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Names and Quantifiers: Bringing Them Together in Classical Logic

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Abstract
Putting individual constants and quantifiers into the same syntactic category within first-order language promises to have far-reaching consequences: a syntax of this kind can reveal the potential of any such language, allowing us to realize that a vast class of noun phrases, including non-denoting terms, can be accommodated in the new syntax as expressions suited to being subjects of sentences. In the light of this, a formal system that is an extension of classical first-order logic is developed here, and is equipped with an appropriate semantics. An ontological interpretation of the new logic is then also provided, with several categories of object and notions of existence being distinguished. Last but not least, a modal version of the logic, with some interesting formal features, is proposed.

Keywords Individual constants · (generalized) quantifiers · General objects · Particular objects · Existence · Modal logic · *De re – de dicto.*

1 Grammatical Versus Logical Treatments of Names and Quantifiers

Traditionally, grammarians have treated proper names, and quantifier expressions such as *something*, *everything*, *somebody*, *everybody*, *some dog* and *every dog*, as expressions of the same category: namely, that of noun-phrases. (Some call it the category of determiner-phrases). The reason is simple: both names and quantifiers can occupy subject positions in simple subject-predicate sentences. Nevertheless, intuitively, there is something about this approach that seems bound to catch our attention. To be sure, the two sentences *Tom is lazy* and *Somebody is lazy* have the same surface structure, which is that of subject-predicate; however, logicians would insist on a different logical structure for each of them, these being Pa and $\exists x\, Px$, respectively. The motivation for

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distinguishing the two logical forms is clear: names are scope-independent, whereas quantifiers are not, names are categorematic whereas quantifiers are syncategorematic expressions.[1]

Let us consider the following sentences:

(1a) Tom is not lazy	(Ia) Somebody is not lazy
(1b) It is not the case that Tom is lazy	(Ib) It is not the case that somebody is lazy
(2a) Tom is lazy and crazy	(IIa) Somebody is lazy and crazy
(2b) Tom is lazy and Tom is crazy	(IIb) Somebody is lazy and somebody is crazy
(3a) Tom loves Mary	(IIIa) Everybody loves somebody
(3b) Mary is loved by Tom	(IIIb) Somebody is loved by everybody
(4a) Tom loves himself	(IVa) Somebody loves himself
(4b) Tom loves Tom	(IVb) Somebody loves somebody

Note, first, that sentences in the same lines have the same grammatical structure. Subject-places and object-places in sentences in the left column are occupied by proper names, but in the right column by quantifier expressions. Apparently, the pairs of sentences (1a) and (1b), (2a) and (2b), (3a) and (3b), and (4a) and (4b) have the same truth values – and therefore, unsurprisingly, their logical structure is rendered by the same formulas, these being $\neg Pa$, $Pa \wedge Qa$, Rab, andRaa, respectively. On the other hand, the sentences in the pairs (Ia) and (Ib), (IIa) and (IIb), (IIIa) and (IIIb), and (IVa) and (IVb) may differ in their truth values, and consequently in their meaning.[2] Logic cannot ignore such differences, and must be capable of explaining them syntactically. This is why the logical forms of pairs of sentences on the right-hand side are distinct: $\exists x \neg Px$ and $\neg \exists x Px$, $\exists x(Px \wedge Qx)$ and $\exists x Px \wedge \exists x Qx$, $\forall x \exists y Rxy$ and $\exists x \forall y Ryx$, and $\exists x Rxx$ and $\exists x \exists y Rxy$.

Hence, it is clear why sentences containing quantifier expressions should require a more complicated syntactical representation than names. It was Frege who introduced the language of predicate calculus and the precise notion of a quantifier, this being undoubtedly one of the greatest achievements in the realm of logical theory.[3] The consequence of the Fregean revolution was that names and quantifiers are assigned to two different categories or types of expression. Montague proposed a type-theoretical

[1] For definition of categorematic-syncategorematic see for example Wikipedia: "In logic and linguistics, an expression is syncategorematic if it lacks a denotation but can nonetheless affect the denotation of a larger expression which contains it. Syncategorematic expressions are contrasted with categorematic expressions, which have their own denotations".

[2] Strikingly, (IIIa) is ambiguous, but one of its meanings does not coincide with the meaning of (IIIb).

[3] The invention of classical quantifiers is sometimes also attributed to Peano and Russell.

approach to natural language based on λ-calculus, in which names and quantifiers are of the same type. This approach is pretty technical, making use as it does of strong formal tools—basically, higher-order logic. Some even claim that it is necessary to use such tools.[4] However, we would like to explore the idea of bracketing together names and quantifiers in classical first-order logic broadly conceived.

We start by unifying names and quantifiers categorially. Since the latter expressions involve more complicated sentence structure, all we should do is let individual constants occupy the same position in sentences as quantifiers—and only this position. Thus, instead of Pa, we will now have $axPx$, and generally, for every formula $A(x)$, $axA(x)$ will be a formula as well. Such a notation may seem awkward and can be replaced by the more familiar the λ-notation, this being $[\lambda x A(x)]a$. However the counterpart of λ-elimination principle does not hold for our (quantificational) notation (i.e. $axA(x)$ is not equivalent to $A(a/x)$). Formulas (1a)-(4b) will now have the forms $ax\neg Px$, $\neg axPx$, $ax(Px \wedge Qx)$, $axPa \wedge axQx$, $axbyRxy$, $byaxRxy$, $axRxx$, and $axayRxy$, respectively.

