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ARTICLE

Structuralism and the Quest for Lost Reality

Bobby Vos[1]

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Abstract
The structuralist approach represents the relation between a model and physical system as a relation between two mathematical structures. However, since a physical system is *prima facie not* a mathematical structure, the structuralist approach seemingly fails to represent the fact that science is about concrete, physical reality. In this paper, I take up this *problem of lost reality* and suggest how it may be solved in a purely structuralist fashion. I start by briefly introducing both the structuralist approach and the problem of lost reality and discussing the various (non-structuralist) solutions that have been proposed in the literature. Following this, I decompose the problem into the *ontological mismatch* and *specification* problems. In response to the former, I present a *metascientific dissolution argument*, according to which the difference in kind between mathematical structures and physical systems poses no deep obstacle to the structuralist approach, and consider some upshots of this argument for our views on representation. By way of conclusion, I argue that the metascientific dissolution argument paves the way for a solution to the specification problem as well.

Keywords Structuralism · Lost reality · Metascience · Inferentialism

1 Introduction

Since the fall of logical empiricism, the structuralist approach to philosophy of science, which seeks to explicate the various aspects of science in terms of *structures*, has been the dominant formal approach within the discipline.[1] Recently, the structuralist approach has been brought to bear on two important, closely related discourses concerned with the relation between science and the world. First, there is the structuralist approach to the study of *scientific representation*, as exemplified by Bartels (2006) and Bueno and French (2011). In addition, structuralist methods have been applied to the *problem of applied mathematics*, i.e. the issue of how it is that mathematics enables us to successfully describe and explain

[1] Although in recent years, the *categorical approach* (Halvorson 2012; Halvorson and Tsementzis 2017; Weatherall 2017) has been gaining significant traction.

✉ Bobby Vos
bvos1992@gmail.com

[1] Department of History and Philosophy of Science, University of Cambridge, Cambridge, Cambridgeshire, UK

empirical phenomena, cf. Pincock (2012) and Bueno and French (2018). Both these discourses have attracted significant interest in recent years, from both the philosophy-of-science community at large and structuralist philosophers in particular.

There is, however, a problem that threatens the structuralist approach to these issues, identified as the *problem of lost beings* by Muller (2011), the *loss of reality objection* by Van Fraassen (2008) and the *bridging problem* by Contessa (2010). Here, I shall refer to the problem as the *problem of lost reality*, or *PLR* for short. The problem has its roots in the following, straightforward observations about science. Scientific representations are representations of things in *concrete, physical reality*. Similarly, when we apply mathematics in scientific enquiry, we apply it to a system in concrete, physical (and very much *non-mathematical*) reality. Naturally, we would like our favourite account of the science-world relation to accommodate this fact.[2] The structuralist approach, however, construes the science-world relation as a relation between *mathematical structures* and thus seems remarkably ill-poised to meet this desideratum. Indeed, if we take the science-world relation to consist of *just* a relation between mathematical structures, then it is entirely unclear where—on the structuralist approach—concrete reality enters into the picture.[3]

In this paper, my aim is to suggest a solution to PLR. More specifically, I advance—in contrast with extant proposals—a *purely structuralist solution* to PLR, i.e., a solution that does not abandon the notion that the science-world relation can be spelled out purely in terms of a relation between structures. (In what follows, I shall denote solutions that are not 'purely structuralist' in this way as *non-structuralist* solutions.)

I begin in Sect. 2 by introducing the structuralist account and PLR in more detail and briefly discussing extant (non-structuralist) attempts at solving PLR. Next, in Sect. 3, I decompose PLR into two sub-problems that any purely structuralist solution must confront. The first of these—the *ontological mismatch problem (OMP)*—asks how the structuralist approach could possibly capture the science-world relation in light of the fact that, *prima facie*, physical systems are not mathematical structures. Assuming, however, that the structuralist can adequately answer OMP, the question remains what *specific* structural relation the science-world relation consists in; I refer to this as the *specification problem (SP).* In Sect. 4, I articulate a solution to OMP. To this end, I first argue in Subsect. 4.1 that there are several ways in which one can be a 'structuralist' in the philosophy of science. First, I distinguish between 'structuralism' in the sense of *structural realism* and 'structuralism' in the sense of (what I call here) *theory-structuralism*, noting that our focus here lies with the latter. Subsequently, I distinguish between theory-structuralism in the 'ontic' (or: 'first-order') sense and theory-structuralism in the 'metascientific' (or: 'second-order') sense.[4] Following this, in Subsect. 4.2, I take up OMP and, in response, present a *metascientific dissolution argument*, according to which OMP all but dissolves once we construe the structuralist approach in the metascientific sense. In Sect. 5, I consider, and reject, a

[2] Since the structuralist account of scientific representation is virtually identical to the structuralist account of applied mathematics—and nothing in the subsequent discussion depends on which of the two we consider—I shall for the remainder of the paper simply speak of the structuralist account of the 'science-world relation'.

[3] The structuralist approach has also been criticized on more general grounds, most notably by those philosophers propounding a deflationist view of the science-world relation, cf. Suárez and Pero (2019). Here, I shall not endeavour to mount a general defence of the structuralist approach, for this would lead us too far from the more specific problem I wish to take up here. Suffice it to say: I hold that the deflationist critique may be accommodated without foregoing the structuralist approach.

[4] Disclaimer: Even though one of the versions of theory-structuralism is referred to as 'ontic theory-structuralism', this is *not* meant to elicit a connotation with (ontic) structural realism.

possible objection to the argument, which will lead us to consider some upshots of the argument for our wider views on *representation* in the philosophy of science. Finally, in Sect. 6, I take stock of our 'quest for lost reality'. In particular, I note that SP remains an outstanding problem, but that, in dissolving OMP, we have implicitly paved the way for a solution to SP as well.

2 Background and Literature

As announced, I start by expounding the structuralist approach and PLR in more detail and by discussing some significant responses to PLR in the literature. However, before we can consider the structuralist approach to the science-world relation, we must first get clear on what the relata of this relation are supposed to be. While we might interpret both 'science' and 'the world' in a variety of different ways, there are two interpretations that are by far the most prevalent. First, we might take the science-world relation to refer to the relation between a *scientific theory* and the class of physical systems to which it is meant to apply. This way of explicating the science-world relation was particularly prevalent in the twentieth century, e.g., in the work of the Munich structuralists (see e.g., Balzer et al. 1987, 38ff).[5] Second, we might shift our focus from theories to *models*, and ask how a given model relates to its target system.[6] This latter conceptualization of the 'science-world relation' has been particularly prevalent in contemporary reflections on scientific representation, as evidenced by the recent survey article by Frigg and Nguyen (2017).

For the purposes of assessing the structuralist approach, we may restrict ourselves to considering the 'science-world relation' in this latter sense. More specifically, we will restrict ourselves to the following question:

Question (Q) Given a physical system P and some corresponding *theoretical model* M of this physical system: what is the nature of the relation between M and P?

That is: we restrict ourselves to those models that can be obtained from applying an underlying scientific theory T to the physical system P in question.

