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On A New Semantics for First-Order Predicate Logic

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Introduction Aldo Antonelli's untimely death is a sad loss to our profession. We have been asked by the editors of the *Journal of Philosophical Logic* to write a short comment on his most recent work, published in this issue, based in part on a referee report by one of us, that, unfortunately, did not reach Aldo in time.

Here is what appeals to us in the innovative work in [4, 5]. There is a long history of attempts to reanalyze the semantics of first-order predicate logic, the most basic system in our field. What many of these attempts have in common is a search for specific parameters in the 'standard semantics' given by Tarski that might be naturally modified or generalized. A further motive has been the issue whether the famous 'undecidability of predicate logic' is truly an intrinsic inescapable property of this system, or a side effect of decisions concerning its semantic design that could

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have gone differently. In our paper [3], we presented one such reanalysis, going back to earlier work in relativized cylindric set algebra (*Crs*), making the choice of variable assignments, or modal 'states', an explicit parameter in first-order models, and modifying the semantics of quantifiers accordingly. This generalized semantics validates a decidable core logic inside standard first-order logic, and we were also able to show that this system is closely tied to the Guarded Fragment, a large decidable slice of first-order logic under its standard semantics. In a recent paper [2], we returned to the issue of generalizing existing semantics via Henkin-style modifications of models, covering second-order moves, algebraic approaches, and others, and we thought that we had pretty much covered all existing strategies.

A New Semantics and Logic Against this background, here is a surprising new angle, overlooked so far to the best of our knowledge – although we will find some precursors later on in work on generalized quantifiers from the 1970s. Antonelli [4] proposes a non-standard semantics for languages with arbitrary generalized quantifiers. Applied to the existential quantifier of first-order logic, this semantics reads as follows:

> $\boldsymbol{M}, s \models Ex.\varphi$ iff (a) there exists an object d in $\boldsymbol{M}$ with $\boldsymbol{M}, s[x{:=}d] \models \varphi$, and (b) the set of all witnesses d of this sort belongs to a family $P^{\mathbf{M}}$ of subsets that has been specified in advance as part of the model.[1]

In what follows, we will use the notation $Qx.\varphi$ for arbitrary generalized quantifiers, $Ex.\varphi$ for the generalized reading of the existential quantifier described just now, and $\exists x.\varphi$ for the standard existential quantifier of first-order logic.

The paper [5] in this volume shows that the induced logic over ordinary first-order syntax is recursively axiomatizable. The author gives a Henkin-style completeness proof, as well as an adequate semantic tableau system, for a proof calculus with respect to his generalized semantics. The calculus derives sequents $\Gamma \vdash \varphi$ that are valid in the usual local sense, from truth of the premises under an assignment to truth of the conclusion under that same assignment. This allows for a standard deduction theorem removing a premise from Γ in order to conditionalize a conclusion φ. The calculus has three principles:

- T All propositionally valid rules of inference.
- EXT Equivalence Rule: If x does not occur free in Γ, from $\Gamma \vdash \alpha \leftrightarrow \beta$, infer $\Gamma \vdash Ex.\alpha \leftrightarrow Ey.[y/x]\beta$, with $[y/x]\beta$ an alphabetic variant of β.
- UG Universal Generalization: If x does not occur free in Γ, from $\Gamma \vdash \alpha$, infer $\Gamma \vdash \neg Ex.\neg\alpha$.

Moreover, it is shown that full first-order logic arises syntactically from this base logic when one adds all instances of the axiom $\varphi \rightarrow Ex.\varphi$. It is also proved that

[1] Technically, Antonelli's semantics uses a function f assigning to every set S a subset $f(S)$ of the full power set $P(S)$. Any quantifier Q then gets its usual denotation taken with respect to a Henkin-style 'generalized power set' $P^{\mathbf{M}}$ for the domain of any model $\boldsymbol{M}$. Specialized to the first-order existential quantifier $\exists$, with standard denotation $\exists(S) = \{X \subseteq S : X \text{ non-empty}\}$, clause (a) then reflects the meaning of $\exists$, while (b) comes from the nonstandard power sets.

first-order logic arises semantically from the base logic by requiring the non-standard power sets in models to be closed under first-order parametrical definability. Finally, the paper proposes a proof for the decidability of the new base logic via an effective translation into the Guarded Fragment.