One may wonder why we are adopting more complicated formal representations of sentences with singular names, instead of the traditional ones. Certainly, the use of a simpler syntax can be viewed as a question of economy. But what is desirable from the point of view of formal elegance and simplicity may not always be preferable when the *adequacy* of the formal representation of natural-language sentential structures is at stake. Moving individual constants to the quantificational position allows us to express, and compare, formal properties of the two kinds of expression.[5] From now on, then, we will call individual constants or quantifiers collectively *terms*, and let t represent any one of these. Furthermore, we will "officially" introduce a language M, consisting of the same symbols as the language of classical first-order logic (without function symbols). The grammar of M will be defined as follows: (1) the individual constants $a, b, a_1, a_2 \ldots$ and the quantifiers $\forall$ and $\exists$ are *terms*; (2) all expressions of the form $Px_1 \ldots x_n$ and $x = y$ are formulas; (3) if A and B are formulas, then $\neg A$ and $(A \supset B)$ are formulas; (4) if A is a formula, and t is a term, then txA is a formula.[6]

Now consider the following scope-involving properties of such terms:

P1 $tx\neg A \supset \neg txA$.
P2 $\neg txA \supset tx\neg A$.
P3 $tx(A \wedge B) \supset txA \wedge txB$.
P4 $txA \wedge txB \supset tx(A \wedge B)$.
P5 $txA \vee txB \supset tx(A \vee B)$.
P6 $tx(A \vee B) \supset txA \vee txB$.
P7 $tx(A \supset B) \supset (txA \supset txB)$.
P8 $(txA \supset txB) \supset tx(A \supset B)$.

[4] See, for example (Akiba 2018).

[5] The idea of treating names as scope dependent expressions and as quantifier-like expressions is not new. In particular it can be found in: (Chin-Mu Yang 2007), (Evans 1982), (Fitting & Mendelsohn 1998), (Hawthorne & Manley 2012), (Justice 2007), (Sainsbury 2005). However, these approaches have basically different goals than our approach. So I am not going to discuss them here.

[6] We can distinguish a category of 'predicates', i.e. expressions of the form xA where A is a formula. Than instead of (4) we may adopt the condition: if t is a term and π is a prediacate, then $t\pi$ is a formula. Such a condition would make explicit the fact that txA is a subject-predicate formula.

P9 $txsyA \supset sytxA$.
P10 $txtyA \supset txA(x/y)$, where $A(x/y)$ is a formula obtained from A by freely substituting every occurrence of y by x.
P11 $txA \supset txtyA(y/x)$, where $A(y/x)$ is a formula obtained from A by freely substituting every or some occurrence of x by y.
P12 $tx(A \supset B) \equiv (A \supset txB)$, where x is not free in A.
P13 $tx(A \supset B) \equiv (\neg tx\neg A \supset B)$, where x is not free in B.
P14 $txA \equiv A$, where x is not free in A.
P15 $txA \equiv tyA^*$, where A^* differs from A in that x is free in A in just those places where y is free in A^*.

Clearly, individual constants, as scope-independent expressions, should have all the syntactic properties listed above. It means that after replacing t or s by a and then every formula $axA(x)$ by the classical formula $A(a/x)$, P1–P15 turn out to be classicaly valid (the replacement makes them tautologies). This amounts to the same thing as the fact that the principle of λ-eliminability holds for names: $[\lambda xA(x)]a \equiv A(a/x)$.

Now let us say that quantifiers fulfill a formula of language M if the formula becomes valid in the sense of classical semantics (i.e. is a classical thesis) after replacing of s and t by a classical quantifier. For instance, $\forall$ (but not $\exists$) fulfills P7 since $\forall x(A \supset B) \supset (\forall xA \supset \forall xB)$ is a classical thesis; $\exists$ fulfills (but not $\forall$) P11 since $\exists xA \supset \exists x\exists yA(y/x)$ is a classical thesis. Quantifiers $\exists$ and $\forall$ in this order fufill P9 since $\exists x\forall yA \supset \forall y\exists xA$ is a classical thesis.

Both quantifiers fulfill P3, P5, P12, P13, P14, and P15. The universal quantifier also fulfills P1, P4, P7, P9 (when s is $\forall$), and P10. Besides those mentioned above, the existential quantifier fulfills P2, P6, P8, P9 (when t is $\exists$), and P11. Clearly, individual constants will have all the syntactic properties possessed by either the universal or the existential quantifier.

The syntactic properties listed above, which are distinct for individual constants and for quantifiers, can be expressed explicitly using the following conditionals:

P*1 $\neg txC \wedge \exists xC \supset (tx\neg A \supset \neg txA)$.
P*2 $txC \wedge \neg\forall xC \supset (\neg txA \supset tx\neg A)$.
P*3 $\neg txC \wedge \exists xC \supset (txA \wedge txB \supset tx(A \wedge B))$.
P*4 $txC \wedge \neg\forall xC \supset (tx(A \vee B) \supset txA \vee txB)$.
P*5 $\neg txC \wedge \exists xC \supset (tx(A \supset B) \supset (txA \supset txB))$.
P*6 $txC \wedge \neg\forall xC \supset ((txA \supset txB) \supset tx(A \supset B))$.
P*7 $txC \wedge \neg\forall xC \supset (sxtyA \supset tysxA)$.
P*8 $\neg sxC \wedge \exists xC \supset (sxtyA \supset tysxA)$.
P*9 $\neg txC \wedge \exists xC \supset (txtyA \supset txA(x/y))$, where $A(x/y)$ is a formula obtained from A by freely substituting every occurrence of y by x.
P*10 $txC \wedge \neg\forall xC \supset (txA \supset txtyA(y/x))$, where $A(y/x)$ is a formula obtained from A by freely substituting every or some occurrence of x by y.

