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Development of a Model to Compute the Extension of Life Supporting Zones for Earth-Like Exoplanets

David Neubauer · Aron Vrtala · Johannes J. Leitner ·
Maria G. Firneis · Regina Hitzenberger

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Abstract A radiative convective model to calculate the width and the location of the life supporting zone (LSZ) for different, alternative solvents (i.e. other than water) is presented. This model can be applied to the atmospheres of the terrestrial planets in the solar system as well as (hypothetical, Earth-like) terrestrial exoplanets. Cloud droplet formation and growth are investigated using a cloud parcel model. Clouds can be incorporated into the radiative transfer calculations. Test runs for Earth, Mars and Titan show a good agreement of model results with observations.

Keywords Life supporting zone · Habitable zone · Radiative convective model · Cloud parcel model

Introduction

Life as we know it uses water as a solvent, which has been common on Earth since the early history of the planet. Its liquid temperature range, its physical buffering capacity and other characteristics make it ideal for the complex chemical interactions of terrestrial biomolecules. But exotic life, based on a different biochemistry (Baross et al. 2007) may be possible. For exotic life the properties of possible solvents like sulfuric acid, ammonia, methane, ethane or mixtures of them (see Table 1) could be as well suited as or even advantageous to those of water (Schulze-Makuch and Irwin 2004). On Titan, as an example

D. Neubauer (✉) · J. J. Leitner
Research Platform: ExoLife, University of Vienna, Türkenschanzstrasse 17, 1180 Vienna, Austria
e-mail: d.neubauer@univie.ac.at

A. Vrtala · R. Hitzenberger
Aerosol Physics and Environmental Physics Group, Faculty of Physics, University of Vienna,
Boltzmanngasse 5, 1090 Vienna, Austria

M. G. Firneis
Institute of Astronomy, University of Vienna, Türkenschanzstraße 17, 1180 Vienna, Austria

Table 1 Physical properties of water and possible alternative solvents (adapted from Schulze-Makuch and Irwin 2004)

Property	H_2O	NH_3	H_2SO_4	CH_4	C_2H_6
Molar mass (g/mol)	18.02	17.03	98.08	16.04	30.07
Density (g/ml)	0.997 (25°C)	0.696 (−33.3°C)	1.831 (25°C)	0.426 (−164°C)	0.572 (−112°C)
Melting Point (°C at 1 bar)	0.0	−77.7	10	−182.5	−183
Boiling Point (°C at 1 bar)	100.0	−33.3	337	−161.5	−89
Range of Liquid (°C at 1 bar)	100	44.4	327	21	94
Critical Temperature (°C)	374	132	NA	−82.6	32.3
Critical Pressure (bar)	215	111	NA	45.5	47.8
Enthalpy of Fusion (kJ/mol)	6.0	5.7	10.7	0.94	2.7
Enthalpy of Vaporization (kJ/mol)	40.7	23.3	61.9	8.2	14.7
Dielectric Constant (ε_r)	80.1 (20°C)	22.5 (−34°C)	101 (25°C)	1.7 (−182°C)	1.9 (−178°C)
Viscosity (10^{-4} Pa·s)	8.9 (25°C)	2.7 (−34°C)	260 (25°C)	1.2 (−164°C)	2.3 (−112°C)
Dipole Moment (10^{-30} C·m)	6.2	4.9	9.0	0.0	0.0

for different environmental conditions, water is not liquid on the surface but methane or ethane could be possible solvents for exotic life.

The classical habitable zone for water is defined (Huang 1959, 1960; Hart 1978; Kasting et al. 1993) as the region around a star where liquid water may exist on the surface of a planet. Energy balance calculations show that the widths and the locations of habitable zones for alternative solvents (Leitner et al. 2010b) can be quite different from those for the classical habitable zone (Leitner et al. 2010a) because of the different temperature ranges for liquid alternative solvents (see Table 1). The life supporting zone (LSZ) is defined (Leitner et al. 2010b) as the zone consisting of all the habitable zones belonging to the considered solvents. To calculate the LSZ a radiative convective model for planetary atmospheres was further developed. For radiative transfer calculations the public domain software 'Streamer' was modified to provide an interface with the cloud model, to increase the spectral range for radiative transfer calculations and to include additional scattering and absorbing gases as well as collision induced absorption. As clouds dominate the radiative transfer when they are present, clouds are incorporated in this model.

