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ASSESSING PRE-SERVICE TEACHERS' SKILLS FOR ANALYZING TEACHING[1]

ABSTRACT. This study investigated the learning-from-practice skills that pre-service teachers possess when they enter teacher preparation programs in the United States. Two subskills were hypothesized to represent, at least in part, what is required to learn from practice: (1) the ability to collect evidence about students' learning in order to analyze the effects of instruction, and (2) the ability to use the analysis to revise the instruction. Because it seems likely that different teaching situations and contexts reveal these learning-from-practice skills in different ways and to different degrees, this study examined the skills that pre-service teachers exhibited under two experimental conditions. Thirty pre-service teachers were asked to analyze the effects of a videotaped mathematics lesson on student learning, to support their analysis with evidence, and to use their analysis to revise the lesson. Based on the results, it appears that many entry level pre-service teachers can carry out a cause-effect type of analysis of the relationships between specific instructional strategies and students' learning, and can use this analysis to make productive revisions to the instruction. However, prospective teachers' ability to collect evidence that supports their analysis appears to be less developed. In addition, the type of analysis that prospective teachers carried out about the effects of instruction on students' learning differed dramatically across the two experimental task conditions.

KEY WORDS: knowledge for mathematics teaching, knowledge for mathematics teacher education, learning to teach, learning from practice

Learning to teach mathematics well is a challenging goal. Pre-service teachers rarely exit their mathematics teacher preparation program as experts. Rather, they must continue to learn while they are teaching. What skills are required to learn from practice? Although there are a variety of kinds of knowledge, skills, and dispositions that certainly would be useful, a central skill needed to study one's own teaching practice and that of others is analyzing teaching in terms of its effects on learning. What do students learn from particular instructional activities and how do the activities facilitate such learning? It is hard to imagine improving one's own teaching in a planned and systematic way without engaging in this kind of analysis.

One promising direction for improving the effectiveness of teacher preparation programs is to include explicit attention to developing these and other skills that enable learning from practice (Hawkins, 1973; Hiebert, Morris, & Glass, 2003; Nemser, 1983). What kinds of instructional activities might facilitate the development of such skills? Because little is known about the precise nature of these competencies, it is premature to design and test teacher education activities that aim to develop them. It is useful, however, to examine specific features of these skills by investigating the extent to which pre-service teachers already possess these skills when they enter teacher preparation programs. Just as mathematics teachers are better equipped to plan appropriate instruction for students when they understand students' entry competencies, so teacher educators will be better equipped to plan preparation programs and activities when they understand pre-service teachers' entry competencies (Ball, 1988). Given the long apprenticeship to teaching that pre-service teachers have served as students (Lortie, 1975; Nemser, 1983), it is likely that they have acquired some skills for observing and analyzing classroom practice.

The goal of this study is to describe the nature of beginning pre-service teachers' skills for analyzing teaching in terms of its effects on learning. What aspects of teaching and learning do beginning pre-service teachers pay attention to when they watch a lesson? How do they use this information to suggest improvements to the lesson? Because previous research suggests that observers of classroom lessons often attend to teachers more than students (e.g., Brown & Borko, 1992; Santagata, Zannoni, & Stigler, 2005), a particular focus of this study was on whether and how beginning pre-service teachers collect evidence about students' learning to inform their recommendations for revising a lesson. By describing the nature of beginning pre-service teachers' analyses, it might be possible to develop some informed conjectures about the kinds of educational activities that would be useful to include in teacher preparation programs.

To guide the investigation, two related subskills were hypothesized to represent, at least in part, what is required to analyze teaching in terms of its effects on learning: (1) the ability to collect evidence about student learning in order to analyze the effects of instruction, and (2) the ability to use the analysis to revise the instruction. An increasing number of teacher learning initiatives that center on teachers systematically studying and improving their practice, such as lesson study, require the two subskills (e.g., Hiebert et al., 2003; Lewis, 2002; Marton & Tsui, 2004; Saunders & Goldenberg, 2005). It is likely that these skills are not all or none—they are more or less developed in individual in-service and pre-service teachers—and that some skills develop before others. It is reasonable to posit, for example, that as these skills develop they show more relevant and focused attention to student thinking (Franke, Carpenter, Fennema, Ansell, & Behrend, 1998; Kazemi & Franke, 2004). This study examined whether and to what extent pre-service teachers exhibited these two subskills.

One way to explore pre-service teachers' skills for analyzing teaching is to provide them with an instance of practice and ask them to analyze it. It is likely, however, that the nature of the lessons pre-service teachers observe and the conditions under which they observe a lesson affect the kinds of analyses they produce. Different lessons in terms of topic, level of expectations, discourse pattern, and so on are likely to prompt different analyses. In addition, there is evidence that when in-service teachers begin their analysis of lessons with the belief that there are some problems to be fixed in the instruction, they bring different orientations and skills to the task (e.g., Fernandez, Cannon, & Chokshi, 2003; Saunders & Goldenberg, 2005). Thus, what pre-service teachers believe about the effectiveness of a lesson before they watch it is likely to influence their analyses. If pre-service teachers expect a lesson will be effective, they might focus on different aspects of the lesson than if they believe it will not be effective. In fact, if a lesson is expected to fail, in at least some respects, observers might attend to students' responses and teacher-student interactions differently in order to identify the instances and causes of the failures.

Because it is impossible, in a single study, to assess the effects of all these variables, the decision was made to hold constant the kind of lesson observed and vary the expectations regarding the effectiveness of the lesson. Two conditions were employed. In both conditions, pre-service teachers were asked to analyze the effects of a lesson on students' learning. In the first condition, pre-service teachers had the freedom to decide whether the lesson was successful, and to decide which instructional activities worked well and which did not. In the other condition, pre-service teachers were told that the lesson was not successful but were free to decide which activities might explain the failed learning. It was hypothesized that these two conditions would prompt different analyses from pre-service teachers with regard to what students in the class learned and why, but no predictions were made regarding the exact nature of these differences. The goal of this study was to investigate, under each condition, pre-service teachers' entry ability to collect evidence about student learning in order to analyze the effects of instruction and to use the analysis to suggest revisions to the instruction.

METHOD

Participants

Thirty pre-service elementary and middle school teachers, from a university in the northeastern region of the United States, volunteered to participate in the study. Their four-year undergraduate program for certification in elementary education (K-8) included general studies (English, science, mathematics, social science, and fine arts), additional courses in a selected discipline area (English, science, mathematics, or social science), professional studies (e.g., human development, professional issues, educational assessment, methods courses), and student teaching. Their preparation program in mathematics consisted of a sequence of three mathematics content courses, followed by a mathematics methods course. The goal was to investigate entry-level competency with respect to the two subskills; therefore participants were recruited from the first of the four courses.

Procedures

Participants were randomly assigned to one of the two conditions, with 15 participants in each. In the CL (Children's Learning) condition, the pre-service teachers had the freedom to decide whether the lesson was successful, and to decide which instructional activities worked well and which did not. In the SP (Sources of the Problems) condition, the task instructions indicated that the lesson was not successful. In both conditions, participants completed two tasks in individual work sessions.

CL (Children's Learning) condition

In the first task, the participants were informed that they would watch a fifth grade lesson[2] on the area of a rectangle and triangle, and that the videotape would also show the students working on the homework assignment for the lesson.[3] The researcher then described the homework assignment: One set of problems showed illustrations of rectangles and triangles partitioned into square units, and students had to find the areas of the figures. The second set showed rectangles and triangles, but in these figures, the heights of the triangles were drawn in, the dimensions of the figures were given, and students had to use the formula for the area of a rectangle or triangle to find the areas. Participants were then instructed to watch the videotape and to "form a hypothesis about what the children have learned and understand (exactly) by the end of the lesson.'' They were also asked to support their hypotheses with evidence. The participants were informed that they were free to form multiple hypotheses, but that if they did so, they should clearly identify each hypothesis, and the corresponding evidence for each hypothesis.

