

# Electromagnetic Ion Cyclotron Waves in Multi-Ions Hot Anisotropic Plasma in Auroral Acceleration Region-Particle Aspect Approach

Soniya Patel · P. Varma · M. S. Tiwari

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**Abstract** Using particle aspect approach, the effect of multi-ions densities on the dispersion relation, growth rate, perpendicular resonant energy and growth length of electromagnetic ion cyclotron wave with general loss-cone distribution function in hot anisotropic multi-ion plasma is presented for auroral acceleration region. It is observed that higher  $\text{He}^+$  and  $\text{O}^+$  ions densities enhance the wave frequency closer to the  $\text{H}^+$  ion cyclotron frequency and growth rate of the wave. The differential heating of  $\text{He}^+$  ions perpendicular to the magnetic field is enhanced at higher densities of  $\text{He}^+$  ions. The waves require longer distances to achieve observable amplitude by wave-particle interactions mechanism as predicted by growth length. It is also found that electron thermal anisotropy of the background plasma enhances the growth rate and reduces the growth length of multi-ions plasma. These results are determined for auroral acceleration region.

**Keywords** Electromagnetic ion cyclotron instability · Auroral acceleration region · Particle aspect analysis · Multi-ions plasma · Thermal anisotropy

## 1 Introduction

Electromagnetic ion cyclotron (EMIC) waves have an important role in the overall dynamics of the Earth's magnetosphere. Particularly, these waves are fundamental in the energization and loss of magnetospheric particles (Kennel and Petschek 1966; Thorne et al. 2006). The thermal anisotropies of the energetic protons and electron serve as a free energy source for the magnetospheric EMIC waves (Anderson et al. 1996; Zhang et al. 2010; Halford et al. 2010).

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The observational evidences of the EMIC wave-associated  $\text{He}^+$  and  $\text{O}^+$  ions are reported at auroral latitudes (Lund 1998; Lund et al. 2000). The polar orbiting satellites carrying energetic ion mass spectrometers detected significant amount of  $\text{O}^+$  ions of ionospheric origin. The high latitude ionosphere is an important source of  $\text{H}^+$ ,  $\text{He}^+$ ,  $\text{O}^+$  and other ions in the auroral acceleration region (Chappell 1988; Andre and Yau 1997). The observational evidences of EMIC waves in auroral ionosphere by Freja satellite are presented by Zakharov and Meister (1999, and references therein). They stated that simultaneous Freja observations of precipitating KeV electrons and electromagnetic emissions around the half the local proton gyrofrequency are analyzed by Oscarsson et al. (1997) showing the interaction between EMIC waves and electrons in resonance at altitudes of several thousands of kilometers above the auroral ionosphere. Cattel et al. (1998) and Reddy et al. (2006) have reported observations of waves near the cyclotron frequencies of  $\text{H}^+$ ,  $\text{He}^+$  and  $\text{O}^+$  ions and associated ion beams and field-aligned currents in night side auroral zone.

In past EMIC wave propagation has been widely investigated (Ahirwar et al. 2006a, b, 2007, 2010; Patel et al. 2011a, b, c, 2012) and its applicability in the auroral acceleration region is emphasized. In the present paper our focus is upon the investigation of electron thermal anisotropy, different densities of multi-ions, on EMIC wave and its suitability for the auroral ionosphere.

In the recent past (Patel et al. 2011a, b) we have considered only most common species ( $\text{H}^+$  ions) of the back ground plasma for the analysis of EMIC waves in auroral acceleration region. In further investigation (Patel et al. 2011c)  $\text{He}^+$  and  $\text{O}^+$  ions were also included to the analysis as these ions affect EMIC instability. The densities of multi-component ions also affect the wave frequency, growth rate and growth length of EMIC wave, therefore in the present investigation the effects of multi-ions component densities are examined on EMIC waves in the presence of ion beam and general loss-cone distribution function in an anisotropic hot plasma focused for auroral acceleration process. Energisation of particles by EMIC wave affected by multi-ions densities is predicted. In the present work we have assumed background plasma consisting of resonant and non-resonant particles. The resonant particles are those ions whose velocity parallel to ambient magnetic field is of the order of phase velocity of EMIC waves propagating in the direction of ambient magnetic field. Thus resonant particles participate in energy exchange with the wave by wave-particle energy exchange mechanism, whereas non-resonant particles support the oscillatory motion of the wave. The multi-ions and  $\text{H}^+$  ion beam may also have velocities parallel to ambient magnetic field of the order of phase velocity of EMIC wave, considered in the analysis. The left hand circular polarization for EMIC waves is considered. We have attempted to explain observations of EMIC waves and related heating phenomena in auroral acceleration region at  $1.4 R_E$  altitude where  $R_E$  is earth's radius.