Model Description

Two models are used: a cloud model and a radiative convective model. The cloud model is used to calculate cloud droplet distributions. The optical properties of these droplet distributions are computed offline using Mie theory (Bohren and Huffman 1983) and used as an input for the radiative convective model. These optical properties depend on the refractive indices of the solvents, which are taken from literature. The radiative convective model then yields the surface temperature of exoplanets.

Cloud Model

A cloud model developed for terrestrial water clouds by Neubauer (2009) is used to investigate cloud droplet formation and growth consisting of water or alternative solvents (e.g. H_2SO_4-clouds). Physico-chemical constants for the solvents available in the literature are used for the computations. As nothing is known about possible cloud condensation nuclei (CCN) in the atmosphere of exoplanets, we assume cloud formation processes follow the same physical principles of solvent condensation from the gas phase on pre-existing aerosol particles formed by the same physical processes as on Earth which are soluble in or at least wettable by the solvent. The model is a cloud parcel model which describes an ascending air parcel containing the droplets (following Kornfeld 1970; Houze 1993; Pruppacher and Klett 1997). The model includes the microphysical processes of nucleation, condensation and coagulation and radiative effects (Chen and Lamb 1994). Entrainment is also considered.

The CCN and the cloud droplets are each categorized into 100 size bins in the model. The scheme of Ochs and Yao (1978) for the method of moments is used to reduce numerical dispersion. A typical background aerosol distribution was employed for Earth. A constant updraft velocity of 0.75 m/s was used for the calculations for Earth and entrainment was implemented by considering the motion of the cloud droplets as a random walk between the upper and the lower cloud boundary (Mason 1952, 1960). The CCN start either with their equilibrium radius on the Köhler curve or if they are larger than a critical radius their initial radius is calculated with a parameterization given by Kogan (1991). The condensational growth uses the algorithm suggested by Chen and Lamb (1994) and accounts for radiative effects (Roach 1975; Brown 1980; Welch et al. 1986; Bott et al. 1990; Harrington et al. 2000; Conant et al. 2002). The continuous and stochastic coagulation equations are solved for a sedimentation and a Brownian diffusion kernel following Ochs and Yao (1978).

The cloud model provides cloud droplet size distributions for different cloud liquid solvent content (e.g. cloud liquid water content) which are stored in a database for use by the radiative convective model.

Radiative Convective Model

The equilibrium temperature profile of the atmosphere and the surface temperature of exoplanets are obtained with a model based on Manabe and Strickler (1964) and Manabe and Wetherald (1967) which we further developed for our purposes.

The atmospheric lapse rate calculated for radiative equilibrium is adjusted (i.e. convective adjustment) not to exceed a given lapse rate (e.g. the applicable dry or moist adiabatic lapse rate). For water vapor a constant absolute humidity, a constant relative humidity (Manabe and Wetherald 1967) or a troposphere fully saturated with water vapor (Kasting 1988; deviations from ideal gas law at high water vapor amounts are implemented) may be chosen. The model computes the horizontally and annually averaged global surface and atmospheric temperatures. The model uses an explicit Euler method with a variable time step for convergence of atmospheric and surface temperatures against thermal equilibrium and hard convective adjustment (Rennó et al. 1994). Several cloud layers with vertically constant optical properties can be inserted in the model atmosphere. Vertical levels in the atmosphere were chosen in constant steps on a logarithmic pressure scale.

Besides the spectral class of the star and the distance between the star and the exoplanets, the surface temperature strongly depends on the amount of atmospheric gases (in particular greenhouse gases), aerosol particles, clouds and surface albedo. Different scenarios (e.g. varying cloud amount, surface albedo, amount of atmospheric gases, etc.) can be investigated for each solvent to calculate the width and the location of the habitable zone belonging to the solvent.