When participants completed the first task, they were given a transcript of the portion of the lesson on the area of a rectangle, and were asked to make revisions to this part of the lesson on the basis of their hypotheses. They numbered places in the transcript where they would make a change, and wrote a narrative that explained ''what [they] would do instead,'' and ''why [they] would do it that way'' for each change.

SP (Sources of the Problems) condition

In the first task, the participants were informed that they would watch a fifth grade lesson on the area of a rectangle and triangle, and that the videotape would also show the students working on the homework assignment for the lesson. The researcher then described the assignment. After the homework assignment was described, the researcher briefly and accurately described the students' behavior on the videotape as they worked on the assignment: The researcher said, ''During the homework assignment, several students asked for the teacher's help and appeared to be confused. Two students asked for assistance on the first set of problems, and many students asked which formula they should use for a particular figure in the second set of problems (the formula for the area of a rectangle or triangle).'' Participants were then asked to watch the videotape and to ''form a hypothesis about the source(s) of the children's difficulty at the end of the lesson'' and to support their hypotheses with evidence. The participants were informed that they were free to form multiple hypotheses, but that if they did so, they should clearly identify each hypothesis, and the corresponding evidence for each hypothesis. The lesson revision task was identical to that for the CL condition.

Both groups watched the entire videotape. Thus both groups of pre-service teachers observed the students' behavior as they worked on the homework.

Coding

Task 1

The pre-service teachers' ability to collect evidence about students' learning in order to analyze the effects of instruction was investigated by identifying the following types of responses in the prospective teachers' written analyses of the lesson: (1) the types of hypotheses that the prospective teachers formed about the effects of the lesson on students' learning, (2) the types of evidence that the prospective teachers used to support their analysis of the effects of the lesson (i.e., their hypotheses), and (3) statements that referred to the students' observable behavior and responses. In each case, categories were developed by reading through all the responses and separating them into qualitatively different groups. Two coders (two university-based mathematics educators) used the established categories to determine the number of participants who gave each type of response. Reliability was calculated by dividing the total agreements by the total number of decisions for each category. All codes had intercoder agreement of .80 or greater, and ranged from .80 to 1.

Task 2

The pre-service teachers' ability to use their analysis of the effects of the teaching on students' learning to revise the lesson was investigated by examining the relationship between the responses to the first and second tasks—did they use their analysis to make the revisions, and how did they use it. Types of lesson revisions were developed on the basis of the lesson revisions and the stated rationales for the revisions. Specific pedagogical approaches that the pre-service teachers used to develop major mathematical concepts in the lesson were also identified. In each case, the categories were developed by reading through all responses and separating them into qualitatively different groups. The two coders then used the established categories to determine the number of participants who gave each type of response. Reliability was calculated by dividing the total agreements by the total number of decisions for each category. All codes had intercoder agreement of .80 or greater, and ranged from .80 to 1. The relationship between the types of hypotheses formed and the types of revisions was then examined. To assess the nature of the revisions, the two coders also evaluated whether the pre-service teachers' lesson revisions had "improved" the lesson and inter-rater reliability was established. A lesson "improved" if the revision explicitly suggested more opportunities for students to develop their understanding of the mathematical concepts and relationships covered in the lesson. The coders assigned a score of 0, 1, 2, or 3 to each participant's lesson revisions; the meaning of the scores is described in the Results section.

The videotaped mathematics lesson

In the videotaped lesson, the teacher uses a teaching approach that many mathematics educators would describe as traditional. The teacher engages in a "recitation" (Fey, 1979; Hoetker & Ahlbrand, 1969), during which he presents brief pieces of information and asks for short answer responses from students. The majority of the questions for students are factual or recall questions. Complex concepts (e.g., the relationship between the area of a non-right triangle and the area of a rectangle) are quickly explained by the teacher and the students do not participate in the development of the concepts (e.g., they do not work with concrete materials to discover the relationship between the area of a non-right triangle and the area of a rectangle). The teacher explains several complex concepts during the lesson, but the majority of the class time is devoted to developing the children's procedural skills. Teacher explanation and questioning is followed by student seatwork on paper and pencil tasks that develop procedural skills. Students work individually and all student talk is directed toward the teacher; there is no student-to-student talk.

The teacher begins by defining area as the amount of "space inside a flat shape." He holds up a square unit (a paper square) and says, "When we ask how much space is inside an object, we are asking how many square units are inside the object." He then holds up a paper rectangle, with eight square units drawn in the figure, and asks, "How much space is in this rectangle?" A student answers, "Eight square units." Two more examples are provided; the teacher holds up rectangles that are partitioned into square units, asks for the areas, and students respond by counting the square units.

The teacher then observes that it is inconvenient to find the area of a rectangle by "drawing lines in a rectangle and counting the squares." He asks the class to find another way to find the area. A student suggests they should "multiply the vertical squares times the horizontal squares." The teacher responds, "That's exactly correct. Who can state it another way?" A student responds, "Times the width times the length." The teacher responds, "So area of a rectangle equals length times width. From just looking at our three samples you know it works every time. The same thing would work on a square, wouldn't it?" He draws a rectangle that is not partitioned into square units, writes '5 cm' and '3 cm' for the length and width, and asks for the area. A student responds, "Fifteen square centimeters." The teacher then writes, "$A = lw$, $A = 5\text{ cm} \times 3\text{ cm}$, $A = 15$ square cm." Another similar example is given.

The teacher then holds up a right triangle, partitioned into square units; because it is a triangle, there are fractions of square units, as well as "whole" square units drawn in the figure. The students can see four square units along one edge and three square units along another. The teacher says, "When you get to triangles, what's the problem? Yes, there are all kinds of pieces." He asks the students to count the square units, and six students offer ideas about the number of square units in the figure. Three of the six correctly give the answer as six square units, but the teacher does not inquire why. The teacher then holds up two triangles, identical to the first, but not partitioned into square units. He puts the triangles together and forms a rectangle. "If I find the area of this rectangle, what is the area of the triangle compared to the area of the rectangle?" A student answers, "Half." The teacher says, "It's half of it, isn't it? So if I can find the area of the rectangle I can find the area of the triangle because it's going to be half. Now the question is, does it work for every triangle?" The teacher holds up a non-right triangle. He then holds up another non-right triangle, identical to the first, but cut into two pieces. He takes the triangle and the two pieces, forms a rectangle, and says, "So every triangle is going to be half of a rectangle." He asks the students what they can do to find the area of a triangle. A student says, "Take the length and the width of the rectangle and divide it in half."

The teacher agrees but says they "need to change a couple of things"; instead of using "length" and "width," they will use "base" and "height." He reviews the definition of a right angle, and explains that they need to draw a perpendicular to the base to obtain the height. He holds up a right triangle, writes the formula, substitutes the values for the base and height, and asks different students how to do each part of the calculation; the associative law, dividing by 2, and multiplying by 1/2 are explained and reviewed by the teacher. He then draws a non-right triangle with the height drawn as a dotted line. He writes the formula, and asks a student for b, another for h, another to calculate 10×8, another for half of 80, and another student "to complete it" (to add "square feet" to the final answer). He says, "Let's do one more. I'm going to leave this pattern here [the prior example on the board]. Because this is the pattern I want you to follow." He draws a non-right triangle with the height drawn as a dotted line, and asks students for the base and height. Students complete the problem at their desks. He provides individual help, writes the solution on the board and observes that, "Most of you remembered to finish it. How did you have to finish it? Square inches." A similar example is given and the process is repeated. The class then begins the homework. As indicated in the task instructions in the SP condition, several students in the videotape ask for the teacher's help and appear to be confused. Two students ask for assistance on the first set of problems, and many students ask which formula they should use for a particular figure in the second set of problems.

