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Introduction

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The usual conception of the philosophy of mathematics is based on the mid-twentieth century configuration of the discipline, giving pre-eminence in an almost exclusive way to the abstract, pure mathematics of set-theoretic structures. It was often not realized how much this presupposition conditioned the framing of philosophical problems and the outline of the usual answers. Much can be gained from reconsideration and historical contextualization, from establishing an interplay between history and philosophy of mathematics (HPS applied to maths). Up until the nineteenth century, mathematics included mechanics (among other topics nowadays classified as 'physics'), and one cannot deny the central role it played in the very definition of mathematical knowledge, throughout the eighteenth century and beyond. In this spirit, in September 2017 we organized a Workshop on *Mathematics and Mechanics in the Newtonian Age* at the Institute of Mathematics of the Universidad de Sevilla (IMUS).[1]

By the 'Newtonian age' we meant not the time of Newton's life, but the period of greatest scientific influence of his theoretical proposals; roughly, 1700 to 1900. During that period, we encounter quite a different field map of the discipline mathematics, which gradually evolves into a more modern classification of subfields, without ever leaving mechanics out. As late as 1889, the *Jahrbuch für die Fortschritte der Mathematik* has, among the twelve main branches of maths, mechanics as no. X, mathematical physics as no. XI, geodesy and astronomy as no. XII. A nineteenth-century mathematician could not spare the study of mechanics; to give just one example out of many, the great two-year arrangement of key mathematical topics that Kummer and Weierstrass established at Berlin included mechanics (taught by Kummer).[2] It could not have been otherwise: in 1887, Kronecker indicated three central concepts of mathematics which require 'philosophische Vorarbeit'—number, space, and time.[3] Notice also that a typical mathematician of the

[1] See https://gecomat1216.wordpress.com/.

[2] See Biermann (1988, 103–104): the core lectures were analytical geometry, theory of surfaces, number theory, mechanics, introduction to analytic functions, elliptic functions (or Abelian functions), applications of elliptic (or Abelian) functions to geometry and mechanics, variational calculus.

[3] The latter points very clearly in the direction of mechanics as a core mathematical topic.

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Newtonian age would work in very different areas, with the subsequent exposure of his views to cross-influences.

A reflection of that state of affairs, in an inverted mirror, so to say, was given by Arnold when he deplored the fact that in the middle of the twentieth century, an attempt was made to separate physics and mathematics: "The consequence turned out to be catastrophic. Whole generations of mathematicians grew up without knowing half of their science and, of course, in total ignorance of any other sciences" (Arnold 1998, 229). Arnold's viewpoint is quite representative of the views of the powerful and influential Moscow School which gathered around Kolmogorov. The mid-twentieth century obsession with purity seems now gone, and there is much philosophical interest in the project to contribute to a broader view of maths that underlines the close relation—both historically and systematically, from today's point of view—of 'mathematics' and 'physics'.

The papers included in this special section were selected by the editors out of the many interesting talks presented in 2017 in Sevilla; the authors have further elaborated their ideas in the final versions gathered here. We shall briefly indicate some of the issues they raise and the philosophical questions they touch upon. Featured problems are the question of the applicability of maths, the role of epistemic values and theoretical virtues in the practice of maths, different or changing configurations of foundational issues, and key methodological and conceptual changes promoting the shift from the 'classical' Newtonian configuration of maths toward so-called 'modern' maths.

The problem of the applicability of maths, and of the origins of mathematical methods, whether concrete or purely abstract, has been discussed countless times. Yet the usual formulation of the problem of 'applicability' presupposes the 'modern' conception of mathematics (emphasizing a reconstruction of pure maths in terms of abstract axiom systems and structures) and cannot be employed for the 'classical' era without questioning. Reference to the map or classification of maths in the Newtonian age, and to the central role of mechanics in particular, immediately highlights the phenomenon of *intended* applications, i.e., of methods intentionally developed to shed light on some concrete kinds of physical configurations. Daniele Molinini studies the method of Lagrange multipliers, which emerged in the context of statics, although it has been developed and generalized into a powerful method for the solution of optimization problems with equality constraints. In his discussion of this question, Molinini highlights the relation between intended and unintended applications, and links it with the attempt to articulate a perspective on the objectivity of mathematical results. Notice that Lagrange's initial justification for his procedure was rooted in physics, although today the Lagrange multipliers can be presented purely mathematically. Since the process of finding a 'purely mathematical' justification or proof is often accompanied with generalization and an increase in flexibility of the presuppositions (for application of the method in question), in many instances there is little wonder that the outcome turns out surprisingly applicable and efficient. At the same time, Molinini argues, the elaboration of different applications, both intended and unintended, serves as a form of 'crosschecking' that contributes significantly to the objectivity of maths.