For radiative transfer calculations we modified a version of the public domain program ‘Streamer’ (Key and Schweiger 1998). ‘Streamer’ accounts for scattering and absorption of radiation by gases and particles. The long wave spectrum consists of 105 wavelength bands between 4 μm and 500 μm and the shortwave spectrum of 24 wavelength bands between 0.2 μm and 5 μm (0.28 μm and 4 μm in the original ‘Streamer’ program). Built-in types of surface albedo as well as other values can be chosen. The radiative transfer equation can be solved in ‘Streamer’ by two different numerical methods to increase the precision of the calculation (Stamnes et al. 1988; Toon et al. 1989). The spectrum of the Sun is built-in in ‘Streamer’. The cloud optical properties calculated offline by the cloud model and Mie theory (Bohren and Huffman 1983) are used as an input for ‘Streamer’ when desired. Rayleigh-scattering is calculated for air in original ‘Streamer’, and was expanded by us to include also the scattering by H_2O, CO_2, CO, CH_4, N_2O, NO, NH_3 and SO_2. Besides H_2O, CO_2, O_2 and O_3 (original ‘Streamer’ absorbing gases) we included other atmospheric absorbing gases in our model: CH_4, NH_3, CO, SO_2, N_2O, NO, NO_2 and HNO_3. The exponential-sum fitting of transmissions-method (ESFT, Wiscombe and Evans 1977) was applied for this purpose and the necessary transmission functions were computed using LOWTRAN 7 (Kneizys et al. 1988) and LBLRTM (Clough et al. 2005). These transmission functions are accurate only for pressures and temperatures typical for Earth’s atmosphere: LOWTRAN 7: temperature range: approx. 215–300 K, pressure range: approx. 1–0.1(0.01 for CO_2, NO, N_2O) atm; LBLRTM: temperatures and pressures inside the normal range of Earth’s atmosphere between 0 and 120 km in altitude. For CH_4 and wavelengths ≤5 μm the transmission functions were computed with absorption coefficients from Karkoschka and Tomasko (2010) and from Irwin (2010) (temperature range: 50–300 K, pressure range: $10–10^{-8}$ atm), for wavelengths >5 μm LBLRTM was used. Continuum absorption by water vapor is included in the original ‘Streamer’. We extended the model to calculate collision induced absorption by N_2-N_2, N_2-H_2, N_2-CH_4, CH_4-CH_4 and CO_2-CO_2 collisions (Borysow and Frommhold 1986a, 1986b, 1987a, 1987b; Borysow and Tang 1993; Gruszka and Borysow 1997; Baranov et al. 2004). The ‘Streamer’ code imposes some limitations: radiative transfer can be computed for only one scattering gas and four absorbing gases. Due to these limits we choose one scattering gas (or a mixture with fixed mixing ratios, e.g. N_2, air) and up to four main atmospheric absorbing gases out of the list (each added gas absorbing in a significant spectral range slows down the model) plus continuum absorption and/or collision induced absorption for each scenario.

First Results

Table 2 shows parameters and atmospheric compositions used in the test runs. A test run for Earth with a surface albedo of 0.1 (Manabe and Strickler 1964) and an average cloud cover (high and low level clouds; Kitzmann et al. 2010) and a critical lapse rate of −6.5 K/km showed a good agreement of the atmospheric temperature profile and surface temperature with observed values. For this test run, atmospheric mixing ratios for N_2, O_2 and CO_2 of 78%, 21% and 0.038% respectively (Bauer et al. 1997; IPCC 2007) and a surface pressure of 1013 mbar were used. The O_3 and H_2O profiles were taken from Manabe and Wetherald (1967) and Manabe and Strickler (1964) respectively. A constant absolute humidity was used for water vapor in this run. H_2O, CO_2, O_3 and O_2 were selected to include the most important greenhouse gases and be able to compute the stratospheric temperature structure. CH_4 and N_2O, though they provide of the order of 3–4 K of greenhouse effect, were

