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CORRECTION

Correction to: Solution to the OK Corral Model via Decoupling of Friedman's Urn

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In this note we acknowledge a mistake made in Section 6 of [1], for the exact probability for the number of survivors in the OK Corral model, given near the bottom of page 275.

Here is the correct (and, in fact, simpler) computation. Let ν be the (random) number of survivors on one of the sides, when the other side is exterminated. Observe that

$$\mathsf{P}(\nu \le S) = 2\mathsf{P}\left(\sum_{k=S+1}^{N} k\xi_k < \sum_{k=1}^{N} k\zeta_k\right) = 2\mathsf{P}(\eta_S > 0)$$

where the factor "2" comes from the fact that each side is equally likely to survive, and

$$\eta_S = \sum_{k=1}^{N} k\zeta_k - \sum_{k=S+1}^{N} k\xi_k.$$

For a λ sufficiently close to 0, we can compute the Laplace transform of η_S as

$$\varphi_{\eta_S}(\lambda) = \mathsf{E}\, e^{\lambda\eta_S} = \prod_{k=1}^{N} \frac{1}{1+\lambda k} \cdot \prod_{k=\sigma+1}^{N} \frac{1}{1-\lambda k} = \sum_{k=1}^{N} \frac{a_k}{1+\lambda k} + \sum_{k=\sigma+1}^{N} \frac{b_k}{1-\lambda k}$$

The original article can be found online at https://doi.org/10.1023/A:1022294908268.

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where the constants

$$a_k = a_k(N, S) = \frac{(-1)^{N-k} k^{2N-S-1} (k+S)!}{(N-k)!(k-1)!(N+k)!}$$

are obtained using the partial fractions decomposition. Since we can invert the above Laplace transform and obtain that the density of η_S is given by

$$f_{\eta_S}(x) = \sum_{k=1}^{N} a_k \frac{e^{-x/k}}{k} \mathbf{1}_{\{x>0\}} + \sum_{k=\sigma+1}^{N} b_k \frac{e^{x/k}}{k} \mathbf{1}_{\{x<0\}},$$

we can compute

$$\mathsf{P}(\nu \le S) = 2\mathsf{P}(\eta_S > 0) = 2\int_0^\infty f_{\eta_S}(x)dx = 2\sum_{k=1}^{N} a_k(N, S).$$

Consequently, since $\mathsf{P}(\nu = S) = \mathsf{P}(\nu \le S) - \mathsf{P}(\nu \le S-1)$, we conclude that

$$\mathsf{P}(\nu = S) = 2\sum_{k=1}^{N} [a_k(N, S) - a_k(N, S-1)] = 2\,S \sum_{k=1}^{N} \frac{(-1)^{N-k} k^{2N-S} (k+S-1)!}{(N-k)!\,k!\,(N+k)!}$$

for $S = 1, 2, \dots, N$.

Note that the formula at the bottom of page 275 for $\mathsf{P}(\nu = 5)$ is actually correct.

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Reference

1. Kingman, J.F.C., Volkov, S.E.: Solution to the OK Corral model via decoupling of Friedman's urn. J. Theor. Probab. **16**(1), 267–276 (2003)