This restriction in scope is both necessary and beneficial. It is necessary because, in line with the well-worn theme of model pluralism, we may note that scientific practice features myriad different kinds of models, and an account of the science-world relation trying to account for all of them far exceeds the scope of the present paper or, indeed, of the structuralist approach itself.[7] It is beneficial because, on the structuralist approach, an account of the *model*-world relation straightforwardly translates into an account of the *theory*-world relation. To see this, we need but note that the structuralist approach to the science-world relation is typically presented as an off-shoot of the *structuralist view* of scientific theories.[8] On this view, a theory is characterized by the class of its theoretical models, with the

[5] More on this strand of structuralist philosophy of science below.

[6] By *target system*, I mean a physical system to which the model (or theory) is meant to apply.

[7] French and Ladyman (1999, 107–110) also consider a more concrete type of scientific model, viz. iconic models. Theoretical models, however, remain the most prevalent kind of scientific model considered in the structuralist framework and will hence be the focal point of this paper.

[8] For the same reasons as Muller (2011, 103), I eschew the more popular label of *semantic view*.

scope of the theory consisting of a class of physical systems. Now, on the non-trivial, but widespread assumption that the relation between these two classes can be defined pointwise (i.e., in terms of the individual relations between their members), it directly follows that an account of the model-world relation can be extended into an account of the theory-world relation.

Now, how does the proponent of the structuralist approach seek to answer Q? To see this, a basic understanding of the notion of a *mathematical structure* is key. (I shall henceforth omit the adjective 'mathematical', whenever it is convenient to do so.) Here, a *structure* means a tuple of the form:

$$\mathfrak{S} = (S_1, \dots, S_{n_0}, R_1, \dots, R_{n_1}, f_1, \dots, f_{n_2}, c_1, \dots c_{n_3}) \tag{1}$$

where each S_{i_0} is some domain and the R_{i_1}, f_{i_2} and c_{i_3} denote some relations, functions and constants defined over the various domains.

Elementary examples of structures abound in mathematics, including:

- $(\mathbb{N}, <)$, the natural numbers with the 'less than' relation;
- $(\mathbb{Z}, +)$, the integers with the function of addition;
- $(V, \mathbb{R}, +, \cdot)$, an arbitrary vector space consisting of a set V and the field of real numbers with the binary functions of vector addition $+$ and scalar multiplication $\cdot$ satisfying the vector space axioms.

With the notion of structure in place, the structuralist approach to understanding the science-world relation can now be expressed as follows:

Structuralist Approach The relation between M and P is given by a *structure-preserving mapping* $h : \mathfrak{S}_P \to \mathfrak{S}_M$ between two corresponding structures $\mathfrak{S}_M$ and $\mathfrak{S}_P$.

Colloquially: structuralists hold that a theoretical model and its target system *share a degree of structure* and it is by virtue of this shared structure that we can use the theoretical model to represent the physical system in question.

The above characterization of the structuralist approach comes with several caveats. First, proponents of the structuralist approach disagree about what we should take to be the structure-preserving mapping in question. While it is generally accepted that isomorphism is too strong a criterion, there is no consensus about what should be its alternative.[9] More generally, opinions vary about the extent to which structure-preserving mappings can account for the science-world relation by themselves, with some assigning a crucial role to the intentional and pragmatic aspects of scientific modelling as well.[10] We need not take a stance on either of these issues here. Regarding the first point of contention: we find PLR emerging as a problem no matter what structure-preserving mapping we use. Regarding the second point: while I am sympathetic to the point that we need to extend the structuralist

[9] For example, Bartels (2006) argues for homomorphism, Van Fraassen (1980; 2008) for isomorphic embeddings, while French and Ladyman (1999) and Da Costa and French (2003) opt for partial isomorphisms.

[10] See e.g., Bartels (2006, 11–13) and Van Fraassen (2008, 22ff).

approach with *some* additional components (cf. Sect. 6), I do not take the inclusion of pragmatic and/or intentional components to be necessary for addressing PLR.[11]

Let us, at this point, pause to consider the plausibility of the structuralist approach. First, we are required to associate to the theoretical model M a corresponding structure $\mathfrak{S}_M$. This requirement seems fairly unproblematic, at least to the extent that we can identify theoretical models with mathematical models.[12] What, however, of the *other* requirement placed on us by the structuralist approach, viz. that we associate to the physical system P a corresponding structure $\mathfrak{S}_P$? The structure $\mathfrak{S}_P$, recall, is supposed to correspond to the physical system *itself*, prior to our modelling it as M. Physical systems, however, are concrete entities existing in physical, non-mathematical reality and are manifestly *not* mathematical structures. Hence, by claiming the science-world relation to consist in a mapping between two mathematical structures, the structuralist is—at least *prima facie*—committing a category mistake. Conversely, *if* we insist on representing the science-world relation as a relation between two mathematical structures, it is unclear how—on the structuralist account—physical reality is supposed to enter into the picture. This, then, is what I refer to as the *problem of lost reality*:

Problem of Lost Reality (PLR) *Prima facie*, physical systems are not mathematical structures. Hence, the structuralist approach seemingly cannot accommodate the fact that theoretical models are about physical systems (rather than mathematical structures).

At first glance, this problem might seem like a deathblow to the structuralist approach. In the following sections, however, I argue that the problem is merely illusory; an illusion that readily vanishes once we get clear on exactly *what kind of account* of the science-world relation we take the structuralist approach to be.

Before doing so, however, let me briefly take stock of the extant solutions that have been advanced in response to the problem and situate my proposal with respect to them.[13] Broadly speaking, we can identify three different kinds of solutions that have been proposed in response to PLR.

Perhaps most famously, van Fraassen (2008, Ch. 11) has advanced a *pragmatic dissolution argument* in response to PLR. Crucial here is the notion of *data model*, i.e. a mathematical model constructed by taking the raw data obtained from measurements on a physical system and applying to it a variety of statistical methods and idealizing assumptions. Rather than a physical system P itself, Van Fraassen notes that what a theoretical model M actually confronts is some data model D of P. And *this* relation—i.e. the relation between M and D—can readily be cashed out as a structural relation between appropriate structures $\mathfrak{S}_M$ and $\mathfrak{S}_D$. But what of the relation between D and P? Does the structuralist not owe us an account of this relation as well? This, then, is where Van Fraassen's pragmatic

[11] This stance is a contentious one, and deserves a more elaborate defence than I can give here. This need not concern us, however: while I hold the solution I sketch from Sect. 4 onward to be both sufficient and necessary for solving PLR (more specifically: the sub-problem OMP), nothing is lost if we take the sketched solution to be merely sufficient and leave open the option for the pragmatically inclined philosopher to advance an alternative approach. Indeed, Van Fraassen does just this. I will argue below, however, that his proposed 'pragmatic dissolution' is unsatisfying on quite general grounds.

[12] Even in cases where the underlying scientific theory—and hence also its theoretical models—are not mathematical in nature, this does not preclude the possibility of representing these theoretical models as mathematical structures, cf. Lloyd (1988) and Thompson (1989) on evolutionary biology.