In this paper we have adopted same theory and method as employed by Patel et al. (2011, b, c). In the investigation of Patel et al. (2011c) we have discussed the EMIC instability with an energetic ion beam and general loss-cone distribution function of the hot core plasma for the polar cusp upper ionosphere at 200 km altitude. The growth rate of the wave, perpendicular heating of ions, transfer of energy by wave particle resonance parallel to magnetic field and marginal instability of the EMIC wave were obtained in a plasma of  $\text{H}^+$ ,  $\text{He}^+$  and  $\text{O}^+$  ions and hot electrons. They have determined the variation in energies and growth rate in hot plasma with the thermal anisotropy of the ions of the core plasma and also discussed the effect of positive and negative ion beam velocity on the growth rate of the EMIC wave. It was predicted that heavier minor ions play significant role in the modification of the EMIC wave spectrum and can be strongly energized by the ion cyclotron waves. In this paper we intend to extend the work of Patel et al. (2011c) in the

auroral acceleration region at different densities of minor ions ( $\text{He}^+$ ,  $\text{O}^+$ ). The growth length of the wave is evaluated to predict the required distance to achieve observable wave amplitude by the wave particle interaction. We studied the effect of multi-ions densities on dispersion relation, growth rate, and perpendicular resonance energy and growth length of the EMIC wave. It is found that  $\text{He}^+$  and  $\text{O}^+$  ions enhance the growth rate of the wave and increase the wave frequency closer to the  $\text{H}^+$  ion cyclotron frequency. The cooling of  $\text{H}^+$  ions perpendicular to magnetic field is found whereas heavier ions ( $\text{He}^+$ ) are heated. Thus the densities of multi-ions should be in consideration in analysis of EMIC wave instabilities in auroral acceleration region.

In Sect. 2 we have derived the basic trajectories. The dispersion relation is derived in Sect. 3, wave energies are explained in Sect. 4. The growth rate is evaluated in Sect. 5. The growth length is evaluated in Sect. 6. The result and discussions with graphical analysis is given in Sect. 7. Finally the conclusion is described in Sect. 8.

### 2 Basic Trajectories

The wave propagating in the direction of ambient magnetic field along the z-axis is considered. The EMIC waves are assumed to start at  $t = 0$  when the resonant particles are not yet disturbed. Taking the particle trajectory in the presence of EMIC waves, the dispersion relation, the change in the charged particle energy and growth rate are then derived in the presence of up flowing anisotropic  $\text{H}^+$  ion beams, in anisotropic core plasma consisting of  $\text{H}^+$ ,  $\text{He}^+$  and  $\text{O}^+$  ions by the particle aspect analysis. We have followed the same procedure as adopted by Misra and Tiwari (1979), Patel et al. (2011a, b) and considered the distribution function  $N_c(V)$  for electron and ion core background plasma of the form

$$N_c(V) = \sum_{\alpha} \frac{n_{c,\alpha} V_{\perp,\alpha}^{2J}}{\pi^{3/2} V_{T\perp,\alpha}^{2(J+1)} V_{T\parallel,\alpha}^J} \exp\left(-\frac{V_{\perp}^2}{V_{T\perp,\alpha}^2} - \frac{V_{\parallel}^2}{V_{T\parallel,\alpha}^2}\right) \tag{1}$$

Where,  $V_{T\parallel,\alpha}^2 = \frac{2T_{\parallel,\alpha}}{m}$ ,  $V_{T\perp,\alpha}^2 = (J + 1)^{-1} \frac{2T_{\perp,\alpha}}{m}$ ,  $\alpha = i$  ( $\text{H}^+$ ,  $\text{He}^+$ , and  $\text{O}^+$ ),  $e$  stands for ions-electron core plasma.  $n_{c,\alpha}$  is the background core density and  $J$  represents the distribution index. The distribution index  $J$  represents effectiveness of loss-cone distribution function as it is powered on perpendicular velocity  $V_{\perp}$ .  $T_{\perp,\alpha}$  and  $T_{\parallel,\alpha}$  are the temperatures perpendicular and parallel to the ambient magnetic field of the respective species.

The drifting general loss-cone distribution function  $N_b(V)$  for ion beam (Gomberoff 1992) is defined as,

$$N_b(V) = \frac{n_b}{\pi^{1/2} V_{T\perp b}^2 V_{T\parallel b}} \exp\left(-\frac{V_{\perp}^2}{V_{T\perp b}^2} - \frac{(V_{\parallel} - V_D)^2}{V_{T\parallel b}^2}\right) \tag{2}$$

where,  $V_D$  is the velocity of drifting beam (Hydrogen ion) relative to the background core plasma.  $V_{T\perp b}$  and  $V_{T\parallel b}$  represent perpendicular and parallel thermal velocities of the ion beam.  $n_b$  is the density of ion beam.

### 3 Dispersion Relation

The dispersion equation for the hot multi-ion plasma for the electromagnetic ion cyclotron wave (Cornwall et al. 1971; Gomberoff 1992; Patel et al. 2011a, b, c) is defined as

$$\frac{c^2k^2}{\omega^2} = \sum_{\alpha} \frac{\omega_{pz}^2}{\Omega_{\alpha}(\Omega_{\alpha} - \omega)} + \frac{\omega_{pe}^2}{\omega^2} A_e + \frac{\omega_{pz}^2}{\omega^2} \frac{n_b/n_c(\omega - kV_D)^2}{\Omega_b(\Omega_b - \omega + kV_D)} \tag{3}$$

where,  $\sum_{\alpha}$  shows the sum of ion species for ( $H^+$ ,  $He^+$ , and  $O^+$ ),  $\omega_{pe}^2 = \frac{4\pi n_e e^2}{m}$  defines the square plasma frequency for the electrons,  $\omega_{pz}^2 = \frac{4\pi n_{\alpha} e^2}{M_{\alpha}}$  defines the square plasma frequency for the ions,  $\Omega_{\alpha}$  is the cyclotron frequency of the respective species,  $A_e = (J + 1) \frac{V_{T\perp e}^2}{V_{T\parallel e}^2} - 1$  is the thermal anisotropy of electrons (Gomberoff and Cuperman 1981) and  $\omega$  is the wave frequency.