Here, $\neg txC \wedge \exists xC$ means that t is not the existential quantifier (i.e. it is an individual constant or the universal quantifier), and $txC \wedge \neg\forall xC$ means that t is not the universal quantifier (i.e. it is an individual constant or the existential quantifier). It follows from P*1-P*10 that if a term does not coincide with the universal or the existential quantifier then it is scope-independent. We can understand these terms as individual constants. Such an approach suggests that if P^*1–P^*10 are stipulated then we need not mention explicitly individual constants while assuming that terms are the basic category of expressions of language M. Formally, the category will consist of term variables ($s, t, t_1, t_2, \ldots$) and two constant terms, namely: $\forall$, $\exists$.

Still proceeding in a systematic way, we can distinguish another group of logical properties common to individual constants and quantifiers. To do so systematically, we will develop an axiomatic system M.

2 System M

The system M consists of the following axioms and rules:

M1 Classical truth-functional tautologies.
M2 $\forall x(A \supset B) \supset (txA \supset txB)$.
M3 $A \supset \forall xA$ provided x is not free in A.
M4 $\forall xA \supset A(y/x)$, where $A(y/x)$ is a formula obtained from A by freely substituting every occurrence of x by y.
M5 $txA \supset tyA(y/x)$ provided y is not free in A.
M6 $\neg\exists xA \supset \forall x\neg A$.
MP if $\vdash A \supset B$ and $\vdash A$, then $\vdash B$.
MG if $\vdash A$, then $\vdash txA$ and $\vdash\neg tx\neg A$.
Additionally, we enrich the M-system with identity:
M7 $x = x$.
M8 $x = y \supset (A \supset A(y\|x))$,
where $A(y\|x)$ is the formula that results from freely substituting all or only some free occurrences of x by y.
Let us mention some easily provable theses of M:
M9 $A \supset \exists yA(y/x)$, where $A(y/x)$ is a formula obtained from A by freely substituting every, or some, occurrence of x by y.
M10 $\forall xA \supset txA$(a version of *dictum de omni*)
M11 $txA \supset \exists xA$(a version of *dictum de singulo,* i.e. existential generalization)

Formulas P3, P5, P12, P13, P14, and P15 are theses of M, and besides those, none of P^*1–P^*10 can be proved in M.[7]

Let us call P^*1–P^*10 collectively as P^*. Now, we can take $M + P^*$ as a logic of sentences in the language of this paper representing individual constants and quantifiers. Notably, we introduce the category of *terms* in order to extract common syntactic properties of individual constants and quantifiers, and properties that serve to differentiate the two categories of expressions. Again, individual constants will be those terms which are neither universal nor existential quantifiers according to P^*1-P^*10.

One can observe a form of duality. For any term t (individual constant or quantifier), there will be a term $\bar{t}$ such that $\bar{t}xA \equiv \neg tx\neg A$. In particular, $\overline{\forall} = \exists$, $\overline{\exists} = \forall$, $\overline{a} =$

[7] However, P^*1-P^*10 are not mutually independent. In particular, P^*1-P^*6 follow from P^*7-P^*10 on the ground of$M$.

$\neg a \neg = a$. It can be easily proved that even system M alone is closed under dual terms: whatever formula $A(t)$ is a thesis, $A(\bar{t})$ is also a thesis. If we assume at the start that the set of terms is closed under the duality operation, then the system M becomes simpler (in particular, MG reduces to: if $\vdash A$ then $\vdash txA$).[8]

3 Semantics for M + P*

The semantics for $M + P*$ closely resembles the semantics for classical first-order logic construed along Tarskian lines. By a model, we mean a pair $m = [D, I]$, where D is a non-empty set called the domain of interpretation, and I is a function defined on predicate symbols and terms and called an interpretation:

(1) For any term t, $I(t) = \{\{d\}\}$ for some $d \in D$ or $I(t) = \{D\}$ or $I(t) = \wp(D) - \{\emptyset\}$,
$I(\forall) = \{D\}$, $I(\exists) = \wp(D) - \{\emptyset\}$.

(2) $I(P) \subseteq D^n$, for some n-argument predicate symbol P.

An assignment in D will be a function V which assigns to every variable an element of D. Given V, by $V[d/x]$ we mean the function which is just like V, except that $V[d/x](x) = d$. The truth conditions for atomic formulas, for negation, and for implication are the same as in the classical semantics.

(3) $\|Px_1 \ldots x_n\|_V^m = 1$ iff $[V(x_1), \ldots, V(x_n)] \in V(P)$; $\|Px_1 \ldots x_n\|_V^m = 0$ otherwise
(4) $\|x = y\|_V^m = 1$ iff $V(x) = V(y)$; $\|x = y\|_V^m = 0$ otherwise
(5) $\|\neg A\|_V^m = 1 - \|A\|_V^m$
(6) $\|A \supset B\|_V^m = \max[1 - \|A\|_V^m, \|B\|_V^m]$
(7) $\|txA\|_V^m = 1$ iff there exists $X \in I(t)$ such that $X \subseteq I_V(xA) = \left\{d \in D : \|A\|_{V[d/x]}^m\right\}$

In particular, the formula $txPx$ is true in m if and only if there exists an $X \in I(t)$ such that $X \subseteq I(P)$.

It should be stressed that txA is to be understood as the subject-predicate formula expressing the basic predication of M, i.e. M-predication. The truth value of the formula is given by condition (7).

A formula is M-valid if and only if it is true in every model with respect to every assignment. Note that interpretations of all terms are of the same type—they are subsets of $\wp(D)$. What is noteworthy is that this is the kind of interpretation that is specific to generalized quantifiers.[9]

[8] If $\bar{t}$ is a term and $\vdash$A, then $\vdash \bar{t}xA$ i.e. $\vdash \neg tx \neg A$. Also, P^*2, P^*4, P^*6, P^*8, and P^*10 become the dual formulas to P^*1, P^*3, P^*5, P^*7, and P^*9 respectively.