[13] I have drawn freely (though not exclusively) on the excellent survey article by Frigg and Nguyen (2017) in compiling this brief literature review.

dissolution comes into play: he argues that it is a pragmatic contradiction (in the manner of Moore's paradox) to assert that M adequately represents D but not P and, hence, PLR is dissolved.

An alternative type of response seeks to solve PLR by an appeal to *realist* sensibilities. This strategy is clearly embraced by, for instance, Contessa (2010), who holds that we can circumvent PLR by becoming metaphysical realists[14]—since doing so would allow us to unproblematically assign an intrinsic structure $\mathfrak{S}_P$ to a physical system P. In a somewhat different vein, Gentile (2017, 454ff), drawing on an analogy with Locke's indirect realism, has argued that, even if the most we know about the 'structure of P' is the structure of the data model D that we obtain from measuring and observing P, "coincidence between measurements and results of observations carried out in different situations... seems to be sufficient basis to infer that the structure of the data model fits into the structure of the [physical system]" (2017, 455).

Lastly, we can identify a third type of response that we can roughly describe as *language-based*. In brief, this type of response takes PLR to show that the science-world relation cannot be understood in purely structural terms and that, indeed, something more is needed in spelling out this relation. The most extreme version of such a proposal is perhaps the one expounded by Muller (2011, 107–109), who argues that in linking up the structure $\mathfrak{S}_M$ to 'the world' we need not introduce a structure $\mathfrak{S}_P$ at all. Rather, Muller argues, we can simply consider the set of sentences made true by $\mathfrak{S}_M$ *qua* Tarskian model ('the linguistic formulation of $\mathfrak{S}_M$') and connect *these sentences* to the world through the general relation of linguistic reference.[15] Other proposals are less extreme. Indeed, Balzer *et al.* (1987, 37–38) still spell out the science-world relation in terms of a structural relation between $\mathfrak{S}_M$ and $\mathfrak{S}_P$, but note that we need to add an 'informal description' to $\mathfrak{S}_P$ to see that $\mathfrak{S}_P$ represents a concrete physical system rather than an abstract mathematical structure. Finally, Frigg (2006, 55–59), Bueno and French (2011, 887) and Nguyen and Frigg (2021) have defended the view that the structure $\mathfrak{S}_P$ is about physical reality by virtue of being an abstraction of the physical system P equipped with some 'structure-generating description'.[16]

In this landscape of prospective solutions to PLR, what does a new proposal still have to offer? In short, I hold that none of the three different types of response manages to solve PLR in a satisfying fashion. Let me briefly explain.

Regarding the pragmatic dissolution argument, we might first ask whether the argument itself is sound. Nguyen (2016) has argued it is not. In addition, bracketing Nguyen's concerns about the cogency of the argument, we might ask whether the argument—if successful—would be enough to dispel PLR. I do not think so. At best, the argument seems to establish some pragmatic constraints *on what we can assert* about the relation between M and D *vis-à-vis* the relation between M and P. But this does not assuage our original concern, i.e. how there can be a structural relation between M and P when P is, *prima facie*, *not* a structure.

[14] Contessa, following Psillos (2000), defines *metaphysical realism* as "the thesis that the world is (largely) independent from our way of representing or describing it" (Contessa 2010, 515).

[15] More precisely, Muller wishes us to consider the class of all equivalent linguistic formulations of $\mathfrak{S}_M$.

[16] Note that, in this proposal, it is the notion of abstraction, rather than the additional linguistic element (i.e., the structure-generating description), that connects the physical system P to the abstract structure $\mathfrak{S}_P$. The structure-generating description, instead, is introduced to address a more specific problem with assigning structure to physical systems, dubbed by Nguyen and Frigg (2021) the *curse of abundance*. In my terminology, this curse becomes one of several issues that we might discuss under the heading of SP ('the specification problem'). Cf. Sect. 4.2 and footnote 27 for a brief comment.

Proposals appealing to realist sensibilities ought, I think, to be rejected out of hand. The reason why is nicely captured by Frigg and Nguyen's (2017) discussion about why a Tegmark-style mathematical Platonism—in which physical reality fundamentally just *is* a mathematical structure, cf. Tegmark (2008)—cannot be invoked to solve PLR. Indeed, Frigg and Nguyen note that what the world is like at a *fundamental* level is irrelevant for many instances of the science-world relation: "When modelling an aeroplane wing we don't refer to the fundamental super-string structure of the bits of matter that make up the wing, and we don't construct wing models that are isomorphic to such fundamental structures" (2017, 76). In the same vein, we can now argue that what the world is like at the *metaphysical* level should be irrelevant for our account of the science-world relation. The upshot, then, is that we cannot solve PLR by adopting a particular position in the realism debate.

The various language-based proposals seem to me the most promising. In contrast to the two above-mentioned proposals, I reject these language-based proposals not because I view them as being unsatisfying *simpliciter*, but rather because I take them to be unsatisfying *from the structuralist perspective*. To substantiate that statement, I will need to elaborate on what I take to be the aim of 'the structuralist approach'—a task that I will take up primarily in Sect. 4. For the moment, however, let me make do with the following argument.

First, let me note that enriching the structuralist approach with linguistic elements need not be unsatisfying by itself. Such a strategy becomes problematic, however, once we exchange the prospect of a *formally* explicated science-world relation for an explication that is partly structural and formal (i.e. by involving the structure $\mathfrak{S}_M$) and partly linguistic *and informal*. This, indeed, is the crux of the matter.

On Muller's proposal, the relation that connects the structure $\mathfrak{S}_M$—via its linguistic formulation—to the relevant physical system P is taken to be the general relation of linguistic reference. No *formal* analysis of this relation is offered, however.[17] Similarly, on the proposal by Balzer *et al.*, the way in which 'physical reality' is brought back into the account is by adding to $\mathfrak{S}_P$ an *informal*, linguistic description, that says that $\mathfrak{S}_P$ represents some concrete physical system P. Finally, on the abstraction-based proposal, the structure $\mathfrak{S}_P$ is taken to represent some part of physical reality by virtue of being an *abstraction* from a physical system P equipped with some structure-generating description. In this case, it is not the added linguistic element—i.e., the structure-generating description—that is problematic, but rather the fact that the proponents of an abstraction-based solution to PLR take the notion of *abstraction* to be a primitive concept.[18] Hence, we see that the abstraction-based proposal similarly foregoes the prospect of a substantial *formal* analysis of the science-world relation in favour of an approach that revolves crucially around an intuitive, primitive concept.[19]

[17] A more specific arguments against Muller's proposals is that by reducing the science-world relation to a generic instance of the more general 'word-world' relation, we seemingly lose track of those features of theoretical models that distinguish them from other kinds of representational tokens (e.g., proper names), such as their capacity to enable surrogative reasoning about their intended targets. In adopting this stance, I go against Callender and Cohen's (2006) view that "there is no special problem about scientific representation". My rejection of their view is for the same reasons as Frigg and Nguyen (2017, 55–57).