In the derivation of the dispersion relation Eq. (3), the ions and electrons are treated as for the hot plasma (Patel et al. 2011a). In our analysis the wave frequency  $\omega$  is treated as real, and therefore, do not consider the imaginary part of the dispersion relation. For the real  $\omega$  one can not expand the dispersion relation in terms of  $i\gamma$  and thus the growth/damping rate are not evaluated. Instead, we use the energy conservation in the resonating wave particle system to obtain the growth/damping rate.

In our analysis we have focused on EMIC  $H^+$  band cyclotron frequencies as the preliminary investigation. However,  $He^+$  band cyclotron frequencies observed at many storm time are significantly important may be the matter of further studies.

### 4 Energy Balance

The perpendicular and the parallel resonant energies of the core plasma are evaluated by Patel et al. (2011a) given as:

$$W_{r\parallel c,\alpha} = \sum_{\alpha} \frac{\sqrt{\pi}}{2} t \frac{B^2 \omega_{pz}^2}{c^2 k^2} \frac{\Omega_{\alpha}^2}{k^2 V_{T\parallel\alpha}} (J + 1) \frac{V_{T\perp\alpha}^2}{V_{T\parallel\alpha}^2} \left( \frac{\omega - \Omega_{\alpha}}{\Omega_{\alpha}} \right)^2 \exp \left( - \left( \frac{\omega - \Omega_{\alpha}}{kV_{T\parallel\alpha}} \right)^2 \right) \tag{4}$$

$$W_{r\perp c,\alpha} = \sum_{\alpha} \frac{\sqrt{\pi}}{2} t \frac{B^2 \omega_{pz}^2}{c^2 k^2} \frac{\Omega_{\alpha}^2}{k^2 V_{T\parallel\alpha}} \left[ (J + 1) \frac{V_{T\perp\alpha}^2}{V_{T\parallel\alpha}^2} \left( \frac{\omega - \Omega_{\alpha}}{\Omega_{\alpha}} \right) + 1 \right] \exp \left( - \left( \frac{\omega - \Omega_{\alpha}}{kV_{T\parallel\alpha}} \right)^2 \right) \tag{5}$$

For ion beam resonant parallel and perpendicular energy is defined as,

$$W_{r\parallel b} = \sum_{\alpha} \frac{\sqrt{\pi}}{2} t \frac{B^2 \omega_{pz}^2}{c^2 k^2} \frac{n_b}{n_c} \frac{\Omega_b^2}{k^2 V_{T\parallel b}} (J + 1) \frac{V_{T\perp b}^2}{V_{T\parallel b}^2} \left( \frac{\omega - kV_D - \Omega_b}{\Omega_b} \right)^2 \exp \left( - \left( \frac{\omega - kV_D - \Omega_b}{kV_{T\parallel b}} \right)^2 \right) \tag{6}$$

$$W_{r\perp b} = \sum_{\alpha} \frac{\sqrt{\pi}}{2} t \frac{B^2 \omega_{pz}^2}{c^2 k^2} \frac{n_b}{n_c} \frac{\Omega_b^2}{k^2 V_{T\parallel b}} \left[ (J + 1) \frac{V_{T\perp b}^2}{V_{T\parallel b}^2} \left( \frac{\omega - kV_D - \Omega_b}{\Omega_b} \right) + 1 \right] \exp \left( - \left( \frac{\omega - kV_D - \Omega_b}{kV_{T\parallel b}} \right)^2 \right) \tag{7}$$

Total resonant energy is given as

$$W_{r\parallel} = W_{r\parallel c,\alpha} + W_{r\parallel b} \tag{8}$$

$$W_{r\perp} = W_{r\perp c,\alpha} + W_{r\perp b} \tag{9}$$

### 5 Growth Rate

The growth rate is obtained as (Patel et al. 2011a, c)

$$\frac{\gamma}{\Omega} = \frac{A + (n_b/n_c)B}{c^2k^2C - \frac{c^2k^2}{\omega^2} \sum_{\alpha} \frac{\omega_{pe}^2}{\omega_{pz}^2} A_e + \sum_{\alpha} \frac{c^2k^2}{\omega_{pz}^2} + \frac{1}{2} \sum_{\alpha} \left( \frac{2\Omega_{\alpha} - \omega}{\Omega_{\alpha} - \omega} \right)^2 + \frac{n_b/n_c}{2} \left( \frac{2\Omega_b - \omega'}{\Omega_b - \omega'} \right)^2} \tag{10}$$

$$A = \sum_{\alpha} \frac{\Omega_{\alpha}}{kV_{T||\alpha}} \left[ \frac{\Omega_{\alpha} - \omega}{\Omega_{\alpha}} (J + 1) \frac{V_{T\perp\alpha}^2}{V_{T||\alpha}^2} - 1 \right] \exp \left( - \frac{(\omega - \Omega_{\alpha})^2}{k^2 V_{T||\alpha}^2} \right)$$