Although it is helpful to know the general make-up and flow of the lesson, it is not essential to assess whether the lesson was effective for helping students achieve the learning goals of the teacher. The study is not designed to tease out differences in pre-service teachers' analyses based on the nature or quality of the lesson. Rather, the study examines the types of analyses of the effects of instruction that beginning pre-service teachers produce, the types of evidence that prospective teachers use to support their analysis, how they use the analysis to suggest revisions to the instruction, and how their analysis is influenced by the conditions under which the lesson is presented. This only requires that the same lesson be presented to all participants. In addition, to aid with the interpretation of pre-service teachers' responses, it is important to know that the lesson shows the teacher providing a number of demonstrations and explanations about particular mathematical topics covered during the lesson and asking the students a number of questions, usually factual or recall questions. Students are seen responding, often with correct and usually brief answers and sometimes with questions and puzzlements.

RESULTS

Pre-service teachers' ability to collect evidence about students' learning in order to analyze the effects of instruction

This section first describes the types of analyses that the pre-service teachers produced about the effects of the instruction on student learning. It then describes the types of evidence that the pre-service teachers collected to support their analysis.

CL group's analysis of the effects of the instruction on students' learning

The CL group responded to the prompt to "form a hypothesis about what the children have learned and understand (exactly) by the end of the lesson." The CL participants' hypotheses about the effects of the instruction on students' learning are presented in Table I. All participants in the CL group formed more than one of the hypotheses shown

TABLE I

Percentage of participants in the Children's Learning condition who generated each hypothesis about the effects of the instruction on students' learning

Hypothesis	Percent
Hypotheses about the effects of specific instructional features on students' learning	
The students understood x because the teacher explained x (x = the idea that the area of a triangle is half the area of a rectangle, the meaning of the formulas, ideas about square units, and/or ideas about the height and base)	27
By breaking down the formula for the area of a rectangle into parts, it made it easier for the students to identify what numbers went where and why they were used. Also by taking it slow and working with the students to develop the formula, the teacher gave the children time to connect the idea of counting square units to the idea of multiplying length times width.	7
Because the students understood the formula for the area of a rectangle and the teacher explained that a triangle is half of a rectangle, the students were better able to understand the formula for the area of a triangle.	7
The drawn out squares on the rectangles and triangles helped the children grasp the idea that a rectangle is split up into square units.	7
The students did not have an opportunity to learn about the relationship between the formula $A = lw$ and the method of counting square units.	7
Some students did not understand the teacher's explanation that two identical triangles make a rectangle and consequently, when finding the area of a triangle we multiply by 1/2. The teacher needed to explain it a little more.	7
Some of the teaching strategies interfered with the students' learning. The teacher criticized students and made them repeat their answers until they gave the correct answer. The teacher should have asked how they obtained their answers.	7
Hypotheses about students' learning that made no references to instructional features	
The students understand what area is.	13
The students understand ideas about square units.	93
The students understand the different methods for finding the area of a rectangle and/or the connections between the methods—i.e., counting square units, multiplying the number of square units in a row by the number of square units in a column, and/or multiplying length times width.	47
The students understand the concepts of length and width and how to identify them on a figure.	27
The students understand that squares are rectangles.	7
The students learned the formulas and/or how to use them.	80
The students understand the concepts underlying the formulas.	54

TABLE I

Continued

Hypothesis	Percent
The students understand that the area of a triangle is half the area of a rectangle, and/or the relationship between the two formulas.	73
The students understand ideas about the base and height. *Example:* The students understand that instead of using length times width, base times height must be used because the height of a triangle is not always perpendicular to the base; in a rectangle, it always is.	67
The students understand how to carry out the arithmetic computations in the formulas.	40
Some of the students could not remember or decide which formula to use.	13
The students don't always know or remember to write the correct type of measuring unit as part of the answer.	13
I don't know if the students understand the concept of area. Because the area of rectangles and triangles are formulas, it is easy for children to memorize the formula without fully understanding the concept.	7
One child asked whether she should divide by 2 to get the area of a rectangle. This tells me that maybe that child didn't understand it completely but when the teacher explained to do that only with triangles, the child seemed to understand better.	7

in Table I (Mean number of hypotheses formed = 7.6, *SD* = 3.1, Range = 3–13).

Two types of analyses of the effects of the instruction on students' learning were evident in the CL group's responses. In the first type of analysis, participants formed a number of hypotheses about the effects of specific explanations, instructional activities, or instructional strategies on students' learning and thinking, and how specific aspects of the teaching facilitated or interfered with the students' learning. Twenty percent of the CL group produced analyses that primarily consisted of hypotheses of this type. A representative response follows:

By breaking down the area formula into small chunks it made it easier for the children to pick out what numbers went where and why they were used. Also by taking it slow and working together it gave [them] time to make their own connections to the material—i.e., why you use length × width because you have to take the amount of units going one way (lengthwise) and multiply them by the ones going the other way (width). Also by fully understanding the rectangle formula they were better able to conceptualize the triangle formula because the teacher explained that a triangle is half a rectangle/square....

In the second type of analysis, participants formed hypotheses about the students' learning that made no references to specific features of the instruction or to the effects of these features on students' learning. Instead their hypotheses about what the students learned consisted of a list or description of all or most of the covered topics in the lesson. The participants who gave this type of response seemed to assume that the students learned and understood what the teacher covered. Eighty percent of the CL group produced analyses that primarily consisted of hypotheses of this type and every participant who gave this type of response claimed the students learned and understood one or more topics for which there was no objective evidence for student learning. A representative response follows:

> The children understand that the height and base are not always in the same places on a triangle. For a rectangle/square, they understand that to find the area of a rectangle you can count all of the square units or you can multiply the vertical times horizontal squares. If you have the width and length values you "plug in" the numbers into the equation ($A = l \times w$).... For a triangle, ... they know that two makes a rectangle. The area of a triangle is half of the rectangle, so every triangle is going to be half of a rectangle. So since rectangle area equals $l \times w$, triangle area would equal $(l \times w)/2$ or in other words base times height divided by two...

For both types of analyses, the majority of the hypotheses indicated the lesson was effective. Only 10% of the hypotheses that were produced by the CL group claimed the children failed to learn or understand a covered idea or skill; there were a total of nine hypotheses of this type, produced by six of the 15 participants. Six of the nine hypotheses referred to observable incidents when the children performed incorrectly or asked for the teacher's help—i.e., to incidents when students failed to write the correct units, used the wrong formula, asked the teacher which formula they should use, or asked questions about the '1/2' in the formula for the area of a triangle.

SP group's analysis of the effects of the instruction on students' learning

The SP group responded to the prompt to "form a hypothesis about the source(s) of the children's difficulty at the end of the lesson." The SP participants' hypotheses about the effects of the instruction on students' learning are presented in Table II. All participants in the SP group formed more than one of the hypotheses shown in Table II (Mean number of hypotheses formed = 5.3, *SD* = 2.3, Range = 3–11).

