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Nothing but Gold: Complexities in terms of Non-difference and Identity. Part 2. Contrasting Equivalence, Equality, Identity, and Non-difference

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Abstract The present paper is a continuation of a previous one by the same title, the content of which faced the issue concerning the relations of coreference and qualification in compliance with the Navya-Nyāya theoretical framework, although prompted by the Advaita-Vedānta enquiry regarding non-difference. In a complementary manner, by means of a formal analysis of equivalence, equality, and identity, this section closes the loop by assessing the extent to which non-difference, the main issue here, cannot be reduced to any of the former. The following sections of this study will focus on the assessment of the eventual possibility of causation and transformation in non-difference.

Keywords Non-difference · Equivalence · Equality · Identity · Nyāya

Abbreviations

a	Primitive term (lowercase italics)
$\_{t}$	Abstraction functor, expressing the Sanskrit suffix *-tva* or *-tā* (e.g., $a_t = a$-hood)
A	Set A (capital)
$\lvert a_t \rvert$	Extension of an abstract; $\lvert a_t \rvert$ = A
R	Relation R (capital italics)
$\boldsymbol{R}$	Relational abstract (bold capital italics)
$R^{(R\prime)}$	Relation R' interpreted as R, *salva veritate*
R[A]	The relation R set of destination; for R: A $\mapsto$ B, dom$R \subseteq$ A, ran$R \subseteq$ B, and R[A] = B
$\urcorner$	*Avacchedaka* operator; identifying the *limitor* of a relational abstract
$\llcorner$	*Nirūpaka* operator; identifying the *conditioner* of a relational abstract

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.	*Niṣṭha* operator; connecting an abstract to a primitive term
$\rightleftharpoons$	*Tadviparyayeṇa* operator (‘vice versa’); expressing a symmetrical relation
$\downharpoonleft\upharpoonright$	*Yathā-tathā* operator (‘just like-so’); capable of expressing the coordination of a relation with its inverse ($R \wedge R^{-1}$). It always preserves the distinction between abstract properties and primitives terms of the *anuyogin* and *pratiyogin* positions
TvN	*Tadvattva-Nyāya* (‘Axiom of Possession’)
SVN	*Samānādhikaraṇa-Viśiṣṭatva-Nyāya* (‘Principle of Coreferential Qualification’)
$*\varphi$	‘It is false that φ’
$^{(t)}$ ‘…’	*Tātparya* (purport of an expression)

As stated in the first part of this investigation (P1), non-difference ($\cancel{2}$)—closely linked to the notion of coreference (*sāmānādhikaraṇya*, *Ṇ*)—cannot be reduced to identity or equality. In the following sections I will try to definitely demonstrate why this is the case, but not before having discussed how non-difference cannot be subsumed to the relation of equivalence, either.[1]

Equivalence

In an axiomatic theory of sets, equivalence (E) is a binary relation capable of formally expressing the naive concept ‘possessing the same property’.[2] In *Nyāya Kośa* (NK), equivalence is described sub voce $tulyatva_{1kha}$.[3] In this manner, x is

[1] For the sake of clarity, formulas numbering follows directly from Anrò 2021, P1, allowing easier intertextual references. The notational system adopted here is in compliance with the ‘Navya-Nyāya Formal Language’ or NL (cf. Anrò, forthcoming); a descriptive table is provided at the end of the article.

[2] Grishin (2014), referring to: N. Bourbaki*, Théorie des Ensembles. Eléments de mathématiques 1*. In this article I chose to use the symbol ‘E’ to express equivalence, in keeping with NL notation (where relations are expressed with italic capital letters). I reserve *tilde* (‘~’) for negation (cf. P1 fn. 23).

[3] Following the indexing proposed by NK, I make explicit the index clue ‘1[*kha*]’ to distinguish this particular sense of the term *tulyatva* from the following ones, instead referable to the concept of ‘equality’. The meanings 1[*ka*] and 1[*kha*] are explicitly reported as analogous to *sādṛśya* (similitude): NK: 333: *tulyatvaṃ*—1[*ka*] *sādṛśyavad asyārtho ’nusaṃdheyaḥ* ǀ. Cf. NK: 991: *sādṛśyam*—[*kha*] *tadbhinnatve sati tadgatabhūyordharmavattvam* ǀ. Although distinct, two objects are said to be ‘similar’ because they share multiple common features. Moreover, in light of the truth conditions laid out (cf. *infra*), I see myself as obliged to introduce some differences in relation to Ingalls’ translation: *samaniyatatva* is ‘equality’ and not ‘equivalence’ here, while ‘equivalence’ is ‘$tulyatva_{1kha}$’. This is because, according to Ingalls: “Equality is a relation between classes. Equivalence is a truth function connecting statements or formulae. Identity is a relation between individuals” (Ingalls 1951, p. 67). Here, on the contrary, equivalence is a relation connecting distinct instances of a given property; equality is a relation connecting statements or formulae (and only in this sense is it, possibly, a relation between classes); and identity is a relation between individuals. According to the theory of sets: “R is an *equivalence relation on* A *iff* R is a binary relation on A that is reflexive on A, symmetric, and transitive” (Enderton 1977, p. 56). Equality and identity, on the other hand, are equivalence relations under more restrictive conditions. This holds true to a great extent in this context as well.

equivalent (*tulya*) to y ($\langle x, y\rangle \in E$) if it shares with y a common property (*dharmavattva*) even while keeping itself distinct from it (*bhinnatva*).[4]

Be it considered, for instance, the indefinite generic statement: *gaur gāṃ janayati* (‘A cow gives birth to a cow’), or the following indefinite non-generic one: *gām ānaya* (‘Fetch a cow’). In all of these cases, by reason of their indefinite character, if a cow (g) possesses the property cow-ness (*gotva*, g_t), then a second cow (g') might be said to be equivalent to g with respect to the property *gotva*. ‘That cow is equivalent to this one’—*so gaur etasya gos tulyaḥ*—will appear in NL as:

[8] $(g' \cdot g_t)\urcorner\, \boldsymbol{E} \llcorner(g \cdot g_t)$
yad tulyatvam idaṃ-go-niṣṭha-gotva(*vattva*)-*āvacchinnaṃ tad adaḥ-go-niṣṭha-gotva*(*vattva*)-*nirūpitam*; ‘Equivalence, conditioned by cow-ness in that cow, is limited by cow-ness in this cow’; iff $(g, g') \in(|g_t| = \mathrm{G})$ (*dharmavattva* = *gotvavattva*; cf. fn. 3: NK: 991) $\wedge\, g' \neq g$ (*bhinnatve sati*; cf. fn. 3: NK, p. 991) $\wedge$ $|g' \cdot g_t| \subseteq |\mathrm{E}\llcorner(g \cdot g_t)|$ (‘Cow-ness in cow g' is a sub-set of What is equivalent to Cow-ness in cow g’); that is, $\langle g, g'\rangle \in E_{gt}$.[5]

[4] NK, p. 334: *tulyatvam*—1[kha] *bhinnatve sati dharmavattvam* | *caitreṇa caitrasya vā tulya ity ādau* | *atra tulyaśabdhārthaniviṣṭe ca bhede tṛtīyādyarthasya pratiyogitvasya dharme cādheyatvasya anvayāt caitratvāvacchinnānyatve sati caitravṛttidharmavān ity arthaḥ* |. Indeed, ‘in possession of a property’ (*dharmavattva*) appears as an excessively vague condition to define equivalence: both pyramids and apples possess at least one property each; it does not follow they they can be said to be equivalent. Nevertheless, NK declares that *tulyatva*$_{1[\mathrm{kha}]}$ is analogous to *sādṛśya*: ‘*bhūyor-dharmavattva*’, ‘possessing multiple [common] properties’ (NK, p. 991; cf. previous fn.). Here, significantly, the term ‘multiple’ (*bhūyas*) is omitted. In fact, ‘possession of multiple properties’ (*bhūyordharmavattva*) appears as either too vague or singularly inappropriate for a technical use of the term ‘equivalent’. If two sisters are said to be alike, then their likeness (*sādṛśya*) must be further articulated: a single feature is picked out and then claimed to be common; say, their nose, their voice, etc. In the sense of ‘possessing [at least a common] property [singularly considered]’, the apparently-lacking definition of *tulyatva*$_{1[\mathrm{kha}]}$ could thus be considered—by virtue of its being connected to *sādṛśya* (*bhūyordharmavattva*)—a case of *lāghava* (lightness in definition). Note also that equivalence can be expressed with either a genitive or instrumental case; yet, NK specifies, the genitive is advisable according to the way grammarians use it: NK, p. 334: 1[kha] *evaṃ caitrena caitrasya vā sādṛśyam ity ādāv api draṣṭavyam* | *atra viśeṣo jñeyaḥ pāṇinīyāḥ tulyopamayor yoge tṛtīyāṃ necchanti iti* |.

[5] Cf. the Axiom of Possession (*Tadvattva-Nyāya*, TvN) formulation, P1.§3: *tadvattvam* (or *taddharma-vattvam*) *tad eva*, ‘What possesses the property of being that, is that’. Thus, in [8] *gotvavattva* = *gotva*, i.e. the property *possessing cow-hood* = *cow-hood*; while, *govattva* = *go*. For this reason, [8] reads the simplified version and *gotva* appears instead of *gotvavattva.* It is well-known that for any equivalence relation R on the set A, it is possible to obtain a *partition* of A. In this sense, we can obtain a partition of the class (*jāti*) cow-ness (*gotva*, g_t) with respect to a particular quality (*guṇa*)—for instance, colour. In set theory, if $x \in \mathrm{G}$ (i.e. x is a cow) and ‘possessing a colour’ is ‘r_t’ (*rāgavattva*; say, *śuklatva*, whiteness), then the class of equivalence of the element x on G, with respect to the equivalence relation (E) ‘possessing the same colour’ (*samarāgavattva*), is $[x]_E = \{y \in \mathrm{G} \mid \langle y, x\rangle \in E_{r\text{-}t}\}$; i.e. the partitions of the cow set G, according their colour. ‘That cow is equivalent to this one, because of their colour’—*so gaur etasya gos tulyaḥ, rāgavattvāt* —in NL: $((g'.\ g_t)\urcorner r_t)\urcorner \boldsymbol{E}\llcorner((g.g_t)\urcorner r_t)$ *yad tulyatvam idaṃ-go-niṣṭha-gotvāvacchinna-(sama)rāga-vattvāvacchinnaṃ tad adaḥ-go-niṣṭha-gotvāvacchinna-(sama)rāgavattva-nirūpita*; ‘Equivalence, conditioned by (same-)colour-ness described by cow-ness in that cow, is limited by (same-)colour-ness described by cow-ness in this cow’; iff $(g, g') \in \mathrm{G} \wedge g' \neq g \wedge |((g' \cdot g_t)\urcorner r_t| \subseteq |\boldsymbol{E} \llcorner((g \cdot g_t)\urcorner r_t)|$, that is, $\langle g, g'\rangle \in E_{r\text{-}t}$. Cf. Enderton (1977, p. 57): “The set $[x]_R$ is defined by $[x]_R = \{t \mid xRt\}$. If R is an equivalence relation and $x \in \mathrm{fld}$ (R) [‘field’], then $[x]_R$ is called the *equivalence* class of x (*modulo* R). […] The status of $[x]_R$ as a set is guaranteed by a sub set axiom, since $[x]_R \subseteq \mathrm{ran}(R)$ [‘range’]. Furthermore, we can construct a set of equivalence classes such as $[x]_R = \{x \mid x \in A\}$, since this set is included in (ran R)”; where, “for any set a, the *power set a* is the set whose members are exactly the subset of a”, Enderton (1977, p. 19).

As shown by the previous example, in general the relation of equivalence appears as necessarily bound to domain multiplicity—setting aside, for the moment, the trivial case of the equivalence of an element with itself (reflexive equivalence). In this respect, let us consider the definition of *jāti* or *sāmānya*: "[…] *sāmānyam iti* | *tallakṣaṇaṃ tu nityatve saty anekasamavetvam*"; "[…] The 'universal'. While its definition is: the property which, being constant, is inherent in many [particulars]" (NSM 1988, pp. 97–98).[6] It follows that all individuals (*vyakti*) belonging to a given *jāti* are by definition equivalent to each other with respect to the *jāti* to which they belong. On the contrary, let us now consider the first *jāti-bādhaka* ('blocker' or 'opposing agent of the universal'): *vyakter abhedaḥ* [*bhedābhāva*], " 'the oneness of the individual' or 'indivisibility of the individual' [or 'radical absence of any possible distinction'], that is when exists only one member of any category, an individual alone" (Pellegrini 2016, p. 79). For instance, according to the Nyāya analysis, *kāla* (time) or *dik* (space) are radically one and therefore, qua singular substances, they cannot have equivalents in the sense that a cow, with respect to another cow, can.