[9] This semantic idea is not entirely new. It goes back to medieval logic, when quantifier expressions were treated as categorematic: i.e. as representing kinds of entities called 'quantifier objects'. At the same time, Frege's view of quantifiers as second-order concepts may also be associated with a categorematic reading of these expressions. Nevertheless, it was Montague who, in his formal treatment of natural language, explicitly proposed a uniform semantic treatment of names and quantifiers within the category of generalized quantifiers; cf. (Montague 1974).

Undoubtedly, the logic $M + P^*$ is equivalent to classical first-order logic.[10] That is why we can say that quantifiers and constants can be treated as expressions of the same category within classical logic broadly conceived. Individual constants are distinguished from quantifiers at the inferential level (not just the linguistic one). But one can ask what is to be gained from such a treatment: isn't $M + P^*$ excessively complicated? This complexity, it seems, does not in fact outweigh its richer logical content. The axioms P^* seem unnatural, and their content may seem unclear. However, it turns out that the logic $M + P*$, even if not considered particularly interesting, can reveal a wider logical perspective.

What about system M itself? If P^* narrows the category of terms to individual constants and classical quantifiers, then one might suspect that M alone allows for some other terms. What kind of terms? The answer will be clear if we equip system M with a semantics. Let us note that there are a lot of subsets of $\wp(D)$ which remain unexploited in the semantics for $M + P*$: i.e. they are neither interpretations of individual constants nor classical quantifiers. So, let us let us extend the interpretations of terms in the following way:

(1*) $I(t) \subseteq \wp(D)$, $I(t) \neq \emptyset$ and $I(t) \neq \{\emptyset\}$, $I(\forall) = \{D\}$, $I(\exists) = \wp(D) - \{\emptyset\}$.

retaining all other semantic conditions unchanged ((2)–(7)). The above, (1^*), means that almost all subsets of $\wp(D)$ may be interpretations of terms. Accordingly, we will not mention 'individual constants', i.e. terms which are scope independent, as separate category of symbols of M. Curiously enough, 'individual constants' (singular names) and some other kinds of noun-phrases can be directly defined in logic M on its deductive level (see: the table in Sect. 4).

So what kind of terms are they supposed to be? What can they represent or stand for? The answer is that the terms are generalized quantifiers, more precisely, monadic generalized quantifiers[11] and that they can be interpreted in various ways in the natural language. However, it is worth noting at this point that M does not accommodate all monadic generalized quantifiers – only those that are increasingly monotonic (see: M2 and (7)). Needless to say, soundness and completeness theorems can be proved for M.[12]

Remarkably, M looks like a generalization of classical first-order logic, and is actually an extension of it: the classical axiomatic framework is explicitly included in the axiomatic system of M. Undoubtedly, this resemblance to classical logic is a

[10] Let us define a translation T of formulas of language M onto language of classical first order language:

(a) $T(A) = A$ if A is an atomic formula.
(b) $T(\neg A) = \neg T(A)$
(c) $T(A \supset B) = T(A) \supset T(B)$
(d) If t is not a quantifier then $T(txA) = T(A(t|x))$
(e) $T(QxA) = QxT(A)$, where Q is a classical quantifier.

Then we can notice that: A is a thesis of $M + P^*$ iff $T(A)$ is a thesis of classical logic. From this the completeness of $M + P^*$ follows in straightforward way.

[11] See the classical paper by Barwise & Cooper (1981) and Westerståhl (1989).

[12] Completeness can be proved in a Henkin-style way; cf. (Paśniczek 1987, 1988). The proof is a little more complicated than the proof of completeness of classical first-order logic. In particular, in order to build the maximally consistent set of formulas we introduce to language not one but two sets of 'witnesses'.

great advantage of M, making it more familiar and easier to learn and use. On the other hand, despite this resemblance, the expressive power of M as displayed in this paper far exceeds that of classical logic, especially in its philosophically relevant respects.[13] First, M provides a category of terms that may represent a rich class of language expressions (proper names, descriptions, noun phrases, quantifiers, etc.). The common form of predication for these expressions, M-predication, is displayed by formula $txPx$ (more generally: txA). Besides, these terms may receive an ontological interpretation and consequently, M can be associated with a rich ontology of objects. Intuitively, according to (7) the object denoted by t possesses property P iff there exists $X \in I(t)$ such that $X \subseteq I(P)$.[14]

4 Philosophical Interpretations of M

We will show now how M can be applied to various kinds of natural language expressions that play the role of subjects in subject-predicate sentences, i.e. the sentences represented by formulas $txPx$, and how the truth value of the sentences are evaluated (according to condition (7)). Let us consider first expression *the Polish Pope.* There are two ways of interpreting it in *M.* First as a term t such that $I(t) = \{thesetof\, PolishPopes\} = \{\{JohnPaulII\}\}$. Then everything that is true of John Paul II (or: Karol Wojtyła) is true of *the Polish Pope* according to semantics *M*. But *The Polish Pope* can also be interpreted as $I(t) = \{the\, set\, of\, Poles, the\, set\, of\, Popes\}$. According to (7), t (or more precisely, the entity represented by t) possesses properties such as being a Pole, being a Pope, being a human being, etc., but not properties such as being born in Wadowice, being a professor, etc. which are properties true of real John Paul II. The first interpretation can be called the *adjunctive* interpretation, the second the *non-adjunctive*[15] one. Notice that the term t in adjunctive interpretation behaves as the classical definite description *the Polish Pope,* however it does not when interpreted non-adjunctively. The classical definite description *the Polish Pope*, and the definite description *the cardinal born in Wadowice*, have the same reference, and consequently share every property truly ascribable to the referent. But this is not the case when the two names are interpreted non-adjunctively, in particular when *the cardinal born in Wadowice* is interpreted as $I(t) =${*the set of people born in Wadowice, the set of cardinals*}. Then t possesses the properties *being born in Wadowice, being a cardinal,* but not *being a Pope* or *being a professor*. What is more, $\cap\{thesetof\, Poles, thesetof\, Popes\} = \cap\{$the *set of people born in Wadowice, the set of cardinals*$\}=$ {*Karol Wojtyła*}, and this may be understood to mean that the terms *the Polish Pope* and *the cardinal born in Wadowice* refer to the same real object.