Table 2 Parameters and atmospheric compositions used in the test runs

Parameter	Earth	Mars	Titan
Number of vertical levels	37	28	28
Surface pressure (mbar)	1013	6.5	1496
Top of the atmosphere-pressure (mbar)	0.2	0.001	0.78
Scattering gas	air	CO_2	N_2-CH_4 mixture
Absorbing gases	H_2O, CO_2, O_2, O_3	CO_2	CH_4
Collision induced absorption	–	CO_2-CO_2	N_2-N_2, N_2-CH_4, CH_4-CH_4, N_2-H_2
High level cloud cover	15%	–	–
Low level cloud cover	39.5%	–	–
Cloud overlap	0%	–	–

omitted as only four absorbing gases can be chosen (see subsection Radiative convective model).

The computations for Mars and Titan were done without clouds in their atmospheres. For Mars atmospheric mixing ratios for CO_2 and N_2 have been taken as 95% and 5% respectively (Taylor 2010; the mixing ratio for N_2 is 2.7% but N_2 represents the rest of the Martian atmospheric gases in this test run). Further absorbing gases have been omitted as their presence does not significantly change the surface temperature. The surface pressure is 6.5 mbar (Taylor 2010). A test run for Mars with a 'dusty' adiabatic lapse rate of −2 K/km (Taylor 2010) showed a good agreement of the atmospheric and surface temperatures with observed temperatures.

For Titan the CH_4 profile given by Niemann et al. (2005) was used with a N_2 atmosphere and a surface pressure of 1496 mbar. C_2H_2 and C_2H_6 could not be included as absorbing gases as we could not obtain the necessary transmission functions. H_2 is included

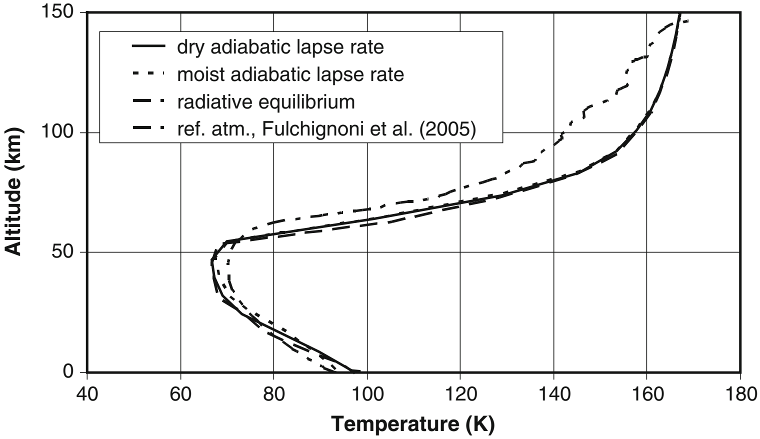

Fig. 1 Temperature profile of Titan for model runs without clouds and for a reference atmosphere (dashed-dotted line; Fulchignoni et al. 2005). Three different approaches for convective adjustment are shown, radiative equilibrium (i.e., no adjustment, dashed line), adjustment with a dry adiabatic lapse rate (solid line) and adjustment with a most adiabatic lapse rate (dotted line). The surface temperature of the run with the moist adiabatic lapse rate adjustment of 93.9 K agrees well with the reference surface temperature of 93.7 K

with a mixing ratio of 0.1% via collision induced absorption by N_2-H_2 collisions. Condensation clouds of CH_4 were omitted in this test run as they only have a small effect on the temperature structure of Titan (McKay et al. 1989). Due to limitations of the radiative transfer program haze was also omitted, but this has no significant impact on surface and tropospheric temperatures (McKay et al. 1989; Lavvas et al. 2008). The model results for the surface temperature are in good agreement with observed values and the atmospheric temperature profile agrees qualitatively with observed temperatures (see Fig. 1). The deviations of the temperatures in the stratosphere might be a result of the absence of haze, C_2H_2 and C_2H_6.

Summary

A radiative convective model was further developed to compute the width and the location of the life supporting zone (LSZ) for different, alternative solvents around the Sun and other main sequence stars. The formation and growth of cloud droplets are investigated for the different solvents with a cloud model. Clouds can be included in the calculation of the width and the location of the LSZ.

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