J Stat Phys (2017) 166:453–455
DOI 10.1007/s10955-016-1687-5

ERRATUM

Erratum to: On a Finite Range Decomposition of the Resolvent of a Fractional Power of the Laplacian

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Published online: 27 December 2016

Erratum to: J Stat Phys (2016) 163:1235–1246 DOI 10.1007/s10955-016-1507-y

The above paper [1] was marred by a number of errors as remarked in Sect. 10 of [2]. I thank G. Slade for bringing this to my attention. These occur in Sect. 3 of [1] where proofs are given for Theorem 1.1 I take this opportunity to point them out and correct them. The references to equations are to those in the above paper. Reference [3] cited in the above paper is cited as Ref. [3] in this erratum. The statement of Theorem 1.1 in [1] has some modifications as a result of these corrections. We also take this opportunity to correct errors about constants. We first state them: If we follow the decomposition in [3], instead of [4], and this will be the case here, then the constants c_p which occur in Sect. 3 should read $c_{L,p}$. As a consequence the constants $c_{p,\alpha}$ which occur in Sect. 3 and Theorem 1.1 should read $c_{L,p,\alpha}$.

The bounds (1.12) and (1.14) of [1] remain true but only for $j \geq 2$ with $0 \leq q \leq j$ and $c_{p,\alpha}$ replaced by $c_{L,p,\alpha}$. (1.13) obviously remains true. For $j = 0, 1$ with $j \geq q \geq 0$ we have to add the additional bound

$$||\partial^p_{\varepsilon_j} \Gamma_{j,\alpha}(\cdot, m^2)||_{L^\infty((\varepsilon_q \mathbb{Z})^d)} \leq c_{L,p,\alpha}(1 + m^2)^{-1} \tag{1.1}$$

Corollary 1.2 (1.17) remains true (with $c_{p,\alpha}$ replaced by $c_{L,p,\alpha}$) but for $j \geq 2$. For $j = 0, 1$ we now have an additional bound in the same Corollary

$$||\partial^p_{\mathbb{Z}^d} \tilde{\Gamma}_{j,\alpha}(\cdot, m^2)||_{L^\infty(\mathbb{Z}^d)} \leq c_{L,p,\alpha}(1 + L^{j\alpha} m^2)^{-1} L^{-(2j[\varphi]+pj)} \tag{1.2}$$

Coarse graining In order to get scale independence in the constants we can pass to a coarser scale L' and redefine fluctuation covariances by summing over the intermediate scales [5]. Let r be a positive integer and let $L' = L^r$ be the coarse scale. For L fixed we can make L'

The online version of the original article can be found under doi:10.1007/s10955-016-1507-y.

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large by making r large. Now redefine for $j \geq 0$ the coarse scale fluctuation covariance as follows:

$$\tilde{\Gamma}'_{j,\alpha}(\cdot, m^2) = \sum_{l=0}^{r-1} \tilde{\Gamma}_{l+jr,\alpha}(\cdot, m^2) \tag{1.3}$$

Then

$$\tilde{\Gamma}'_{j,\alpha}(x-y, m^2) = 0, \ |x-y| \geq (L')^{j+1} \tag{1.4}$$

Now it is easy to prove that for a fixed L the coarse grained fluctuation covariances satisfy the same bounds as above with new constants $c'_{L,p,\alpha}$ that are independent of the coarser scale L' for $0 < \alpha < 2$ and all $d \geq 2$. The essential reason is that the scaling dimension $[\varphi] = (d-\alpha)/2$ remains strictly positive in these cases. The coarse scale L' independence with respect to the bounds in [3] was found earlier in R. Bauerschmidt (unpublished data) for $d \geq 3$ but in $d = 2$ an additional $\log L'$ dependence was found.

Remark I emphasise that the $(1+m^2)^{-1}$ term in the bounds only occurs for the $j = 0, 1$ terms and *not* for $j \geq 2$ in contrast to [2] where it occurs for all terms.
We now turn to the corrections to be made in the proof supplied in Sect. 3 of [1].

1. In Appendix A of [3] interior regularity estimates (like those of Nirenberg and Agmon in the continuum) were obtained for the solution of a lattice Dirichlet problem for the minus lattice laplacian plus a mass squared parameter (called $a \geq 0$). This is called s in the present paper. As part of this estimate a linear decay in the mass squared parameter was given and this sufficed for the purposes of [3]. However at the end of Appendix A [3] an exponential type decay in the mass parameter was sketched following an Agmon type argument. However on a lattice this will not be true for an arbitrarily large mass parameter. Therefore the bound $e^{-cs^{\frac{1}{2}}}$ occuring in Eqs. (3.4), (3.6) and (3.7) of [1] cannot be used when integrating over s up to infinity in (3.13), (3.14), (3.15) and (3.16) of [1]. However the exponential estimates are not necessary as we will now see. We shall replace it by a weaker power law decay which will suffice for our purpose.
2. For $j \geq 2$ replace $e^{-cs^{\frac{1}{2}}}$ in (3.4) by $(1+s)^{-2}$, so that (3.4), [1] now reads for $j \geq 2$ with $0 \leq q \leq j$

$$||\partial^p_{\varepsilon_j} \Gamma_j(\cdot, s)||_{L^\infty((\varepsilon_q \mathbb{Z})^d)} \leq c_{L,p}(1+s)^{-2} \tag{1.5}$$

   This is a slight improvement of (5.11), Theorem 5.5 of [3] plus Sobolev embedding. In Theorem 5.5 of [3] the decay rate $(1+s)^{-1}$ was given. The symbol s above was called a in [3].