In the utterance *vaidyam ānaya* ('Fetch a doctor!'), the implied meaning appears, reasonably enough, to be: *mad-rogamukti-viśaya eka-kuśala-vaidyasya anya-kuśala-vaidyas tulyaḥ*; *atha kuśala-vaidyam ānaya* ('In order to heal my disease, a skilled physician is equivalent to another one, provided that he is a skilled physician as well; so, fetch one!'). So—for v : *vaidya*, a physician; v_t : *vaidyatva*, the property *being a physician*; V: the set Physicians; and for $(v, v')\in V$ and $|v_t|=V$—we could obtain the meaningful assertion: $[8_b]$ $(v'.v_t)\neg\boldsymbol{E}\llcorner(v.v_t)$, for $v'\neq v$. However, $[8_c]$ $(v'.v_t)\neg\boldsymbol{E}\llcorner(v'.v_t)$, for $v=v$, that is, *etat-kuśala-vaidyasya etat-kuśala-vaidyas tulyaḥ* ('This very physician is equivalent to this very physician') is true either in the secondary and here pointless sense of an individual being equivalent to himself; or even, taken as a negation, with a completely opposite meaning. This last sentence, in fact, could be interpreted as *etad-vaidyasya na kaścit tulyaḥ, tasya uttamattvāt*: 'This physician has no equivalent, because he is the best' or $*[8_{c\text{-}1}]$ $(v'.v_t)\neg\boldsymbol{E}\llcorner(v.v_t)$, which is nevertheless a patent contradiction for $(v, v')\in(V=|v_t|)$ (*='false'; cf. P1). The truth values thus suggest that the properties involved cannot be the same. Instead, for instance v_t = 'Being an average physician', and v'_t = 'Being the finest physician', with a significative change in the truth values: *a*) $v'_t\neq v_t$, $(v'\in|v'_t|)\subseteq(v\in|v_t|)$; *b*) the domain of $|v_t| = V$ will be now greater than one ($\mathrm{card}(V)\geq 2$) if we assume that there exists at least one average doctor apart from the outstanding one; and *c*) the domain of $|v'_t| = V'$ will necessarily be equal to one, because there is by definition only one utmost exemplar ($\mathrm{card}(V') = 1$; for the concept of cardinality of a set, cf. *infra*). In sum, two distinct elements *a* and *b*, both belonging to the generic set $Z = |z_t|$, could be said to be equivalent to the generic property z_t. This is impossible with respect to the two distinct properties z_t and z'_t, however, because *x* and *y* now no longer belong to the same reference domain; i.e. an average physician is not equivalent to the best physician. Moreover, a reflexive equivalence could be either true but trivial or paradoxically negative and highly

[6] Along the same lines: TrS (2007): *nityam ekam anekānugataṃ sāmānyam* |; "The universal is constant, one and recurrent in many particulars". Śivāditya (1934, p. 50): *sāmānyaṃ nityam ekam anekāsamavetam* || 62; Udayanācārya (1989, p. 120): *nityam ekam anekāvṛtti sāmānyam*. Quotations and translations from Pellegrini (2016, p. 76). Cf. also NS 2.2.67-68 (2009, pp. 522–523).

context-sensitive, although still formally true—e.g. 'This physician is only equivalent to himself, because he has no equivalent'.

Equivalence appears, in light of the above, to be closely linked to domain multiplicity. Yet what could multiplicity mean in this context? In modern times, Navya-Nyāya—and Raghunātha Śiromaṇi (c. 1510), in particular—moved beyond the theory of number as an inherent quality (*guṇa*) in "adjectival function" (Ganeri, 1996, p. 111), through the logically primitive relational concept of *paryāpti-sambandha* in the sense of 'completion'.[7] This new conception "bears a close resemblance to the recent concept in Western logic of number as a class of classes" (Ingalls 1951, p. 76).[8] Framed in this way, number becomes an imposed property (*upādhi*) related by *paryāpti* to the set of objects being numbered: indeed, "*paryāpti* is the relation by which numbers reside in wholes rather than the particulars of wholes", so that "the *loci* of two-ness and of three-ness are mutually exclusive" (Ingalls 1951, p. 77). In this manner, "a trio of men, for example, is an instance of number 3, and the number 3 is an instance of number; but the trio is not an instance of number [...; because] a number is something that characterises certain collections, namely, those that have that number" (Russell

[7] 'Completion', 'thoroughness', or 'wholeness', as translated in Ingalls (1951, pp. 76–77) and Guha (1979, pp. 50–56). Also: *paryavasāna* or *sākalya.* In other words, *paryāpti* is "a one-to-many relation [...]. It relates numbers to pluralities of objects, but not to objects taken individually" (Ganeri 1996, p. 113). Number is thus a *vyāsajya-vṛtti-dharma*, that is, a "property that occur in loci (e.g., $a \cup b$) whose parts (a, b) adhere to each other (i.e., are inseparable)" (Ingalls 1951, p. 78); or a "property which occurs jointly [and thus not distributively]" or a "collective property" (Ganeri 1996, p. 115). Regarding the flaws of the number-as-*guṇa* account (in particular, *self-inherence* and *cross-categoricity*) cf. Ganeri 2001, pp. 414–418. Phillips (1997, p. 361): "Numbers larger than one are cognition-dependent in a strong sense, in that they are created and last only by the act of counting". See also Shaw (1982) and Jha (1992, pp. 49–60). About Frege's criticism on the adjectival account, cf. Dummett (1991, pp. 72–81) and Frege (1953: § 22, 28; § 29, pp. 39–40). Acceptance of *paryāpti-sambandha* was far from unanimous; for a synthetic description of Raghunātha's innovations and the associated debate, see Ingalls (1951, pp. 76–77); Ganeri (2011, pp. 181–199); and Guha (1979, pp. 169–201) about 'the technique of the insertion of *paryāpti*'. For Raghunātha, "[*paryāpti*] is a special kind of self-linking relation" (*svarūpa-sambandha-viśeṣa*), thus not reducible to inherence; translated by Ganeri (1996, pp. 112–113), quoting Jagadīśa (1977, pp. 38–39).

[8] Cf. also: "I would like to observe a point of similarity between the Nyāya theory and Russell's definition of the number *n* as the class of all classes of *n* objects". Russell (1919, p. 14): "It is clear that number is a way of bringing together certain collections, namely, those that have a given numbers of terms. We can suppose all couples is one bundle, all trios in another, and so on. In this way we obtain various bundles of collections, each bundle consisting of all the collections that have a certain number of terms. Each bundle is a class whose members are collections, *i.e.* classes; thus each is a class of classes". Ganeri (1996, p. 120) notes that, in Nyāya approach, "numbers are relations taken in *intension*, not in extension. This means that the Nyāya has no need for Russell's 'axiom of infinity', the postulate that there are infinite objects in the universe"; for a first survey on the Axiom of Infinity (Axiom des Unendlichen) in Zermelo-Fraenkel, cf. Jech (2006, pp. 12–13); for its formulation: Zermelo (1907, pp. 266–7). Nevertheless, the issue appears even more nuanced. For Russell's own admission: "Of these two kinds of definitions [definition of a number by extension or by intension], the one by intension is logically more fundamental. This is shown by two considerations: (1) that the extensional definition can always be reduced to an intensional one; (2) that the intensional one often cannot even theoretically be reduced the extensional one. [...] We wish to define 'number' in such a way that infinite numbers may be possible; thus we must be able to speak of the number of terms in an infinite collection, and such a collection must be defined by intension, *i.e.* by a property common to all its members and peculiar to them"; Russell (1919, pp. 12–13). See also Frege (1953, § 46, p. 59): "The content of a statement of number is an assertion about a concept". For a slightly different translation, cf. Dummett (1991, p. 88): "The content of an ascription of number consists in predicating something of a concept".

1919, pp. 11–12).[9] Thereby, numbers could be conceived as "*n*-fold relations of mutual distinction: 'The planets are (at least) three' is 'logically equivalent' to: $(\exists x)$ $(\exists y)$ $(\exists z)$ [Planet (x) & Planet (y) & Planet (z) & $x \neq y$ & $y \neq z$ & $z \neq x$]".[10] In a nutshell, the condition laid down by the NK definition of equivalence—*bhinnatve sati*—requires that the cardinality of the reference domain be greater than one (*condition-a*) and, therefore, not trivially reflexive (*condition-b*). Equivalence in a full sense thus only exists between two distinct elements of a given set, which in turn is the reference domain of that property to which these elements are declared equivalent.

For the limited purposes of this article, we are dealing exclusively with natural numbers (viz., not negative integers); I thus propose to express the *paryāpti* relation in NL through the natural numbers symbol ('ℕ'), leaving the possibility of expanding the system open to further investigation.[11] Consequently, being 'two' linked to the property *two-ness* (*dvitva*, 2_t; cf. NK, p. 381), the statement *dvau gāvau* ('Two cows') could be expressed in NL as:

> [9] $((g, g') \bullet g_t)\urcorner\ \mathbb{N}\ \llcorner 2_t$
> *yat paryāptitvaṃ dvi-go-niṣṭha-gotvāvacchinnaṃ tad dvitva-nirūpitam*; 'The relational abstract completion-ness, conditioned by two-ness, is limited by cow-hood in two cows'.[12]

Now, NK explicitly states that *tulyatva* means sharing a given property (*dharma-vat-tva*) in the context of a mutual distinction (*bhinnatva*). However, this very distinction cannot but imply multiplicity—as we have seen, an "*n*-fold relations of mutual distinction". Therefore, equivalence can only be conceived as a relation the cardinality of which is strictly greater than one: $card(E)>1$.[13] Thus—being the

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[9] Cf. also, Russell (1919, p. 13): "In the first place, numbers themselves form an infinite collection […]. In the second place, the collections having a given number of terms themselves presumably form an infinite collection: it is to be presumed, for example, that there are infinite collections of trios in the world […]". Yet, see also Ingalls, *Introductory Note* to Guha (1979, p. xi).

[10] Bigelow (1988) quoted by Ganeri (1996, pp. 120–121). Cf. also: "$(\exists x_1)$ $(\exists x_2)$… $(\exists x_n)$ $(Fx_1$ & Fx_2 & Fx_n & $x_1 \neq x_2$ & $x_2 \neq x_3$ & $x_1 \neq x_3$ & $x_{n\text{-}1} \neq x_n)$" in Ganeri (2001, p. 418), quoting Sainsbury (1991, p. 160). And again: "numbers are *n*-place relations holding jointly between *n* distinct objects" (Ganeri 1996, p. 114), whose extension is thus "the class of all ordered *n*-tuples" (Ganeri 1996, p. 120).

[11] On natural numbers, cf. Russell (1919, pp. 1–19); Quine (1981, pp. 237–50); Jech (2006, pp. 27–36). A blackboard bold (or double struck) capital N ('ℕ') is commonly used to symbolise natural numbers.

[12] For brevity's sake, only the *tātparya* (purport, $^{(t)}$) of this particularly cumbersome expression will hereafter be referred to as card(R) = n ('The cardinality of the relation R is equal to n'). Regarding the relational abstract *paryāptitva*, cf. Jha (2001, p. 263): "the state of being the relation of *paryāpti*".