As Antonelli notes, we have the beginnings of a new program here, studying the spectrum of logics and semantics in between his base logic and full first-order logic. Indeed, the more we looked at his system, the more several interesting things started striking us. In what follows, we make a few observations and suggestions strengthening this general perspective – though we have a qualification about the claimed results that we will explain in due course.

System Variations For a start, analyzing the above syntax and semantics suggests a few natural variations. We list a few. By *'basic A-logic'* we mean Antonelli's complete logic given above. It retains an essential feature of standard first-order semantics: variables are independent of each other. The truth value of a formula φ in a model $\boldsymbol{M}$ under an assignment s only depends on the objects in $\boldsymbol{M}$ assigned by s to the free variables in φ. This is reflected in allowing alphabetic variants in the Equivalence Rule.

Typically, this independence fails in generalized semantics of the above-mentioned modal or *Crs* type: values to individual variables outside of the formula may matter since not all variable assignments may be admissible in a model. In semantics of the latter kind, the Equivalence Rule only holds in the weaker version

EXT' from $\vdash \alpha \leftrightarrow \beta$ to $\vdash$ Ex. $\alpha \leftrightarrow Ex.\beta$.

This is the basic rule of replacing provable equivalents in standard algebraic logics – being the minimum required for a compositional semantics of the quantifier.

It would be of interest to merge the two lines of extending first-order semantics: generalized power sets, and allowing dependencies, but we will not do so in any detail.[2] Another proof system, and arguably the base logic for Antonelli's style of analysis, arises if we drop even the (UG) rule, and merely retain (T) and (EXT). Then we get basic classical propositional logic with an added generalized quantifier $Qx.\varphi$ interpreted by any family of subsets:

$\boldsymbol{M}, s \models Qx.\varphi$ iff $\{d \text{ in } \boldsymbol{M} \mid \boldsymbol{M}, s[x{:=}d] \models \varphi\} \in \mathrm{P}^{\mathbf{M}}$

The earlier truth condition for the existential quantifier *Ex* then refers to a generalized quantifier satisfying the further condition that all its subsets are non-empty. In particular, we see this reflected in the rule of Universal Generalization in basic *A*-logic. In the presence of his Equivalence Rule, (UG) amounts to just adding one special axiom

$\neg Ex.\bot$[3]

[2]Uniformity is achieved in [4] by using the same generalized quantifier for all variables. A weaker version would give each variable x its own quantifier, moving to a neighborhood version of the modal *Crs*-style semantics in [3]. (This suggestion was made by Wes Holliday.) We will return to the matter of uniformity of the Antonelli semantics below.

[3](T) and (UG) prove $\neg Ex.\bot$, using an empty set of assumptions. Conversely, if we can prove φ from a Γ not containing x free, then we can prove $\neg\varphi \leftrightarrow \bot$ by (T), and then $Ex.\neg\varphi \leftrightarrow Ex.\bot$ by the Equivalence Rule, whence we get $\Gamma \vdash \neg Ex.\neg\varphi$ using the formula $\neg Ex.\bot$ as an axiom, by applying the (T) rule.

In all, an interesting landscape of new weak first-order logics is opening up here, starting from very weak systems, and then progressively adding further features such as monotonicity or distributivity. This landscape lends itself to comparative analysis in terms of deductive power, but also in terms of translations, relative interpretations, or other connections.