[18] See Nguyen and Frigg (2021, 5954n).

[19] I will, however, return briefly to the notion of an abstraction-based solution to (part of) PLR in Sect. 6. While the focus of this paper lies primarily with expounding a dissolution of the *ontological mismatch problem (OMP)*, I take the notion of abstraction to be crucial for solving the related, but distinct *specification problem (SP)*; the decomposition of PLR into OMP and SP is clarified and motivated in Sect. 3.

In sum, then, we see that pragmatic and realists strategies for dealing with PLR should be rejected on quite general grounds and that the various language-based proposals—while promising—are nevertheless disappointing for a proponent of the structuralist approach, because they introduce informal elements in the analysis of the science-world relation. This paves the way, then, for the proposal I wish to expound below.

3 Towards a New Proposal

As announced in the Introduction, I shall now propose a *purely structuralist* solution to PLR, i.e. a solution that aims to show that we can explicate the science-world relation as a relation between two structures $\mathfrak{S}_M$ and $\mathfrak{S}_P$.[20]

To this end, it will be beneficial to decompose the original problem into two parts. First, we might consider the *ontological mismatch problem (OMP)*. This is the problem of how, given the fact that physical systems and mathematical structures are *prima facie* two very different kinds of entities, we could possibly construe the science-world relation as a relation between two structures. Indeed, we might say that for many commentators—e.g. Van Fraassen (2008) or Contessa (2010)—OMP simply *is* PLR. Strictly speaking, then, a solution to OMP would already suffice to salvage the tenability of the structuralist approach. It seems reasonable, however, to expect more from a solution that purports to be *purely structuralist*. More specifically, it seems reasonable to expect such a solution to show us not only *that* the science-world relation can be construed as a relation between structures, but also *what kind* of structural relation this is supposed to be. Let us, then, call this latter challenge the *specification problem (SP)*.

In what follows, I will—for the most part—be concerned exclusively with OMP. This restriction is primarily motivated by space limitations. However, as noted, it is also the case that for many commentators PLR consists entirely in OMP, making the lack of a detailed discussion of SP in the present paper somewhat more tolerable. The few remarks I do have on SP will be confined to the Conclusion. As we will see, in spelling out a solution to OMP in Sect. 4, we implicitly impose some constraints on what a satisfying solution to SP should consist in. By way of closing remarks, then, I will reflect on how the structuralist approach may be further developed so as to satisfy these constraints.

4 On Ontological Mismatch

As noted in the Introduction, we can situate the structuralist approach to the science-world relation within a wider school of 'structuralist philosophy of science', or more compactly: *structuralism*, even if not all proponents of the structuralist approach would necessarily view themselves as belonging to such a wider school. The central claim I want to make in this section is that 'structuralist philosophy of science' can be construed in two distinct ways— *ontic* (or: *first-order*) and *metascientific* (or: *second-order*)—and that OMP,

[20] I should emphasize, however, that I do not take such a structuralist solution to be inherently superior to non-structuralist solutions. Indeed, a purely structuralist solution is desirable and interesting, *to me*, because such a solution will *a fortiori* also be purely formal, i.e. will not rely on the introduction of informal elements (linguistic or otherwise) into the structuralist approach. So much for my own motivation. One may, of course, have independent (stronger) reasons for desiring a purely structuralist solution to PLR. No matter what one's motivation, I hope for the following proposal to be of use.

while posing a genuine problem for structuralists in the ontic sense, all but dissolves on the metascientific construal. Accordingly, I call this the *metascientific dissolution argument*.[21]

I shall proceed as follows. First, in Subsect. 4.1, I will explicate the two main ways in which one can be a structuralist in the philosophy of science and will—as mentioned—distinguish between ontic and metascientific construals of structuralism. Next, in Subsect. 4.2, I will argue that, indeed, OMP dissolves on a metascientific construal of the structuralist approach. Following this, in Sect. 5, I defend the argument against a possible objection, and in doing so elaborate on the view of *metascientific representation* underpinning the argument. This completes the metascientific dissolution argument and shows that the structuralist approach is a viable account of the science-world relation.

4.1 Structuralism as Metascience

'Structuralism', even when we consider that term only within the context of philosophy of science (as I will do henceforth), can be used to refer to several different positions within several different discourses. Accordingly, some preliminary comments are in order about the kind of structuralism under consideration. The ontic and metascientific construals mentioned above will then be two different construals of a *single kind* of structuralism.

Let me start by mentioning the kind of structuralism I wish to consider here. The structuralism I have in mind is the the approach to philosophy of science that emerged in the wake of the structuralist view of scientific theories, as pioneered by (among others) Patrick Suppes (1960) and Evert Willem Beth (1960). As noted at the start of Sect. 2, the structuralist approach to the science-world relation has often been presented as an off-shoot and/or extension of the structuralist view of theories. In what follows, let us refer to this kind of structuralism—i.e. the structuralist view of theories plus its various extensions—as *theory-structuralism*.

Now, theory-structuralism should be distinguished from the 'structualism' that has emerged in the context of another—arguably more famous—philosophy-of-science debate. Indeed, the *other* discourse where we find the notion of structure play a prominent role is the scientific realism debate. I am, of course, alluding here to the family of positions known as *structural realism*, i.e. the view that "scientific theories tell us only about the form or structure of the unobservable world and not about its nature" (Ladyman 2019). However, while structural realism is (in its epistemic guise) historically significant—being retroactively attributed to such luminaries as Duhem, Cassirer, Carnap and Russell (Gower 2000)—and (in its ontic guise) a contemporary 'hot topic', it is not the kind of structuralism with which we shall be presently concerned.

Let us now consider theory-structuralism in more detail. In particular, let us consider what we may take to be the *aim* of theory-structuralism *qua* philosophical project. Unfortunately, we cannot answer this question straightforwardly, due to the many different versions of the structuralist view that exist in the literature. Complicating matters further is the fact that the proponents of these various accounts are themselves not always clear on what kind of account it is that they are proposing. I now contend that we can construe these different construals of theory-structuralism in two different ways: the previously announced *ontic* and *metascientific* construals.

Before doing so, however, it will be useful to first say a few more words on the nature of theory-structuralism itself. In Sect. 2, I noted that the structuralist view characterizes a

[21] My terminology here is, of course, inspired by that of van Fraassen (cf. Sect. 2).

scientific theory with the class—let us call it C_T^m—of all its theoretical models. The *additional* assumption underlying the structuralist view is now that this class C_T^m of theoretical models is in some sense 'given' by a class of mathematical structures. That is: we assume C_T^m is given by a class

$$C_T^s := \{\mathfrak{S} \mid \tau_T(\mathfrak{S})\}, \tag{2}$$

where each $\mathfrak{S}$ is a structure in the sense of (1) and τ_T denotes the set-theoretic predicate that every structure in C_T^s should satisfy. Lastly, let us note that proponents of theory-structuralism typically do not *just* hold that C_T^m is given by C_T^s, but that, indeed, each individual theoretical model M in C_T^m is 'given' by a structure $\mathfrak{S}_M$ in C_T^s. And with this, we have of course returned to one of the crucial assumptions of the structuralist approach to the science-world relation, viz. that to any theoretical model M we can associate a structure $\mathfrak{S}_M$. Indeed, this illustrates nicely why the structuralist approach to the science-world relation is so often expounded against the backdrop of theory-structuralism.