[13] The cardinality of a generic set A (i.e., the number of elements belonging to A) is defined as its equivalence class under equinumerosity; and assuming that: "a set A is equinumerous as the set B (written A≈B) iff there is a one-to-one function from A onto B" (Enderton, 1977, p. 129). Jech (2006, p. 26): "Two sets X, Y have the same cardinality, |X|=|Y|, if exists a one-to-one mapping of X onto Y". Enderton (1977, pp. 136–137): "For any set A we will define a set card A ['cardinality'] in such a way that: (a) For any sets A and B, card A = card B iff A ≈ B. (b) For a finite set A, card A is the natural number for which A ≈ *n*. […] We define a cardinal number to be something that is card A for some set A. […] Any natural number *n* is also a cardinal number, since *n* = card *n*. […] In general, for a cardinal number *k*, there will be a great many set A of cardinality *k*, i.e., sets with card A = *k*. […] In fact, for any nonzero cardinal *k*, the class K_k = {X | *card* X = *k*} of sets of cardinality *k* is too large to be a set. But all of the sets of cardinality *k* look, from a great distance, very much alike—the elements of two such a sets may differ but the number

condition ‘greater than one’ expressible as *dvitvādi* (‘two, etc.’, or $\geq 2_t$)—our first example concerning equivalence to *gotva* now begs a further truth condition which was previously solely implicit. This means that the reference domain must possess more than one element (i.e., there is more than one cow; *condition-a*); and that the relation involves two distinct elements of this multiple reference domain ($g' \neq g$; i.e., we are talking about two different cows; *condition-b*). In NL:

> $[8_a]$ $((g'.\ g_t)\urcorner\ \boldsymbol{E}\ \llcorner(g.\ g_t))\urcorner\ \mathbb{N}\ \llcorner(\geq 2_t)$
> *yat paryāptitvaṃ go-niṣṭha-tulyatvāvacchinnaṃ tad dvitvādi-nirūpitam, etad eva tulyatvaṃ ca idaṃ-go-niṣṭha-gotvāvacchinnam adaḥ-go-niṣṭha-gotva-nirūpitaṃ ca*; ‘Equivalence, conditioned by cow-ness in that cow, and limited by cow-ness in this cow, for card(E) ≥ 2’.[14]

In general—being ‘Possessing a particular property’ expressible as *taddharmavattva* (td_t , cf. fn. 3)—the statement ‘Equivalence between the generic element a and b is a relation whose cardinality is strictly greater that one’ will now appear in NL as:

> [10] $(b\ .\ td_t\urcorner\ \boldsymbol{E}\ \llcorner(a\ .\ td_t))\urcorner\ \mathbb{N}\ \llcorner(\geq 2_t)$
> *yat paryāptitvaṃ tulyatvāvacchinnaṃ tad dvitvādi-nirūpitam, etad eva tulyatvaṃ ca idam-niṣṭha-taddharmavattva-avacchinnam adaḥ-niṣṭha-taddharmavattva-nirūpitaṃ ca*; ‘The relational abstract completion-ness, conditioned by two-ness, etc., is limited by equivalence, which is in turn conditioned by a particular property in a generic element, and limited by the same property occurring in another element’; iff $(a \neq b) \wedge ((a, b) \in |td_t|) \wedge \mathrm{card}(E) \geq 2$.

The truth conditions and cardinality of a *tulyatva* relation undoubtedly show that this cannot, except in a secondary sense, concern the relation of non-difference (*abheda*). A gold crown is undeniably one and, in this sense, the crown (m) is non-different from the gold (h). This state of affairs has been provisionally expressed in P1 through the relation of *sāmānādhikaraṇya* ($\underset{\cdot}{N}$): $[2_a]$ $(\boldsymbol{h.h_t})\urcorner\underset{\cdot}{\boldsymbol{N}}\llcorner(\boldsymbol{m.m_t})$. In all evidence, the cardinality of $[2_a]$ is equal to one (there is but one crown) and thus incompatible with the requested cardinality of *tulyatva* expressed in [10]. Moreover —in violation of the definition of both *tulyatva* (cf. *dharmavattva*) and *condition-a* (cf. *supra*)—a well-formed equivalence formula cannot be provided, by substitution, starting from $[2_a]$. The same properties do not appear on both sides of the

Footnote 13 continued
of elements is always k”. Enderton (1977, p. 132): “Equinumerosity has the property of being reflexive (on the class of all sets), symmetric, and transitive. But it cannot be represented by an equivalence relation, because it concerns *all* sets”. Enderton (1977: 133): “A set is *finite* iff it is equinumerous to some natural number. Otherwise it is *infinite*. Here we rely on the fact that in our construction of [i.e., infinite], each natural number is the set of all smaller natural numbers. For example, any natural number is itself a finite set”. Cf. also Moschovakis (2006, pp. 7–18). Thereby, $card(R) > 1 \leftrightarrow_{\varphi} \langle x, y\rangle R \wedge (x \neq y)$; i.e, the cardinality of a generic relation R is strictly greater than one if and only if the two relata x and y stand in relation R and x is different from y. Conversely, if $\langle x, y\rangle R \wedge x{\neq}y \therefore card(R) > 1$; i.e., if x stands in relation R with y and x is distinct from y, therefore ($\therefore$) the cardinality of relation R is greater than one.

[14] Note that $[8_a]$ is a composed relation; that is, there appears a chief relation ($\mathbb{N}$) whose limitor (*avacchedaka*) is another relation (E), in turn composed of its own limitor and conditioner. Parenthesis highlight in NL chief relations.

equivalence relation, that is, the two *relata* do not belong to the same reference domain: *[2_b] *$(h.h_t)\neg\boldsymbol{E}\llcorner(m.m_t)$, false because $h \in |h_t|$ (an instance of gold belongs to the set Gold), but $m \in |m_t|$ (a crown belongs to the set Crowns). So, *hāṭakasya na mukuṭaṃ tulyam* ('A crown is not equivalent to gold', $\langle h, m\rangle \notin E$).

In a further countercheck, we could state that crown and gold are nonetheless equivalent: *mukuṭasya hāṭakaṃ tulyam*. What could be the meaning implied here? Firstly, that they are two. This immediately gives rise to a second question: with respect to what property? Uttered by a merchant, it could mean that they are equivalent to their value (*mūlya*) or their 'purchasing power' (*krayaṇa*): *krayaṇāya hāṭaka-mukuṭa-bhūṣaṇasya piṇḍa-rūpa-hāṭakaṃ tulyam* ('For the purpose of purchasing, a golden accessory, such as a crown (m), is equivalent to raw forms of gold, such as a nugget (p)'). In this area NL precludes misinterpretations and reshapes (including visually: note the symmetry of properties on both sides of the equivalence; in this case: *being gold*) every possible hypothesis into a well-formed formula (in observance of *conditions a & b*):

> [11] $(m \,.\, h_t)\neg\,\boldsymbol{E}\,\llcorner(p \,.\, h_t)$
> *yad tulyatvaṃ mukuṭa-niṣṭha-hāṭakatva-avacchedakāvacchinnaṃ tad piṇḍa-niṣṭha-hāṭakatva-nirūpitam*; 'Equivalence, conditioned by *gold-ness* in a nugget, is limited by *gold-ness* in a crown'; iff $((m, p) \in |h_t|) \wedge (m \neq p) \wedge |(m.h_t)| \subseteq |\boldsymbol{E}\llcorner(p.h_t)|$, that is, $\langle g, p\rangle \in E_{ht}$.

It follows that there is at least a crown and a nugget, and that both of them are equivalent to the set to which they belong, defined by the property 'gold-ness': [11_a] $((m.h_t)\neg\boldsymbol{E}\llcorner(p.h_t))\neg\mathbb{N}\llcorner(\geq 2_t)$, iff card($E$) > 1. If this interpretation is in perfect compliance with the aforesaid equivalence conditions, in all evidence it again fails to match the non-difference truth conditions, the cardinality of which is strictly one (card ([2_a]) = 1). The same conclusion is reached for any property whatsoever. Is there any way to force '*mukuṭasya hāṭakaṃ tulyam*' to be true without appealing to any further property? Only against the '*condition-b*', that is, in the reflexive form of equivalence with cardinality equal to one: [12] $(m.h_t)\neg\boldsymbol{E}\llcorner(m.h_t)$, iff $m = m$, that is, just as with the derivate form we saw to be either pointless—lacking in any informative value—or directly contradicting the relation itself: *anuttara-hāṭaka-mukuṭaḥ*, 'An unparalleled gold crown'.

Equality

"Equality gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In [his] *Begriffsschrift* [Frege] assumed the latter", and here I do as well.[15] According to NK, the relation of equality (*samaniyatatva*)[16] consists in a mutual pervasion or invariable

[15] Frege (1966, p. 56): *On Sense and Reference*, first published in *Zeitschrift für Philosophie und philosophische Kritik*, vol. 100, 1892, pp. 25–50. The reference is to: *Begriffsschrift, eine der aritmetichen nachebildete Formelsprache des reinen Denkens*, Halle, 1879.

concomitance (*vyāpti*) in which the pervaded (*vyāpyatva*) is also the pervader (*vyāpakatva*): *vyāpyatve sati vyāpakatvam*.[17] NK advances the classical example concerning *cow-hood* (*gotva*) and *possessing dewlap, etc.* (*sāsnādimattva*): these properties must be said to be equal since every instance of the former is an instance of the latter, and vice versa, and because—according to the *Axiom of Extensionality* (AE)—if two sets have exactly the same members then they are equal.[18] The very concept is expressed in NK *sub voce* '*tulyatva*$_{2ka\text{-}kha}$': *anyūnānatirikta-vyaktikatvam*, "*x* is equal to *y* when *x* has all the manifestations (*vyakti*) of and no other manifestation than *y*" (Ingalls 1951, p. 67); as in the case of *ghaṭatva* and *kalaśatva*, both translatable as *potness* and whose manifestations are nothing but pots; or as in the further case of *buddhitva* (intellection) and *jñānatva* (cognition).[19] *Tulyatva*$_{2ka\text{-}kha}$ is explicitly mentioned by NK as a 'blocker' (*bādhaka*) impeding the establishment of distinct general properties; it follows that the same individual manifestations (*vyakti*) cannot but point to the very same *jāti*, even if they are expressed with different terms.

16 NK, p. 957. Correspondingly: "*niyata-tva*, the state of being pervaded", Jha (2001, p. 224). The term '*niyata*'—closely related to '*niyama*', 'restriction'—is a *kta-pratyaya* (past passive participle; cf. *kṛt-pratyaya* or primary derivates) from the root *ni-*√*yam* ('to restrict'). In case of an invariable concomitance (*vyāpti*), a pervaded (*vyāpya*, e.g. smoke) is related to a pervader (*vyāpaka*, e.g. fire), while the reverse relation is usually not allowed (*vyabhicāra*; lit. 'deviating'). The *vyāpaka* (fire) is said to be *adhika-deśa-vṛtti*; viz., occurring in a greater number of instances; the *vyāpya* (smoke), on the contrary, *nyūna-deśa-vṛtti*, occurs in a smaller number of instances. This is the case of a *viṣama-niyama*, an unequal distribution of occurrences between *vyāpaka* and *vyāpya*. However, in case of *samavyāpti*, *samaniyama*, *samaniyata* (lit., 'equal restriction'), *sāhacarya-niyama*, or *sāhacarya-niyata*, both *vyāpaka* and *vyāpya* occur in the same number of instances or *loci*; i.e., a co-extension of pervader and pervaded is given. Cf. NK, p. 964: *samavyāptitvam—samaniyatatvam*|. NK, p. 1017: *sāhacaryam*—[1] *sāhityam*| [2] *sāmānādhikaraṇyam*| [3] *samabhivyāhāraḥ*|; cf. Govardhana, *Nyāyabodhinī* (TrS, p. 92): *sāhacaryaṃ nāma sāmānādhikaraṇyam*. Potter (1968, p. 717) translates *samaniyatatva* as "co-extensiveness" and significantly connects it to *samavyāpti* or "equal pervasion", a proposal perfectly suited to the the interpretation outlined here. Cf. also Matilal (1964, p. 87): "The word *samaniyata* contains the notion of *niyama* which is usually explained as a *vyāpti*-relation (cf. *niyamaś cātra vyāpakatā*). Thus, *samaniyatatvam* has been analysed by the Nayāyikas as follows: *x* is *samaniyata* with *y* if and only if *x* is pervaded by *y* and also the pervader of *y* (*tatsamaniyatatvaṃ tad-vyāpyatve sati tad-vyāpakatvam*)".