Lagrange was a master of analytical methods, who famously boasted that his work on analytical mechanics was free of any reliance on figures. Yet the second paper, dealing with a relatively unknown aspect of the work of Chasles, focuses on the elaboration of a *geometrical* theory of the attraction of ellipsoids. As Nicolas Michel shows, Chasles's preference for synthetic geometry over analytical calculations raises the question of the role of epistemic values in the actual development of maths. During the nineteenth century, and especially around the development of projective geometry, there was a significant debate concerning the merits of the analytical versus the synthetic method;[4] authors like Chasles tried to give form to purely synthetic methods that could be as powerful as the analytical methods developed since Descartes's revolutionary work. In this case, one can study the interplay between mechanics and pure geometry, but in the process, Michel finds epistemic values centrally at work—the search for explanation, simplicity, and generality, which according to Chasles reinforced each other. The promotion of synthetic methods over the opacity of convoluted analytical calculations was based precisely on the epistemological virtues of those methods.

The interactions between geometry and mechanics are also the matter studied in the third contribution. María de Paz and J. Ferreirós pay attention to some of the factors leading to a relevant epistemological shift, the demotion of the basic principles of maths from their former status of absolute, a priori truths—a question that affected not only the foundations of maths, but the nature and character of scientific knowledge as a whole. This time, the interplay is between the categories applied to basic principles of mechanics and geometry, and the argument revolves around the importance of the oral dimensions of a mathematical culture (especially once professionalization and deep institutionalisation are underway, as happened in Europe by the mid-nineteenth century). Jacobi presented Newton's laws as *conventional* in lectures of 1847/48 (decades before the work of Poincaré), Riemann applied to them—and to the axioms of geometry—the category of *hypothetical*. De Paz and Ferreirós argue that both moves may have been interconnected, and that the entourage of Gauss (a network of scientists linked with the famous Göttingen astronomer by correspondence and apprenticeship) facilitated the transmission of such innovative views. Yet they aim to show that presenting the changing status of the principles of mechanics as a mere epiphenomenon of the emergence of non-Euclidean geometries is inaccurate, that there were back-and-forth relations between the two subdisciplines, richer than what is usually pictured in the literature.

The question of attention to practice is an old one, emphasized among others by the constructivists—Brouwer, Bishop—but also by Poincaré: "Let us then watch geometers at work to catch a glimpse of their methods" (Poincaré 1902, 9). This question has now become a leitmotiv, and it requires paying attention to the study of actual cases, to changing historical contexts. The three papers in this special section are clear examples. As Michel writes, the history of Newtonian mechanics is not merely that of a succession of results, or of additions and improvements upon Newton's initial results: it was in fact shaped by fluctuations in disciplinary boundaries and identities. Besides the historical and philosophical interest of these questions, it seems relevant to add that they are of current concern since we are living changes in the understanding of maths as a whole. Images of mathematics do change, and the extreme purism of the mid-twentieth century has given way to a more balanced image, where the pure and the applied feature merely as different sides of a single coin (see Bottazini and Dahan-Dalmedico 2013).[5] Thus to revisit past developments can be enlightening for those mainly interested in understanding the current situation.

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[4] See, e.g., Gray (2010) or Ostermann and Wanner (2012).

[5] Notice also the topics of recent meetings at the Mathematisches Forschungsinstitut Oberwolfach (Epple et al. 2013; Gray et al. 2015).

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