

Correction to: An Itô Formula in the Space of Tempered Distributions

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The following corrections are required in Theorem 4.7 of [2].

Recall that the following condition is included in the assumptions (see [1, Chapt. 6, Sect. 2]) made on the coefficient \bar{F} ; for example, there exists $K > 0$ such that

$$\int_{0 < |x| < 1} |\bar{F}(y, x)|^2 \nu(dx) \leq K (1 + |y|^2), \forall y \in \mathbb{R}. \quad (1)$$

Consequently, we have

$$\begin{aligned} & \int_0^t \int_{(0 < |x| < 1)} |\bar{F}(X_{s-}, x)|^2 \nu(dx) ds \\ & \leq \int_0^t K (1 + |X_{s-}|^2) ds \leq K \left(1 + \sup_{s \in [0, t]} |X_{s-}|^2 \right) t, \end{aligned}$$

and hence, we get a.s.

$$\int_0^t \int_{(0 < |x| < 1)} |\bar{F}(X_{s-}, x)|^2 \nu(dx) ds < \infty, \forall t \geq 0. \quad (2)$$

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The integrability condition (2) was originally stated as an assumption.

However, we now make a new integrability assumption. a.s.

$$\int_0^t \int_{(0 < |x| < 1)} |\bar{F}(X_{s-}, x)|^4 \nu(dx) ds < \infty, \quad \forall t \geq 0. \quad (3)$$

Note that, if \bar{F} is bounded, using (2), (3) is satisfied.

Correction to Proof of Theorem 4.7 Equation (4.9) is to be read as

$$\Delta X_t = \bar{F}(X_{t-}, \Delta N_t) \mathbb{1}_{(0 < |\Delta N_t| < 1)} + \bar{G}(X_{t-}, \Delta N_t) \mathbb{1}_{(|\Delta N_t| \geq 1)}.$$

As such, the first observation mentioned in the article does not hold. However, due to (2), (3) and eq. (4.1), we still get

$$\begin{aligned} & \sum_{s \leq t} [\tau_{X_s} \phi - \tau_{X_{s-}} \phi + \Delta X_s \partial \tau_{X_{s-}} \phi] \\ &= \int_0^t \int_{(0 < |x| < 1)} \left(\tau_{\bar{F}(X_{s-}, x)} - Id + \bar{F}(X_{s-}, x) \partial \right) \tau_{X_{s-}} \phi \tilde{N}(ds dx) \\ &+ \int_0^t \int_{(0 < |x| < 1)} \left(\tau_{\bar{F}(X_{s-}, x)} - Id + \bar{F}(X_{s-}, x) \partial \right) \tau_{X_{s-}} \phi \nu(dx) ds \\ &+ \int_0^t \int_{(|x| \geq 1)} \left(\tau_{\bar{G}(X_{s-}, x)} - Id + \bar{G}(X_{s-}, x) \partial \right) \tau_{X_{s-}} \phi N(ds dx). \end{aligned}$$

The proof now can be continued as before. \square

References

1. Applebaum, D.: Lévy Processes and Stochastic Calculus, 2nd edn. Cambridge Studies in Advanced Mathematics, vol. 116. Cambridge University Press, Cambridge (2009)
2. Bhar, S.: An Itô formula in the space of tempered distributions. J. Theoret. Probab. **30**(2), 510–528 (2017)