[13] Conspicuously, translations of formulas P^*1-P^*10 (and many other M-formulas) are classical theses but, as we mentioned earlier, the formulas are not theorems of *M*.

[14] Here we may conceive of objects as bundles of properties, where 'bundle' and 'property' are interpreted extensionally.

[15] Roughly, adjunctiveness means that if object denoted by a term possesses extensionally properties P and Q than it possesses the complex property '*P and Q'*. Non-adjunctiveness means that such an inference does not hold. Conspicuously, the distinction applies to complex predicates that can be displayed as conjunction of simpler ones.

Interestingly enough, the identity of referents is not only expressible on the semantic level, but on the syntactic level of M as well.[16] *The red car* treated as a generic term and interpreted non-adjunctively would refer to the entity possessing the properties of being red, being a car, being a vehicle, etc. But it is neither two-wheel drive vehicle, nor four-wheel drive vehicle. *The present king of France* can be interpreted non-adjunctivelly as {*the set of living people, the set of kings of France*}, and as such, *the present king of France* would be a living person and would be the king of France, but would be neither bald nor non-bald.[17] If it is interpreted adjunctively as {*the set of present kings of France*} than *the present king of France* would possess all properties whatsoever (since the empty set is included in every set).

The issue of non-existence or the related problem of empty names is certainly one of the most recalcitrant challenges where ontology and philosophy of language are concerned, and is especially difficult in relation to logic. Thus, in particular, the theory of descriptions, free logics, modal logics, and—more recently—logics of a Meinongian kind, have all been devised to cope with this.[18] Unlike the classical theory of descriptions, which makes any predication involving nonexistent objects false and thus trivial, in M, some predications are true and some are false, closely matching in this respect our intuitions (see the examples given earlier). That is why M may also be classified as a free logic.[19] But most of all, perhaps, M is akin in its ontological spirit to the Meinongian theory of objects and, indeed, can be regarded as a basic instance of a so-called 'Meinongian logic'.[20] Roughly speaking, according to Meinong, every set of properties constitutes an object and the object possesses a property if the property is a member of this set. Meinongian objects may be incomplete or impossible: an object will be incomplete if and only if it does not possesses a pair of complementary properties, and will be inconsistent if and only if it possesses a pair of contradictory properties. We can see that Meinong's principle is fulfilled extensionally in our semantics in the following way: every set of properties represented by sets of individuals

[16] Namely, by the following complicated formula: $\exists x((\neg sy(x \neq y) \wedge \neg ty(x \neq y) \wedge \forall yz(\neg sx(x \neq y) \wedge \neg tx(x \neq z) \supset y = z))$. One may also note that the difference in meaning between *the Polish Pope* and *the cardinal born in Wadowice* can be rendered using possible-world semantics: in some worlds, the Polish Pope can be a different person than the cardinal born in Wadowice. However, let us consider another example: *the number greater than ten and smaller than twelve* and *the prime number consisting of two identical digits.* These two definite descriptions refer in all possible worlds to the same number, 11. Thus, they are indistinguishable with respect to their intensions when construed on the basis of this semantics. On the other hand, when interpreted as {*the set of numbers greater than ten, the set on numbers less than twelve*} and {*the set of prime numbers, the set of numbers consisting of two identical digits*}, the expressions have different meanings: e.g., it is true of the first one that it is less than twelve but the same isn't true of the second one.

[17] Conspicuously, our interpretation of description 'the present king of France' essentially differs from the classical one for according to the latter all predication involving this description as the subject is false.

[18] Such logics inspired by Meinong's theory of objects were developed by Jacquette (1996), Parsons (1980), Paśniczek (1998), Routley (1980), and Zalta (1983, 1988).

[19] M is free according to the definitions of free logics given by Lambert (2001), Bencivenga (1986), Morscher and Simons (2002) if we agree that what the authors mean by 'terms' are our 'terms' (with objectual quantification in M being over existing individuals). M can be made even "more free" if MG is replaced by the classical rule of generalization: if $\exists x(Px \wedge Qx) \vdash A$ then if$\vdash \forall x A$. The M10 and M11 are no longer theses. Instead, only weaker forms of these formulas are provable: $txB \supset (\forall x A \supset txA)$; $txB \supset (txA \supset \exists x A)$. Cf. (Paśniczek 1998, 2001).

[20] See (Paśniczek 1993, 1998).

