the Supervenience of Reference to Contexts: Where Γ is the context that pertains to an utterance u of demonstrative δ , the fact that it pertains and the fact that $Den_{\Gamma}(\delta) = \Omega$ supervene on the structure of the situation in which u takes place.

My argument against reflexive reference to contexts will require a further assumption, namely, that supervenience relations must be *intelligible*. By this I mean that if we claim that fact A supervenes on a set of facts S , then we are obligated to be able to identify in a non-question-begging way the set of facts S on which fact A supervenes. So we ought to be able to settle the issue, raised two paragraphs back, concerning the determinants of demonstrative reference and then say, in each particular case, what the facts are, about the circumstances of utterance, on which the reference supervenes. Applying this lesson to reference to contexts, if an utterance u of a demonstrative δ refers to a context Ω , then it should be possible to identify the facts about the situation in which u occurs that make it the case that u refers to Ω , and it should be possible to identify those facts in a way that does not presuppose that u refers to Ω .

So suppose, for a reductio, that context Γ is such that (a) Γ pertains to some utterance u of δ and (b) $Den_{\Gamma}(\delta) = \Gamma$. By the Principle of the Supervenience of Reference to Contexts, this fact (both (a) and (b)) supervenes on the structure of the situation in which u takes place. By the intelligibility of supervenience, we are obligated to be able identify the facts about the situation in which u occurs on which these facts supervene. By the Principle of Referring to Contexts, in identifying these we will confront the question how the fact that (c) u refers to Γ supervenes on the structure of the situation in which u occurs. But since the content of Γ includes the specification that $Den_{\Gamma}(\delta) = \Gamma$, identifying the facts on which the fact (c) supervenes will take us back to the question of what the facts are, about the situation in which u occurs, on which the fact that $Den_{\Gamma}(\delta) = \Gamma$ supervenes. That question has to be answered in order to explain why it is that what u refers to is specifically Γ and not some other context. But the question we began with was, in part, how the fact that $Den_{\Gamma}(\delta) = \Gamma$ supervenes on the situation in which u occurs. So we are caught in a circle of obligations that we cannot fulfill. But that is absurd. So we were mistaken in supposing both that Γ pertains to some utterance and that $Den_{\Gamma}(\delta) = \Gamma$. Reflexive reference to contexts must be forbidden for contexts that pertain to some utterance. But, as always, we do not want to place any restrictions on denoting, relative to a context, the members of the context domain. So if Γ pertains to some utterance, then it is not a member of its own context domain.

Having concluded that if a context pertains to some utterance, then it does not belong to its own context domain, we may generalize and conclude that no context whatsoever is a member of its own context domain. It is not just contexts that pertain to some actual utterance that cannot be members of their own context domains. If any context could possibly pertain to some possible utterance, then, by the same argument, reformulated in counterfactual mode, it could not be a member of its own context domain. But we can exclude from the set of all contexts those contexts that cannot pertain to any possible utterance, since these are of no interest. So we may define the set of contexts in such a way that no context can be a member of its own context domain. Since the relation of *pointing* is defined recursively in terms of membership in the context domain (see the definition in the previous section), the argument can be generalized, thus: Contexts cannot point to themselves either.

This argument yields some of the restrictions on the structure of contexts that we need in order to block the paradoxes of truth-in-context-X. However, it does not, as far as I can see, justify the requirement that every maximal chain of contexts bottoms out, and consequently also does not justify the full well-foundedness conception of contexts. We might have an infinitely descending sequence of contexts ordered under the \succ -relation, and one of those contexts might pertain to some utterance, so that each context lower down in the sequence indirectly pertains as well. But that would not mean that any attempt to explicate the supervenience of the context on the circumstances of utterance would lead us into a circle of obligations.