Algebraic Content and Proof Analysis Playing with basic A-logic reveals more combinatorial content than might show at the surface. Thus, in assessing this system, syntax and proof theory play a role in addition to semantic considerations about its models. This is why we will include a few formal derivations in what follows. In particular, we find it instructive to look for algebraic equations that the quantifiers satisfy in the Lindenbaum-Tarski algebra of the logic. Basic A-logic obviously lacks laws of monotonicity or distribution over disjunction, but it does validate, for instance, this basic prenex distribution law:

$Ex.\ (\alpha \wedge Ex.\ \beta) \leftrightarrow (Ex.\ \alpha \wedge Ex.\ \beta)$[4]

Furthermore, with this useful principle in place, it is easy to show, for instance, how all the further laws of a standard proof system for first-order logic, such as monotonicity or distribution of E over $\vee$, become explicitly derivable from basic A-logic when we add the earlier-mentioned axiom $\varphi \rightarrow Ex.\varphi$ of Existential Generalization.[5] Incidentally, the interest in this syntactic exercise is not so much in quickly retrieving standard first-order logic from basic A-logic, but rather to get a concrete feeling of what precise proof power returns us to the undecidable system that we started with.

As for other system observations, it also makes sense to go down in power from basic A-logic, rather than up. For instance, it is easy to see that

Fact $Ex.\ Ex.\ Ex.\ \alpha \leftrightarrow Ex.\ Ex.\ \alpha$ is valid in basic A-logic, but no longer valid in the weaker system of basic Qx-logic. But $Qx.\ Qx.\ Qx.\ Qx.\ \alpha \leftrightarrow Qx.\ Qx.\alpha$ is still valid in this weaker system.[6]

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[4]Here is a proof by the above rules. (a) From the (T)-valid $\{Ex.\ \beta\} \vdash \alpha \leftrightarrow (\alpha \wedge Ex.\ \beta)$, using (EXT), derive $\{Ex.\ \beta\} \vdash Ex.\ \alpha \leftrightarrow Ex.\ (\alpha \wedge Ex.\ \beta)$. It follows by (T) that $\vdash (Ex.\ \alpha \wedge Ex.\ \beta) \rightarrow Ex.\ (\alpha \wedge Ex.\ \beta)$. (b) By using (T) once more, $\{Ex.\ \beta\} \vdash Ex.\ \alpha \leftrightarrow Ex.\ (\alpha \wedge Ex.\ \beta)$ also implies that $\{Ex.\ \beta,\ Ex.\ (\alpha \wedge Ex.\ \beta)\} \vdash Ex.\ \alpha$ (c) Next, from the (T)-valid $\{\neg Ex.\ \beta\} \vdash (\alpha \wedge Ex.\ \beta) \leftrightarrow \bot$, using (EXT), derive $\{\neg Ex.\ \beta\} \vdash Ex.\ (\alpha \wedge Ex.\ \beta) \leftrightarrow Ex.\ \bot$, and using (T) plus the earlier proof of $\neg Ex.\ \bot$, derive that $\vdash Ex.\ (\alpha \wedge Ex.\ \beta) \rightarrow Ex.\ \beta$. Putting (b) and (c) together, by applications of (T), we derive that $\vdash Ex.\ (\alpha \wedge Ex.\ \beta)) \rightarrow (Ex.\ \alpha \wedge Ex.\ \beta)$.

[5]These facts are easy to prove using the auxiliary inference rule: "from $\vdash \phi \rightarrow Ex.\ \psi$ to $\vdash Ex.\ \varphi \rightarrow Ex.\ \psi$" that can be derived from our prenex law.

[6]For a change, we give a semantic proof for these claims. Let $\boldsymbol{M}$ be a model and s an assignment of objects to variables. For any formula φ, set $X(\varphi, s) :=\{d \in \boldsymbol{M}:\boldsymbol{M}, s[x :=d] \models \varphi\}$. Now, the set $X(Ex.\ \varphi, s)$ is always the whole domain of $\boldsymbol{M}$ or the empty set $\emptyset$, by the truth definition. Thus, quantifier iterations with the same variable only involve two issues: whether Q accepts $\emptyset$, and whether it accepts $\boldsymbol{M}$. Checking all 4 combinations for this, it is easy to see that 3 of them (including the Antonelli quantifier itself) satisfy the equivalence $Ex.\ Ex.\ \alpha$- $Ex.\ Ex.\ Ex.\ \alpha$. However, the fourth quantifier Q with $\emptyset \in P^{\mathbf{M}}$ and with $\boldsymbol{M} \notin P^{\mathbf{M}}$ keeps switching its truth values for iterations in each round, validating only $Ex.\ Ex.\ \alpha$- $Ex.\ Ex.\ Ex.\ Ex.\ \alpha$. Indeed, the latter principle holds in all 4 cases. We could even go further with these prima facie somewhat unusual principles, and classify generalized quantifiers by their behavior on the preceding iteration laws.