17 NK, p. 957. Ingalls (1951, p. 67): "a relation of *x* to *y* such that *x* pervades *y* and is pervaded by *y*; *x* and *y* may belong to any category". Ingalls (1951, p. 86): "Gaṅgeśa defines 'pervasion of *x* with *y*' in the *Pañcalakṣaṇī* of *TC* as 'non-deviation of *x* with respect to *y*', which is further explained as 'non-occurrence of *x* in the locus of absence of *y*'". Cf. Matilal (1968, pp. 79–80): " 'pervasion of *x* with *y'* is '*x*'s concurrence with such a *y* as is not the counterpositive of an absence which occurs in the locus of *x'* (see: "*hetuman-niṣṭha-virahāpratiyoginā sādhyena hetor aikādhikaraṇyaṃ vyāptir ucyate*", Viśvanātha, *Bhāṣāpariccheda*, v. 69)". See also Matilal (1964, p. 87).

18 NK, p. 957: *yathā lakṣyatāvacchedakasamaniyato dharmaḥ asādhāraṇadharmaḥ ityādau gor lakṣaṇasya sāsnādimattvasya lakṣyatāvacchedakībhūtagotvasamaniyatatvam* |. Regarding AE, see Jech (2006, p. 3): "*1.1. Axiom of Extensionality* [Axioms of Zermelo-Fraenkel]. *If* X *and* Y *have the same elements, then* X=Y". Cf. also Enderton (1977, p. 2): "If A and B are sets such that for every object *t*, $t \in A$ iff $t \in B$, then A=B". In standard notation, with respect to the generic properties *P* and *Q,* $(\forall x)\ (P(x) \leftrightarrow Q(x)) \rightarrow (P(x) = Q(x))$. *Samaniyatatva* is therefore a binary, reflexive, symmetric, and transitive relation ruled by the logical biconditional (↔), in the sense of 'both or neither'—as in the case of the properties 'being an equilateral triangle' and 'being an equiangular trilateral'.

19 NK, p. 335: *tulyatva*—2[*ka*] *anyūnānatiriktavyaktikatvam* | *yathā nyāyamate buddhitvajñanatvayor ghaṭatvakalaśatvayor vā tulyatvam* | *idaṃ tu ghaṭatvakalaśatvādīnāṃ bhede bhinnajātitve vā bādhakam iti bodhyam* |. 2[*kha*]: *tulyavyaktivṛttitvam* | *svabhinnajātisamaniyatatvam iti phalito 'rthaḥ* | *yathā ghaṭatvakalaśatvayos tulyatvam* |. Cfr. Mahādeva Bhaṭṭa, *Dinakarī*, NSM, p. 103-104: *tulyatvaṃ tulyavyaktivṛttitvaṃ ghaṭatvakalaśatvādīnāṃ jātīnāṃ bhede*. See also Jha (2001, p. 182).

"[*Vyakter*] *tulyatvam*, the sameness [of the individual]" operates as a blocker; therefore, "the substrate (*adhikaraṇa*) of the first property is nothing but the substrate of the second one, and *viceversa*".[20] More precisely: *tulyatvaṃ ca na jātibādhakam* | *kintu jātibhedabādhakam*; "sameness [of the individuals] is not an universal blocker, but a blocker of the difference between universals", which are thus, *stricto sensu*, equal.[21] What follows (*phalita*) is the very same extension (*samaniyatatva*) of properties which differ only linguistically.

If *samaniyatatva* and *tulyatva*$_{2ka\text{-}kha}$ define the relation of equality between distinct expressions both of which can refer to the same property, then: *iyaṃ gau iti iyaṃ sāsnāmatī iti vā, samaniyatatvāt* ('This *cow* or this [animal] *possessing dewlap*, by virtue of equality') or *ghaṭatvakalaśatvayos tulyatvam* ('*Pot-ness* is equal to *pitcher-ness*'). Equality expresses an identity of reference (*vācya* or *artha*) between distinct signs and expressions (*vācaka* or *pada*). *Samaniyatatvaṃ vāgālambanaṃ nāmādheyaṃ vā*: equality is a matter of words; it is a mere verbal difference regarding names or denominations. Thus, there is equality between signs and expressions, but identity regarding the object. In Frege's words: "Equality. I use this word in the sense of identity and understand '$a = b$' to have the sense of 'a is the same as b' or 'a and b coincide' " (Frege 1966, p. 56, fn. *). Along the same lines, Quine opportunely notes that "confusion and controversy have resulted from the failure to distinguish clearly between object and its name. […] The trouble comes […] in forgetting that a statement about an object must contain a name of the object rather than the object itself" (Quine 1981, p. 24). It is thus necessary to plainly distinguish between "Use versus Mention" (Quine 1981: §4, pp. 23–26; cf. also 1987, pp. 231–235). "The name of a name or other expression is commonly formed by putting the named expression in single quotation marks […]. We mention x by using a name of x; and a statement about x [inescapably] contains a name of x" (Quine 1981, p. 23). In this sense, in defining the relation of identity as $x = y$ iff $(z{=}x) \leftrightarrow \varphi\ (z{=}y)$, Quine himself makes use of three different names for the object under examination—while '*the object under examination*' constitutes a fourth expression. Only the names of x (i.e., its mentions) are distinct, however, because: 'x' ≠ 'y' ≠ 'z' ≠ '*the object under examination*' (all in single quotation marks); while the use of the names, stricto sensu, allows the affirmation that $x = y = z$ = the object under examination (all without single quotation marks).[22]

[20] Pellegrini (2016, pp. 79–80), on differentiating *jāti* and *upadhi* by means of the concept of 'blocker' (*bādhaka*). [*Vyakter*] *tulyatvam* is traditionally counted as the second *bādhaka*. Cf. also, Phillips (1997, pp. 60–63).

[21] *Setu* commentary on *Kiraṇāvalī*, quoted by Śāstrī (cf. Udayana 1980, pp. 323–324, fn. 2); cf. Pellegrini (2016, p. 79, fn. 30). Apart from adding the square brackets, I also changed the translation slightly, substituting 'sameness' for 'equivalence'.

[22] It reads: 'x is equal to y if and only if, for every z, z is equal to x and to y'. Quine's definition might not be immediately intelligible for the reader not conversant with his notation; that is why I have chosen to roughly simplify his account. I am aware that the proposed provisional definition is boldly circular, defining '=' via '=' (and not via '∈' as Quine does). I hope the reader will understand the point of this simplification. Nevertheless, here is Quine's original text: "We turn now to the problem of defining '$x = y$', in terms of '∈' and our other primitives, that it will carry the intended sense x and y are the same object' […]: $(z)\,(z \in x \,.\!\equiv .\, z \in y)$, when x and y are classes, since classes are the same when their members are the same"; Quine (1981, p. 134). "Let us use 'ζ', 'η' […] to refer in general to any terms. […] The general definition of identity [is thus expressed] as follows […]: $\ulcorner \zeta = \eta \urcorner$ for $\ulcorner (\alpha)\alpha \in \zeta \,.\equiv .\, \alpha \in \eta \urcorner$"; Quine (1981, pp. 135–6).

Words—variously: *pada, śabda, vācaka*, or *nāman*—are said to possess a peculiar *primary referential power* (*śakti*; together with its related abstract, *śakyatā*) by virtue of which they stand solely for certain defined entities (*sattva*) or meaning-relata (*artha, vādya* or *vācya*) and not others. The issue is particularly complex and surely beyond the scope of this paper; yet, roughly speaking, the *pada* 'go' refers to its *artha*—the animal called 'cow'—and not to a chair precisely because of that *śakti*: the power to point at the specific quality which distinguishes cows from chairs, that is, the *pravṛtti-nimitta*, the basis or grounds for using that term and not another.[23] In this sense, two different expressions in possession of the very same grounds for use (*pravṛtti-nimitta*) could be said to be equal: *vaṭavṛkṣa* = *nyagrodhapādapa* because their primary referential power (*śakti*) points at the very same referent or *artha* (i.e., in a third expression, *ficus benghalensis*). In other terms, I assume that equality, in its proper sense, concerns first and foremost the *padapadārtha-sambandha*. *Samaniyatatva* must be conceived as a matter of *śakyatā* because it provides information about the use of the names of x (viz. about 'x', or about its mentions)—while establishing relations of co-extensionality, co-reference or synonymity (*samabhivyāhāra*; cf. NK, p. 957) between expressions.[24] Consequently, I suggest that identity, stricto sensu, must concern the referent in question and not its names—being a statement about x and not about 'x' (cf. *infra*, § 7.).

Let us now analyse the NK example in NL involving the non-symmetric relation of invariable concomitance or pervasion (*vyāpti*) (cf. Anrò, forthcoming §4.4-5). Let *samaniyatatva* the relational abstract of equality ($\boldsymbol{Q}$); *gotva* (g_t) the property *cow-hood* relative to the set Cows ($|g_t|$=G); and *sāsnāmattva* (s_t) the property *possessing-dewlap* referred to the set Living beings possessing-dewlap ($|s_t|$=S).[25]

[23] NK, p. 855 *śaktiḥ*—[ṅa] *padapadārthayor vādyavācakabhāvaniyāmakaṃ saṃbandhāntaraṃ śaktiḥ*; "The primary referential power is another [kind of] relation, which defines the relation between term and referent, that is, between expression and what is to be expressed". NK, p. 580 *pravṛttinimittam*—[ka] *padaśakyatāvaccedakam* | *yathā ghaṭatvaṃ ghaṭapadasya pravṛttinimittam* |; "the ground for use is the limitor of the primary meaningfulness of a term. Thus, the grounds for use of the term 'pot' is pot-ness". NK, p. 860 *śakyatvam*—1. *viṣayatāsaṃbandhena śaktyāśrayatvam* | *yathā gavāder arthasya gopadaśakyatvam* |; "primary meaningfulness is the property 'being the locus' of the primary referential power, in virtue of the relation of content-ness. Thus, the primary meaningfulness of the term 'cow' is the referent cow, etc."; cf. Govardhana, *Nyāyabodhinī* (TrS, p. 129): *viṣayatāsaṃbandhena śaktyāśrayatvaṁ śakytvam*. For a general survey, cf. Ganeri (2006, pp. 9–48).

[24] Nonetheless, as starting points for a discussion of the delicate issue related to interchangeability and cognitive synonymy, cf. among the others: Quine (1951), Carnap (1955). For a purely nominalist and extensional account, where no two different expressions in a language are synonymous, cf. also: Goodman (1949).

[25] A cow is usually defined as '*sāsnādimat*', 'possessing dewlap, etc.'; in order to avoid possible confusion, I prefer here to omit '*ādi*' (lit. 'beginning from'; e.g. 'possessing cloven hoofs'), focusing on the property 'possessing dewlap' only. Thus: '*sāsnāmat*' (*sāsnāmatī gauḥ*).

The equality of expressions and the identity of their reference could thus be conveyed as:

[13] $((g_t \urcorner \boldsymbol{Q} \llcorner s_t) \wedge (s_t \urcorner \boldsymbol{Q} \llcorner g_t)) \leftrightarrow ((g_t \urcorner \underset{.}{\boldsymbol{N}} \llcorner s_t) \wedge (s_t \urcorner \underset{.}{\boldsymbol{N}} \llcorner g_t))$
yadi sāsnāmattvaṃ gotvaṃ vyāpnoti evaṃ gotvaṃ sāsnāmattvaṃ vyāpnoti, tarhi sāsnāmattvagotve samaniyate ; 'If cow-ness pervades possessing-dewlap-ness and possessing-dewlap-ness pervades cow-ness, then cow-ness and possessing-dewlap-ness are equal'. Or, in full expression: *yadi yad vyāptitvaṃ gotva-avacchinnaṃ tat sāsnāmattva-nirūpitam evam yat vyāptitvaṃ sāsnāmattva-avacchinnaṃ tad gotva-nirūpitam, tarhi yad yat samaniyatatvaṃ gotva-avacchinnam tat tat sāsnāmattva-nirūpitam, athavā yad yat samaniyatatvaṃ sāsnāmattva-avacchinnaṃ tat tad gotva-nirūpitam.*

NL calls for a further operator here to express a symmetrical—that is, reversible—relation. For this purpose, be introduced the symbol '$\rightleftharpoons$' in the straightforward meaning of: *tadviparyayeṇa* ('vice versa', hereafter '&vv').[26] Consequently, [13] will now turn into:

[14] $(g_t \rightleftharpoons \boldsymbol{Q} \llcorner s_t) \leftrightarrow (g_t \rightleftharpoons \underset{.}{\boldsymbol{N}} \llcorner s_t)$
yadi sāsnāmattvaṃ gotvaṃ vyāpnoti tadviparyayeṇa ca, tarhi ete samaniyate; 'If the property cow-ness pervades the property possessing dewlap, &vv, then these properties are equal'. Iff $(G \subseteq S) \wedge (S \subseteq G) \therefore (G = S)$.