5 The question of semantic closure

If a language contains all of the vocabulary that must be used to explicate the semantic properties of that same language, then we may say that it is semantically closed. If we “cannot talk about the context we are in”, then that means that a language cannot be completely closed semantically. To that extent, at least, we must lower our expectations. But apart from that concession, is there any other respect in which a language must fall short of the ideal of semantic closure? If we adopt the well-foundedness conception of the set of contexts, then, as far as I can see, there is not (despite Gupta’s and Belnap’s extended attack on the idea (1993, Ch. 7)).

A question about the semantic properties of sentence s relative to a context can devolve into a question about the semantic properties of some other sentences relative to contexts if s is itself a sentence about the semantic properties of sentences relative to contexts. Likewise, questions about the semantic properties of those other sentences relative to contexts can turn into questions about the semantic properties of still other sentences relative to contexts. And so on. But given the well-foundedness conception of contexts, this series of questions about the semantic properties relative to contexts is bound to come to an end when we reach the point of evaluating sentences about the semantic properties of sentences relative to contexts that have empty context domains. Thus we can expect the semantic properties of each sentence relative to each context will be assignable in a way that is consistent with the assignment of semantic properties to every other sentence-context pair.

What one would like to see at this point is a proof-of-concept consisting of the following two components: First, we would define a language that includes at least the truth-in-context- X and the falsehood-in-context- X predicates and quotation names for every sentence of that same language. This language should also contain resources sufficient for defining in the language the set of all contexts according to the well-foundedness conception of contexts. Second, we will need to prove that there is no sentence and no context such that the sentence is both true and not-true in that context or both false and not-false in that context or both true and false in that context. This proof would involve a simultaneous induction over three hierarchies, the hierarchy of sentences, the hierarchy of quotation names of sentences and the hierarchy of contexts. I will not provide that proof-of-concept here, but I will lay out some of the key ideas that such a project might involve. For simplicity, I will work with a substitutional interpretation of the quantifiers, but I understand that as only a crutch that we would eventually need to discard. I will also assume for this illustration that even nonsemantical sentences may in some contexts fail to be true or false.

The first step is to define the components of a context. Suppose that singular terms are either individual constants or quotation names of sentences. An atomic sentence consists of a nonsemantic predicate followed by an appropriate number of singular terms. (So $[a = [s \text{ is true in } \delta]]$ is an atomic sentence, but $[s \text{ is true in } \delta]$ is not one.) Where B is any set of sentences, let us say that two singular terms a and b are *identity-linked* in B if and only if either $[a = b]$ or $[b = a]$ is a member of B or

there is a singular term c such that a is identity-linked to c in B and c is identity-linked to b in B . Let us say that two atomic sentences ϕ and ψ are *identity-linked* in B if and only if ψ results from ϕ by substituting, for zero or more occurrences of the singular terms in ϕ , singular terms that are identity-linked in B to those singular terms in ϕ . Say that a set B_Γ is the *base* for a context Γ if and only if (a) B_Γ is a nonempty set of atomic sentences and negations of atomic sentences that is consistent in the sense that if ϕ is an atomic sentence, then not both ϕ and $\neg\phi$ are in members of B_Γ and if ϕ and ψ are atomic sentences that are identity-linked in B_Γ , then not both ϕ and $\neg\psi$ are members of B_Γ , and (b) distinct quotation names are never identity-linked in B_Γ . The base for a context need not be maximally consistent. (So there may be atomic sentences such that neither they nor their negations are members.)

Further, a context will contain an *object domain*, which is a set of individual constants, not a set of objects such as the individual constants might denote. The object domain must contain every individual constant that occurs in any sentence in the base, and may contain other individual constants in addition. The object domain does not contain quotation names. (In this illustration quantification over sentences is not provided for.)

In view of these components, we can write conditions on truth- and falsehood-in-context-X such as the following. As usual, the expression $\Phi n/v$ stands for the result of substituting the singular term n for the variable v wherever v occurs free in Φ .

If an atomic sentence p belongs to the base of Γ , then p is true in Γ .