All members of the SP group produced the first type of analysis described above: The participants formed some claims about the

TABLE II

Percentage of participants in the Sources of the Problems condition who generated each hypothesis about the effects of the instruction on students' learning

Hypothesis	Percent
Hypotheses about the effects of specific instructional features on students' learning	
The teacher did not adequately develop the meaning of area so the students did not have a good understanding of the concept of area.	20
The teacher's development of the concept of square units had negative effects on student learning. *Examples:* The teacher's failure to develop the concept of square units led to misconceptions (specific misconceptions were described), lack of understanding, forgetting to write "square units" as part of the answer, and/or inability to solve problems.	40
The teacher did not develop the connections among the different approaches for finding area so the students did not understand the connections among the approaches.	27
The teacher's development of the meaning of the formulas had negative effects on students' learning. *Examples:* The teacher just relied on examples of the formulas and the students did not learn the underlying concepts. This may lead to forgetting the formulas, inability to reconstruct the ideas, or lack of understanding of when to use the formulas and why to use the formulas.	27
Students had insufficient understanding of the area of a rectangle before they were required to move to the related topic of the area of a triangle.	40
The teacher's development of the idea that the area of a triangle is half the area of a rectangle, and/or the relationship between the two formulas, had negative effects on students' learning. *Examples:* The teacher's demonstrations that the area of a triangle is half the area of a rectangle did not adequately help the students understand why there is a 1/2 in the formula, or the relationship between the formulas. Consequently the students might have developed various misconceptions (specific misconceptions were described), had problems finding the area, and/or did not know which formula to use.	47
The students did not have enough practice identifying l and w, or h and b on figures, and plugging these numerical values into the relevant formula. Thus they had trouble identifying the dimensions of figures, plugging values into the formula, and/or selecting the correct formula.	13
The teacher did not provide enough examples or practice. If the students had more examples or practice for each formula, type of triangle, idea, etc., they would know which formula or approach to use, would understand better, and/or could independently solve the problems.	20

TABLE II

Continued

Hypothesis	Percent
The teacher's development of the concepts of base and height had negative effects on students' learning. *Examples:* The teacher changed "length" and "width" to "base" and "height" in the formula for the area of a triangle. This made the students think the length times the width is the same as the base times the height. This could lead to various misconceptions or errors (specific misconceptions and errors were described), or could interfere with their ability to select the correct formula, or to correctly identify the height.	60
Too much information was introduced without allowing children to understand and/or master each topic. Therefore the children lost interest, did not have enough time to process the information, were not able to apply the material, did not remember the material, and/or did not understand subsequent related topics.	60
The teacher did not allow or ask the students to reason and figure things out for themselves. Consequently they forgot the material, did not understand why things work the way they do, and/or could not reconstruct the ideas.	27
The teacher did not address students' incorrect ideas or questions. Thus many children were left behind, did not understand the material, and/or were not able to clarify ideas.	20
Children need to actively participate in order to learn, remember, and/or understand. Because they did not actively participate in the lesson, the children got lost, could not focus and pay attention, and/or misjudged their level of comprehension.	20
The children experienced difficulty because they did not have any manipulatives or concrete materials to help them learn or understand. If children worked with concrete materials, they could cut up a triangle without a right angle in order to figure out the area, could invent ways for finding the area of rectangles and triangles themselves, and/or could use the concrete materials to develop an understanding of the formulas or to solve the problems.	20
The teacher coached the students through each step of the problems. Because of the coaching, students did not know how to start a problem, did not learn, had trouble focusing, got lost, mixed up the formulas, and/or did not remember what to do.	20
Children need to work with a new concept right after it is introduced, and the teacher did not provide for this. Therefore the children forgot the material, lost interest, became confused, and/or did not develop an understanding of the material.	13

TABLE II

Continued

Hypothesis	Percent
The teacher did not make the content meaningful to the children—i.e., explain or ask why things are true, why we would want to know something. Therefore the children get confused with the different steps of the procedure whereas if they were shown why they must complete the steps, they would have more of a conceptual understanding of the material.	7
The teacher assumed that everyone remembered concepts from previous lessons, such as ''perpendicular,'' and did not refresh their memories. This interfered with learning new material that depended on these concepts.	7
Hypotheses about students' learning that made no references to instructional features	
Some of the students' basic mathematics skills were not very good. If students lack basic skills, then it does not matter if they know the formulas because their answers will be incorrect.	7

effects of specific explanations, instructional activities, or instructional strategies on students' learning and thinking, and how particular aspects of the teaching facilitated or interfered with the students' learning. As shown in Table II, these hypotheses focused on the effects of the teacher's development of the mathematical concepts and skills, relationships among concepts, or connections across different parts of the lesson, and the effects of the pedagogy.

The hypotheses claimed the teacher did not adequately develop the mathematical concepts and relationships of the lesson, including the meaning of area, the concept of square units, connections among the different approaches to finding area (counting square units, multiplying the number of square units in a row of a rectangle by the number of square units in a column, and the formulas), the meaning of the formulas, the concept that the area of a triangle is half the area of a rectangle, the relationship between the two formulas, the concepts of base and height, and the relationship between base and height and length and width. Other hypotheses claimed the students did not have enough practice identifying the dimensions of figures or relating these quantities to the variables in the formulas. The pre-service teachers also formed hypotheses about the types of misconceptions that might result from these features of the instruction, and other possible effects on children's learning. A representative response follows:

> I don't know if the kids grasped the concept of base and height. The children may have gotten confused when the teacher changed "length" and "width" to "base" and "height." I think that the children may have the impression that the base and height are the same exact thing as the length and width. This would cause major problems.... [One] problem I see is that a child might mistake one of the sides of the triangle as the height.

The hypotheses also claimed there were insufficient opportunities for children to think or reason independently about the content of the lesson, to work with ideas immediately after they were introduced, to participate actively, or to work with concrete materials. Participants claimed the teacher coached too much, presented too much information, and failed to address children's ideas, questions, or prior knowledge. The prospective teachers also formed conjectures about the effects of these pedagogical features on children's learning. A representative response follows:

> [The] children needed visual and concrete references like the teacher had to show how to find the area of a triangle which they didn't have on their homework.... [S]howing the triangle as one half of a rectangle was a good idea but then when the children are working through the problems alone, they don't have two triangles to put together to show a rectangle. This could be especially important when trying to figure out the area of a triangle without a right angle. They can't cut it up [like the teacher did]....

Conclusions

The CL and SP conditions appeared to affect the level of analysis of the effects of the lesson on students' learning, as shown by the types of hypotheses that were formed. Most hypotheses by the CL participants appeared to be based on an assumption that students learned what the teacher explained. Under this condition, pre-service teachers did not usually try to identify and establish relationships between specific instructional moves and students' thinking and learning. However, when pre-service teachers began their analysis with the belief that there were problems to be fixed, as in the SP condition, many of them attended to the critical elements of classroom lessons (students' thinking and learning, mathematical content, and pedagogical approaches) and carried out a cause-effect type of analysis of the relationships among these elements.

CL group: Types of evidence used to support their analyses

The CL group primarily used four types of evidence to support their claims about the effects of the instruction on students' learning. Types of evidence are labeled Type A, B, C, and so forth to facilitate comparison across the conditions. Representative responses are provided for each type.

Type A: Evidence that referenced the teacher only
One third of the pre-service teachers in the CL group supported at least one hypothesis about students' learning with evidence that referred only to the teacher's explanations and statements.

(a) "The children understand a triangle is half the area of a square/rectangle. [My evidence is] [t]hey know this from his demonstration of placing two triangles together and when he cut the one and made them into a rectangle."
(b) "The children understand that the height and base are not always in the same places on a triangle. [My evidence is] because the teacher said that a triangle's height must always form a right angle with the base."

Type B: Evidence that referenced the students' correct performance
The students' correct responses to the teacher's questions and tasks were used to support claims about what the students learned or understood. Eighty-seven percent of the participants offered this type of evidence for at least one hypothesis.

(a) "The children understand that area is the amount of space in an object. [My evidence is] they were able to count up the number of squares to find the area [when the teacher held up paper rectangles that had square units drawn in the figures]."
(b) "The children understand that they always need a right angle to have a base and height. [My evidence is] the children give the base and height [when the teacher asks for the base and height of a figure on the board, and the height of the figure has been drawn in by the teacher as a dotted line]."
(c) "The children were able to find the area on their own which tells me that they understand the lesson."