$$B = \frac{\Omega_b}{kV_{T||b}} \left[ \frac{\Omega_b - \omega'}{\Omega_b} (J + 1) \frac{V_{T\perp b}^2}{V_{T||b}^2} - 1 \right] \exp \left( - \frac{(\omega' - \Omega_b)^2}{k^2 V_{T||b}^2} \right)$$

$$C = \sum_{\alpha} \left[ \frac{1}{(\Omega_{\alpha} - \omega)^2} + \frac{(n_b/n_c)(\omega')}{\Omega_b} \left\{ \frac{\omega\Omega_b - kV_D\omega + kV_D\Omega_b + k^2V_D^2}{\omega^2(\Omega_b - \omega + kV_D)^2} \right\} \right]$$

where, summation stands for the sum of ion species  $\alpha$  ( $H^+$ ,  $He^+$ , and  $O^+$ ),  $\omega' = \omega - kV_D$ ,  $(J + 1) \frac{V_{T\perp\alpha}^2}{V_{T||\alpha}^2} - 1$  is the effective thermal anisotropy of core plasma and  $(J + 1) \frac{V_{T\perp b}^2}{V_{T||b}^2} - 1$  is the effective thermal anisotropy of beam,  $\Omega_b$  is cyclotron frequency of hydrogen beam.

Here it is noticed that the ion beam ( $V_D$ ) has affected the growth rate and change in energy for the electromagnetic ion cyclotron waves propagating parallel to the magnetic field with a general loss-cone distribution function for the core plasma.

### 6 Growth Length

The growth length of the electromagnetic ion cyclotron wave is derived from (Tiwari et al. 2008; Patel et al. 2011b)

$$L_g = \frac{V_g}{\gamma} \tag{11}$$

Where,  $\gamma$  is growth rate,  $V_g$  is group velocity of the wave.

$$V_g = \frac{d\omega}{dk}$$

$$\frac{d\omega}{dk} = \frac{M}{N} \tag{12}$$

Where

$$M = \sum_{\alpha} \omega_{pz}^2 (\omega^2 V_D + 2\eta k V_D^2 \Omega_{\alpha} - 2\eta k V_D^2 \omega - 2\eta \omega V_D \Omega_{\alpha} + 2\eta \omega^2 V_D)$$

$$+ \omega_{pe}^2 (A_c \Omega_{\alpha}^2 V_D + A_c \Omega_{\alpha} \omega V_D) + (-3c^2 k^2 \Omega_{\alpha} V_D \omega + c^2 k^2 \Omega_{\alpha}^2 V_D + 2c^2 k^2 \Omega_{\alpha}^3$$

$$- 4c^2 k \Omega_{\alpha}^2 \omega + 2c^2 k^2 \Omega_{\alpha} \omega^2)$$

$$N = \sum_{\alpha} \omega_{pz}^2 (-2\omega \Omega_{\alpha} + 3\omega^2 - 2\omega \Omega_{\alpha} k V_D - 2\eta \omega \Omega_{\alpha} + 3\eta \omega^2 + \eta k^2 V_D^2 + 2\eta k V_D \Omega_{\alpha} - 4\eta \omega k V_D)$$

$$+ \omega_{pe}^2 (2A_c \Omega_{\alpha}^2 - 2A_c \Omega_{\alpha} \omega + A_c \Omega_{\alpha} k V_D) + (-2c^2 k^2 \Omega_{\alpha}^2 + 2c^2 k^2 \Omega_{\alpha} \omega - c^2 k^3 \Omega_{\alpha} V_D)$$

Where,  $\eta = \frac{n_b}{n_c}$ , is the ratio of beam density to core density.

$$\gamma = \frac{P}{Q} \tag{13}$$

Where  $P = \sum_{\alpha} \Omega_{\alpha}(A + (n_b/n_c)B)$

$$Q = c^2k^2C - \sum_{\alpha} \frac{c^2k^2 \omega_{pe}^2}{\omega^2 \omega_{p\alpha}^2} A_c + \sum_{\alpha} \frac{c^2k^2}{\omega_{p\alpha}^2} + \sum_{\alpha} \frac{1}{2} \left( \frac{2\Omega_{\alpha} - \omega}{\Omega_{\alpha} - \omega} \right)^2 + \frac{n_b/n_c}{2} \left( \frac{2\Omega_b - \omega'}{\Omega_b - \omega'} \right)^2$$

Thus growth length is obtained from using Eqs. (11, 12, 13) as

$$L_g = \frac{M/N}{P/Q} \tag{14}$$

The above equation represents the expression for growth length, which shows the length required for wave excitation in hot plasma in presence of multi-ions.

