

Erratum to: An integer linear programming formulation and heuristics for the minmax relative regret robust shortest path problem

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Let the *critical scenario* r of a path $P \subseteq A$ from $s \in V$ to $t \in V$ be the one, among all the possible scenarios, in which arc costs are set to $c_{ij}^r = u_{ij}$ for all $(i, j) \in P$, and $c_{ij}^r = l_{ij}$ for all $(i, j) \in A \setminus P$. The authors in [3] proved that the maximum regret of P over all scenarios occurs in its critical scenario. Therefore, the maximum regret of P is given by $R_P = C_P^r - C_S^r$, where S is the shortest path between s and t in the scenario r , and $C_P^r = \sum_{(i,j) \in P} c_{ij}^r$ and $C_S^r = \sum_{(i,j) \in S} c_{ij}^r$ are the costs of P and S in the scenario r .

This work focuses on finding the minmax *relative* regret of P in critical scenarios, as it is an intuitive bad situation [2]. The problem addressed aims at finding the path $P \subseteq A$ from s to t with the smallest value of $\widehat{R}_P = \frac{C_P^r - C_S^r}{C_S^r} = \frac{C_P^r}{C_S^r} - 1$. This robust optimization criterion has been studied by other authors in the scientific literature [1, 4, 5]. The motivation of using the minmax *relative* regret over the minmax regret criterion can be found in Sect. 1. The study considers that the maximum *relative* regret of P occurs in its critical scenario, as it

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is the case for other robust optimization criteria. However, we show below that this is not necessarily true. Nevertheless, we prove that the version of the robust shortest path problem studied is NP-Hard and point out that this version of the problem is suitable to applications sensitive to critical scenarios.

Proposition 1 *the maximum relative regret of P does not necessarily occur in its critical scenario.*

Proof The proof is given by a counterexample. Figure 1 shows an example of a Karasan instance [3] with six vertices and a path $P = [s, 1, 3, t]$ depicted with a dotted line. Figure 2 depicts the critical scenario r induced by P . The relative regret of P in r is $\frac{(40+30+10)}{(2+3+10)} - 1 = 4.33$. However, if the cost of arc $(3, t) \in P$ is set to 1, as shown in Fig. 3, one can obtain a scenario where the relative regret of P increases to $\frac{(40+30+1)}{(2+3+1)} - 1 = 10.83$, despite the fact that the cost of P is smaller in this scenario. Therefore, the maximum relative regret of P does not necessarily occur in its critical scenario. \square

In order to prove that the minmax *relative* regret in critical scenarios is NP-Hard, we first prove Lemma 1. This result is then used in the proof of Proposition 2 below. Let P_1 and P_2 be any two paths from s to t , and r_1 and r_2 be the critical scenarios of P_1 and P_2 , respectively. Besides, let also S_1 and S_2 be the shortest path from s to t in the scenarios r_1 and r_2 , respectively.