(except for $\emptyset$ and $\{\emptyset\}$) can be an interpretation of a term—i.e. it can be understood as an object. In particular, *the round square* as interpreted non-adjunctively by the set $\{thesetofcircles, thesetofsquares\}$ possesses the properties of being round, being a square, and being a geometrical figure, but it possesses neither the property of having a 1m^2 nor the property of having different than 1 m^2 area. As such the object will be an incomplete and inconsistent. Importantly, M is not ontologically committed to 'Meinongian' objects: objectual quantification is restricted to individuals, members of D – i.e. M has no more commitments than classical first-order logic. The semantics for M at its very bottom assumes the same non-empty domain of individuals as classical extensional semantics. Thus, according to Quine's famous criterion of ontological commitment, M has no more commitments than does classical logic. Objects which are represented by terms are not quantified over! Nevertheless, there are theses that imitate quantification over such objects. Here, one may point to M10 and M11, which mimic M4 and M9, respectively. Certainly, this feature of M will come as a welcome one indeed for nominalistically minded philosophers, who are reluctant to talk about nonexistent objects.[21]

Let us now consider a different kind of terms. These are various logical generalized quantifiers.[22] The set $\{X \subseteq D : |X| \geq 5\}$ can be interpreted as *at least five things*, whereas the set $\{X \subseteq D : |X| \geq |D - X|\}$ as Rescher's quantifier *most things,* and the set $\{X \subseteq D{:}|X| \geq \aleph_0\}$ as *infinitely many things.* Thus, at least some terms, including the two classical quantifiers, may be understood as generalized quantifiers, more precisely, monadic generalized quantifiers.[23] Yet not all generalized quantifiers are definable in M (were they to be so, M would not be a first-order logic according to Lindström theorem).

The expressive power of some formulas involving identity can be displayed in the following table:

Formula	Semantic intepretation	Language intepretation	Ontological interpretaton
$\exists y(tx(x = y) \wedge \neg tx(x \neq y))$	$I(t) = \{\{d\}\}$, for a certain $d \in D$	t is an singular name like 'Socrates'	t is an *individual*
$\exists y(\neg tx(x \neq y) \wedge$ $\wedge \forall z(\neg tx(x \neq z) \supset y = z))$	$\bigcap I(t) = \{d\}$, for a certain $d \in D$	t is a definite description in our sense, e.g. 'the Polish Pope'	t is an *existent individual* (*individual existence*)

[21] The status of terms in M is analogous to that of expressions representing sets in the simple theory of sets that merely 'go proxy' for sets. Cf. (Quine 1963) for his "theory of virtual classes and relations".

[22] These quantifiers fulfill a condition requiring them to be invariant with respect to the bijection of D onto D, as opposed to the examples of the terms discussed above, which need not fulfill that condition. See (Westerståhl 1989).

[23] Some other examples of interpretations of noun-phrases in M: I("every dog") = $\{$*the set of dogs*$\}$, I(„some dog") = $\{X{:}\ X \cap$ *the set of dogs* $\neq \varnothing\}$, I(„John and Peter") = $\{\{$*John, Peter*$\}\}$, I(„John or Peter") = $\{\{$*John*$\},\{$*Peter*$\}\}$, I(„John and a woman") = $\{\{$*Jan*$\} \cup X{:}\ X \subset$ *the set of women* $\wedge\ X \neq \varnothing\}$.

Formula	Semantic intepretation	Language intepretation	Ontological interpretaton
$\exists y \neg tx(x \neq y)$	$\bigcap I(t) \neq \emptyset$	t is a non-empty name, e.g. 'the red car'[24]	*t exists* (*existence*)[25]
$tx\neg ty(x \neq y)$	$I(t)$ is a principal filter	t is a universal quantifier (possibly restricted to a subset of D)[26]	t is a *general object*
$\neg tx\neg ty(x = y)$	$I(t)$ is a principal ideal	t is an existential quantifier (possibly restricted to a subset of D)[27]	t is a *particular object*

The terminology adopted in the right column is conventional. Yet it meets some formal properties of the defined objects. To see this let us list some theses of M that will characterize more closely the meaning of the identity-involving formulas.

M12 $tx(x = y) \wedge \neg tx(x \neq y) \supset (txA \equiv A(y/x))$.
M13 $tx\neg ty(x \neq y) \supset (tx\neg A \supset \neg txA)$
M14 $tx\neg ty(x \neq y) \supset (txA \wedge txB \supset tx(A \wedge B))$
M15 $tx\neg ty(x \neq y) \supset (tx(A \supset B) \supset (txA \supset txB))$
M16 $tx\neg ty(x \neq y) \supset (sxtyA \supset tysxA)$.
M17 $tx\neg ty(x \neq y) \supset (txtyA \supset txA(x/y))$, where $A(x/y)$ is a formula obtained from A by freely substituting every occurrence of y by x.
M18 $\neg tx\neg ty(x = y) \supset (\neg txA \supset tx\neg A)$.
M19 $\neg tx\neg ty(x = y) \supset (tx(A \vee B) \supset (txA \vee txB))$.
M20 $\neg tx\neg ty(x = y) \supset (tx(A \vee B) \supset (txA \vee txB))$.
M21 $\neg tx\neg ty(x = y) \supset (tysxA \supset sxtyA)$.
M22 $\neg tx\neg ty(x = y) \supset (txA \supset txtyA(y/x))$, where $A(y/x)$ is a formula obtained from A by freely substituting every or some occurrence of x by y.
M23 $tx\neg sy(x \neq y) \supset (sxA \supset txA)$.
M24 $tx\neg sy(x \neq y) \wedge sx\neg ty(x \neq y) \supset (sxA \equiv txA)$.
M25 $\neg sx\neg ty(x = y) \supset (sxA \supset txA)$.
M26 $\neg sx\neg ty(x = y) \wedge \neg tx\neg sy(x = y) \supset (sxA \equiv txA)$.
M27 $\exists y(tx(x = y) \wedge \neg tx(x \neq y)) \equiv tx\neg ty(x \neq y) \wedge \neg tx\neg ty(x = y)$.

[24] This is simply because there exist red cars.