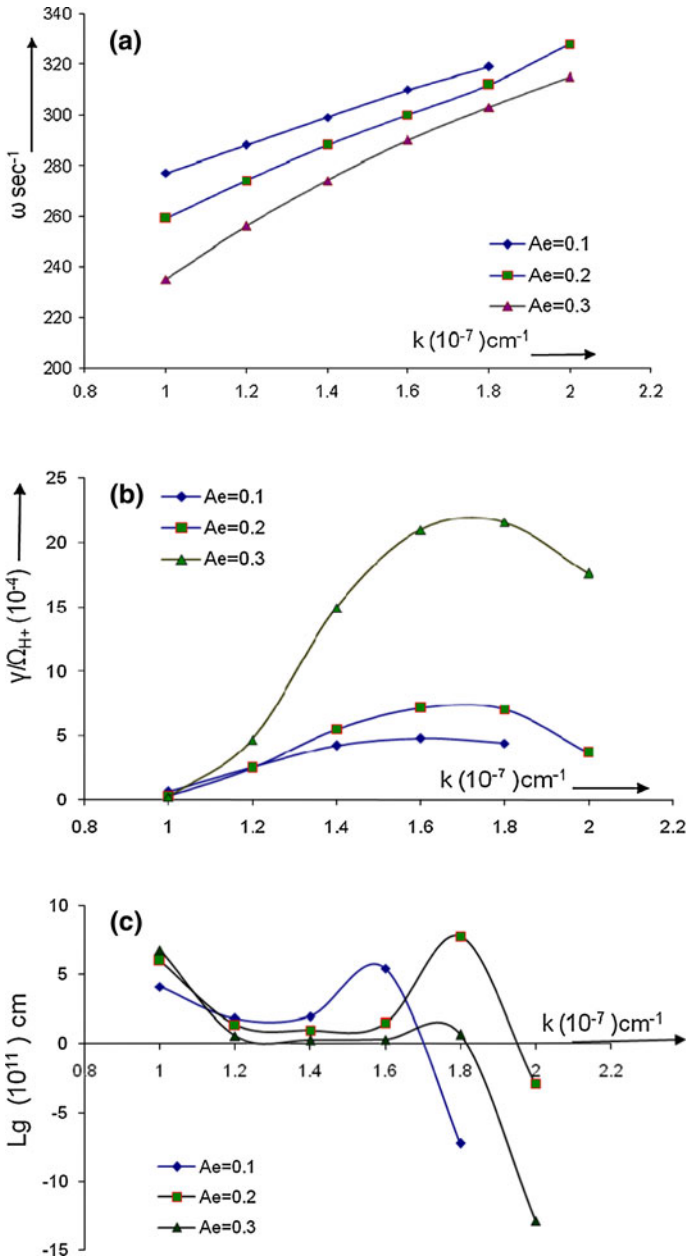
### 7 Result and Discussion

In the numerical calculation of the frequency, growth rate of the wave, perpendicular heating (perpendicular resonant energies of particles), and growth length of the wave, we have used the following parameters for the auroral acceleration region at 1.4 R<sub>E</sub> altitude (R<sub>E</sub> is the earth’s radius) (Mishra and Tiwari 2006; Patel et al. 2011a; Ahirwar et al. 2010; Gomberoff and Cuperman 1981) as

$$\begin{aligned} B_0 &= 4,300 \text{ nT} & \Omega_H^+ &\approx \Omega_b = 412 \text{ s}^{-1} & \Omega_{He}^+ &= 102.5 \text{ s}^{-1} & \Omega_O^+ &= 25.625 \text{ s}^{-1} \\ A_e &= \frac{V_{T\perp e}^2}{V_{T\parallel e}^2} = 0.1 - 2 & A_i &= \frac{V_{T\perp i}^2}{V_{T\parallel i}^2} = 10 - 50 & T_{\perp e} &= 25 - 50 \text{ eV}, & V_{T\parallel i} &= 6.41 \times 10^8 \text{ cm/s} \\ \omega_{pe}^2 &= 3.18 \times 10^8 \text{ s}^{-2} & \omega_{pHe^+}^2 &= 2.156 \times 10^5 \text{ s}^{-2} & \omega_{pO^+}^2 &= 2.156 \times 10^4 \text{ s}^{-2} & \eta = \frac{n_b}{n_c} &= 0.04 \end{aligned}$$

The Equations 3, 8, 9, 10, and 14 have been evaluated numerically using Matlab software to solve the growth rate and energy exchange between wave and particles. We have considered isotropic ion beam in the numerical calculations.

Figure 1a–c show the variation of wave frequency, growth rate and growth length of the wave versus wave vector k at distribution index J = 2 for different values of electron thermal anisotropies for multi-ions hot anisotropic plasma. Here we considered the density of ions as  $n_{H^+} = 0.9$ ,  $n_{He^+} = 0.05$ , and  $n_{O^+} = 0.05 \text{ cm}^{-3}$ . It is observed that the effect of increasing values of  $\frac{V_{T\perp e}^2}{V_{T\parallel e}^2}$  of the background plasma is to enhance the growth rate and to reduce the growth length that may be due to the free energy of anisotropic electron core plasma. It is also observed that the growth length is reduced at lower value of the wave vector k. At  $k > 1.4 \times 10^{-7} \text{ cm}^{-1}$  growth length is tending towards the maximum, and decreases at the higher values of wave vector k. The decrease in wave frequency by electron thermal anisotropy is noted as seen in Fig. 1a. A similar effect of thermal anisotropy was discussed by Gamayunov et al. (1993). The calculation of  $L_g$  involves the group velocity  $\frac{d\omega}{dk}$  which can be estimated from Fig. (1a), since variations in  $\omega$  and  $\gamma$  with respect to k are not on a linear scale, it is typical to predict the relation for  $L_g$  based upon the figures.