If p is an atomic sentence and $[\neg p]$ belongs to the base of Γ , then p is false in Γ .
For any sentence P and singular terms a and b , if P and either $[a = b]$ or $[b = a]$ are true in Γ , then the result of substituting b for an occurrence of a in P is true in Γ .

For any sentence P and singular terms a and b , if P is false in Γ and either $[a = b]$ or $[b = a]$ is true in Γ , then the result of substituting b for an occurrence of a in P is false in Γ .

For any sentence P , if P is false in Γ , then $[\neg P]$ is true in Γ .

For any sentence P , if P is true in Γ , then $[\neg P]$ is false in Γ .

For any variable v , any formula Φ , if for every individual constant n in the object domain of Γ , $\Phi n/v$ is true in Γ , then $[\forall v\Phi]$ is true in context Γ .

For any variable v , any formula Φ , if for some individual constant n in the object domain of Γ , $\Phi n/v$ is false in Γ , then $[\forall v\Phi]$ is false in context Γ .

The clauses for disjunction and conjunction will be just those one would expect (modeled on the weak Kleene scheme).

A context will also contain a context domain, which is either the empty set or a set of other contexts. A context Γ will also contain a function Den_Γ that maps some, but not necessarily all, context constants into members of the context domain and maps no context constant into any context not in the context domain. Den_Γ also maps each quotation name of a sentence into the sentence quoted. We can collect all of the context constants that are mapped into a context in the context domain into a set called the *context constant domain* for the context. In other words, the context

constant domain for a context Γ is $\{\delta \mid \text{For some context } \Delta \text{ in the context domain for } \Gamma, \text{Den}_\Gamma(\delta) = \Delta\}$. (If any context in the context domain is not denoted in the context by any context constant, then it will not be represented in the context constant domain.) The total set of contexts conforms to the well-foundedness conception of contexts. So there will be a base layer of contexts that satisfy the descriptions of contexts so far given but which have empty context domains. The rest of the total set will be built up from this base in accordance with the definition of the well-foundedness conception in Sect. 3.

For each context Γ and each context constant δ , $\text{Den}_\Gamma(\delta)$ is either undefined or a member of the context domain for Γ . Think of “ Π ” as forming a universal quantifier that ranges over contexts in the context domain (albeit in the substitutional manner via the context constants in the context constant domain). Then we can add to the list of truth- and falsehood-in-context-X conditions the following truth- and falsehood-in-context-X conditions for sentences about truth- and falsehood-in-context-X:

If $\text{Den}_\Gamma(\delta)$ is defined and $\text{Den}_\Gamma(s)$ is true in $\text{Den}_\Gamma(\delta)$, then $[s \text{ is true in } \delta]$ is true in Γ .

If $\text{Den}_\Gamma(\delta)$ is defined and $\text{Den}_\Gamma(s)$ is not true in $\text{Den}_\Gamma(\delta)$, then $[s \text{ is true in } \delta]$ is false in Γ .

If $\text{Den}_\Gamma(\delta)$ is defined and $\text{Den}_\Gamma(s)$ is false in $\text{Den}_\Gamma(\delta)$, then $[s \text{ is false in } \delta]$ is true in Γ .

If $\text{Den}_\Gamma(\delta)$ is defined and $\text{Den}_\Gamma(s)$ is not false in $\text{Den}_\Gamma(\delta)$, then $[s \text{ is false in } \delta]$ is false in Γ .

If for every term δ in the context constant domain of Γ , $\Phi\delta/v$ is true in Γ , then $[\Pi v\Phi]$ is true in Γ .

If for some term δ in the context constant domain of Γ , $\Phi\delta/v$ is false in Γ , then $[\Pi v\Phi]$ is false in Γ .

The total list of sufficient conditions on truth/falsehood in Γ is followed up by a closure clause asserting that under no other condition is a sentence true/false in Γ .