As illustrated by the prior examples, the evidence that referenced the children's correct responses frequently appeared to be marginally related to the claims. For at least one of the covered concepts, 87% of the CL group accepted correct student responses that provided little information about student understanding as evidence for understanding of the mathematical concept. For example, 47% of the CL group accepted the students' ability to select the correct formula to find the area of a triangle, to correctly use the formula, or to correctly identify the values of the height and base of a triangle when the height was drawn as a dotted line, as evidence for student understanding of one or more of the following concepts: the concepts of base and height, the concept that the base and height are perpendicular, the concept that a right angle must be made in order to find the base and height, the idea that drawing the height creates two right angles, or the concept that the height of a triangle "is not just the other side."

Type C: Evidence that referenced the students' incorrect performance or queries for help

The students' incorrect responses to the teacher's questions and tasks, and queries for the teacher's help, were used to support claims that students did not learn or remember a covered topic. Thirty-three percent of the pre-service teachers offered this type of evidence for at least one hypothesis.

(a) "[My hypothesis is] the students may have a harder time remembering [to change formulas] than actually solving the problem with the formula given. [My evidence is] [t]hey keep asking when to use the formula on each problem."
(b) "[My hypothesis is] they don't always know to use the different types of measuring units. [My evidence is] some of the children would just put 'square units' and not the actual units from the problem."

Type D: Evidence that described the teacher's explanations/tasks/actions, included at least one reference to the students' observable responses or behavior, and posited how the instruction was affecting students' thinking, learning, or understanding

Thirteen percent of the group offered this type of evidence for at least one hypothesis.

By breaking down the area formula into small chunks it made it easier for the children to pick out what numbers went where and why they were used. Also by taking it slow and working together it gave [them] time to make their own connections to the material (i.e., why you use length × width because you have to take the amount of units going one way (lengthwise) and multiply them by the ones going the other way (width)).... [My evidence for this hypothesis is] the teacher broke the formula down first by not even introducing it. He first showed a picture of a rectangle to the children and had them count the number of square units. He did this a few times and made sure that his students understood the concept that there are all these square units in whatever shape they were looking at. Next he had the children come up with another way of finding the area without counting all the blocks. To do this, the children had to understand how they were finding the blocks in the first place (i.e., counting over horizontally and then down vertically and then back horizontally and so on). The children could make the connection that because of the way they were counting the blocks that it was the same as multiplying those numbers....

In this type of response, participants attended to both the teacher and the students, and attempted to establish relationships between the specific instructional activities or strategies and students' responses and learning. However, responses of this type included little objective evidence for the claims about student learning, and it was difficult to distinguish the hypotheses from the evidence. For example, in the response above, the participant writes, "Next he had the children come up with another way of finding the area without counting all the blocks. To do this, the children had to understand how they were finding the blocks in the first place .... The children could make the connection that because of the way they were counting the blocks that it was the same as multiplying those numbers." This is an insightful hypothesis about the possible effects of the instruction. However, there was little empirical evidence that students made the connection, and the participant does not offer any empirical support. The majority of the "evidence" consists of more hypotheses about the effects of the instruction.

SP group: Types of evidence used to support their analyses

The SP group primarily used three types of evidence to support their hypotheses about the effects of the instruction on students' learning.

Type C: Evidence that referenced the students' incorrect performance or queries for help

Students' incorrect responses to the teacher's questions and tasks, and queries for the teacher's help, were used to support claims that students did not learn or understand a covered topic. Fifty three percent of the participants offered this type of evidence for at least one hypothesis.

[I]n their individual work the students have trouble picking which formula to use. This indicates to me that they didn't have a strong grasp on what the variables represent (l and w) and/or how to identify those variables on a rectangle or square.

Type D: Evidence that described the teacher's explanations/tasks/actions, included at least one reference to the students' observable responses or behavior, and posited how the instruction was affecting students' thinking, learning, or understanding

Sixty percent of the group offered this type of evidence for at least one hypothesis.

I noticed that the teacher ... failed to explain why? For instance, why would they want to know the area of a square or triangle (he doesn't make it meaningful to them)... . [W]hy do we write square units to represent the product of an area problem?.... Therefore I hypothesize that the children get confused with the different steps of the procedure ... whereas if they were shown why they must complete the steps the children would have more of a conceptual understanding when solving the homework problems. [My evidence for this hypothesis is:] .... [T]he teacher [mentions] properties, such as the associative property, and the fact that the height of the triangle must be perpendicular to the width, but never asks the children why they think the calculation must be performed this way. For instance, he ... simply draws [a non-right triangle] with a line down the center [the height] but doesn't explain why they must do that.... The children's reactions to his questioning during the lesson relate to the formula but never entail descriptions as to why we must multiply length × the width, such as that they represent the number of square units within the shape. I feel like they are robots simply spitting out numbers in order to fill their set formula. As a result, when they get to the homework problems, ... they may forget the formula and due to the fact that they don't understand the concept behind the formula they are unable to solve the problem.... When children are not shown why and perhaps forget the formula ... they have nothing to base their solution on and nothing to refer back to....

As in the CL group, SP participants who used Type D evidence referred to both observable teacher actions and observable student responses, and attempted to connect the student behaviors and responses with the instructional events. However, in both conditions, responses of this type frequently included little objective evidence for the claims about student learning, and it was difficult to distinguish the hypotheses from the evidence.

Type E: Evidence that described the teacher's explanations/tasks/actions, and posited how the instruction was affecting students' thinking, learning, or understanding

This type of evidence was like Type D evidence, but included no references to the students' observable responses or behavior. Forty percent of the participants offered this type of evidence for at least one hypothesis.

[My hypothesis is] [the teacher's explanation of] the 1/2 concept of the triangle formula was confusing [to the children].... [My evidence is] I personally got pretty confused when the teacher had the three triangular shapes and showed how these three shapes fit nicely into one rectangle.... A kid in his class may get confused when trying to apply that to his or her triangle formula. They may think of 3 parts of a rectangle and possibly use 1/3 instead of 1/2 because the teacher's visual can be perceived different ways without the correct explanation.

References to students' responses and behavior in the evidence
In order to further examine the pre-service teachers' ability to collect evidence about students' learning to analyze the effects of instruction, statements in the participants' evidence that referred to the students' behavior and responses were identified. Table III shows the percentages of pre-service teachers who referred to various student behaviors and responses.

Table III suggests that when the pre-service teachers collected evidence about the students' responses and behavior in order to analyze the effects of instruction, the CL group collected evidence that primarily referred to the students' correct responses to the teacher's tasks and questions, while the SP group collected evidence that focused on the students' limited opportunities to learn concepts, students' incorrect responses, and the level and nature of the students' participation. Table III also shows that many members of the SP group "observed" what was not occurring in the classroom—i.e., what the students were not doing; for example, members of the SP group observed that the students were primarily learning procedures and not concepts, were not learning or answering questions about why something was true, were not reasoning or figuring things out for themselves, and were not handling the concrete materials that the teacher was using in his lecture. Table III suggests that the CL group may not have made these kinds of observations.

Table III also shows, however, that the participants in both groups did not refer to specific events in the videotape that best revealed student thinking. For example, six students in the videotaped classroom offered ideas about the number of square units in the paper triangle that the teacher held up. The pre-service teachers did not refer to this event. Table III also suggests that both groups may have focused on a subset of the students' responses and behaviors; there is very little overlap in the groups' observations.