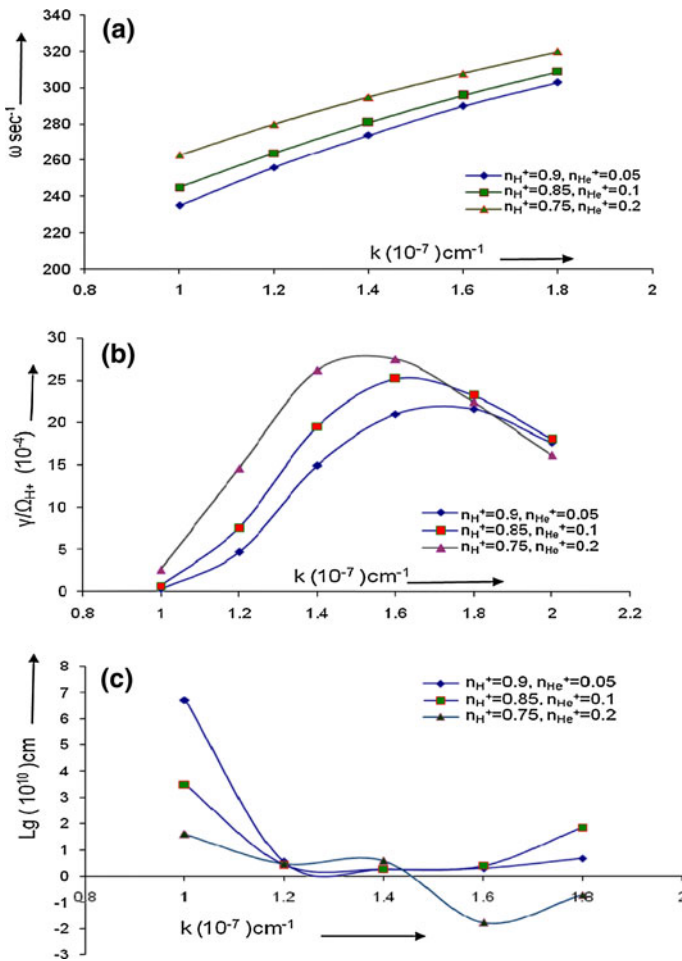
**Fig. 1** (a–c) Variation of wave frequency ( $\omega$ ), Growth rate ( $\gamma/\Omega_{H^+}$ ) and growth length ( $L_g$ ) respectively versus  $k$  at fixed values of  $V_D = -2 \times 10^7$  cm/s,  $\eta = 0.04$ , ion ( $H^+$ ,  $He^+$ ,  $O^+$ ) densities at different values of electron thermal anisotropies  $A_e$  for hot plasma at distribution index  $J = 2$

We have taken  $A_e \sim 3 \times 10^{-1}$  assuming that electrons are accelerated along the auroral field lines and their perpendicular acceleration is small. Thus thermal anisotropy is very small for core of electrons, however the opposite is true for ions as the ions are mainly

accelerated perpendicular to the auroral field lines. We have taken large thermal anisotropy for ions as perpendicularly heated ions are always reported in auroral acceleration region (Lund et al. 2000). However perpendicularly heated electrons are rarely observed, therefore, electron thermal anisotropy is taken small. Thus  $A_i$  was taken much larger (up to 20) whereas  $A_e$  much smaller of the order of 0.3.

In this paper we have considered EMIC wave around hydrogen ion cyclotron frequency. Thus  $H^+$  ions are in cyclotron resonance with the wave and participate more effectively in energy exchange process as compared to  $He^+$  ions. The definition of growth length evolves the group velocity which is negative at higher  $k$  as  $\omega$  is the decreasing function  $k$  whereas growth rate is positive. This results negative values of growth length which may be unphysical.

Figure 2a–c show the variation of wave frequency, growth rate and growth length of the wave versus wave vector  $k$  at distribution index  $J = 2$  for different values of helium ion



**Fig. 2** Variation of wave frequency ( $\omega$ ), Growth rate ( $\gamma/\Omega_{H^+}$ ) and Growth length ( $L_g$ ) versus  $k$  at fixed values of  $V_D = -2 \times 10^7$   $cm/s$ ,  $\eta = 0.04$ ,  $A_e = 0.3$  and  $n_{o^+} = 0.05$   $cm^{-3}$  for different  $He^+$  ion densities with respect to  $H^+$  ion densities for hot plasma at  $J = 2$