With [14] we have definitely clarified that g_t and s_t have the same extension. Consequently—lest they not mean what they mean—they are in possession of the same ground of use (*pravṛttinimitta*), which is the limitor of their property of primary meaningfulness (*śakyatā*, $\boldsymbol{\acute{S}}$).

[15] 'g' $\urcorner \boldsymbol{\acute{S}} \llcorner g_t$
yā śakyatā go-pada-avacchinnā sā gotva-nirūpitā, 'The primary meaningfulness is limited by the term cow, while conditioned by cow-ness'. Iff $|`g'| = g \in (|g_t| = G)$, where single brackets in formulas such as [15] do mean the word (*pada*) x; thereby, 'The extension of the word 'cow' is a cow, qua instance of cow-ness and belonging to the set Cows'. Analogously, for '*sāsnāmat*': 's' $\urcorner \boldsymbol{\acute{S}} \llcorner s_t$, for $|`s'| = s \in (|s_t| = S)$.

However, in [14]: $(G = S)$, and in [15]: $(|`g'| = (g \in G)) \wedge (|`s'| = (s \in S)) \therefore (|`g'| = |`s'|)$. Therefore, we can conclude that equality relation—as *padapadārtha* (or *vācyavācaka*) *sambandha* and with respect to the terms '*go*' and '*sāsnāmat*' (*sāsnādimat*)—might be fully interpreted as:

[26] For an example of the use of the locution '*tadviparyayeṇa*', cf. the incipit of *Śaṅkara's Brahma-sūtra-bhāṣya*: […] *tadviparyayeṇa viṣayiṇas taddharmāṇām ca viṣaye 'dhyāsaḥ* […] |; VM-B, pp. 7–9. For alternative formulations in the same meaning, consider also: *vilomata, mithas, viparyak, anyonya(-tas)*; cf. Bö. (VI: 117; V: 78; VI: 102; I: 67). It goes without saying that equivalence (E) is a symmetric relation as well. Extensionally, operator '$\rightleftharpoons$' thus has the meaning of: *sāmānādhikaraṇya* or *anyonyādhikṛtatva*, respectively 'coreferentiality' or 'being reciprocally sustained one on another' (*gotva-sāsnāmattve samānādhikaraṇe bhavataḥ*). Formulas [8]–[12], §1, could therefore be improved and rephrased by means of the '$\rightleftharpoons$' operator. For this usage, cf. § 4–5.

> [16] $(\text{‘}g\text{’}\neg\acute{\boldsymbol{S}}\llcorner g_t) \rightleftharpoons \boldsymbol{Q}\llcorner\ (\text{‘}s\text{’}\neg\acute{\boldsymbol{S}}\llcorner s_t)$
> *yad samaniyatatvaṃ go-pada-avacchinna-gotva-nirūpita-śakyatā-avacchedaka-avacchinnaṃ tat sāsnāmad-pada-avacchinna-sāsnāmattva-nirūpita-śakyatā-nirūpitam, tadviparyayeṇa ca*; ‘Equality, conditioned by primary meaningfulness described by the property possessing dewlap and limited by the expression ‘possessing dewlap’, is limited by primary meaningfulness described by the property cow-hood and limited by the word ‘cow’, &vv’.

In light of the above, the cardinality of the relation of equality will be greater than or equal to one ($card(Q) \geq 1$). Firstly, because of the intrinsic plurality of manifestations of a general term (cf. *supra*). Secondarily, because a term could clearly refer to a singular, as in cases such as ‘*dik*’ (‘space’, cf. § 1) or in sentences such as ‘*ayodhyā-kumāro rāmaḥ*’.[27] It follows that, being the condition ≥ 1 expressed as *ekatvādi* ($\geq 1_{t,}$ lit., ‘oneness, etc.’), the equality between ‘*gotva*’ and ‘*sāsnāmattva*’ needs its cardinality truth condition to be made explicit, that is: $[16_{\mathrm{a}}]$ $((\text{‘}g\text{’}\neg\acute{\boldsymbol{S}}\llcorner g_t) \rightleftharpoons \boldsymbol{Q}\llcorner(\text{‘}s\text{’}\neg\acute{\boldsymbol{S}}\llcorner s_t))\neg\mathbb{N}\llcorner(\geq 1_t)$, for ($card\ Q$) ≥1. In more general terms, the equality between this (*etat*; ‘*a*’) and that (*tat*; ‘*b*’) generic expression—in relation to their common grounds of use (*pravṛttinimitta*), expressed by the same generic property (*taddharmavattva*, $td_{t;}$ cf. fn. 3)—as a symmetric relation whose cardinality is greater than or equal to one, will now appear in NL as:

> $[17_a]$ $(\text{‘}a\text{’}\neg\acute{\boldsymbol{S}}\llcorner td_t) \rightleftharpoons \boldsymbol{Q}\llcorner\ (\text{‘}b\text{’}\neg\acute{\boldsymbol{S}}\llcorner td_t))\neg\mathbb{N}\llcorner(\geq 1_t)$
> *yat paryāptitvaṃ samaniyatatva-avacchinnaṃ tad ekatvādi-nirūpitam; etad eva samaniyatatvaṃ etat-pada-avacchinna-taddharmavattva-nirūpita-śakyatā-avacchedakāvacchinnaṃ tad-pada-avacchinna-taddharmavattva-nirūpita-śakyatā-nirūpitaṃ, tadviparyayeṇa ca*; ‘Equality, conditioned by primary meaningfulness described by a particular property and limited by that linguistic expression, is limited by primary meaningfulness described by the same property and limited by this linguistic expression, &vv, for $card(Q) \geq 1$’.

Identity

“Identity, we will say, is the relation that each thing has to itself and nothing else. […] The concept of identity is so basic to our conceptual scheme that it is hopeless to attempt to analyse it in terms of more basic concepts” (Hawthorne 2003, p. 99). The problem is that, “roughly speaking, to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing at

[27] Thereby: ‘*dik*’ ≠ ‘*ākāśa*’ ≠ ‘*vyoman*’, but *dik* = *ākāśa* = *vyoman*. In the same manner: ‘*Rāma*’ ≠ ‘*ayodhyā-kumāra*’, but Rāma = *ayodhyā-kumāra* (‘Rāma is the prince of [the city of] Ayodhyā’, or ‘Rāma, the prince of Ayodhyā’).

all".[28] A first move in the attempt to figure out this puzzle could be recognising that "a thing is identical with itself and with nothing else", however obvious it may sound; consequently, to admit that "the identity relation comprises all and only the repetitious pairs, $\langle x, x\rangle$"; nevertheless, and this is the key point, "$\langle x, x\rangle$ is still not to be confused with x" (Quine 1987, pp. 89–90). Along exactly the same lines, NK defines the relation of identity—sub voce '*tādātmya*$_2$'—as referring to a singularity (*aikya*) that cannot but be declared identical to itself precisely because it is that very singularity.[29] Vācaspati Miśra (VM) likewise seems to accept this definition of identity: in negative terms, where there is not difference there is unit or singularity (*ekatva*): *na cet, ekatvam evāsti, na ca bhedaḥ* (cf. fn. 49 and P1). Similarly, *tādātmya*$_{1kha}$ suggests that identity could also be conceived as an idiosyncratic feature (*dharma*) by virtue of being 'not-common' (*asādhāraṇa*) and 'self-referring' (*svavṛtti*); thus, radically singular (*ekamātra*).[30] This idiosyncratic feature has individuality (*vyaktitva*) as its form (*rūpa*). Thereby, in case of a blue pot, identity—grammatically expressed through the notion of *sāmānādhikaraṇya*—is precisely that particular individuality in (*niṣṭha*) that very pot.[31] In this sense, identity could thus be defined as a relation the cardinality of which is strictly equal to one.[32] Obviously, I am not arguing here that the concept of unit completely parallels that of identity. Rather, I propose that identity is usefully describable through the cardinality one of the ordered couple it consists of; consequently, cardinality one must compose the definition of identity as a decisive factor.

Bhāsarvajña (c. 950)[33] maintains that numbers stand for relations of identity (*abheda*) and difference (*bheda*). "Identity and difference depend on sameness [*svātmāpekṣā*] and distinctness [*parātmāpekṣā*] in colour and so on, and so are not considered to be qualities [*guṇa*]. Further, it is a tautology [*paryāya*] to say 'the one is identical' [*ekam abhinnam*] or 'the many are different' [*anekaṃ bhinnam*]".[34]

[28] Wittgenstein (2001, 5.5303); or, as Quine puts it: "evidently to say of anything that it is identical with itself is trivial, and to say that it is identical with anything else is absurd" (Quine 1987, p. 90). Cf. also Potter (1977, p. 54): "Strictly speaking, identity cannot be a relation within the [*Nyāya-Vaiśeṣika*] system, since the system may contain no two identical things […]. A relation must relate two distinct things, and it must be distinct from them" (cf. P1 fn. 32).

[29] NK, p. 328 2 [*tādātmyam*] *aikyam*. Straightforwardly: 'identity is singularity'.

[30] Enterton (1977, p. 40): "Let be the set {0, 1, 2, …}, […]. The identity relation on is $I_\omega = \{\langle n, n\rangle \mid n \in \omega\}$".

[31] NK, p. 328 1[*kha*] [*tādātmyam] svavṛttyasādhāraṇo dharmaḥ* | *tādṛśyadharmas tadvyaktitvādirūpaḥ* | *yathā nīlo ghaṭa ity ādau prathamāvibhakter abhedārthakatvamate nīlādiniṣṭhatadvyaktitvam eva nīlapadottaraprathamavibhaktyarthas tādātmyam* | *atrāsādhāraṇyaṃ caikamātravṛttitvam* |; cf. Gadādhara (2005, p. 37): *satyam—abhedas tādātmyam* | *tac ca svavṛttyasādhāraṇo dharmaḥ* | *asādhāraṇaṃ ca ekamātravṛttitvam* |.

[32] Given the generic set A, the identity relation consists in the Cartesian product $A \times A = \{(x, x) \mid x \in A\}$ whose cardinality is equal to one. Cf. *supra*, Quine's quotation (1987, pp. 89–90). Ingalls interprets *identity*—also expressed as '*x y-svarūpa*', '*x y-tādātmya*', '*x y eva*'—as a form of *equality* referring to individuals. The expressed concept is clear, all things considered; however, it seems to me that the chosen lexicon is highly misleading. Cf. Ingalls (1951, p. 68, 67 fn. 40)

[33] "Bhāsarvajña is a radical Naiyāyika who rejects the classical Vaiśeṣika theory that numbers are qualities [*guṇa*]"; Ganeri (2001, p. 418).

[34] *abhedabhedau ca svātmaparātmāpekṣau rūpādiṣv api bhavata iti na tayor guṇatvādikalpaneti* | *yathā caikam abhinnam iti paryāyas tathānekaṃ bhinnam iti ca paryāyas tataś ca dvitvādir apy anekaparyāyaḥ* […] |; Bhāsarvajña (1968, p. 159), as translated by Ganeri (2001, p. 418); square brackets are mine.

"The statement '*a* and *b* are one' is synonymous with '$a = b$'. […] On the other hand, the statement '*a* and *b* are two' asserts that $a \neq b$. […] Indeed, it is now standard to formalise sentences of the form 'there are *n* *F*s' by means of non-identity' […]" (Ganeri 2001, p. 418). In short, "number is but another name for *diversity*. Exact identity is unity, and with difference arises plurality"[35].

If *x* and *y* are meant as identical, "the intended sense [is that] '*x* and y are the same object'" (Quine 1981, p. 134). Therefore, being that very object, *x* and *y* are one. However, we have already seen that the definition of identity, according to Quine, likely sounds like: '*x* is *y* iff *x* is *z* and *z* is *y*'. Apparently, defining or even simply talking about identity—which is oneness—necessarily implies a panoply of multiple symbols and expressions; that is, any discourse about the identity of *x* makes use of the relation of equality between the different names for *x*—for instance, *x* is *y*; then *x* is *y* via *z*, etc. And yet, what about the relation between *x* and *z*, used as a medium between *x* and *y*? Multiplicity and the proliferation of names and relations are therefore paradoxically introduced where there was nothing but oneness.