This way of specifying truth- and falsehood-in-context-X conditions, via a series of sufficient conditions and a closure clause, rather than via a series of biconditionals, one for each type of sentence, allows us to accommodate the fact that sentences can qualify as true in a context (false in context) because they result from identity substitutions in sentences that are independently qualified as true in the context (false in the context). (So the truth of an atomic sentence can be demonstrated in various ways.) It should be noted that the falsehood in context of $[s \text{ is true in context } \delta]$ is defined in terms of the lack of truth in context, not in terms of falsehood in context, and that the falsehood in context of $[s \text{ is false in context } \delta]$ is defined in terms of the lack of falsehood in context, not in terms of truth in context. This way of writing the clauses ensures that we can maintain that a sentence of the form $[\neg(s \text{ is true in } \delta) \wedge \neg(s \text{ is false in } \delta)]$ can be true in a context. (It will be true in Γ if and only if s is neither true nor false in $\text{Den}_\Gamma(\delta)$.)

For examples, we can construct contexts Ω and Δ , with non-empty context domains, such that $[\lambda = [\lambda \text{ is not true in } \Gamma]]$ is true in both Ω and Δ , but $[\lambda \text{ is not true}$

in Γ is true in Ω and *not* true in Δ . First, let $[\lambda = [\lambda \text{ is not true in } \Gamma]]$ be a member of the bases of contexts Ω and Δ . Second, suppose $Den_{\Delta}([\Gamma]) = \Omega$, $Den_{\Omega}([\Gamma]) = \Theta$, and $Den_{\Theta}([\Gamma])$ is undefined (which ensures that the context domains of Ω and Δ are non-empty). By the clause for the truth of atomic sentences, $[\lambda = [\lambda \text{ is not true in } \Gamma]]$ is true in Δ and Ω . We can show as follows that $[\lambda \text{ is not true in } \Gamma]$ is true in Ω (line 6) but is not true in Δ (line 10). (I will employ the closure clause without explicit mention.)

1. $[\lambda \text{ is true in } \Gamma]$ is not false in Θ (by the clause for the falsehood of sentences about truth, given that $Den_{\Theta}([\Gamma])$ is undefined).
2. $[\lambda \text{ is not true in } \Gamma]$ is not true in Θ (by the clause for truth of negations).
3. $[\lambda \text{ is not true in } \Gamma]$ is not true in $Den_{\Omega}([\Gamma])$ (by an identity substitution).
4. $[[\lambda \text{ is not true in } \Gamma] \text{ is true in } \Gamma]$ is false in Ω (by the clause for the falsehood of sentences about truth).
5. $[[\lambda \text{ is not true in } \Gamma] \text{ is not true in } \Gamma]$ is true in Ω (by the clause for truth of negations).
6. $[\lambda \text{ is not true in } \Gamma]$ is true in Ω (by the clause for the truth of identity substitutions).
7. $[\lambda \text{ is not true in } \Gamma]$ is true in $Den_{\Delta}([\Gamma])$ (by an identity substitution).
8. $[[\lambda \text{ is not true in } \Gamma] \text{ is true in } \Gamma]$ is not false in Δ (by the clause for the falsehood of sentences about truth).
9. $[[\lambda \text{ is not true in } \Gamma] \text{ is not true in } \Gamma]$ is not true in Δ (by the clause for truth of negations).
10. $[\lambda \text{ is not true in } \Gamma]$ is not true in Δ (by the clause for the truth of identity substitutions).

What this sketch of truth- and falsehood-in-context-X conditions is supposed to illustrate is the possibility of a language in which we can write the semantic theory for that very language and which is provably consistent in the sense that it contains no sentences that are both true and not true or both false and not false or both true and false in a single context. If there is an independent one-place truth predicate such as I described in Sect. 2, then we should be able formulate a semantic theory for it without running the risk of creating revenge paradoxes at the level of the semantic metalanguage. There will be no paradoxes at the level of the semantic metalanguage because the language contains no sentences that are both true and not true or both false and not false or both true and false in a single context.