Semantic Correspondence Analysis As for the landscape of logics suggested by Antonelli, it is intuitively clear how various intermediate axioms express conditions on generalized quantifiers lending themselves to immediate semantic analysis, now reading formula variables in a second-order sense as ranging over all subsets of the model. Construed in this way, for instance, the additional axiom of

Existential Generalization $\varphi \rightarrow Ex.\varphi$

says that all non-empty sets belong to the Antonelli generalized quantifier, which, with the non-emptiness for the base logic, makes that quantifier the standard one. Similar analyses work for monotonicity or distribution laws for the existential quantifier.

Two Broader Perspectives Let us now look at two streams of work in the earlier literature that connect with the program of exploring weak predicate logics sketched here. In each case, we only make a few observations, mostly without proof. We believe that an interesting conglomerate of topics is coming to light here concerning decidable semantics and decidable fragments for first-order logic, that we will address at greater length in a follow-up paper.

Logics of Generalized Quantifiers Logics with added generalized quantifiers have been studied extensively since the 1950s, although these systems largely consisted of systems *FOL(Q)*, that is, first-order logic in its standard interpretation with some new generalized quantifier *Q* added. Going one step further, [9] considers systems that add the generalized quantifier to a propositional base without the standard first-order quantifiers present – though still retaining the standard assumption, coming already from Mostowski and Lindström, that the sets in the quantifier be closed under isomorphism, making the quantifier express essentially a numerical criterion. In particular, [1] showed that the weakest such logic is decidable, using a semantic tableau technique. We cannot survey this work here, but merely note that it shows formal resemblances to Antonelli's program, even though the generalized quantifiers in Antonelli's semantics are not necessarily closed under isomorphic images.[7]

These remarks are just a start. We believe that many existing results and themes from current generalized quantifier theory (cf. the survey [8]) can be brought to bear on generalized semantics for weak predicate logics.

Intermezzo: Antonelli's Proof of Decidability This is a good point to mention a worry that occurred to us in thinking through Antonelli's decidability proof for his logic via reduction to the Guarded Fragment. We believe that the result is correct, but that the argument as stated is flawed – though in an interesting way.

[7] Anapolitanos and Väänänen [1] do consider non-isomorphism-closed models for generalized quantifiers as an intermediate stage in their argument before giving an ingenious construction that guarantees permutation invariance. Conversely, with the same specialization in mind, one could investigate versions of closure under isomorphism for Antonelli's semantics.

Antonelli's proof gives a translation *tr* from the basic quantifier language that involves the following essential clause:

$$tr(Qx.\varphi(x,\boldsymbol{y})) =$$
$$\exists u(G^{+}(x,u,\boldsymbol{y}) \wedge \forall z(G^{+}(z,u,\boldsymbol{y}) \rightarrow tr(\varphi(z,\boldsymbol{y})) \wedge \forall z(G^{-}(z,u,\boldsymbol{y}) \rightarrow \neg tr(\varphi(z,\boldsymbol{y})))$$

It is easy to show that, if a formula in Antonelli's language has a model as described above, its translation is in the Guarded Fragment, and it has a two-sorted standard model that treats subsets as new objects, with an obvious interpretation of the two guard predicates. Next, one shows that, if a guarded formula $tr(\varphi)$ has an arbitrary standard model, this model induces what Antonelli calles a 'multi-dimensional model' for φ where the generalized quantifier now consists of a set of *finite tuples* of objects: these sets arise because of the dependence of the guard predicates on the tuples of objects assigned to the parameters $\boldsymbol{y}$. Interpretation of $Qx.\ \varphi(x,\ \boldsymbol{y})$ then checks whether the set of witness tuples $(e,\ \boldsymbol{d})$, with $\boldsymbol{d}$ the fixed objects assigned to the parameter variables $\boldsymbol{y}$, belongs to the quantifier.