Conclusions

(1) Prospective teachers supported hypotheses about students' learning with evidence that made no references to the students' responses or behavior (Type A and Type E evidence for the CL and SP groups respectively), included additional hypotheses about the effects of the instruction in their evidence (Type D evidence for the CL group, and Type D and Type E evidence for the SP group), supported hypotheses about students' learning with evidence that referred to student responses that were marginally related to the

TABLE III

Percentage of participants in each condition who referred to particular types of student responses and behaviors in their evidence

Types of observations	Condition	
	SP	CL
The students gave the formula when the teacher asked for it and/or correctly solved problems involving the formulas.	7	73
The students correctly said or wrote 'square units' after their answers.	0	67
The students correctly identified the dimensions of figures.	0	53
The students said that multiplying by 1/2 and dividing by 2 were the same thing, correctly multiplied by 1/2 or divided by 2, and/or said that to find half of something one should divide by 2.	0	47
The students counted the square units in the teacher's examples and/or gave the number of square units as the area.	7	40
The students developed or helped to develop the formula(s).	0	27
The students correctly applied the associative property.	0	13
The students were involved, participated, or continually answered questions.	13	13
The students performed incorrectly, were unable to answer the teacher's questions, or needed help from the teacher.	53	33
The students are answering questions about, or learning about procedures, not concepts and/or they do not answer questions about, or learn why something is true or done in a particular way.	47	0
The students are not thinking or reasoning or are not asked to figure out a particular idea for themselves.	40	0
The students are just imitating or copying the teacher's solutions, or are being led along by the teacher.	33	0
The students are not using concrete materials.	20	0
The students are not allowed to ask questions, or their incorrect answers are ignored or not addressed.	20	0
The same students are answering the teacher's questions each time and/or a small number of students are answering the questions.	13	0
The students are distracted and not engaged and/or student participation decreases over the course of the lesson.	13	0
The students begin to work on problems several minutes into the lesson, after a great deal of information is introduced.	13	0

claims, and attributed a wide range of understandings to students on the basis of little or no objective evidence.

(2) Pre-service teachers who were encouraged to believe the lesson was unsuccessful focused on a range of student responses and

behaviors. They focused on students' opportunities to learn concepts, students' incorrect responses, and the level and nature of the students' participation. Pre-service teachers who were allowed to form their own evaluation of the success of the lesson appeared to focus on the students' correct performance. It appeared that only the SP group "observed" what the students were not doing. Both groups failed to refer to student responses that provided the most access to students' thinking.

(3) The SP group was more likely to collect evidence that included observations of both teacher actions and student behaviors and responses, and to attempt to connect the student behaviors and responses with the instructional events.

Pre-service teachers' ability to use their analysis of the effects of instruction on students' learning to revise the instruction

In the second task, participants were given a transcript of the portion of the lesson on the area of a rectangle, and asked to make revisions to this part of the lesson on the basis of their hypotheses. The participants' responses to the second task are summarized in Tables IV and V. Table IV shows the percentages of participants who attempted to make particular types of lesson revisions. Table V shows specific pedagogical approaches that the pre-service teachers used to develop two of the major mathematical concepts in the lesson: (1) the concept of square units, and (2) the meaning of the formula $A = l \times w$.

Pre-service teachers' ability to use their analysis of the effects of the instruction on students' learning to revise the lesson was investigated by examining the relationship between the responses to the first and second tasks—did they use their analysis to make the revisions, and how did they use it. The types of lesson revisions (Tables IV and V) were closely related to the types of hypotheses from the first task (Tables I and II).

In general, prospective teachers aligned their revisions with the analyses they had just completed. If participants in the SP condition hypothesized that some aspect of the development of the mathematical content or pedagogy had negative effects on student learning, then they usually attempted to address the problem in their lesson revisions. For 47 of the 56 hypotheses of failed or limited learning, there was at least one corresponding lesson revision that attempted to address the problem. For example, if a member of the SP group hypothesized that the children's problems were attributable to the teacher's failure to

TABLE IV

Percentage of participants in each condition who made particular types of lesson revisions

Types of lesson revisions	Condition	
	SP	CL
Revisions that focused on the treatment of the mathematical content		
Develops the meaning of area	33	13
Develops the concept of square units, and/or measuring area with square units	87	53
Develops the idea that "the units of area are squared"	33	13
Develops the connections among the different approaches for finding area	27	0
Develops the concepts underlying the formula, the meaning of the formula	73	20
Teacher reminds students to keep the units the same throughout the problem solution, to write the correct units in the answer	0	20
Develops the idea that the order of the variables in the formula is irrelevant	13	7
Asks students to measure the length and width of rectangles	13	0
Develops the ability to identify the length and width of rectangles	27	7
Teaches students how to plug specific numbers in for specific letters in the formula	13	0
Provides more student practice (e.g., using the formula, counting square units)	40	0
Includes more examples	27	0
Develops idea that squares are rectangles, or how the area of a square and rectangle are related	13	13
Develops the idea that the formula can be used "for both a square and a rectangle"	20	0
Introduces the formula before introducing square units or introduces the formula earlier	13	7
Teacher states, "This is the formula you will always use when finding the area of a rectangle."	0	7
Revisions that focused on pedagogy		
Includes activities that are intended to help students master a topic more completely so they do not become confused, know how to apply the information, can move to the next topic	40	0
Includes tasks that require students to reason and figure things out for themselves	40	13
Plans to address children's ideas or questions, or to use children's ideas in the lesson	53	20
Increases the amount of student participation and involvement	53	27

TABLE IV

Continued

Types of lesson revisions	Condition	
	SP	CL
Students use concrete objects or drawings to help them learn, reason, or understand	40	20
Ensures students can solve problems on their own without the teacher's guidance	13	0
Provides an opportunity to apply a new concept right after introducing the concept	20	0
Attempts to make the material meaningful and relevant to the children's lives	27	7
Tries to build on prior knowledge	27	13

connect the different approaches for finding the area of a rectangle, then the participant made at least one revision to develop the connection in her lesson revisions. In the CL group, if participants hypothesized that students had learned or understood an idea or had mastered a skill of a given type, then they seldom made a revision involving the treatment of the concept or skill; this was true for 52 of the 61 hypotheses of this type. When CL participants hypothesized that students did not learn or understand a concept or had not mastered a skill, they usually made a revision involving the treatment of the concept or skill; this was true for 4 of the 7 hypotheses of this type.

Ninety three percent of the SP group attempted to improve the development of one or more concepts in the lesson revision task and 67% changed the pedagogical approaches in some way. The larger percentages for the SP group for the revisions that focused on pedagogy in Table IV indicate that the SP group used a wide variety of pedagogical approaches, including alternative teaching approaches, but Table V shows their revisions to develop the major concepts of the lesson were often limited to additional teacher explanation of the concept.

The data suggested the CL group's analyses were less helpful for making revisions, and that their analysis of the effects of the instruction on students' learning suggested the need for very few revisions. The CL group made very few lesson revisions and significantly fewer types of revisions than the SP group (Mean number of types of revisions = 2.7 and 7.6 for the CL and SP conditions respectively, SD = 2.3 and 3.8 for the CL and SP conditions respectively, $t(28) = -4.31$, $p < .001$).

TABLE V

Percentage of participants in each condition who used particular pedagogical approaches to develop major mathematical concepts in the lesson

Pedagogical approach	Condition	
	SP	CL
Development of the concept of square units		
(1) Teacher explanations of the concept of a square unit *Examples:* Teacher explains that if different sized units are used to measure two different rectangles, the rectangle with the smaller area can have a larger numerical value for its area. Teacher shows two identical rectangles partitioned into different sized units to show that different numbers can represent the area of a rectangle. Teacher explains that in a partitioned rectangle, each square unit represents 'one.' Teacher puts paper square units into a rectangle to show how many fit in, to illustrate the idea of measuring with square units.	67	27
(2) Students are asked to independently reason about or use the idea of square units in the context of concrete materials *Examples:* Students are asked to draw what a rectangle with k square units would look like. Students measure a rectangle with different sized measuring units: What can they conclude? Teacher tells students to take out a sheet of paper and to divide the paper into square units–as many as they want–and to determine the area and defend their response. Teacher presents an irregular shape that can be divided into "full" square units, and asks, "Can you find the area?"	40	13
(3) Students place square units into a rectangle or partition a rectangle into square units under the direction of the teacher.	7	13
(4) Teacher explains that "square unit" means "unit squared"	33	13
Development of the meaning of the formula		
(1) Students independently develop the formula by working with concrete objects or drawings *Examples:* After measuring and/or building rectangles with square units, students are asked to figure out a formula. After eliciting the formula from the students for partitioned rectangles, the teacher shows them an unpartitioned rectangle with the width and length labeled. Teacher asks, "How could we solve for the area in this problem?" The teacher shows the students an unpartitioned rectangle with dimensions 4 and 3. Teacher asks, "How can we find the area by drawing in individual square units?" After the students solve the problem, the teacher focuses the students' attention on the 3 groups of 4 units and 4 groups of 3 units to help them develop the idea of multiplying in the formula.	40	7