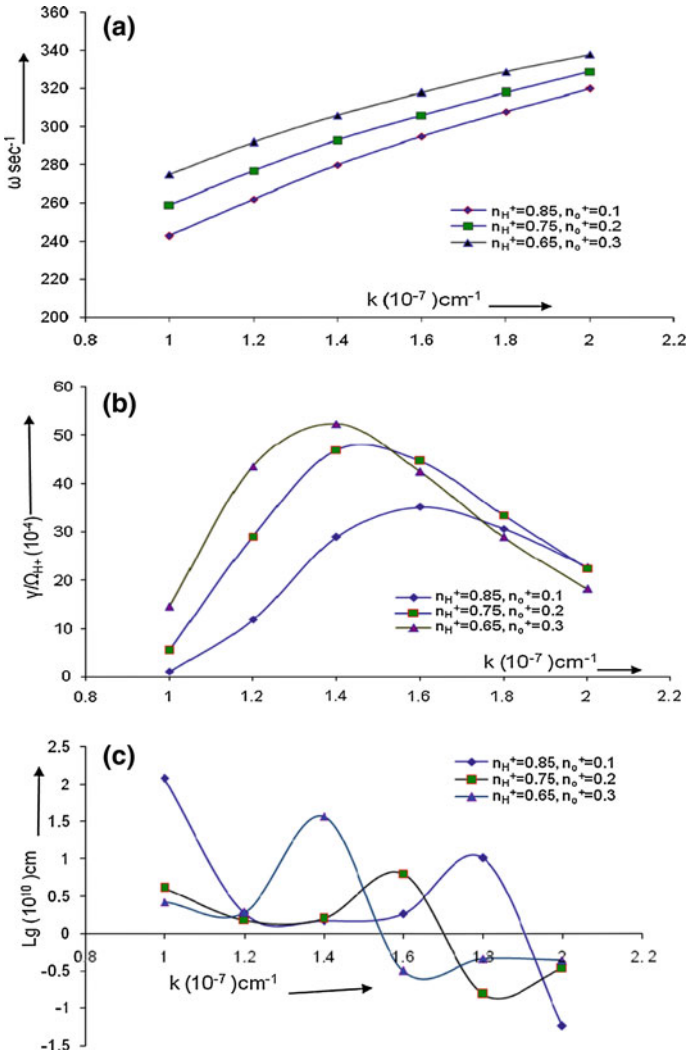


density balanced with the hydrogen ion density at fixed value of oxygen ion density  $n_{O^+} = 0.05 \text{ cm}^{-3}$ . In this study we observed that the effect of helium density on wave frequency, growth rate and on growth length for multi-ions hot anisotropic plasma. It is noticed that the effect of increasing the values of  $n_{He^+}$  of the background plasma is to enhance the growth rate and to reduce the growth length. The frequency is increasing with increasing the values of  $He^+$  ion density. Ahirwar et al. (2010) have stated that the FAST (Fast Auroral Snapshot) satellite has detected a class of ions conic events in which  $He^+$  is more strongly accelerated than  $H^+$  or  $O^+$  ions. The helium ions in these events typically have 3–30 times the energy of simultaneously observed protons, unlike most conic in which no strong preferential acceleration occurs (Khazanov et al. 2006).

EMIC waves are widely investigated (e.g., Thorne and Horne 1997) in the high altitude low geomagnetic field equatorial region in a different plasma environment to those seen in the high latitude auroral acceleration region. The transverse acceleration of ions is a usual and normal process in the high latitude auroral ionosphere that is approximately coincident with the auroral oval. In past transversely accelerated ions and their association with EMIC waves have been reported by the analysis of FAST satellite observations (Lund et al. 2000). The steep loss-cone affects the heating rate of the transversely accelerated ions through the EMIC instability in the auroral acceleration region. The auroral acceleration region accelerates both ions upward along magnetic field lines, and electrons downward, and is, of course, better known because it creates the visible aurora. Ions that do precipitate in the energy exchange process are predominantly due to the direct injection into the loss cone in the region, we consider here as the auroral acceleration region. The heating and acceleration of heavier ions and their contribution towards precipitation in association with ion beam may interpret different observations as compared to the equatorial region.

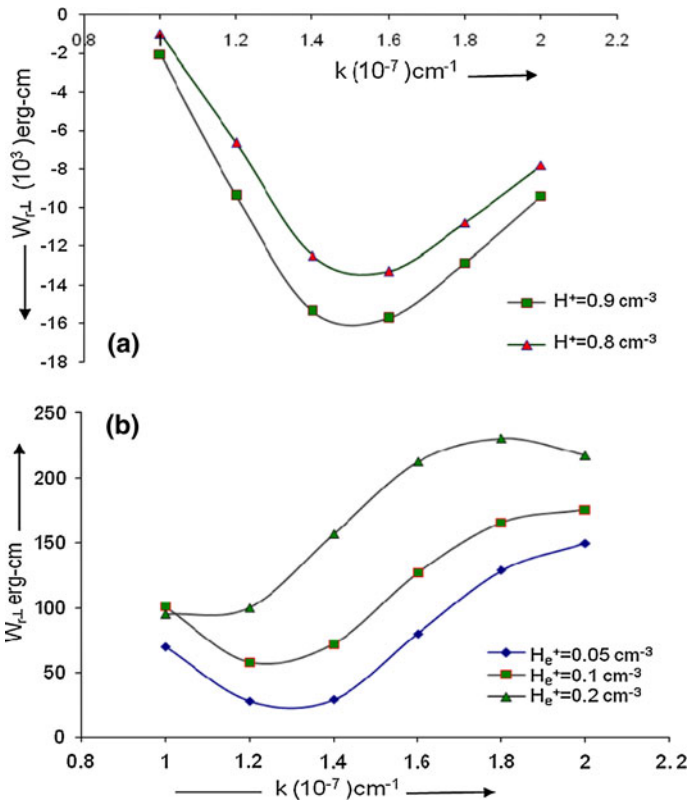
Figure 3a–c show the variation of wave frequency, growth rate and growth length of the wave versus wave vector  $k$  at distribution index  $J = 2$  for different values of oxygen ion density balance with the hydrogen ion density at fixed value of helium ion density  $n_{He^+} = 0.05 \text{ cm}^{-3}$ . In this study we observed that the effect of oxygen ion density on wave frequency, growth rate and growth length for multi-ions hot anisotropic plasma. It is noticed that growth rate is increasing and growth length is decreasing with the increase in the  $O^+$  ion density. We observed that the growth rate is higher for  $O^+$  ions density as compared to the  $He^+$  ions density. During great magnetic storms, when a sufficiently high mixture of ionospheric  $O^+$  ions is observed, a switch to a stronger instability with  $\gamma \propto (n_{O^+})^{1/2}$  is possible (Lund 1998). The effects of density of  $O^+$  ions on wave frequency as shown in Fig. 3a is to increase the frequency at lower wave numbers but less effective for larger  $k$ . The growth length  $L_g$  varies with wave vector  $k$  and depends upon the group velocity and growth rate of the wave. It is smaller and minimized to zero at some values of  $k$  showing that wave has achieved enough growth rates on the scale of the problem. However, the higher growth length at various  $k$ , the growth rate is very small and cannot predict the excitation of wave on the auroral scale length. We have considered the homogeneous plasma for simplicity; however extensive study is required for inhomogeneous plasma of auroral acceleration region.

