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ERRATUM

Erratum to: Return Probabilities for the Reflected Random Walk on $\mathbb{N}_0$

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In the original publication of this paper, we fix a constant $\mathbf{K} > 1$ and consider the set $\mathcal{K}(\mathbf{K})$ of functions $K : \mathbb{Z} \to \mathbb{R}^+$ satisfying the following conditions:

$$\forall x \in \mathbb{N}_0 \quad K(x) \geq 1, \quad \mathcal{R}K(x) \leq 1 \quad \text{and} \quad K(x) \sim \mathbf{K}^x. \tag{1}$$

Unfortunately, the two first conditions readily imply $K = 1$ since the operator $\mathcal{R}$ is markovian, so that the three above conditions cannot be satisfied simultaneously.

In fact, we will simply consider the function $K : s \mapsto \mathbf{K}^x$. The only one reason for the condition $\mathcal{R}K(x) \leq 1$ appeared in the proof of Fact 4.4.1, where the peripherical spectrum of the operators $\mathcal{R}_s$ for $|s| = 1$ and $s \neq 1$ is controlled. With this new choice of function K, one gets

Fact 4.4.1 *For* $|s| = 1$ *and* $s \neq 1$ *one gets* $\|\mathcal{R}_s\|_K < 1$; *in particular, the spectral radius of* $\mathcal{R}_s$ *on* $(\mathbb{C}_0^{\mathbb{N}}, \|\cdot\|_K)$ *is* < 1.

Proof We could adapt the proof proposed in the paper and show that $\|\mathcal{R}_s^{2n}\|_K \leq C\rho_s^n$ for some $\rho_s < 1$ when $s \neq 1$. We propose here another simpler argument.

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Recall that $\mathcal{R}_s$ acts from $(\mathbb{C}^N, |\cdot|_K)$ into $(\mathbb{C}^N, |\cdot|_\infty)$ and that the identity map is compact from $(\mathbb{C}^N, |\cdot|_\infty)$ into $(\mathbb{C}^N, |\cdot|_K)$. Consequently, the operator $\mathcal{R}_K$ is compact on $(\mathbb{C}^N, |\cdot|_K)$ with spectral radius ≤ 1 since it has bounded powers.

Let us fix $s \in \mathbb{C}\backslash\{1\}$ with modulus 1 and assume that $\mathcal{R}_s$ has spectral radius 1 on $(\mathbb{C}^N, |\cdot|_K)$; since it is compact, there exists a sequence $\mathbf{a} = (a_x)_{x\in\mathbb{Z}} \neq 0$ and $\theta \in \mathbb{R}$ such that $\mathcal{R}_s\mathbf{a} = e^{i\theta}\mathbf{a}$, i.e.

$$\text{for all } x \in \mathbb{Z}: \quad \sum_{y\in\mathbb{Z}} \mathcal{R}_s(x, y)a_y = e^{i\theta}a_x. \tag{2}$$

It follows that $|a_y| = |a_0| \neq 0$ for any $y \in \mathbb{Z}$ since $\sum_{y\in\mathbb{Z}} |\mathcal{R}_s(x, y)| \leq \sum_{y\in\mathbb{Z}} \mathcal{R}_s(x, y) = 1$; without loss of generality, we may assume $|a_y| = 1$, i.e. $a_y = e^{i\alpha_y}$ for some $\alpha_y \in \mathbb{R}$. The equality (2) may be thus rewritten

$$\text{for all } x \in \mathbb{Z}: \quad \sum_{y\in\mathbb{Z}} \mathcal{R}_s(x, y)e^{i\alpha_y} = e^{i\alpha_x}.$$

By convexity, using again the inequality $\sum_{y\in\mathbb{Z}} |\mathcal{R}_s(x, y)| \leq \sum_{y\in\mathbb{Z}} \mathcal{R}_s(x, y) = 1$, one readily gets $e^{i\alpha_y} = e^{i\theta}e^{i\alpha_x}$ for any $x, y \in \mathbb{Z}$; consequently, $e^{i\theta} = 1$, the sequence $\mathbf{a}$ is constant and $R_s(x, y) = \mathcal{R}(x, y)$ for any $x, y \in \mathbb{Z}$, which implies in particular $s = 1$. This is a contradiction. □

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