TABLE V

Continued

Pedagogical approach	Condition	
	SP	CL
(2) Students work with concrete materials to find the values that they need to substitute into the formula, or to check the value that they obtained from the formula *Example:* After using the formula to find the area of a rectangle, students check their answers by using a ruler to draw square units in the rectangle.	13	0
(3) Teacher explanations of the formula *Example:* Teacher explains why we multiply length times width and demonstrates with manipulatives.	27	13
(4) Teacher introduces the counting of square units and the formula simultaneously because it makes it easier for students to understand the formula, so students can check the formula by counting the square units, and/or because it prepares them for the situation in which there are no square units drawn in the rectangle.	13	0

(The types of revisions are shown in Table IV.) Sixty percent of the prospective teachers in the CL group attempted to improve the development of one or more concepts in their lesson revisions and 33% changed the pedagogical approaches in some way. The lower percentages for the CL group for the revisions that focused on pedagogy in Table IV and the data in Table V show that the CL group rarely used alternative teaching approaches.

There were some (unsolicited) comments written on the CL group's lesson revisions that seemed to explain the small number of revisions and traditional teaching approaches. Some representative comments follow:

(1) Other than that [two lesson revisions described in the pre-service teacher's lesson plan] I would not change much of what the teacher did. All of the children ended with a complete understanding of square units and how exactly to find area using formulas that they were able to come up with. If the kids did not understand the lesson then I would change more. I think the teacher did a great job and got the results he was looking for. He gave the students plenty of examples which definitely helped them in the end. Each child has a full understanding of the entire concept.
(2) In all I would not change much of the lesson. Besides the two small changes stated above [two revisions described in the pre-service

teacher's lesson plan] I believe the teacher very successfully taught area to the children. I think he explained well and reiterated his points of the lesson successfully. It seems to me that his method of explaining area was very good and in all I would use it to teach area myself.

To assess the nature of the revisions, two coders evaluated whether the pre-service teachers' revisions had "improved" the lesson. As explained in the Methods section, a lesson "improved" if the revision explicitly suggested more opportunities for students to develop their understanding of the mathematical concepts and relationships covered in the lesson. The coders assigned a score of 0, 1, 2, or 3 to each participant's revisions. Percent agreement across the coders for the lesson ratings was 83%. The meanings of the scores are described below, and a representative example is given for each score.

0 (no improvement): The pre-service teacher did not make any revisions that developed mathematical concepts or relationships.

What you would do instead: [When the teacher introduces the formula], I would just add that this is the formula you will always always use when finding the area of a rectangle.

Why would you do it that way?: The students did not seem to be completely aware when to use the formula $A = lw$ and when to use the formula $A = 1/2\ bh$.

1 (small improvement): The pre-service teacher developed one or two concepts in her lesson revisions, primarily through a teacher explanation.

What you would do instead: I would explain that a square is a rectangle, just a special form of it. I would explain why by saying that both have 4 right angles (4 pairs of perpendicular lines), [and] 2 pairs of parallel lines. Also, the pairs of parallel lines are equal to each other. The square is just special because both pairs of parallel lines are the same. Therefore you can actually call a square a rectangle.

Why would you do it that way?: I would do this because I noticed that one of the boys wasn't sure which formula to use for a square. He knew that $A = lw$ is the formula for a rectangle, but didn't realize that a square is a rectangle and would use the same formula.

2 (some improvement): The pre-service teacher (a) developed multiple concepts primarily through teacher explanations, and/or (b) designed one or two instructional activities that developed a concept(s), and used pedagogical approaches that appear to support concept learning in the activities (e.g., NCTM, 2000; NRC, 2001)—for example, the pre-service teacher designed an instructional activity that engaged students actively in the conceptual development of the topic; included cognitively demanding tasks that required a higher level of student engagement than the original lesson; designed an instructional activity that included teacher talk and questions for students that emphasized explanation, conceptual understanding, and the development of meaning; planned how to use student solutions and ideas in the development of a concept; or required students to construct mathematical arguments or explanations involving a concept, to respond to others' arguments/explanations, and to reconcile different arguments/explanations.

What you would do instead: [Activity 1:] Show a huge 10 by 20 rectangle with 200 little squares in it. Say, "There are a lot of square units in this rectangle. Can anyone think of a more effective way than counting each square that we could use to find the area?" Wait for a student to say, "Multiply." Say: "Yes, multiplication would be very useful. Can anyone tell me why we can use multiplication?" Allow any responses. Say: "Multiplication works because, as we have said before it is a fast way to add the same number many times. What numbers would we multiply to solve this problem?" Wait for someone to say, "10×20." Say, "Yes 10×20 would work. There are 10 rows of 20 squares so there are 10×20 squares. So what is the area of this rectangle?" Wait for students to say "200 square units." [Activity 2:] Pass out [a worksheet with three rectangles on it] to each student. The first rectangle is 10 by 30 and has 300 square units drawn in the rectangle with no numbers on the edges [i.e., the numbers 10 and 30 are not written along the length and width of the rectangle]. The second rectangle is 20 by 40, has 800 square units drawn in the figure, but no numbers 40 and 20 on the edges. The third rectangle says, "10 units" and "15 units" along the length and width, but has no square units drawn in the rectangle. Say, "Okay now I would like you to find the area of these three rectangles." Give students time to find the areas of all three, walk around to make sure everyone gets at least the first two right. Say, "Let's look at rectangle number 3. What do you think the area of this rectangle is?" Call on someone that you noticed has the correct answer and ask them how they got it. Say, "That's right. Even though the [square units] aren't drawn in we still know how many groups we have because we are told how many units are along each edge. Does anyone have any questions about how student x did this problem?" Answer any questions students might have about how knowing the number of units along the side equals the number of boxes.

Why would you do it that way?: [Activity 1:] This approach draws the correlation between the counting method and the multiplication method based on previous knowledge the children have. [Activity 2:] The worksheet allows them to practice the new technique they learned and then attempt to generalize it for times when the square units are not drawn in.

3 (more significant improvement): The pre-service teacher designed three or more instructional activities that developed concepts, and used pedagogical approaches that appear to support concept learning in the activities (e.g., NCTM, 2000; NRC, 2001).

TABLE VI

Percentage of participants receiving each score for lesson revisions

Condition	Percent receiving each score				Mean score	*SD*
	0	1	2	3		
CL	40	40	13	7	0.87	0.92
SP	7	33	40	20	1.73*	0.88

*$t(28) = -2.64$, $p = .013$

As Table VI shows, mean scores for the revisions for both groups were not high. The mean score was significantly higher for the SP group than the CL group ($t(28) = -2.64$, $p = .013$).

Conclusions

There was a clear relationship between the types of hypotheses that were formed about the effects of instruction on students' learning and the types of revisions made. When prospective teachers began their analysis with the belief that the lesson was unsuccessful, they were able to use their analysis of the effects of instruction on students' learning to make modest improvements in the lesson. When the pre-service teachers had the freedom to decide whether the lesson was successful, and to decide which instructional activities worked well and which did not, their analysis of the effects of the instruction on students' learning suggested the need for very few revisions. The CL group made very few revisions and their revisions received significantly lower scores.