In the above summary of theory-structuralism, I have been ambiguous in two ways. Namely, I noted that a theory T was in some sense 'characterized' by the class C_T^m and that this class, in turn, was 'given' by the class C_T^s. This terminology, however, leaves it unclear what the precise relation is between T and C_T^m on the one hand and between C_T^m and C_T^s on the other. For present purposes, I wish to focus on this latter ambiguity.[22]

My claim is now the following: by looking at the different ways in which proponents of theory-structuralism take M to be 'given' by a structure $\mathfrak{S}_M$, we can learn more about what these proponents take the *general aim* of theory-structuralism to be. This will, as mentioned, yield two different construals of theory-structuralism.

Perhaps the most straightforward construal of the relation between M and $\mathfrak{S}_M$ is that it is simply the identity relation. On this construal, theory-structuralism would involve the claim that theoretical models *really* are mathematical structures. This would seem to be the view held by Suppes when he argued that "the meaning of the concept of model is the same in mathematics and the empirical sciences" (Suppes 1960, 289).[23] Similarly, Van Fraassen, while speaking of theoretical models, notes that "[t]hese models *are* mathematical entities, so all they have is structure..." (van Fraassen 1997, 528–529), italics added. The upshot is now that theory-structuralism, thus understood, is an account that makes *ontic, first-order claims* about science, e.g. that that theoretical models simply *are* mathematical structures. Accordingly, we can refer to this as an *ontic* (or: *first-order*) construal of theory-structuralism:

Ontic ('First-Order') Theory-Structuralism Theory-structuralism, as developed by Suppes and (at times) van Fraassen, is *ontic* in spirit: it aims to make first-order claims about what science (scientific theories, models, representation, etc.) is *really* like.

[22] Regarding the former ambiguity, French (2010) has argued that while proponents of theory-structuralism typically take the relation between T and C_T^m to be one of identity, we should, in fact, take this relation to be one of representation. In particular, French argues that we can—as philosophers of science—*represent* T by the class of its theoretical models, while adopting a quietist position on the 'true' nature of T. Regarding the latter ambiguity, French and his collaborators seem to hold an extremely similar view, as we will see in a moment.

[23] Note that 'models in mathematics' refers here to models as found in the field of model theory, in which a 'model' *by definition* is a mathematical structure $\mathfrak{S}$. (Or, more accurately, we define in model theory a *V-model* to be a structure $\mathfrak{S}$ together with an *interpretation mapping* $I : V \to \mathfrak{S}$ sending each symbol in the non-logical vocabulary V to some relation/function/individual constant in $\mathfrak{S}$ of appropriate arity.) See Hodges (1993) or any other introduction to model theory for more details.

The observation that theory-structuralism in this sense makes claims about what science is like, and is therefore 'ontic', might strike one as rather anti-climatic. After all, is not the *point* of doing philosophy of science to say some (hopefully insightful) things about the nature of science? In this light, one might wonder what the relation between M and $\mathfrak{S}_M$ could possibly be, if not the identity relation. This, however, is where a structuralist of a different stripe might step in. Indeed, consider the following, alternative construal of the relation between M and $\mathfrak{S}_M$. Rather than saying that M simply *is* $\mathfrak{S}_M$, the structuralist might take $\mathfrak{S}_M$ to be a *formal representation* of M, useful for the purposes of the metatheoretical study of scientific theories and models.

It is comparatively rare to see this view of the relation between M and $\mathfrak{S}_M$ being articulated explicitly. As noted by Frigg and Nguyen (2017, 67), proponents of the *partial-structures approach* to scientific theories (a variant of the structuralist view) have probably done the most to promote this conception of the theory-structuralism. The point is perhaps mostly powerfully articulated by Bueno and French: [A]dvocates of the [structuralist] account need not be committed to the ontological claim that models *are* structures... Set-theoretic structures provide a useful representational... device at the meta-level of the philosophy of science. What theories and models *are, qua* objects, is then a further matter" (Bueno and French 2011, 890), original italics.[24] A similar attitude is expressed, albeit less prominently, by the various exponents of the school of *Munich structuralism* (an especially rigorous and well-developed version of the structuralist view), with Balzer et al. (1987) emphasizing the "representational nature" of their account (1987, xvii) and Stegmüller (1976) repeatedly referring to his method as one of "rational reconstruction" (1976, 5ff) and "explication" (1976, 8ff). I follow the latter author in referring to this type of philosophical project, i.e. the construction of a formal representation of the process and products of scientific enquiry, as *metascience*.[25] Accordingly, let me refer to this second construal of theory-structuralism as *metascientific* (or: *second-order*) theory-structuralism:

Metascientific ('Second-Order') Theory-Structuralism Theory-structualism, as developed in the partial-structures approach as well as the school of Munich structuralism, is *metascientific* in spirit: it aims to make second-order claims about science, i.e. claims about the best way to *formally represent* various aspects of science.

In sum, we find ourselves with two rather different construals of (the aims of) theory-structuralism. Ontic theory-structuralism seeks to make claims about what science—in particular scientific theories and models—is *really* like. By contrast, metascientific theory-structuralism does not seek to make first-order claims at all, but merely has as its goal the

[24] The same view is also prominently expressed in French's recent monograph (French 2020, 95–96, 179, 235–239), more on which in Sect. 5 below. I thank an anonymous reviewer for alerting me to the existence of French's monograph, which appeared just as an earlier version of this paper had been completed.

[25] The term 'metascience' has been used to denote several, rather different areas of study. On one construal, the term is a synonym for the 'science of science'. On another construal, it is an umbrella term for history of science, philosophy of and science studies, cf. the journal *Metascience*. Recently, the term has also been used in a way to elicit a connotation with metaphysics, cf. the *MetaScience* project (2018–2023) at the University of Bristol. On the present use, 'metascience' is meant to elicit an association with metamathematics: in the same way that 'metamathematics' may be used to describe the formal study of mathematics, so too may we use 'metascience' to denote the formal study of science.

fruitful *formal representation* of science. Having distinguished these two construals, we are now well-poised to deliver the promised dissolution of OMP.[26]

4.2 The Problem Dissolved

My contention so far has been that by taking the structuralist approach to be an account within metascientific rather than ontic theory-structuralism, OMP straightforwardly dissolves. Let me now expound this argument in more detail, thereby presenting—what I have termed—a metascientific dissolution argument.