Finally, Antonelli claims that we can go one step further: basic A-logic as defined above is also sound for the broader class of multi-dimensional models, and hence φ is consistent, and hence by his completeness theorem, φ has an intended model.

A Problem We believe that the final claim of soundness for multi-dimensional models is incorrect, since it breaks down on a delicate point of notation. The rule (EXT) is indeed valid in the multi-dimensional semantics, if the premise $\Gamma \mid\!- \alpha \leftrightarrow \beta$ involves formulas *with the same free variables* $x,\ \boldsymbol{y}$, as is assumed in Antonelli's soundness proof. However, the general rule of basic A-logic does not assume this equality of variables, and it should not – since in general, we need the inhomogeneous case, say to prove the validity

$$Qx.Px \leftrightarrow Qx.(Px \wedge y{=}y)$$

But in the latter case, there is no guarantee that the two quantifiers introduced in the conclusion, referring to sets of tuples of different arities, support an equivalence – and one can also see that Antonelli's guarded translations do not yield equivalent formulas here.

Even so, we believe that Antonelli's decidability result is correct, and that the preceding difficulty can be fixed by several methods: changing the translation to one going into the larger decidable 'loosely guarded fragment', 'preprocessing' the formulas first modulo validity to improve performance of the translation, or reducing to generalized quantifier results like those for the generalized semantics of [1]. However, we also see the result as provable by just using natural direct techniques for establishing decidability, such as the 'mosaics' of [7]. We defer a proof to a follow-up paper.

Local Generalized Semantics However this may be, the above difficulty can also be turned into a positive point, since there is independent interest to the case where the proof does work. Antonelli's guarded translation does establish the following

Fact A-validity is decidable over multi-dimensional models.

This modified observation concerns a natural 'local' variant of the original uniform semantics where the model had just one generalized quantifier for the Q in all contexts. In contrast, multi-dimensional models have a family of quantifiers $Q^{\mathbf{d}}$ depending on the tuples of objects interpreting the free variables in formulas $Qx.\varphi(x, \mathbf{y})$. This distinction between natural local and uniform variants will return below.

We conclude with two comments. First, we believe that Antonelli's semantics in either uniform or local variants has the Finite Model Property. Moreover, this raises the interesting issue of finding the exact computational complexity for either local or uniform A-logic.

Next we turn to a final related perspective, that connects up with both Antonelli's semantics and generalized quantifier theory in a natural way.

Modal Neighborhood Models Several of the preceding points suggest one more analogy that may be fruitful in thinking about Antonelli's program, namely, with *modal logic*.

For a start, the preceding correspondence results for generalized quantifier axioms are reminiscent of modal correspondence theory for axioms over frames. Also, the spirit of the *Crs* semantics or its equivalent generalized assignment semantics – that we have mentioned as a useful comparison case – is modal. Typically, *Crs* models validate monotonicity and distribution over disjunctions, just as in the minimal modal logic.

But if there is a modal angle here, why are the latter principles absent from basic A-logic? The analogy we see here is with a well-known semantic move toward generality in modal logic, from relational graph models to *neighborhood models* where each world has a family of neighborhoods attached to it. In such models,

> A box modality $\Box\varphi$ is true at world s if φ holds throughout some neighborhood of s – or in a still more minimal version that drops even upward monotonicity, if the set of all worlds where φ is true is a neighborhood of s.

Now note that a family of neighborhoods as used here is, essentially, just an arbitrary generalized quantifier. Thus the conceptual step from *Crs* models to Antonelli's generalized models seems similar to that from relational to neighborhood models in modal logic. In this light, one might see modal neighborhood languages as fragments of the full first-order language with respect to the generalized semantics discussed here.

We believe that this analogy may be a fruitful one, including the taking of themes from the quite active area of modal neighborhood semantics to our current setting, such as the model theory of appropriate generalized notions of bisimulation, or the introduction of richer modal languages suggested by neighborhood models.