To express the difficulties language encounters in dealing with identity—a structurally binary relation, by virtue of the very fact of being a *relation*, although radically converging on one—what could come to our aid is Frege's premise about the problem of unit, as expressed in the context of his scrutiny of unit as the building-block of numbers and as the alleged result of *abstraction* (my glosses about identity appear in square brackets): "If we try to produce the number by putting together different distinct objects [*or, in our case, to express identity from the combination of distinct expressions; e.g. 'Scott = author of Waverley'*], the result is an agglomeration in which the objects contained remain still in possession of precisely those properties which serve to distinguish them from one another [*and, similarly, we obtain but an agglomeration once again comprising exactly those properties that differentiate the distinct expressions we used: 'Scott' and 'author of Waverley'*]; and this is not a number [*or identity*]. But if we try to do it in the other way, by putting together identicals [*or, in our case, if we reaffirm the identity by a combination of identical expressions; e.g. Scott = Scott*], the result runs perpetually together into one and we never reach a plurality [*or, this constantly coalesces into trivial tautology, and we never achieve any informative expression*]. […] The word 'unit' is admirably adapted to conceal this difficulty [*and so is the term 'identity'*]".[36]

How, then, to solve this conundrum? A negative, counterfactual, formulation could be attempted. Be '$\nexists$' the relation of 'constant or absolute absence'

[35] A quotation by W.S. Jevons in Frege (1953, p. 46; § 35).

[36] Frege (1953, p. 50, § 39). Dummett (1991, p. 86): "[A] number will be independent of the particular objects counted, being determined, as it ought to be, solely by how many of those objects there are […]. It seems to be possible to guarantee this only if no trace of individuality is retained by the units […]". The problem is that "if every unit is identical with any (other) unit, there can only be one unit". Cf also, Frege (1953: § 35, 46) quoting W.S. Jevons: "It has often been said that units are units in respect of being perfectly similar to each other; but though they may be perfectly similar in some respect, they must be different in at least one point [for Jevons: "the empty form of difference", cf. § 44, 56], otherwise they would be incapable of plurality". Regarding the ambiguity of one, cf. also: Frege (1953: § 29). Bhāsarvajña's recursive definition of number (1968, p. 159) faces the same difficulties in distinguishing the unit: "So it is said that one is the initial integer [*abhinna*], two is that [one] together with another identical, four is those [three] together with another identical, and so on"; trans. Ganeri (2001, p. 419).

(*atyantābhāva*), 'constant absence-hood' (*atyantābhāvatva*; $\nexists$) its relational abstract, and 'constant absentee-hood' (*atyantābhāvīya-pratiyogitā*; $\nexists^{-1}$) the inverse of this latter—where, in general: $(\nexists \llcorner v) = (v \neg \nexists^{-1})$.[37] Consequently, the traditional example *bhūtale ghaṭo na* ('There is no pot on the ground') can be expressed in NL as:

[18] $g_t \neg \nexists^{-1} \llcorner (\boldsymbol{L}^{-1} \llcorner b)$
yā atyantābhāvīya-pratiyogitā ghaṭatvāvacchinnā sā ādheyatā-nirūpitā, saiva ādheyatā bhūtala-nirūpitā; 'The constant absentee-hood, with respect to the property *being over* (L^{-1}) a ground (b), is limited by pot-ness (g_t)'; iff $|g_t| \cap |\boldsymbol{L}^{-1} \llcorner b| = \varnothing$ ('The intersection of the set Pots and the set Superstrata of a certain ground is empty'); in s.n. $(\exists x, \forall y \mid Bx, Gy)\ (\langle x, y \rangle \notin L)$.

Let us now use the same approach to analyse a second classical assertion: *ghaṭaḥ paṭo na* ('A pot is not a cloth'). To avoid any confusion with the relation of equality, concerning expressions, I will introduce here a specific notation for identity (I) and its negation ($\not{I}$), absolutely abandoning the equality-identity overlap and radically embracing the account according to which "*Nyāya* conceives of identity as obtaining between objects, not between symbols" (Matilal 1968, p. 46). So, let *anyonyābhāva* ($\not{I}$) be the symmetrical relation of mutual absence; *anyonyābhāvatva* ($\boldsymbol{\not{I}}$) its relational abstract, i.e. the mutual absent-hood; and *anyonyābhāvīya-pratiyogitā* ($\boldsymbol{\not{I}}^{-1}$) the converse of the latter, i.e. mutual absentee-hood. Accordingly, *ghaṭaḥ paṭo na* will turn into:

[19] $p \rightleftharpoons \boldsymbol{\not{I}}^{-1} \llcorner g$
yā anyonyābhāvīya-pratiyogitā paṭa-niṣṭhā sā ghaṭa-nirūpitā, tadviparyayeṇa ca; 'Mutual absentee-hood, conditioned by a pot, is limited by a cloth, &vv'; iff $(g \in G) \wedge (p \in P) \wedge (G \cap P = \varnothing)$.

The relation of difference or mutual absence can easily be transformed into a 'negation of identity between the relata': *anuyogi-pratiyogi-tādātmya-pratiṣedha*. Thus, for the same truth conditions, [19] can be rephrased in: [20] $\nexists(p \rightleftharpoons \boldsymbol{\not{I}}^{-1} \llcorner g)$, where the identity relation (I) between g and p is said to be absent. In accordance with [18], it could be stated that:

[21] $p \rightleftharpoons \nexists^{-1} \llcorner (\boldsymbol{\not{I}}^{-1} \llcorner g)$
yā paṭa-niṣṭha-atyantābhāvīya-pratiyogitā sā tādātmyatā-nirūpitā, saiva tādātmyatā ghaṭa-nirūpitā, tadviparyayeṇa ca; 'Constant absentee-hood, limited by a cloth, is conditioned by mutual absentee-hood, in turn conditioned by a pot, &vv'; iff $(G \cap P = \varnothing)$.

[37] Cf. Matilal (1968, pp. 52–61). Regarding the expression of [18] in fourteen different NL permutations, see Anrò (forthcoming). Moreover, relations [19]–[21]—for *paṭaḥ pratiyogī* and *ghaṭo 'nuyogī* in $\not{I}^{-1}$, and the opposite in $\boldsymbol{\not{I}}^{-1}$—could be symmetrically construed, with the same results, i.e. for *ghaṭaḥ pratiyogī* and *paṭo 'nuyogī* in $\not{I}^{-1}$, and the opposite in $\boldsymbol{\not{I}}^{-1\,-1}$. However, paying homage to the syntax of the sentence (*vākyamaryādā*)—which reads '*ghaṭaḥ paṭo na*' and not '*paṭo ghaṭo na*'—the former reading could be considered 'verbally intelligible' (*śabdalabhya*), while this latter is only implicit (*tātparyalabhya*); see Pellegrini (2015, pp. 152–153).

Keeping in mind the elements laid out in these introductory examples, let us now move to the analysis of the counterfactual definition of identity. As mentioned above, identity is defined in terms of oneness.[38] Now, Gadādhara (c. 1650) maintains that "the meaning of 'one *F*' [*eka-śabda*] is: an *F* qualified by being-alone [*kaivalya*; i.e. 'being a unit'], where 'being-alone' [or 'being a unit'] means 'not being the counterpositive of a difference resident in something of the same kind' [*svasajātīya*]".[39] This 'uniqueness' (*kaivalya*), Gadādhara overtly states, radically excludes multiplicity: *kaivalya* in the meaning of *svasajātīya-dvitīya-rāhitya*, 'being devoid of a second one of the same kind'. If a second one of the same kind were presumed here, the postulated relation would collapse into equivalence—as in the case of two manifestations of the same property. Therefore, the expression " 'one *F*' is to be analysed as saying of something which is *F* that no *F* is different to it. If this is paraphrased in a first order language as Fx & $\neg(\exists y)$ (Fy & $y \neq x$), then it is formally equivalent to a Russellian uniqueness clause: Fx & $(\forall y)$ $(Fy \rightarrow y = x)$" (Ganeri 2001, p. 419). In other words, "to deny that an object a is numerically different from an object *b* is tantamount to saying that *a* is identical with *b*" (Matilal 1968, p. 46; cf. also, NK, p. 186, *ekatva*).

Let us proceed step by step. The definition opens by claiming that this 'unit' is the *pratiyogin* of a relation of difference (*bheda*). Accordingly, a single pot *g*, e.g., must appear in the *pratiyogin* position with respect to difference or mutual absence relation (*bheda* = *anyonyābhāva*, $\not{I}$; and therefore as *anuyogin* in $\not{I}^{-1}$), just as in the assertion *paṭo ghaṭo na*: $g \cdot \not{I}^{-1} \llcorner p$ ('A cloth is not a pot').[40]

Now, this difference could be said to momentarily occur (*niṣṭha*) in 'Something which is the same' (*svasajātīya*): let us call it g'. Consequently: $*(g.\ \not{I}^{-1} \llcorner g')$, *'Something which is the same is different from this (e.g., a pot)'. As the third step, this relation is subsequently negated; because the object under examination must not be the *pratiyogin* of such a relation: "*a-pratiyogin*", the text states. Thus: $\nexists(g.\ \not{I}^{-1} \llcorner g')$. In light of the above examples, this last assertion can easily be transformed into:

[22] $g \rightleftharpoons \nexists^{-1} \llcorner (\not{I}^{-1} \llcorner g')$
yā ghaṭa-niṣṭhā-atyantābhāvīya-pratiyogitā sā anyonyābhāvīya-pratiyogitā-nirūpitā, saiva anyonyābhāvīya-pratiyogitā svasajātīya-ghaṭa-nirūpitā,

[38] Cf. also, Russell (1919, p. 181): "Number 1 is to be defined as the class of all unit classes, i.e. of all that have just one member, as we would say but for the vicious circle".

[39] *ekaśabdasya kaivalyādiviśiṣṭe śaktiḥ, kaivalyañ ca svasajātīya-dvitīyarāhityaṃ, tac ca svasajātīyaniṣṭhabhedāpratiyogitvaṃ, svasajātīyañ ca uddeśyaviśeṣyavācakaikaśabdāt kaivalyaghaṭakatvena prakṛtavidheyavattvarūpaṃ pratīyate*; Gadādhara (1929, p. 167), as translated by Ganeri (2001, p. 419); square brackets are mine. As a general rule: "*yasyābhāvaḥ sa pratiyogī* (counterpositive is that whose absence [is spoken of]); Matilal (1968, p. 52, fn. 2).

[40] In our example, we are talking about a pot (*g*). Now, Gadādhara states that 'oneness' (*ekatva*) = 'uniqueness' (*kaivalya*) = 'Being devoid of a second of the same kind' (*svasajātīya-dvitīya-rāhityatva*) = 'Not being the counterpositive of a difference with respect to something which is the same' (*svasajātīya-niṣṭha-bhedāpratiyogitva*) (cf. previous fn.). Now, where do all these more and more defined properties occur? It is easy to understand that singularity cannot but occur (*niṣṭha*) in the pot (*g*) we are talking about: because 'this pot is this pot' ($\langle p, p\rangle \in$I, cf. *supra* Quine (1987, p. 98–90)). Consequently, the remaining three properties cannot but concern and be referred to this very object. Thus, this pot (*g*) is not the *pratiyogin* of the claimed relation of difference, because the property 'not being the counterpositive' (*a-pratiyogitva*) occurs in this.

tadviparyayeṇa ca; $^{(t)}$‘This pot is the limitor of the constant absence of the mutual absence with respect to something which is the same, &vv’.