First, let us recall how we arrived at the general characterization of 'ontic' and 'metascientific' theory-structuralism. Recall from Subsect. 4.1 that my strategy was to look at how proponents of different versions of theory-structuralism construed the relation between M and $\mathfrak{S}_M$. The attitude displayed towards *this* relation was, then, taken to be indicative of the more general view held about the kind of philosophical project theory-structuralism is supposed to be. Hence, on the one hand, authors that seemed to construe the relation between M and $\mathfrak{S}_M$ as the identity relation were taken to hold a view of theory-structuralism in which theory-structuralism is an account making claims about what science is *really* like. And, on the other hand, authors that seemed to construe the relation between M and $\mathfrak{S}_M$ as one of formal representation, were taken to hold a view of theory-structuralism as an account that formally represents the process and products of scientific enquiry.

Having extrapolated two possible different construals of the M-$\mathfrak{S}_M$ relation (identity, formal representation) to two different attitudes towards the nature of theory-structuralism *in general* (ontic, metascientific), let us now apply these general attitudes to the relation between a physical system P and the corresponding structure $\mathfrak{S}_P$—for, recall, it was *this* relation that seemingly gives rise to OMP.

On the ontic construal, the aim of the structuralist approach is to make first-order claims about what, for a given theoretical model M and target system P, the M-P relation is *actually* like. In particular, this means that if we—*qua* structuralists—render the physical system P as a mathematical structure $\mathfrak{S}_P$, we are also—in line with the ontic construal—committing ourselves to the claim that P is *actually* structural in nature. In doing so, however, we open up the structuralist approach to OMP.

Of course, by saying that the target system P is 'structural in nature', we are not *yet* committed to the additional, stronger claim that P actually *is* the mathematical structure $\mathfrak{S}_P$. (If we were to subscribe to this additional claim, then we would immediately be led to OMP.) Indeed, the relation between P and $\mathfrak{S}_P$ need not be the identity relation in order to identify the M-P relation with the $\mathfrak{S}_M$-$\mathfrak{S}_P$ relation. As noted by Frigg and Nguyen (2017, 74), we could also hold that P is not itself a structure, but that P *instantiates* a structure. That is: P instantiates $\mathfrak{S}_P$. Adopting this line would, indeed, present us with a valid strategy for staving off OMP, were it not for the fact that the 'instantiation strategy' faces several, more

[26] It has been suggested to me that the distinction I draw between ontic and metascientific theory-structuralism is ill-defined: ontic theory-structuralism, the thought goes, makes claims about science and can, hence, also be said to be 'metascientific' in nature. Note, however, that I use the term 'metascience' in a specific way, i.e. as referring to the construction of formal representations of (various aspects of) science. To qualify as *metascience*, it is thus not enough for an account to make claims about science *simpliciter*. Admittedly, the term 'metascience' has been used in different ways as well (cf. footnote 25), but those are not senses in which I use the term here.

specific challenges of its own, such as Nguyen and Frigg's *curse of abundance* (Nguyen and Frigg 2021).[27]

Let us now, however, consider the bite of OMP on the *metascientific* construal of the structuralist approach. On *this* construal, the structuralist approach no longer involves the claim that the relation between *M* and *P actually* consists in a structural relation between $\mathfrak{S}_M$ and $\mathfrak{S}_P$. In particular, the structuralist approach now no longer involves the claim that *P* itself either *is* a structure or is in some other sense 'ontologically related' to a structure (e.g. through the relation of instantiation). Rather, the claim to which the structuralist approach is now committed is that the relation between *M* and *P* can be fruitfully *represented* as a structural relation between appropriate structures $\mathfrak{S}_M$ and $\mathfrak{S}_P$.

The upshot, of course, is that $\mathfrak{S}_P$ can now simply be taken to be a *formal representation* of *P*, without thereby subscribing to any ontological claims about the nature of physical systems. Hence, there is now no longer any ontological mismatch between *P* and $\mathfrak{S}_P$ (*prima facie* or otherwise), for the simple reason that there are simply no ontological claims being made about *P*. OMP, therefore, does not pose a genuine problem to the structuralist approach, provided we construe the account in a metascientific—rather than an ontic—vein. I conclude that OMP is straightforwardly dissolved.[28]

Before moving on, it is worth emphasizing the way in which the above argument is a new one. Indeed, the idea that adopting (what I have termed) a metascientific attitude can help safeguard theory-structuralism from some of its criticisms has been explored in the literature. As previously mentioned, such a metascientific attitude has already been adopted with regard to the relation between *M* and $\mathfrak{S}_M$—explicitly by French and his collaborators and implicitly by the Munich structuralists. My argument, in turn, consists in taking this attitude towards the *M*-$\mathfrak{S}_M$ relation, noting that we can extend that attitude to the *P*-$\mathfrak{S}_P$ relation and arguing that, by doing so, we can dissolve OMP. These latter two steps are, to my knowledge, not explicitly articulated by the aforementioned authors. And I take *this* to be the main contribution of this paper.[29]

5 Upshots of the Metascientific Stance

Let me now present some upshots of the metascientific dissolution argument just offered. In particular, let me mention an upshot of the argument, not just for our views on scientific representation, but for our views on *metascientific* representation as well. To this end, let me first sketch a possible objection to the line of argument given in Sect. 4. I argued that in *formally representing P* as a structure $\mathfrak{S}_P$, we were not committing ourselves to any ontological claims about *P*. In science, however, formal representations (e.g. mathematical models) would appear to be used *exactly* with the aim of finding out more about what their representational targets (typically some physical system) are *really* like. We might, therefore, be tempted to conclude that formal representations, if they are to be at all insightful,

[27] While Nguyen and Frigg do, in that same paper, offer a solution to their curse, I do not think—for the reasons mentioned in Sect. 2—that their solution is a satisfying one.

[28] Incidentally, I could also have presented this argument without first situating the structuralist approach within the wider landscape of theory-structuralism. Doing so, however, would have led us to ignore the *precedent* that exists in the theory-structuralism literature to construe our account in a metascientific vein. Situating the structuralist approach within theory-structuralism therefore serves to show why construing the structuralist approach in a metascientific vein is not simply ad hoc.

[29] See, however, my comments on French's (2020) eliminativist position in Sect. 5.

come along with ontological commitments about their targets after all. How seriously one should take this challenge depends on one's underlying view on the nature of representation. Let me, therefore, present my view on representation and, following this, explain why I take it to render the above challenge harmless.

On the construal I have promoted here, the structuralist approach is an account that aims to expound a formal, metascientific representation of the science-world relation. (The *formal* nature of the representations in the structuralist approach is irrelevant for the subsequent argument.) But what is the nature of the representation relation that holds between our representational token (the $\mathfrak{S}_M$-$\mathfrak{S}_P$ relation) and our representational target (the *M*-*P* relation)? In particular, we might ask, what is the nature of the representation relation between $\mathfrak{S}_P$ and *P*?

My answer, here, is to adopt an *inferentialist* stance. 'Inferentialism'—as the position is sometimes called (Frigg and Nguyen 2017, 76ff)—is a deflationary view on representation, according to which (successful) representation consists in nothing more than the capacity of the token to support surrogative reasoning about the target. While inferentialism was originally articulated as a stance towards *scientific* representation (Suárez 2004), I propose to extend it to *metascientific* representation as well.

