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ERRATUM

Erratum to: Asymptotic Estimates of the Distribution of Brownian Hitting Time of a Disc

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In my paper [1] it is purported stated that Theorem 1 immediately follow from Lemmas 4 and 5, which statement is incorrect (at least for x fixed the error estimate of Theorem 1 does not follow from Lemmas 4 and 5). For it to be correct, the following slight modification of Lemma 4 is sufficient.

Lemma 4′ *Uniformly for* $|x| > r_0$, *as* $t \to \infty$

$$p_{r_0,x}(t) - q_x(t) = O\left(\frac{1+\lg^+|x|}{t^2(\lg t)^2} \wedge \frac{1}{|x|^4\left(1+\lg^+|x|\right)}\right)$$

and the difference $p_{r_0,x}(t) - q_x^c(t)$ *admits the same estimate.*

If $t^\delta < |x| < t^{1/2}$ for some $\delta > 0$, this is the same as Lemma 4 of [1]. Hence for the proof of Lemma 4′ we can suppose that $|x| \le t^{1/4}$ and it suffices to prove

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{2iu}{g(iu)} w(u) K_0(|x|\sqrt{2iu})e^{itu}\,du = O\left(\frac{1+\lg^+|x|}{t^2(\lg t)^2}\right) \tag{1}$$

in place of Eq. (16) (in [1]). (Here $g(z) = -\lg(2^{-1}e^{\gamma} r_0\sqrt{2z})$ and w is a smooth function that equals 1 in a neighborhood of the origin and vanishes outside a finite

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interval.) Let $r_0 = 1$ for simplicity. The leading term of $K_0(|x|\sqrt{2iu})$ is $g(ix^2u) = -\lg|x| + g(iu)$ and its contribution to the above integral equals

$$\frac{-\lg|x|}{2\pi}\int_{-\infty}^{\infty}\frac{2iu}{g(iu)}w(u)e^{itu}du + O\bigl(t^{-N}\bigr) = \frac{4\lg|x|}{t^2(\lg t)^2}\bigl(1+o(1)\bigr) \quad (t\to\infty),$$

where N may be any positive constant and the last equality may be derived by using Eq. (35), which implies $\int_{-\infty}^{\infty}(\lg iu)^{-1}e^{itu}\,du = [2\pi/t(\lg t)^2](1+o(1))$.

Put $h(z) = K_0(\sqrt{2z}) - g(z)$ and $V(u) = h(i|x|^2u)$. We must obtain a uniform bound for the integral $I := \int[w(u)uV(u)/g(iu)]\,e^{itu}\,du$. Noting that $h(z) = O(z\lg z), h'(z) = O(\lg z)$ and $h^{(j)}(z) = O(z^{-j+1})$ $(j = 2, 3, 4)$ for $z = iy$ with $y \in \mathbf{R}\setminus\{0\}$, we have

$$\begin{aligned}\bigl|V^{(j)}(u)\bigr| &\le C|x|^2\bigl[|u|^{-j+1}\bigl(1+\bigl|\lg\bigl(|u||x|^2\bigr)\bigr|\bigr)\bigr] \quad (j=0,1) \quad \text{and}\\ \bigl|V^{(j)}(u)\bigr| &\le C|x|^2|u|^{-j+1} \quad (j=2,3,4).\end{aligned} \tag{2}$$

We can integrate by parts twice the integral that defines I, transforming it into

$$I = -\frac{1}{t^2}\int_{-\infty}^{\infty}\frac{d^2}{du^2}\left[\frac{w(u)uV(u)}{g(iu)}\right]e^{itu}\,du.$$

Then we split the range of this integral at $|u| = 1/t$ and apply to the integral on $|u| > 1/t$ integration by parts twice more, which with the help of (2) gives for it the bound $O(x^2/t)$ valid uniformly for $|x| \le \sqrt{t}$ (cf. [2], Lemma 2.2), so that $I = O(x^2/t^3)$, a bound sufficient for the required estimate. This completes the proof of Lemma 4′.

In addition there are simple errors on p. 456: in the second formula on the fifth line from the bottom of the page the bound $O(y\wedge 1)$ must be replaced by $O(\lg(1+y))$ and also in the next line $O(e^{-2\sqrt{y}})$ by $O(y^{-1/4})$. These errors only require a few simple modifications to the arguments (tacitly) involved there.

References

1. Uchiyama, K.: Asymptotic estimates of the distribution of Brownian hitting time of a disc. J. Theor. Probab. **25**, 450–463 (2012). doi:10.1007/s10959-010-0305-8
2. Uchiyama, K.: The first hitting time of a single point for random walks. Electron. J. Probab. **16**(71), 1160–2000 (1911)