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ERRATUM

Erratum to: Relativistic Hardy Inequalities in Magnetic Fields

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A sign misprint in the statement of Proposition 3.1 in [1] has a consequence in the statement of the main Theorem 1.1 in [1]. We write a short note in which we present the correct statements.

The correct version of Proposition 3.1 in [1] is the following one.

Proposition 0.1 *Let* $A : \mathbb{R}^3 \to \mathbb{R}^3$. *For any* $\phi = \phi(x) \in \mathcal{C}_0^\infty(\mathbb{R}^3; \mathbb{C}^2)$, *the following identity holds*

$$\begin{aligned}\int_{\mathbb{R}^3} r\,|\sigma \cdot \nabla_A \phi|^2 = \int_{\mathbb{R}^3} r \left|\partial_r^A \phi\right|^2 dx \\ + \int_{\mathbb{R}^3} r \left|\frac{1}{r}(\sigma \cdot L_A + 1)\,\phi\right|^2 dx - \int_{\mathbb{R}^3} \frac{|\phi|^2}{r}\,dx \\ + \int_{\mathbb{R}^3} \langle \sigma \cdot [\partial_r (x \wedge A)]\phi, \phi\rangle\,dx - \int_{\mathbb{R}^3} \langle \sigma \cdot (x \wedge \nabla A_r)\,\phi, \phi\rangle\,dx\end{aligned} \tag{0.1}$$

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where $r := |x|$, $\partial_r^A := \frac{x}{r} \cdot \nabla_A$, *and* $A_r := A \cdot \frac{x}{r}$.

The main difference between Proposition 3.1 in [1] and the proposition above is the negative sign in from of the last term in (0.1).

We also underline that the identity (0.1) follows by the identities (3.7) and (3.11) in [1], whose proof given in [1], is correct.

In order to present a correct version of Theorem 1.1 in [1] we first need the following version of Proposition 2.1 in [1].

Proposition 0.2 *Let* $B : \mathbb{R}^3 \setminus \{0\} \to \mathbb{R}^3$, *and assume that there exists a vector potential* $A : \mathbb{R}^3 \to \mathbb{R}^3$ *such that the following holds*

$$A \cdot x = 0, \qquad A(\lambda x) = \lambda^{-1} A(x), \qquad curl\, A(x) = B(x), \tag{0.2}$$

for all $x \in \mathbb{R}^3 \setminus \{0\}$, $\lambda \neq 0$. *Then we have*

$$\partial_r\,(x \wedge A) - x \wedge \nabla \left(A \cdot \frac{x}{r}\right) = 0 \tag{0.3}$$

for $\in \mathbb{R}^3 \setminus \{0\}$.

Proof The proof is quite immediate. Due to the first condition in (0.2), one has $x \wedge \nabla\,(A \cdot x/r) = 0$. Moreover, since A is homogeneous of degree -1, then $x \wedge A$ is homogeneous of degree 0, hence $\partial_r(x \wedge A) \equiv 0$. □

We can now state the new and correct version of Theorem 1.1 in [1].

Theorem 0.1 *Let* $B : \mathbb{R}^3 \setminus \{0\} \to \mathbb{R}^3$, *and assume that there exists a vector potential* $A : \mathbb{R}^3 \to \mathbb{R}^3$ *such that the following holds*

$$A \cdot x = 0, \qquad A(\lambda x) = \lambda^{-1} A(x), \qquad curl\, A(x) = B(x), \tag{0.4}$$

for all $x \in \mathbb{R}^3 \setminus \{0\}$, $\lambda \neq 0$. *Then, for any* $\phi = \phi(x) \in \mathcal{C}_0^\infty(\mathbb{R}^3; \mathbb{C}^2)$, *the following inequality holds*

$$\mu_1 \int_{\mathbb{R}^3} \frac{|\phi|^2}{|x|}\, dx \leq \int_{\mathbb{R}^3} \frac{1}{|x|}\, |(\sigma \cdot L_A + 1)\, \phi|^2\, dx \leq \int_{\mathbb{R}^3} |x|\, |\sigma \cdot \nabla_A \phi|^2\, dx, \tag{0.5}$$

where $\mu_1 = \mu_1(A) = \inf\{spec(\sigma \cdot L_A + 1) \cap [0, \infty)\}$.

Proof The proof is an immediate corollary of Propositions 0.1 and 0.2, and the usual radial Hardy inequality. □

References

1. Fanelli, L., Vega, L., Visciglia, N.: Relativistic Hardy inequalities in homogeneous magnetic fields. J. Stat. Phys. **154**, 866–876 (2014)