In this setting, an earlier point returns in an illuminating form. Neighborhood models for modal logic have local families of sets depending on the current world. This is a special case of the above-mentioned 'local semantics' for generalized quantifiers $Q^{\mathbf{d}}$.[8] Accordingly, basic neighborhood logic is local, and this is reflected in

[8] In this connection, note also that *Crs* semantics has variable assignments as its basic items, not the individual objects themselves as in [4] (cf. also the point made in Footnote 2) – and this difference mattered, e.g., to treating dependence and independence of variables.

its validities, which are axiomatized by a minimal proof system that just contains replacement of equivalents and propositional inference – with upward monotonicity of the modality added for the monotonic version.

If we want a uniform version with only one family of neighborhoods (generalized quantifier) throughout, new principles of inference are needed, such as the rule that (EXT) is always allowed from sets of boxed premises. We will not pursue these analogies with A-semantics here, except to note that they go through even in small details.[9] We end with a few more general points about the program considered here.

What is the Right Language? Often generalized semantics suggest richer languages with more distinctions than the original language over the initial models. For instance, the above-mentioned *Crs* semantics has new 'polyadic quantifiers' $\exists xy\ldots\ \varphi$ introducing tuples of objects in a way that is no longer definable from iterated single quantifiers, though the base logic of polyadic quantifiers remains decidable. Antonelli, too, discusses such quantifiers, but it is not clear to us if these represent a substantial extension to what might be the natural formalism for his generalized quantifier models. What logical language best fits these models?[10]

A less radical approach would look at basic A-logic, or its underlying more general quanti-fier Q-logic, adding the standard existential quantifier $\exists$ and perhaps others with their usual meanings, the same way we kept the standard Boolean operators fixed in his logic. This richer language allows us to move some of the earlier semantic observations (e.g., those on correspondence) into the object language. For example, the non-emptiness condition in the semantics of the quantifier *Ex* is expressed by the first-order formula

$$Ex.\varphi \rightarrow \exists x.\varphi$$

We forego further exploration of this multi-quantifier system.

Wider Semantics, or Narrower Fragments? A general theme in our own work has been a search for precise correspondences between two perspectives: (a) generalizing a semantics for a whole logical language, and (b) sticking to standard semantics for a suitably chosen matching *fragment* of that language.

What fragment of first-order logic then matches basic A-logic? One answer is the subset of the Guarded Fragment that one gets through Antonelli's translation for the local semantics (modulo the above qualifications). Are there more perspicuous matching fragments? And what about the weaker base logic that we obtained by restricting the Equivalence Rule to its algebraic essentials – or the still weaker logic of one arbitrary generalized quantifier?[11]

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[9]For instance, Antonelli's guarded translation shows similarities with the embedding of the basic neighborhood logic into standard relational polymodal logic in [6].

[10]One candidate are relational generalized quantifiers of the Lindström type $Qxy.\ \varphi$ and higher, as a way of controlling polyadic quantification in first-order models. Other candidates would be extended modal languages for neighborhood models.

[11]For instance, define the following 'Sandwich-Guarded Fragment' *SGF* of first-order logic: (i) Atomic formulas are in *SGF*, (ii) *SGF* is closed under Boolean connectives, (iii) $\exists x[S(x, \mathbf{y}) \wedge \forall v(G(x, v, \mathbf{y}) \rightarrow \varphi(x, v, \mathbf{y})) \wedge \forall v(H(x, v, \mathbf{y}) \rightarrow \neg\varphi(x, \mathbf{y}))$ is in *SGF* whenever $\varphi(x, \mathbf{y})$ is in *SGF*. Does *SGF* correspond to the above weaker local base-logic, or does it have other nice properties of its own?

Summary With this brief note, we hope to have shown that Antonelli's generalized first-order semantics opens up new lines of inquiry that are well-worth investigating, while it also ties in naturally with two major existing research programs: logics for generalized quantifiers, and modal logics with neighborhood semantics.

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