6 Conclusion

Once we recognize that the truth predicate that we will use to characterize the semantic properties of sentences is the two-place truth-in-context-X predicate, we should recognize that the paradoxes that threaten our semantics are not the paradoxes concerning the one-place truth predicate but the paradoxes of the truth-in-context-X predicate. These, I have claimed, can be avoided by placing restrictions on the structure of contexts. It suffices to adopt the well-foundedness conception,

according to which any series of contexts such that the second belongs to the context domain of the first, the third belongs to the context domain of the second, and so on, must eventually bottom-out in a context with an empty context domain.

However, several issues remain open. One is whether the well-foundedness conception is the most permissible definition of the set of contexts that suffices to block the paradoxes of truth-in-context-X. Another is how, apart from the need to block the paradoxes, the full well-foundedness conception can be justified. Finally, it remains to prove in a general way that we can have a language that is semantically closed and in which the truth value of each sentence relative to each context can be univocally evaluated.

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References

- Barwise, J., & Etchemendy, J. (1987). *The liar: An essay on truth and circularity*. Oxford University Press.
- Burge, T. (1979). Semantical paradox. *Journal of Philosophy*, 76, 169–198.
- Field, H. (2008). *Saving truth from paradox*. Oxford University Press.
- Gauker, C. (2003). *Words without meaning*. MIT Press.
- Gauker, C. (2005). Semantics for deflationists. In J. C. Beall & B. Armour-Garb (Eds.), *Deflationism and paradox* (pp. 148–176). Oxford University Press.
- Gauker, C. (2006). Against stepping back: A critique of contextualist approaches to the semantic paradoxes. *Journal of Philosophical Logic*, 35, 393–422.
- Gauker, C. (2008). Zero tolerance for pragmatics. *Synthese*, 165, 359–371.
- Gauker, C. (2010). Contexts in formal semantics. *Philosophy Compass*, 5, 568–578.
- Gauker, C. (2012). Semantics and pragmatics. In D. Graff Fara & G. Russell (Eds.), *Routledge companion to the philosophy of language* (pp. 481–563). Routledge.
- Gauker, C. (2014). How many bare demonstratives are there in English? *Linguistics and Philosophy*, 37, 291–314.
- Gauker, C. (2019). Against the speaker-intention theory of demonstratives. *Linguistics and Philosophy*, 42, 109–129.
- Glanzberg, M. (2001). The liar in context. *Philosophical Studies*, 103, 217–251.
- Glanzberg, M. (2004). A contextual-hierarchical approach to truth and the liar paradox. *Journal of Philosophical Logic*, 33, 27–88.

- Grover, D. (1992). *A prosentential theory of truth*. Princeton University Press.
- Gupta, A., & Belnap, Nuel. (1993). *The revision theory of truth*. MIT Press.
- Kaplan, D. (1989). Demonstratives. In J. Almog, J. Perry, & H. K. Wettstein (Eds.), *Themes from Kaplan* (pp. 481–563). Oxford University Press.
- King, J. C. (2014). Speaker intentions in context. *Noûs*, 48, 219–237.
- Koons, R. C. (1992). *Paradoxes of belief and strategic rationality*. Cambridge University Press.
- Kripke, S. (1975). Outline of a theory of truth. *Journal of Philosophy*, 72, 690–716.
- Murzi, J., & Rossi, L. (2018). Reflection principles and the liar in context. *Philosopher's Imprint*, 18, 1–18.
- Parson, C. (1974). The liar paradox. *Journal of Philosophical Logic*, 3, 381–412.
- Simmons, K. (1993). *Universality and the liar: An essay on truth and the diagonal argument*. Cambridge University Press.
- Simmons, K. (2018). *Semantic singularities: Paradoxes of reference, predication and truth*. Oxford University Press.
- Williamson, T. (1998). Indefinite extensibility. *Grazer Philosophische Studien*, 55, 1–24.
- Yablo, S. (1993). Paradox without self-reference. *Analysis*, 53, 251–252.

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