This assertion is true iff $g \notin |\not{I}^{-1} \llcorner g'|$, because the constant absence (*atyantābhāva*, $\nexists$) of g occurs in $|\not{I}^{-1} \llcorner g'|$. And yet $|\not{I}^{-1} \llcorner g'| = \overline{\mathrm{G}'}$, i.e. the set Everything which is not g', in which G′ is a singleton containing g′ solely: i.e. $\mathrm{G}' = \{g'\}$. Therefore, if $g \notin \overline{\mathrm{G}'}$, then $g \in \mathrm{G}'$; but $\mathrm{G}' = \{g'\}$, so: $g = g'$ or, better, $\langle g, g' \rangle \in I$ (i.e. the two linguistically different expressions ‘*g*’ and ‘*g*′’ refer to the very same singular extension). In other words, if *g* does not belong to the set Everything which is not g' (i.e. $\overline{\mathrm{G}'}$), then *g* cannot but belong to G′; however, G′ is a singleton whose unique element is g'. Thereby, *g* and g' are one and the same. In brief: $g = g'$, even if ‘*g*’ ≠ ‘*g*′’ (name vs. mention); $\langle g, g' \rangle \in I$; and $(g, g') \in \mathrm{G}'$ (as imposed by the definition: *svasajātīya*) for card(G′)=1. So: *ghaṭo aneko na bhāsate, ghaṭaikatvāt*, ‘No pot-multiplicity appears, because there is but one pot’. That is, *ghaṭa-svasajātīya-ghaṭayos tādātmyam*, ‘Identity between this pot and what is the same thing as this pot’, because these two expressions point to the very same pot. Therefore, if [22], then:

[23] $(g \rightleftharpoons \boldsymbol{I} \llcorner g')\neg\mathbb{N}\llcorner(1_t)$
yat tādātmyatā-avacchedaka-avacchinna-paryāptitvaṃ tad ekatva-nirūpitam, ghaṭa-kaivalyād; saiva ghaṭa-niṣṭhā-tādātmyatā svasajātīya-ghaṭa-nirūpitā, tadviparyayeṇa ca; $^{(t)}$‘*Pot*′ is identical to *pot*, &vv, for *card*(*I*) = 1’; iff ($\langle g, g' \rangle \in I$).[41]

The question potentially remains, why must the cardinality necessarily be equal to one? Firstly, for textual reasons: because Gadādhara himself imposes this condition when discussing the meaning of ‘the term one’ (*ekaśabda*). Secondly, for logical reasons. Indeed, what if the above analysis (cf. [21]–[23]) were repeated in terms of general properties, e.g. pot-ness (g_t)? The result would then be:

[24] $g_t \rightleftharpoons \nexists^{-1} \llcorner(\not{I}^{-1} \llcorner g_t')$
yā ghaṭatvāvacchinna-atyantābhāvīya-pratiyogitā sā anyonyābhāvīya-pratiyogitā-nirūpitā, saiva anyonyābhāvīya-pratiyogitā svasajātīya-ghaṭatva-nirūpitā, tadviparyayeṇa ca; whose purport is: $^{(t)}$‘Pot-ness (g_t) identical to pot-ness′ (g_t'), &vv’.

Formula [24] is true iff $(|g_t| = \mathrm{G}) \wedge (|g_t'| = \mathrm{G}') \wedge (|g_t| \cap |\not{I}^{-1} \llcorner g_t'| = \varnothing)$; but, $|\not{I}^{-1} \llcorner g_t'| = \overline{\mathrm{G}'}$; therefore, $\mathrm{G} \cap \overline{\mathrm{G}'} = \varnothing$. It follows that $\langle ‘g_t’, ‘g_t'’ \rangle \in Q$ and $\langle \mathrm{G}, \mathrm{G}' \rangle \in I$, i.e. the expression ‘*pot-ness*’ is equal to the expression ‘*pot-ness*′’ and the set Pot-ness is identical to the set Pot-ness′ because they are the very same set (AE; cf. fn. 18). Now, if we chose to distinguish g_t and g_t' call them *ghaṭatva* and *kalaśatva*, from a linguistic perspective the application of Gadādhara’s definition to a general property

41 It is also worth noting that, if card(*I*) =1, nonetheless, card($\not{I}$) >1. Indeed, $|\not{I}^{-1} \llcorner g'| = \overline{\mathrm{G}'}$ is the set containing everything but g' (which is identical to *g*), that is, innumerable if not infinite elements: *etadghaṭo ’nyapadāthebhyo bhinnaḥ* (‘This pot is distinct from whatever else anything is’). Thus: $(g \rightleftharpoons \not{I}^{-1} \llcorner p)\neg\mathbb{N}\llcorner(\geq 2_t)$, *ghaṭa-paṭayor bhinnatve sati, yad anyonyābhāvīya-pratiyogitāvacchinna-paryāptitvaṃ tad dvitvādi-nirūpitam, saiva ghaṭa-niṣṭhā-pratiyogitā paṭa-nirūpitā, tadviparyayeṇa ca*; iff $(g \neq p) \wedge (\mathrm{card}(\not{I}^{-1}) \geq 2)$.

such as g_t collapses significantly into equality (Q; cf. § 2). If, on the contrary, the same name, say *ghaṭatva*, were retained, this would be just a reflexive case of equality. If equality primarily concerns different names with the same reference, identity should first and foremost concern reference and not its names, otherwise the one would collapse into the other. What could identity mean with regards to a set, if it is not a matter of names? Once again, the key is to think in terms of relations on the Cartesian plane. What is at stake here is the set of ordered couples belonging to the relation $\langle \mathrm{G}, \mathrm{G}\rangle \in I$ (i.e. $\mathrm{G}\times\mathrm{G}$ according to relation I), in which every single element of G (g^1, g^2, …, g^n) stands in relation I to itself: $\langle g^n, g^n\rangle \in I$. In other words, the extensional interpretation of identity, with respect to a general property such as ghaṭatva (g_t), turns out to be the set of the ordered couples stating the identity of all the elements of the dominion with themselves. It goes without saying that each of these couples clearly has a cardinality equal to one, as expressed in [23].

The cases of *dik* and Rāma have already been discussed in § 2: '*dik*' ≠ '*ākāśa*' ≠ '*vyoman*', and 'Rāma' ≠ '*ayodhyā-kumāra*', are meant as different names (*nāman, śabda*, or *pada*) for the same reference, the space and the hero called Rāma. In light of § 3, it is now clear that from a linguistic point of view (*śabdataḥ*), referring to the same *artha* (object or reference), the name '*dik*' is said to be equal to '*ākāśa*' and 'Rāma' to '*ayodhyā-kumāra*'; however, from an extensional point of view (*arthataḥ*) there is nothing but space, or nothing but Rāma. Thereby, *arthataḥ*, and starting from the assertion *rāmo 'yodhyā-kumāro naiti na* ('It is false that Rāma is not the prince of Ayodhyā'):

> [25] $((\textit{ayodhyā-kumāra}) \rightleftharpoons \nexists^{-1} \llcorner (\boldsymbol{I}\!\!\!/^{-1} \llcorner (\textit{Rāma})))\neg\mathbb{N}\llcorner(1_t)$
> *yad atyantābhāvīya-pratiyogitā-avacchedakāvacchinna-paryāptitvaṃ tad ekatva-nirūpitam, saiva ayodhyā-kumāra-niṣṭha-atyantābhāvīya-pratiyogitā anyonyābhāvīya-pratiyogitā-nirūpitā, saiva anyonyābhāvīya-pratiyogitā rāma-nirūpitā, tadviparyayeṇa ca*; (t)'The prince of Ayodhyā is not the counterpositive of a difference occurring in Rāma, &vv; for card=1'.

That is, *rāmo 'yodhyā-kumāraḥ* in the meaning of *rāmāyodhyā-kumārayos tādātmyam* ('Rāma is identical to the prince of Ayodhyā'), because: $\langle$*Rāma, ayodhyā-kumāra*$\rangle \in I$, or $\langle$*Rāma, Rāma*$\rangle \in I$, or $\langle$*ayodhyā-kumāra, ayodhyā-kumāra*$\rangle \in I$, and card(I) radically equal to one. In parallel, *śabdataḥ* (cf. *supra* [17]):

> [26] $(\text{‘}\textit{Rāma}\text{’}\neg\boldsymbol{\acute{S}}\llcorner \textit{etatpuruṣa}) \rightleftharpoons \boldsymbol{Q}\llcorner (\text{‘}\textit{ayodhyā-kumāra}\text{’}\neg\boldsymbol{\acute{S}}\llcorner \textit{etatpuruṣa}))\neg\mathbb{N}\llcorner(\geq 1_t)$
> *yat samaniyatatva-avacchedakāvacchinna-paryāptitvaṃ tad ekatvādi-nirūpitam, tatra yad eva rāma-pada-avacchinna-etatpuruṣa-nirūpita-śakyatā-avacchedakāvacchinna-samaniyatatvaṃ tat ayodhyā-kumāra-pada-avacchinna-śakyatā-nirūpitaṃ, eṣaiva śakyatā etattva-avacchinna-etatpuruṣa-nirūpitā, tadviparyayeṇa ca*. Whose purport is: (t)'Two distinct verbal expressions—'Rāma' and 'prince of Ayodhyā'—referring to the very same individual (card($Q^{\mathrm{sub}[26]}$)=1)'.

In conclusion, the constant counterpositive-ness (*atyantābhāvīya-pratiyogitā*) of identity (*tādātmya*) has proved to be mutual absence (*anyonyābhāva*) or diversity (*bheda*). Conversely, the constant counterpositive-ness of mutual absence is nothing but identity.[42] What has been obtained in this section is thus a counterfactual redefinition of identity in terms of oneness, mutual exclusion and constant absence; or, from a purely extensional perspective, its redefinition in terms of membership relation, complement and the cardinality of a set.[43]

Interpreting Non-difference

VM has openly stated that the relation of non-difference (*abheda*, *abhinna*; $\cancel{2}$) is linguistically expressible in terms of *sāmānādhikaraṇya* ($\underset{.}{N}$), syntactical homogeneity or coreferentiality (cf. P1). Yet, how to interpret in detail this relation? Could non-different *relata* be also said at once equivalent, equal, or identical? In the light of previous paragraphs, it will be argued that none of these interpretations is viable.

Interpreting a given relation means here to explore when this is true with respect to other relations. In this sense, if two relations are at once true, in every possible cases, they are one and the same, as e.g. syntactical homogeneity and coreferentiality are: $(\forall x)\ (\langle x, y\rangle \in R \wedge \langle x, y\rangle \in R') \rightarrow (R{=}R')$. In parallel, two relations are completely distinct if they never are simultaneously true: $(\forall x)\ (\langle x, y\rangle \in R \wedge \langle x, y\rangle \notin R') \rightarrow (R \neq R')$. If a relation is always true when a second one is true, but not vice versa, the former is included in the latter: if $(\forall x)\ (\langle x, y\rangle \in R' \rightarrow \langle x, y\rangle \in R) \wedge (\exists x)\ (\langle x, y\rangle \in R \wedge \langle x, y\rangle \notin R')$, then $R' \subseteq R$. In a fourth case, two relations could share some common pairs and, in this sense, they could be said just 'resembling' ($\cong$): $(\exists x)\ (\langle x, y\rangle \in R \wedge \langle x, y\rangle \in R') \wedge (\exists x')\ (\langle x', y\rangle \in R \wedge \langle x', y\rangle \notin R') \rightarrow (R \cong R')$; or $R \cap R' \neq \varnothing$.[44]

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[42] Cf. the analogous counterfactual definitions: Viśvanātha's NSM 12: *anyonyābhāvatvaṃ tādātmya-sambandhāvacchinna-pratiyogitākābhāvatvam* (quoted by Ingalls as an instance of 'essential identity'; Ingalls 1951, p. 68 fn. 134); NK, p. 328: 1[*ka*]: *tādātmya-sambandhāvacchinna-pratiyogitāko yaḥ abhāvaḥ so 'nyonyābhāvaḥ*; cf. TrS (2007, p. 172): *tādātmya-sambandhāvacchinna-pratiyogitāko 'nyonyābhāvaḥ*. See also *Bālavyutpattiḥ* (2012, p. 13; cf. also Pellegrini 2015, pp. 152–153): *tādātmya-sambandhāvacchinna-pratiyogitākābhāvatvam anyonyābhāvasya lakṣaṇam*. The same definition is also discussed in Matilal (1968, pp. 46–47).

[43] Ingalls (1951, p. 71) defines identity in the same manner: "If x occurs whenever y occurs and vice versa, then x and y are essentially identical"; in Ingalls' notation: '$-\ \dot{-}\ x \doteq x$', i.e. constant absence of mutual absence (*bhedābhava*) of x, identical ($\doteq$) to x. About *bhedābhava* as a matter of controversy, cf. Ingalls (1951, pp. 71–72).