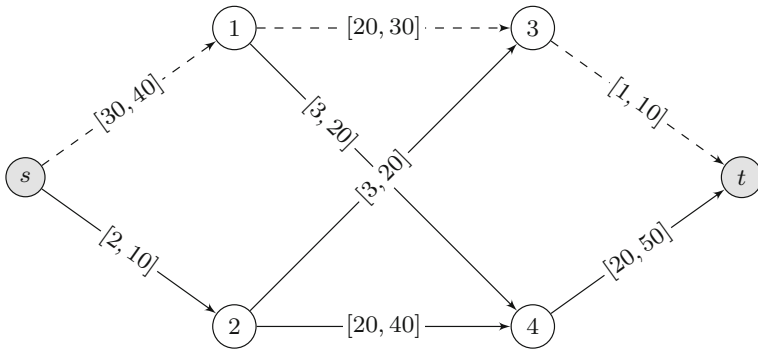


Fig. 1 Karasan instance with a dashed path P

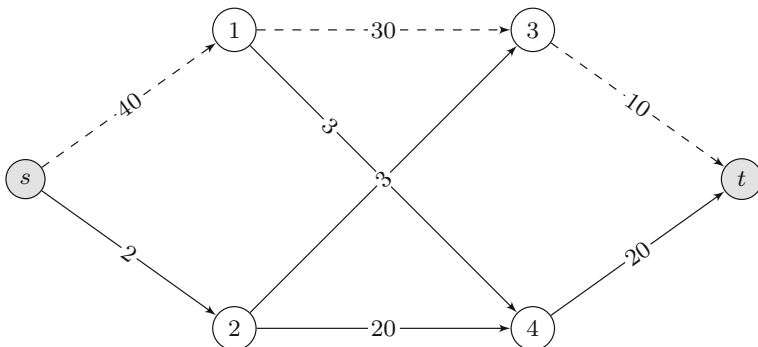


Fig. 2 Critical scenario induced by P

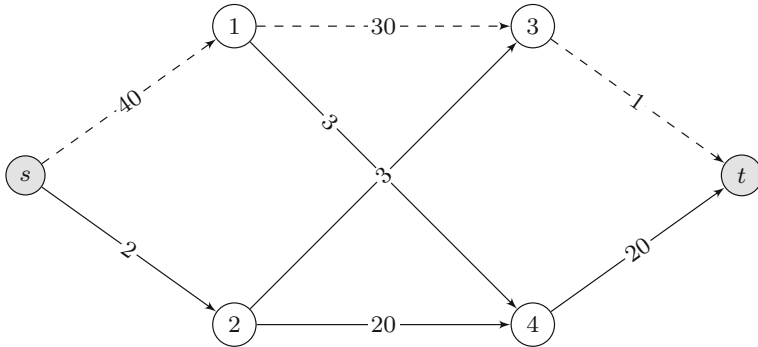


Fig. 3 An arbitrary scenario where the relative regret of P is larger than the one in its critical scenario

Lemma 1 Given two paths P_1 and P_2 , Eq. (1) holds for $M \geq (|V| \cdot U)^2$, where $U = \max_{(i,j) \in A} u_{ij}$.

$$C_{P_2}^{r_2} - C_{S_2}^{r_2} > C_{P_1}^{r_1} - C_{S_1}^{r_1} \implies \frac{C_{P_2}^{r_2} + M}{C_{S_2}^{r_2} + M} > \frac{C_{P_1}^{r_1} + M}{C_{S_1}^{r_1} + M}, \tag{1}$$

Proof We have that

$$\frac{C_{P_2}^{r_2} + M}{C_{S_2}^{r_2} + M} - \frac{C_{P_1}^{r_1} + M}{C_{S_1}^{r_1} + M} = \frac{(C_{S_1}^{r_1} + M) \cdot (C_{P_2}^{r_2} + M) - (C_{S_2}^{r_2} + M) \cdot (C_{P_1}^{r_1} + M)}{(C_{S_2}^{r_2} + M) \cdot (C_{S_1}^{r_1} + M)},$$

which can be rewritten as (2).

$$\frac{M \cdot (C_{P_2}^{r_2} - C_{S_2}^{r_2} - C_{P_1}^{r_1} + C_{S_1}^{r_1}) + C_{P_2}^{r_2} \cdot C_{S_1}^{r_1} - C_{P_1}^{r_1} \cdot C_{S_2}^{r_2}}{(C_{S_2}^{r_2} + M) \cdot (C_{S_1}^{r_1} + M)} \tag{2}$$

From the left-hand side of (1), we have that $C_{P_2}^{r_2} - C_{S_2}^{r_2} - C_{P_1}^{r_1} + C_{S_1}^{r_1} > 0$. Besides, we have by definition that $l_{ij} \in \mathbb{Z}_+^*$ and $u_{ij} \in \mathbb{Z}_+^*$ for all $(i, j) \in A$. Therefore, Eq. (3) holds.

$$C_{P_2}^{r_2} - C_{S_2}^{r_2} - C_{P_1}^{r_1} + C_{S_1}^{r_1} \geq 1. \tag{3}$$

Equation (3) together with $M \geq (|V| \cdot U)^2 > C_{P_1}^{r_1} \cdot C_{S_2}^{r_2}$ implies that (2) is greater than zero, which completes the proof. \square

Proposition 2 Solving the minmax relative regret RSP in the critical scenarios is NP-hard.

Let $f : \{(G, l, \widehat{u}, s, t)\} \mapsto \{(G', l, u, s', t)\}$ be a function that transforms an instance I of minmax regret RSP in an instance \widehat{I} of minmax relative regret RSP in critical scenarios, as defined by (4)–(7), where $M = (|V| \cdot U)^2$.

$$V' = V \cup \{s'\} \tag{4}$$

$$A' = A \cup \{(s', s)\} \tag{5}$$

$$G' = (V', A') \tag{6}$$

$$l_{(s',s)} = u_{(s',s)} = M \tag{7}$$

The worst case complexity of the transformation $I \rightarrow \widehat{I}$ is clearly polynomial. Given an optimal solution $\widehat{P}_2 \subseteq A$ to \widehat{I} , the path $P_2 = \widehat{P}_2 \setminus (s', s)$ is also optimal for I . Otherwise,

there would exist a solution P_1 to I with lower robust cost than the one of solution P_2 . However, from Lemma 1, $\widehat{P}_1 = P_1 \cup (s', s)$ would have a robust relative cost smaller than \widehat{P}_2 , which is a contradiction. Therefore, P_2 is indeed optimal for I . \square

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