We can now appeal to this stance towards representation to resolve the above challenge. In both the scientific and metascientific cases, (successful) representation now does not commit us to the view that our token is similar, structurally or otherwise, to its target. Instead, the deflationary character of the inferentialist stance allows us to remain silent on what our target is *really* like and, as a result, we incur no ontological commitments about our target. Hence, we see that—on this view of representation—metascientifically representing the physical system *P* as a mathematical structure $\mathfrak{S}_P$ does *not* implicitly commit us to the view that *P* is structural in nature after all.

A reader acquainted with the scientific representation literature might be surprised by my above endorsement of the inferentialist approach. In particular, they might ask how I can coherently adopt an inferentialist approach to representation, while simultaneously defending the structuralist approach to the science-world relation. After all, in the contemporary literature, the relation between a theoretical model *M* and target system *P* is typically taken to be an instance of the scientific representation relation, with the structuralist approach then being an account *of scientific representation*—indeed, a rival of the inferentialist approach.[30]

The key to resolving this apparent inconsistency lies in making a distinction quite similar to the distinction between ontic and metascientific construals of theory-structuralism, in the sense that we need to distinguish between a (first-order) stance concerning the *actual* nature of representation and a (second-order) stance concerning the best way to *represent* representation. Indeed, when asked about what either scientific or metascientific representation actually consists in, I will reply along inferentialist (i.e. deflationary) lines. By contrast, when asked about the most fruitful way of metascientifically representing the relation of scientific representation, I will adopt a structuralist stance. To put the same point in more general terms: We can hold a deflationary view about what representation consists in at the *object level*, while still aiming for a substantive (e.g. structuralist) account of representation at the *meta-level*. See Figure 1 for a schematic summary.

There is much more that could be said about the nature of metascientific representation, and the way in which it is related to our views about scientific representation. Doing

[30] See, for instance, McCullough-Benner (2020) for a recent and illuminating critique of the structuralist approach in favour of an inferentialist approach.

so would, however, take us beyond the scope of the present paper. For the time being, let me make due with the following remarks, which might be taken as a point of departure for further discussion:

Pluralism about representation? First, note that the above strategy for dealing with the objection presupposes that it makes sense to be *pluralist* about representation. More specifically, we are assuming that we can simultaneously adopt an inferentialist stance towards *metascientific* representation and a structuralist stance towards *scientific* representation. But is this, in fact, a coherent position to hold or are we here unwittingly engaging in an exercise of naive philosophical doublethink? *Prima facie*, the prospect of a pluralist stance towards representation does not strike me as particularly radical or problematic, and it may interact in interesting ways with recent work on model pluralism. Moreover, the fact that the inferentialist stance towards metascientific representation is adopted at the *object level*, whereas the structuralist stance towards scientific representation is adopted at the *metalevel*, might further help to assuage any concerns about incoherence. Nevertheless, those keen on taking the 'naturalistic turn' in the philosophy of science, be it in the sense of Quine (1969), Giere (1985) or Ladyman et al. (2007), might have their reservations about a representational pluralism motivated by a hard science/metascience divide. A more elaborate discussion is required to address these points in detail.

Tenability of inferentialism In subscribing to an inferentialist stance towards metascientific representation, I implicitly presupposed that inferentialism can indeed eschew ontological commitments in the case of *scientific* representation. This latter presupposition, however, has been questioned by e.g. Chakravartty (2009), who notes that "putative cases of scientific representation in the absence of similarity are few and far between" (2009, 201). If inferentialism *does* turn out to bring in ontological commitments in the case of scientific representation, then it would presumably also bring in such commitments in the case of *metascientific* representation. But this would, then, lead us right back to the worry which started this subsection, namely: Does construing the P-$\mathfrak{S}_P$ relation as a metascientific representation relation not implicitly bring in ontological commitments about the nature of P after all? Such a rejoinder would, it seems, pose a genuine challenge to the metascientific dissolution argument presented in Subsect. 4.2. At the same time, though, I note that this challenge would go well beyond the metascientific dissolution argument in particular and would, indeed, call into question the core of the inferentialist approach itself. Accordingly, a defence of the metascientific dissolution argument would, in all likelihood, involve a general defence of inferentialism as a position in the representation debate: a task that lies outside the scope of this paper.

Deflationism vs functionalism Let us assume that the inferentialist approach can indeed overcome the above challenge. Inferentialism would then remain a viable alternative to the structuralist approach as an object-level view of metascientific representation. However, in presupposing the inferentialist and structuralist approaches to be two opposing views, do we not fall foul of Chakravartty's (2009) plea for irenicism between "informational" approaches and "functional" approaches to scientific representation?[31] My response, briefly stated, would be to disentangle the *deflationist* and *functionalist*

[31] Here, we may consider the structuralist approach to be a particular kind of informational approach and the inferentialist approach to be a particular kind of functional approach.

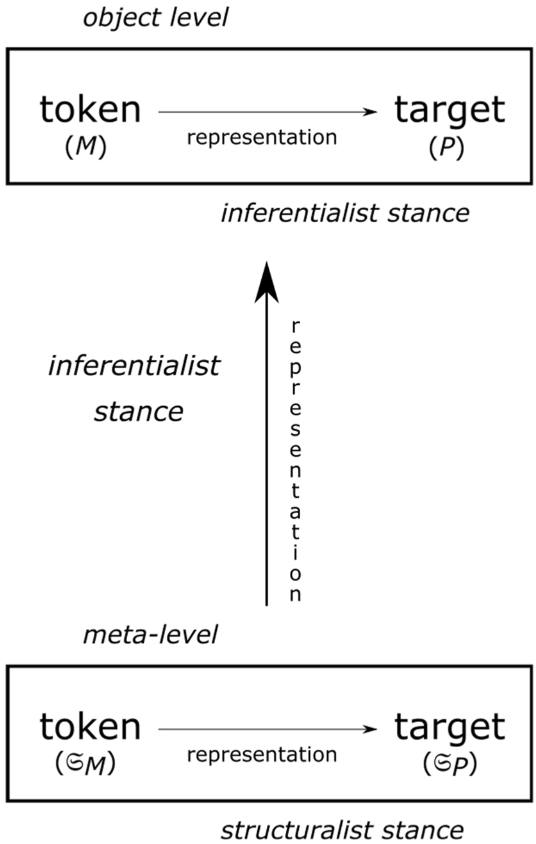

Fig. 1 Object-level versus meta-level views on representation

aspects of the inferentialist approach. Subsequently, I would argue that we can maintain the opposition between the structualist approach and inferentialism *qua* deflationary informational position, while endorsing wholeheartedly (as I do) Chakravartty's plea for the compatibility of the structuralist approach and inferentialism *qua* substantive functionalist position. Again, I note that a more elaborate discussion falls outside the scope of this paper.