In past (Gomberoff and Neira 1983; Horne and Thorne 1993) have shown that the generation and propagation characteristics of EMIC waves in a multi-ion ( $H^+$ ,  $He^+$ ,  $O^+$ ) plasma are profoundly controlled by ion fractional composition. Guha and Ruby (1990) has stated that Multi-ion species plasma exists in space as well as in laboratory plasmas (Dash et al. 1984; Sharma 1986; Mille and Krstic 1987).



**Fig. 3** (a–c) Variation of wave frequency ( $\omega$ ), Growth rate ( $\gamma/\Omega_{H^+}$ ) and Growth length ( $L_g$ ) versus  $k$  at fixed values of  $V_D = -2 \times 10^7$   $\text{cm/s}$ ,  $\eta = 0.04$ ,  $A_e$  for different  $O^+$  ion densities with respect to  $H^+$  ion densities at  $n_{He^+} = 0.05 \text{ cm}^{-3}$  for hot plasma

Figure 4a, b shows the variation of perpendicular resonant energy (perpendicular heating) of the particles versus wave vector  $k$  at distribution index  $J = 2$  for different values of hydrogen ion density (Fig. 4a) and helium ion density (Fig. 4b). In this study we observed that the effect of hydrogen ion density on energy transfer is greater than the helium ion density. We also observed that the resonant energy for helium ion is positive whereas for same scale perpendicular resonant energy for hydrogen is negative. Thus  $H^+$  ions are cooled and  $He^+$  ions are heated by the interaction of EMIC waves. The effect of their densities is to increase in the cooling/heating process.



**Fig. 4** (a, b) Variation of perpendicular energy ( $W_{\perp}$ ) versus  $k$  for different values of  $H^+$  ion density and in second for different values of  $He^+$  ion density at  $J = 2$

In past electromagnetic ion-cyclotron instability has been analyzed by many workers (e.g. Kennel and Petschek 1966; Cornwall et al. 1971; Gomberoff 1992) by considering resonant electrons of much higher energies and neglecting thermal anisotropies of electrons. In the auroral ionosphere there are accelerated electron parallel to the magnetic field and ionospheric plasma is cold enough (Wygant et al. 2000, 2002) and thus electron also contribute towards the dispersion properties of EMIC waves (Patel et al. 2011a). Considering anisotropic distribution function and detailed analysis of plasma dispersion function to higher orders the contribution of electron anisotropies is also noticed (Patel et al. 2011a), which may be suitable for discussion of EMIC wave growth.

According to Omidi et al. (2010) the presence of multiple ion species modifies the dispersive properties of EMIC waves and thus they occur in distinct frequency bands between the multi-ion gyrofrequencies. The dispersion relation, and hence propagation characteristics are dependent upon the relative concentrations of the various heavy ion species. They have stated that these ions also affect reflection of the EMIC waves in the inner magnetosphere, and the coupling between the left and right hand polarized modes. In addition to affecting the linear properties of EMIC waves, the presence of heavy ions can lead to generation or suppression of wave growth in certain bands due to nonlinear wave-particle interactions (Thorne and Horne 1997).

## 8 Conclusions

Present investigation describes a useful mathematical model developed for hot multi-ion ( $H^+$ ,  $He^+$  and  $O^+$ ) plasma for studies of the electromagnetic ion cyclotron waves in auroral acceleration region. The effect of increasing electron thermal anisotropies is to enhance the growth rate due to the interaction of electrons with EMIC waves. The fractional densities of heavier ions enhance the wave frequency and growth rate but reduce the growth length significantly. The minimum values of growth length are significant for auroral scale considered in the analysis. Thus, the densities of multi-ions should be in consideration in the analysis of EMIC wave instabilities in the auroral acceleration region. The effect of ions ( $H^+$ ,  $He^+$  and  $O^+$ ) densities on perpendicular heating of the plasma particles in hot plasma is predicted. The cooling of  $H^+$  ions perpendicular to magnetic field was found whereas heavier ions  $He^+$  are heated. Present investigations show that EMIC waves are probably not generated in the auroral acceleration region as the growth rates are smaller, so the situation considered is one where EMIC waves generated elsewhere are passing through auroral acceleration region. The conclusion of the work is that the auroral acceleration region has very little effect on EMIC waves and the processes mentioned do affect particle population in the auroral acceleration region.

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