[25] Intuitively, an object 'exists' in this sense if it exits but it need not be one (like *a red car*). It also can be incomplete – it is neither a two-wheel drive vehicle, nor a four-wheel drive vehicle.

[26] E.g. "everything", "every dog".

[27] E.g. "something", "some dog".

We can see that identity plays a highly important role in M. It enables us to categorize terms with respect to their syntactic and semantic properties, and categorize objects represented by terms accordingly (see M12-M27). The fact that t is an individual constant (and represents an *individual*) is expressed by the formula $\exists y(tx(x = y) \wedge \neg tx(x \neq y))$, and the fact that this *individual* coincides with the value of the variable y is expressed by the formula $tx(x = y) \wedge \neg tx(x \neq y)$. This means that in the sense of M-predication *individual t* possesses the same properties as the object y in the sense of classical predication (cf. M15). The formula $tx\neg ty(x \neq y)$ expresses the fact that the term t possesses the same syntactic properties as the universal quantifier (see M13-M17). We shall call the corresponding entities—i.e. the entities correlated with such terms—*general objects.* The identity of these objects will be rendered by the formula $tx\neg sy(x \neq y) \wedge sx\neg ty(x \neq y)$ (see M24). *General objects* are basically incomplete, but are consistent in the sense of M13.[28] Meanwhile, the formula $\neg tx\neg ty(x = y)$ expresses the fact that the term t possesses the same syntactic properties as the existential quantifier (see M18-M22), and in this case we shall call the corresponding entities *particular objects*. The identity of these objects will be rendered by the formula $\neg sx\neg ty(x = y) \wedge \neg tx\neg sy(x = y)$ (see M18). *Particular* objects are basically inconsistent, but they are complete in the sense of M21. An object will be an *individual* if and only if it is both *general* and *particular* (see M27), so it must be complete and consistent. Apart from general and particular objects, further categories of entities can also be distinguished; however, the notion of identity can only be defined in M for these two categories. Note, also, that two kinds of existence can be defined in M, and that *individuality* entails *individual existence* and the latter entails *existence. Existent individual* possess only properties possessed by respective *individual;* compare *existent individual*: *The Polish Pope* interpreted as {*the set of Poles, The set of* Popes} and *individual* per se: *John Paul II* interpreted as $\{\{JohnPaulII\}\}$. Notice that neither *the round square,* nor *the present king of France* exists in any sense of existence.[29]

5 Notes on Modal M

Let us now mention some further consequences of bracketing together names and quantifiers within first-order logic. The opposition *de re – de dicto* is certainly highly important within modal logic, but this opposition can only be expressed in classical logic by means of universal and existential quantifiers. Yet in the language M we can express it directly in relation to any term: $tx\Box A$, $\Box txA$ (or, correlatively, in relation to any object). Thus, the formulas $tx\Box A$ and $\Box txA$ need not be logically equivalent. We will now sketch a semantics for a modal version of M, and show some possible advantages of this extension of it.[30]

[28] The term 'general object' was used by members of Lvov-Warsaw School. In particular, they stresses the incompleteness of such objects.

[29] For more on philosophical applications of M, see (Paśniczek 1998).

[30] We will apply here S5 as the modal base for M.

Let $\langle \boldsymbol{D}, \mathbb{W} \rangle$ be a modal frame, where $\boldsymbol{D}$ is a nonempty set of possible individuals and $\mathbb{W}$ is a nonempty set of possible worlds. And let $\boldsymbol{M} = \boldsymbol{D}, \mathbb{W}, I$ be a model of the modal M-language, where I is an interpretation function such that:

(a) $I(P)(w) \subseteq \boldsymbol{D}^{\mathrm{n}}$ for $w \in \mathbb{W}$ and n-argument predicate symbol P,
(b) $I(t) \in (\wp(\wp(\boldsymbol{D})))^{\mathbb{W}}$

Let V be a valuation function which assigns to variables elements of $\boldsymbol{D}^{\mathbb{W}}$. V_i^x differs possibly from V, in that $V_i^x(x) = i$, where $i \in \boldsymbol{D}^{\mathbb{W}}$. $\|A\|_V^{\boldsymbol{M},w}$ will be the logical value of A in $\boldsymbol{M}$ in possible world w with respect to the valuation$V$.

(c) $\|Py_1 \dots y_n\|_V^{\boldsymbol{M},w} = 1$ iff $[V(y_1)(w), \dots, V(y_n)(w)] \in I(P)(w)$,
$\|Py_1 \dots y_n\|_V^{\boldsymbol{M},w} = 0$ in the other case;
(d) $\|x = y\|_V^{\boldsymbol{M},w} = 1$ iff $V(x)(w) = V(y)(w)$,
$\|x = y\|_V^{\boldsymbol{M},w} = 0$ in the other case
(e) $\|\neg A\|_V^{\boldsymbol{M},w} = 1 - \|A\|_V^{\boldsymbol{M},w}$,
(f) $\|A \supset B\|_V^{\boldsymbol{M},w} = \max\left\{1 - \|A\|_V^{\boldsymbol{M},w}, \|B\|_V^{\boldsymbol{M},w}\right\}$.
(g) $\|txA\|_V^{\boldsymbol{M},w} = 1$ iff $there\,exists X \in I(t)(w)\, and\; X \subseteq \{i(w) : \|A\|_{V_i^x}^{\boldsymbol{M},w} = 1\}$
(h) $\|\Box A\|_V^{\boldsymbol{M},w} = 1$ iff $forall w \in \mathbb{W}$, $\|A\|_V^{\boldsymbol{M},w} = 1$.