For both $j = 1$ with $0 \leq q \leq j$ as well as $j = q = 0$ we use the bound in (3.5), [1] namely

$$||\partial^p_{\varepsilon_j} \Gamma_j(\cdot, s)||_{L^\infty(\mathbb{Z})^d} \leq c_{L,p} \frac{1}{1+s} \tag{1.6}$$

Proof The bound (1.6) is part of Theorem 5.5 of [3]. So only the bound in (1.5) needs to be considered.

The bounds (3.6) and (3.7) in [1] now read

$$||\partial^p_c \Gamma_{c*}(\cdot, s)||_{L^\infty(\mathbb{R}^d)} \leq c_{L,p}(1+s)^{-2} \tag{1.7}$$

and for $j \geq 2$

$$||\partial_{\varepsilon_j}^p \Gamma_j(\cdot, s) - \partial_c^p \Gamma_{c*}(\cdot, s)||_{L^\infty((\varepsilon_q \mathbb{Z})^d)} \leq c_{L,p}(1+s)^{-2} L^{-\frac{j}{2}} \tag{1.8}$$

3. We will first prove (1.12) of Theorem 1.1 for $j \geq 2$. In the sentence before (3.13) of [1] replace $j > q \geq 0$ by $j \geq 2$ with $0 \leq q \leq j$. In (3.13), (3.14), (3.15) and (3.16) replace $e^{-cs^{\frac{1}{2}}}$ by $(1+s)^{-2}$. Then all integrals converge for $0 < \alpha < 2$ and the argument leading to (3.18) goes through and proves (1.12) of Theorem 1.1 for $j \geq 2$, $0 \leq q \leq j$ with the constant $c_{p,\alpha}$ replaced by $c_{L,p,\alpha}$. (1.14) of Theorem 1.1 is similarly proved for $j \geq 2$ with the constant replaced as before.
4. For $j = 0, 1$ we proceed otherwise to prove the additional bound (1.1) stated at the beginning of this erratum.
We replace (3.13) of [1] by

$$||\partial_{\varepsilon_j}^p \Gamma_{j,\alpha}(\cdot, m^2)||_{L^\infty((\varepsilon_q \mathbb{Z})^d)} \leq c_{L,p} c_\alpha \int_0^\infty ds \, \frac{s^{\alpha/2}}{s^\alpha + m^4} \frac{1}{1+s} \tag{1.9}$$

Define the integral above as

$$F_0(m^2) = \int_0^\infty ds \, \frac{s^{\alpha/2}}{s^\alpha + m^4} \frac{1}{1+s} \tag{1.10}$$

The integral converges for $0 < \alpha < 2$. This is a continuous monotonic decreasing function for increasing m^2 and is well defined for $m^2 = 0$ in the above range of α. For $m^2 \neq 0$ we obtain after some changes of variables

$$\begin{aligned} F_0(m^2) =& \frac{1}{m^2}(m^2)^{\frac{2}{\alpha}} \frac{2}{\alpha} \int_0^\infty dx \, \frac{e^{\frac{2}{\alpha}x}}{e^x + e^{-x}} \frac{1}{1 + ((m^2)^{\frac{2}{\alpha}} e^{\frac{2}{\alpha}x}} \\ \leq& \frac{1}{m^2} \frac{2}{\alpha} \int_0^\infty dx \, e^{-x} \\ \leq& \frac{2}{\alpha} \frac{1}{m^2} \end{aligned}$$

By continuity at $m^2 = 0$ we have for some constant c_α

$$F_0(m^2) \leq c_\alpha \frac{1}{1+m^2}$$

Using this bound in (1.9) proves the additional bound (1.2) for $j = 0, 1$. □

References

1. Mitter, P.K.: On a finite range decomposition of the resolvent of a fractional power of the Laplacian. J. Stat. Phys. **163**, 1235–1246 (2016)
2. Slade, G.: Critical exponents for long range $O(n)$ models below the upper critical dimension, arXiv:1611.06169
3. Brydges, D., Guadagni, G., Mitter, P.K.: Finite range decomposition of Gaussian processes. J. Stat. Phys. **115**, 415–449 (2004)
4. Bauerschmidt, Roland: A simple method for finite range decomposition of quadratic forms and Gaussian fields. Probab. Theory Relat. Fields **157**, 817–845 (2013)
5. Brydges, D.C.: Lectures on the renormalisation group. Statistical Mechanics. IAS/Park City Mathematics Series, vol. 16, pp. 7–93. American Mathematical Society, Providence (2009)