French's eliminativism Lastly, let me situate my comments here with respect to some of the views recently expounded in French's 2020 monograph *There Are No Such Things as Theories* (French 2020),[32] in which he develops an *eliminativist* view on the nature of scientific theories. French's view, as suggested by his chosen title, is that—ontologically speaking—there are no such things as scientific theories or, indeed, models. Moreover, French argues that statements involving scientific theories/models in everyday, scientific and/or philosophical contexts are made true or false, not by virtue of the properties of some abstract

[32] An earlier version of some of these views may already be found in French and Vickers (2011).

objects known as *theories/models*, but by the practices involving the use of theories/models in the corresponding contexts.

French's line of argument is a rich one, drawing on considerations from philosophy of science, metaphysics and philosophy of art. What matters for present purposes is the upshot of French's eliminativism for his view on the science-world relation. I have already noted in Sect. 4 that French and his collaborators are among the few philosophers of science to have taken up an explicitly metascientific stance towards theory-structuralism and, indeed, towards philosophy of science as a whole. In fact, French even seems to hint, in passing, to a similar metascientific dissolution of OMP: "If you are an advocate of the Semantic Approach [in my terms: structuralist approach], then, likewise, you 'represent' the theory at the *meta-level of the philosophy of science* in terms of set theory and also 'represent' the way the theory latches onto the world via the formal notion of (partial) isomorphism, which of course then relates to the formal representation of the target system (in order to sidestep protests that set theoretical notions cannot relate mathematical structures and physical ones)" (2020, 235–236), italics added. And solidifying the similarities even further, French also notes that we can subscribe to a deflationary, inferentialist account of scientific representation at the object level, while maintaining a substantive (formal) account of scientific representation at the meta-level (2020, 233–235).

The above similarities notwithstanding, French's views diverge significantly from the views on representation presented here (cf. Fig. 1). Consider in particular the upshot of French's eliminativism for the notion of *metascientific* representation. Since French rejects the notion that there exists a theoretical model M (either as an abstract entity or abstract artefact) at the object level, the metascientific representation relation between M and $\mathfrak{S}_M$ would lack one of its prerequisite relata. French instead argues that we should view $\mathfrak{S}_M$ as "a *meta-level construction* that we, as philosophers of science..., deploy when we endeavour to make sense of scientific practice and its implications for our understanding of the world" (2020, 235), original italics.[33]

What are we to make of French's views, in relation to both (I) the metascientific dissolution argument itself and (II) the view on scientific/metascientific representation as given in Fig. 1? Regarding (I), let us note that, while the metascientific stance towards theory-structuralism is arguably a necessary component of French's eliminativism, the converse is not the case. In other words, we can still subscribe to the metascientific dissolution argument for OMP, without necessarily subscribing to the eliminativist position defended by French. This means, for example, that we can follow Halvorson (Forthcoming) in wondering whether "practices [can] bear the ontological weight that French needs them to in order to ground claims about theories", without thereby problematizing the metascientific stance towards theory-structuralism.

Regarding (II), let me note that Fig. 1 presents us with a way of conceptualizing the relation between the object and meta-levels in the philosophy of science, without requiring us to abandon the view that philosophers of science are engaged in *representational* activities. This is particularly beneficial when we consider the status of the structure $\mathfrak{S}_M$ *vis-à-vis* the structure $\mathfrak{S}_P$. On French's account, the former structure is designated a 'meta-level construction' (the exact nature of which remains opaque), while the latter structure would—presumably—still qualify as a metascientific *representation* of the target system P. Such a stance, however, seems particularly undesirable, since it leads us to the counter-intuitive

[33] The quoted text actually refers to (meta-level constructions of) theories rather than (meta-level constructions of) models, though this makes no difference for the point being expressed.

conclusion that the way philosophers of science arrive at the structure $\mathfrak{S}_M$ is substantially different from the way they arrive at the structure $\mathfrak{S}_P$.

It is not my goal here to assess the veracity of French's eliminativism. Indeed, if one is already convinced of the eliminativist proposal, both of my above two points would be rendered moot.[34] What matters, for present purposes, is the observation that neither the metascientific dissolution argument, nor the object level/meta-level distinction in philosophy of science, should be viewed as being exclusively tied to an eliminativist view of theories and models.

6 Conclusion: The Quest Continues

In this paper, I have argued for the viability of the structuralist approach to the science-world relation, by showing how one of that account's most serious challenges, viz. PLR, may be overcome. More specifically, I have argued that the part of PLR that turns on the *prima facie* ontological mismatch between physical systems and mathematical structures—the problem I referred to as OMP—is straightforwardly dissolved, once we adopt a metascientific construal of the structuralist approach (along with an inferentialist view on the nature of metascientific representation).

I have, of course, yet to say anything about *what kind* of structures $\mathfrak{S}_M$ and $\mathfrak{S}_P$ actually are, how we are to formally construct these structures given our (informally specified) theoretical model M and physical system P and, perhaps most importantly, about what kind of structural relation should hold between $\mathfrak{S}_M$ and $\mathfrak{S}_P$. As noted in Sect. 3, I relegate these issues to SP ('the specification problem')—a problem falling outside the scope of this paper. In presenting my dissolution of OMP, however, I have already implicitly placed some constraints on what a satisfying solution to SP should look like.

More specifically, on the metascientific constual of the structuralist approach, a satisfying solution to SP is one that specifies all significant features of the *M*-*P* relation in terms of features of its formal representation (i.e. the $\mathfrak{S}_M$-$\mathfrak{S}_P$ relation). Two features of the *M*-*P* relation seem especially significant, namely (I) the fact that M enables surrogative reasoning about P and (II) the fact that M is more tractable than P itself. Accordingly, a satisfying solution to SP should at least involve:

(SP1) the specification of a structural property by virtue of which $\mathfrak{S}_M$ enables surrogative reasoning about $\mathfrak{S}_P$, and
(SP2) the specification of a structural property by virtue of which $\mathfrak{S}_M$ is more tractable than $\mathfrak{S}_P$.

While proponents of the structuralist approach have already paid much attention to criterion SP1,[35] criterion SP2 has been relatively neglected.

I would argue, though I cannot expand upon this point here, that SP2 requires an extension of the structuralist approach with a structuralist account of *abstraction*. To my knowledge, there currently exists no such account: while both Suppe (1989) and Haase (1996) discuss abstraction in the context of theory-structuralism, neither expounds the notion of

34 On my part, I remain attached to what French calls the *abstract artefact* view of theories and models (cf. French 2020, Ch. 5) and accordingly do *not* subscribe to his eliminativist proposal.

35 See, for instance, the various proposed structure-preserving mappings mentioned in footnote 9.

abstraction in a formally substantive way.[36] It is exactly in articulating such a formally substantive account of abstraction that a satisfying solution to SP is to be found.

But that is a project for another day. For now, let us be content with the present waypoint on our quest for lost reality.

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[36] I do not, however, follow Pero and Suárez (2016) in their judgment that the structuralist approach is intrinsically incapable of incorporating abstractions, as their argument—it seems to me—rests on an overly narrow conception of abstraction as well as the structuralist approach itself; a more elaborate reply, unfortunately, exceeds the scope of this paper.

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