Let us consider two formulas whose contents are of interest:

(*) $\Box tx\Box\neg ty(x \neq y)$
(**) $\Box\neg tx\neg\Box ty(x = y)$

As regards the above, (*) expresses the fact that t is a (possibly restricted) universal quantifier **the same** in all possible worlds. Meanwhile, (**) expresses the fact that t is a (possibly restricted) particular quantifier the same in all possible worlds. In other words, (*) and (**) define 'rigid' quantifiers. Notice also that if t is 'rigid' than its converse $\bar{t}$ is also 'rigid':

$$\Box tx\Box\neg ty(x \neq y) \equiv \Box\neg\bar{t}x\neg\Box\bar{t}y(x = y) \text{ and,}$$

$$\Box\neg tx\neg\Box ty(x = y) \equiv \Box\bar{t}x\Box\neg\bar{t}y(x \neq y)$$

In ontological terms, (*) means that t refers to the same general object in all possible worlds, (**) means that t refers to the same particular object in all possible worlds. Both (*) and (**) express the fact that t refers to an individual object that is **the same** in all possible worlds, making t a truly rigid designator.[31] So we see that in modal M we can define cross-world identities for some kinds of objects.

It can also be verified that (*) entails both the *generalized Barcan formula* and the *generalized converse of the Barcan formula*, while (**) entails only the latter:

$$\Box tx\Box\neg ty(x \neq y) \supset (tx\Box A \equiv \Box txA)$$

[31] Actually, we can define within modal M rigidity of other terms, in particular definite and indefinite descriptions.

$$\Box\neg tx\neg\Box ty(x = y) \supset (tx\Box A \supset \Box tx A)$$

Because the proposed semantics is constant domain semantics, $\Box\forall x\Box\neg\forall y(x \neq y)$ and $\Box\neg\exists x\neg\Box y\exists(x = y)$ are valid, consequently, the classical Barcan formulas $\forall x\Box A \equiv \Box\forall x A$ and $\exists x\Box A \supset \Box\exists x A$ will also be valid.[32]

6 Conclusions

The present paper has sought to demonstrate the great potential of classical first-order logic. As we have seen, certain relatively small changes to the language of the predicate calculus (i.e. the axiomatic system and the semantics involved), when made, lead to a logic M which is much stronger in its expressive power, while the latter can still be considered a first-order logic—one that, on the syntactic level, does not involve higher-order logic. We have then tried to argue that M furnishes an efficient tool for addressing various linguistic and ontological issues, including those pertaining to modality.

References

Akiba, K. (2018). Montague's treatment of determiner (or quantifier) phrases: A philosophical introduction, *Philosophical Compass*, March 2018.

Barwise, J., & Cooper, R. (1981). Generalised quantifiers and natural language. *Linguistics and Philosophy, 4*, 159–219.

Bencivenga, E. (1986). Free logics. In D. M. Gabbay & F. Guenthner (Eds.), *Handbook of philosophical logic.* (Vol. 3). Reidel.

Evans, G. (1982). *Varieties of reference*. Oxford University Press.

Fitting, M., & Mendelsohn, R. L. (1998). *First-order modal logic*. Kluwer Academic Publishers.

Hawthorne, J., & Manley, D. (2012). *The reference book*. Oxford University Press.

Jacquette, D. (1996). *Meinongian logic: The semantics of existence and nonexistence*. Walter de Gruyter & Co.

Justice, J. (2007), Unified semantics of singular names" *The Philosophical Quarterly,* 57(228).

Lambert, K. (2001). Free logics. In L. Goble (Ed.), *Philosophical logic.* Blackwell Publishers.

Montague, R. (1974). *Formal philosophy: Selected papers of richard montague*. Yale University Press.

Morsher, E., & Simons, P. (2001). Free logic: A fifty-year past and an open future. In E. Morsher & A. Hieke (Eds.), *New essays in free logic.* Kluwer Academic Publishers.

Parsons, T. (1980). *Nonexistent objects*. Yale University Press.

Paśniczek, J. (1987). The Meinongian Version of Classical Logic, *Studies in Logic, Grammar, and Rhetoric,* Vol. VI.

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Paśniczek, J. (1988). *Meinongowska wersja logiki klasycznej. Jej związki z filozofią języka, bytu i fikcji.* Lublin: Wydawnictwo Uniwersytetu Marii Curie Skłodowskiej.

Paśniczek, J. (1993). The simplest meinongian logic. *Logique Et Analyse, 143–144*, 1993.

Paśniczek, J. (1998). *The logic of intentional objects. A meinongian version of classical logic*. Kluwer Academic Publishers.

Paśniczek, J. (2001). Can mainongian logic be free? In E. Morscher & A. Hieke (Eds.), *New essayes in free logic. In honour of Karel Lambert.* Kluwer Academoc Publishers.

Quine, W. V. (1963). *Set theory and its logic*. Harvard University Press.

Routley, R. (1980). *Exploring Meinong's Jungle and Beyond.* Canberra: Department Monograph #3, Philosophy Department, Research School of Social Sciences, Australian National University.

Sainsbury, R. M. (2005). *Reference without referents*. Clarendon Press.

Westerståhl, D. (1989). Quantifiers in formal and natural languages. In D. M. Gabbay & F. Guenthner (Eds.), *Handbook of philosophical logic.* (Vol. 4). Springer.

Yang, C.-M. (2007). Proper names as quantifiers: A neo-fregean account of the sense of names. *EurAmerica, 37*(2), 1–44.

Zalta, E. (1983). *Abstract objects: An introduction to axiomatic metaphysics*. D. Reidel Publishing Company.

Zalta, E. (1988). *Intensional logic and the metaphysics of intentionality.* MIT Press.