DISCUSSION

This study investigated the learning-from-practice skills that pre-service teachers possess when they enter teacher preparation programs. In particular, the goal was to investigate, under two conditions, pre-service teachers' entry ability to collect evidence about student learning in order to analyze the effects of instruction, and to use the analysis to revise the instruction.

The performance of the SP group suggests that, under the right conditions, beginning pre-service teachers attend to both teachers and students and can develop some claims, although somewhat elementary, about how teaching and learning might be connected. The SP group formulated conjectures about the effects of specific instructional activities and strategies on student learning, and many were able to use the analysis to suggest productive revisions to the instruction. Beginning prospective teachers were less able to collect evidence that supported each conjecture about the effects of the instruction, and particular types of deficiencies in the pre-service teachers' evidence-gathering were apparent.

The conditions under which the lesson was presented dramatically influenced the type of analysis that prospective teachers carried out about the effects of instruction and the types of evidence that they used to support their analysis. The SP group was more likely to collect evidence that included observations of both the teacher and the students, and to attempt to connect the student behaviors and responses with the instructional events.

The effects of the conditions can be explained by assuming that the SP condition prompted the prospective teachers to begin shifting their attention from the teacher to the students. The finding that most CL participants produced an analysis of the effects of instruction that consisted of a list of the covered topics suggests that they kept their attention primarily on the teacher. This type of analysis appeared to be based on an assumption that "students learn what the teacher explains." If one assumes that students learn what the teacher explains, attention may be primarily directed toward the teacher when one is trying to assess what students learned from a lesson. As the prospective teachers in the CL condition watched the videotape, they saw a teacher giving explanations and children giving correct responses. Thus they concluded that the children understood the teacher's explanations, and made minimal revisions to the lesson.

The results of this study suggest that perceiving a lesson to be problematic encourages pre-service teachers to look more closely at students, perhaps because they look for places where the lesson did not work well and they need to watch students to find these places, and perhaps because they are asking themselves why the lesson might not have worked well. Whereas pre-service teachers who were allowed to form their own evaluation of the success of the lesson focused on the students' correct performance, the SP condition apparently prompted participants to attend more carefully to a range of student behaviors and responses, and to attempt to connect the student behaviors and responses with the instructional events. Pre-service teachers in the SP condition focused on students' opportunities to learn concepts, the level and nature of the students' participation, and students' incorrect responses, and attempted to "observe" what the students were not doing. Focusing some attention on the students opens new opportunities to examine the teaching-learning links in a lesson, and student responses and behaviors can provide information that suggests how to improve instruction. Doing this analysis well requires additional skills (pre-service teachers do not automatically function at high levels when attending to students) but shifting attention to students allows skills for analyzing teaching to emerge and develop.

One of the goals of this study was to provide information that can be used by teacher educators to design instructional activities that build on, and further develop beginning pre-service teachers' skills for analyzing teaching in terms of its effects on learning. The nature of the prospective teachers' responses—what they did well and what they failed to do—suggests some conjectures about the kinds of subskills and dispositions that are needed to learn from practice. If the conjectures are confirmed in future studies, they would provide appropriate learning goals for teacher preparation programs. Although it is not yet possible to specify the optimal instructional activities, it is possible to develop some informed conjectures about the kinds of educational activities that might help to develop these subskills and dispositions.

(1) *The ability and tendency to analyze the effects of specific instructional activities or strategies on students' learning and responses*: Beginning prospective teachers produced two types of analyses of the effects of teaching. Under one condition, prospective teachers focused on the effects of specific instructional activities and strategies on students' responses, and posited how particular aspects of the teaching facilitated or hindered student learning; they were able to use this type of analysis to make productive revisions to a lesson (e.g., getting students more actively engaged in the development of major concepts in the lesson). Under another condition, pre-service teachers tended to produce an analysis of students' learning that consisted of a list of the covered topics, a type of analysis that is less helpful for suggesting improvements to instruction. The findings suggest that it could be beneficial to develop pre-service teachers' disposition to carry out the first type of analysis, as well as their ability to carry out this type of analysis well. The findings also suggest that activities that involve observing examples of practice, and that also provide external support or a compelling reason for trying to understand the causal connections between specific teaching moves and students' learning (e.g., the task in the SP condition), would allow pre-service teachers' analysis skills to emerge and develop, and help to focus their attention on students' learning and responses. The performance differences across conditions suggest that it could be beneficial to develop prospective teachers' disposition to critically analyze the teaching-learning links in every lesson, as learning from

practice in a systematic and continuous way appears to require this kind of stance towards one's practice (e.g., Hiebert et al., 2003).

(2) *The ability to identify student responses that provide information about students' learning*: Conducting empirical observations to learn from practice requires recognizing that evidence on students' learning is needed to assess the effects of teaching, and the ability to distinguish between student responses that provide information about students' learning, and those that do not. In this study, prospective teachers supported hypotheses about students' learning with evidence that included no references to students' responses, referred to student responses that were marginally related to the claims, attributed a wide range of understandings to students on the basis of little or no objective evidence, and failed to refer to student responses that provided the most access to students' thinking. These findings suggest that activities that develop pre-service teachers' ability to distinguish between student responses that do and do not provide relevant information about students' learning could be beneficial. For instance, examples of student responses (videotaped or transcribed) that provide and do not provide evidence about their achievement of the learning goal of a classroom lesson can be presented to pre-service teachers, and the pre-service teachers can be asked to evaluate what, if anything, the responses reveal about students' achievement of the learning goal.

(3) *The ability to support conjectures (or claims) with evidence, and to distinguish conjectures (or claims) and evidence*: Analyzing teaching in terms of its effects on learning requires making conjectures or claims about what the students have learned during an instructional episode and how instruction might have facilitated the learning. These conjectures are justified by descriptions of the instruction and students' responses. In the CL condition, prospective teachers appeared to focus on the correctness of a student response more than the connection between the content of the student response and the claim they were making about student learning on the basis of the response; their conjectures about student learning, and the student responses that they offered as evidence for the conjecture, did not appear to be connected. Prospective teachers in both groups supported hypotheses about students' learning with evidence that referred only to the teacher's actions. These results suggest that it could be beneficial to develop pre-service teachers' ability to assess how well a claim about the effects of teaching is justified by the evidence of students' responses and descriptions of the instruction. For instance, examples of claims and supporting evidence can be presented to pre-service teachers, and they

can be asked to evaluate whether the evidence substantiates the claim. Because the pre-service teachers' evidence frequently included additional conjectures about the effects of the instruction, and because they frequently failed to provide evidence for each conjecture, it could also be beneficial to develop their recognition of the distinction between conjectures (or claims) and evidence, and the role of each in analyzing the effects of instruction on students' learning (see Fernandez et al. (2003) for a similar finding with in-service teachers).

Although the data from this study suggest that programs of teacher preparation can realistically aim to develop these subskills and dispositions by building on the entry competence of pre-service teachers, the data do not address the effectiveness of the instructional activities just described. These are conjectures for how teacher educators might build on the entry competencies found among the pre-service teachers in this sample, but further work that tests these conjectures will reveal more about the skills themselves and about pre-service teachers' abilities to acquire them.

As pre-service teachers move through preparation programs designed to facilitate the ability to analyze and improve practice, they will need to develop skills for revising, implementing, and testing increasingly effective versions of classroom lessons. These skills are likely to require competencies beyond those of the initial diagnostic and proposed revision skills addressed in this study. In addition, analyzing and improving someone else's lesson is different than analyzing and improving your own lesson. The data reported in this article are best viewed as a first step on a long and potentially rich program of research.

NOTES

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[2] In the U.S., students in the fifth grade are usually 10 to 11-years old.

[3] In U.S. mathematics lessons, it is very common for teachers to allow some time during the lesson for students to begin the homework (cf., Stigler & Hiebert, 1999).

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