[44] E.g., the relation R ('Having the same number of sides') and R' ('Having the same number of vertices') identify the very same ordered pairs of two-dimensional polygons; thus, $R{=}R'$. On the contrary, the relation Q ('Having the same squared root') and Q' ('Having the same shoe size'), on domains Numbers and Humans, do not apparently share any pair; thus, $Q \neq Q'$. Yet, on the domain Kids, the relations Z ('Cohousing'), Z' ('Sibling'), and Z'' ('Having the same surname') reasonably could be said to resemble each other, albeit modulated by conventions and contingencies. Thus, $Z \cong Z' \cong Z''$, also assuming here, for the sake of the discussion, that Z' and Z'' resemble each other more than Z and Z'. The concept of relational resemblance ($\cong$) aims to highlight *resemblances* (common instances) between relations; these *resemblances* may then make it possible to interpret one relation via another. Relational interpretation aims to act as a tool in defining non-difference by means of overlapping and distinction with other relations. On resemblance as a tool to detect overlapping similarities and crisscrosses (e.g. games resemblances), cf. *family resemblances* (*Familienähnlichkeit*), Wittgenstein (2009: § 66).

Let us consider again the case of a golden crown. In [3] $h.h_t \neg \boldsymbol{V}^{(\underset{.}{N})} \llcorner m.m_t$ (ruled by SVN, *Samānādhikaraṇa-Viśiṣṭatva-Nyāya* or ‘Principle of Coreferential Qualification’; cf. P1), it has been shown that every case of coreference ($\underset{.}{N}$) can properly be interpreted in terms of qualification (*viśeṣya-viśeṣaṇa-saṃsarga*, V). Moreover, assertions [2]–[3] can stand as proper interpretations of *mukuṭa-hāṭakayor abhedaḥ* (‘Non-difference between crown and gold’), because ostensibly: ($\not{2}$) $\subseteq$ ($\underset{.}{N}$) $\subseteq$ (V).[45]

[27] $h \,.\, \not{\boldsymbol{2}} \llcorner m$
yā abhinnatā hāṭaka-niṣṭhā sā mukuṭa-nirūpitā & yā abhinnatā hāṭaka-niṣṭha-hāṭakatvāvacchinnā sā mukuṭa-niṣṭha-mukuṭatva-nirūpitā; ‘Non-difference-ness in gold, conditioned by a crown’. Iff $\langle m, h\rangle \in \not{2} \wedge \langle m, h\rangle \in \underset{.}{N} \wedge \langle m, h\rangle \in V$.

In assertions *[2_{b}] and [11]–[12] (cf. §2), it has already been shown that [2]–[3]—and consequently [27]—cannot be interpreted as instances of equivalence on the model of [10]. Neither could they be interpreted as instances of equality on the model of [17]: *[17_{a}] $*((\text{‘}m\text{’} \neg \boldsymbol{\acute{S}} \llcorner m_t) \rightleftharpoons \boldsymbol{Q} \llcorner (\text{‘}h\text{’} \neg \boldsymbol{\acute{S}} \llcorner h_t)) \neg \mathbb{N} \llcorner (\geq 1_t)$, which is false because: $(|h_t| \nsubseteq |m_t|) \wedge (|m_t| \nsubseteq |h_t|)$, therefore G≠M. It follows that only its negation can be true, i.e.:

[28] $((\text{‘}m\text{’} \neg \boldsymbol{\acute{S}} \llcorner m_t) \rightleftharpoons \nexists^{-1} \llcorner (\boldsymbol{Q} \llcorner (\text{‘}h\text{’} \neg \boldsymbol{\acute{S}} \llcorner h_t))) \neg \mathbb{N} \llcorner (\geq 1_t)$
yad atyantābhāvīya-pratiyogitā-avacchedakāvacchinna-paryāptitvaṃ tad ekatvādi-nirūpitam, tatra yā mukuṭa-pada-avacchinna-mukuṭatva-nirūpita-śakyatā-avacchedaka-avacchinna-atyantābhāvīya-pratiyogitā sā samaniyatatva-nirūpitā, tad eva samaniyatatvaṃ śakyatā-nirūpitam, saiva śakyatā hāṭaka-pada-avacchinnā hāṭakatva-nirūpitā, tadviparyayeṇa ca.

It is clear that $^{(\mathrm{t})}$‘The term gold is not equal to crown, simply because that which is a crown is not indifferently called gold, &vv’. Let us consider the assertions: ‘Gold is mined’ or ‘In the periodic table, the chemical element known as gold has the atomic number 79’. Here, any substitution would clearly be nonsense because crowns are not mined, nor are they chemical elements in the periodic table, nor do they have an atomic number.[46] The grounds for the use (*pravṛtti-nimitta*) of the terms ‘*mukuṭa*’ and ‘*hāṭaka*’ is plainly distinct, thus the two terms cannot be coextensive.

Moreover, equality is unquestionably a symmetrical relation since it identifies coreferentiality between terms, as in formulas such as [16] and as expressly stated

[45] For instance, the assertion *daṇḍī puruṣaḥ* (‘A staff holder’) qualifies (V) a man by means of a staff, though that does not imply that there is a relation of coreference ($\underset{.}{N}$) between the two relata—despite the fact that it is linguistically expressed as a case of syntactic homogeneity. The same goes for *ghaṭavad-bhūtalam*, ‘A ground qualified by a pot’ (lit. ‘A pot-possessing ground’) or *kākavad-gṛham*, ‘A house qualified by a crow [on its roof]’. Since there are cases in which V is true but $\underset{.}{N}$ is false, qualification appears to be more general and co-reference a more specific interpretation of the former (e.g., excluding all instances of qualification by contact, *saṃyoga-sambandha*).

[46] The substitution—in every assertion and also in [17]—would instead be perfectly sound with truly coextensive terms such as ‘*suvarṇa*’, ‘*kanaka*’, ‘*kāñcana*’, etc. or, say, with the chemical symbol ‘*Au*’. It is well known that the analysis could be pushed forward as advanced, among others, by Putnam in his ‘Twin Earth thought experiment’ about the analogous case of ‘water’ and ‘H_2O’; cf. Putnam (1973). For the present purposes, these further issues are voluntary set aside. Regarding Substitutivity test, cf. fn. 50.

by the operator ‘$\rightleftharpoons$’ (*go* = *sāsnādimat* as well as *sāsnādimat* = *go*). Since |‘g_t’|=G ∧ |‘s_t’| =S ∧ (G=S), equality is a relation having set G—that is S—as its reference domain and range (i.e., $Q^{sub[16]}$: G $\mapsto$ G or S $\mapsto$ S). The same is not true for *V* and $\cancel{2}$, which are consequently not symmetric. Consider the case of ‘A smiling man’ (*smayan puruṣaḥ*): while this man is qualified by his smile, it is harder to accept that a smile is qualified by this man who smiles—just as in the case of blueness qualifying a pot, which simply cannot be qualified by pot-ness. Thus, relation *V* openly appears to be not-symmetric and requires its proper inverse (V^{-1}) to be reversed.[47] Syntactic homogeneity (*sāmānādhikaraṇya*, *Ṇ*) is, on the contrary, too vague a notion to be considered symmetrical or not. In fact, its possible symmetry depends on its interpretation: if *Ṇ* means equality—as in the sentence *sāsnādimatī gauḥ*—then it will be transitive and symmetric. As shown, however, if it was interpreted as a general instance of qualification, it could no longer be said to be either symmetrical or transitive—just as in *nīlo ghaṭaḥ* (cf. also fn. 47). The issue might not be quite so predictable with regard to non-difference. In the golden crown case, if $\cancel{2}$ is interpreted as a *viśiṣṭa-jñāna*—in which the crown is non-different ($\cancel{2}$) from the gold by which it is qualified (*V*)—then *abheda* will clearly be non-symmetrical. Moreover, if non-difference were then further interpreted as ‘consisting of’ or ‘being made of’, it would be newly non-symmetrical. Indeed, it can safely be stated that a pot is ultimately clay (cf. *Chāndogya* Up. 6.1.4–6), but it is harder to accept that clay is a pot or consists of a pot. Along the same lines, VM’s interpretation explicitly puts *abheda* in contact with causation (*kāryakāraṇabhāva*, *K*) in general and with material cause (*upādānakāraṇa*, ^{u}K) in particular (VM-B, p. 72–73). Thus, if *k*⌐$^{u}\boldsymbol{K}$⌞*r*, *yā upādānakāraṇatā kāraṇa-avacchinnā sā kārya-nirūpitā* (‘Material causeness, conditioned by the effect (*r*), occurring in the cause (*k*)’); its symmetric form is clearly false: **r*⌐$^{u}\boldsymbol{K}$⌞*k*, **yā upādānakāraṇatā kāryāvacchinnā sā kāraṇa-nirūpitā* (*‘*Material causeness*, conditioned by the cause, occurring in the effect’). Then, the effect (*kārya*, *r*) could be said, once proved, to be non-different from the cause (*kāraṇa*, *k*) from which it derives: *k*⌐ $\boldsymbol{\cancel{2}}$ ⌞*r*. However, merely switching the relata is nothing but nonsense here as well: **r*⌐$\boldsymbol{\cancel{2}}$ ⌞*k* (*‘The cause is non-different from the effect’). A negation of symmetry could be also achieved by interpreting non-difference as a case of ‘part and whole relation’, since what possesses parts (*avayavin*) might be conceived as non-different from the parts (*avayava*) it possesses, but not vice versa. Thus, while it is reasonable to say that ‘A horse is not different from a limb of itself’, ‘A limb is not-different from a horse’ sounds slightly stranger in some way. In the form of joke, one of the Buddha’s teeth is not the Buddha.[48]

[47] Thereby: ⟨*pot, blueness*⟩∈*V*, i.e. ‘A pot qualified by blueness’, true for *V*: Pots $\mapsto$ Properties of Pots; while ⟨*blueness, pot*⟩∈V^{-1}, i.e. ‘Blueness qualifying a pot’, true for V^{-1}: Properties of Pots $\mapsto$ Pots. In other words, be it considered that the relation *B* (‘*x* is brother of *y*’); *B* is clearly symmetrical, because: ⟨*x*, *y*⟩∈*B* is true as well as ⟨*y*, *x*⟩∈*B*, having the set Brothers as its domain and range (*B*: Brothers $\mapsto$ Brothers). Consider now that the relation *F* ‘x is the father of *y*’; *F* is patently not symmetric, because ⟨*x*, *y*⟩∈*F* is true but *⟨*y*, *x*⟩∈*F* is false (*y* is not the father of *x*, simply because *x* is the father of *y*). The only way to make *⟨*y*, *x*⟩∈*F* true is to construe its inverse relation F^{-1}: ‘*y* is son of *x*’. Thereby: ⟨*x*, *y*⟩∈*F*, true for *F*: Fathers $\mapsto$ Sons; while, ⟨*y*, *x*⟩∈F^{-1}, true for *F*: Sons $\mapsto$ Fathers. The relation $\cancel{2}$ has to be treated in the same way (cf. *infra*).

[48] While without hooves or pectoral muscles there is no horse (and therefore ‘A horse is not-different from its hooves or pectoral muscles’), pectoral muscles are not a horse. Similarly, a pot is non-different

Moreover, while equality is a transitive relation, non-duality is not—and neither is *V*. If *hāṭaka* = *suvarṇa* and *suvarṇa* = *kanaka*, then *hāṭaka* = *kanaka* (cf. fn. 46), since these *padas* have one and the same grounds of use. And yet, being b_t the property *kaṭakatva* (‘bracelet-hood’, for $|b_t|$=B), given $h.\underset{\cdot}{\boldsymbol{N}}\llcorner b$ (A golden bracelet) and [2] $h.\underset{\cdot}{\boldsymbol{N}}\llcorner m$ (A golden crown)—or $h.\ \not{\boldsymbol{2}}\llcorner b$ (A bracelet not-different from gold) and [27] $h.\ \not{\boldsymbol{2}}\llcorner m$ (A crown not-different from gold)—it patently does not follow that $*b.\underset{\cdot}{\boldsymbol{N}}\llcorner m$ (A crown which is a bracelet) or $*b.\ \not{\boldsymbol{2}}\llcorner m$ (A crown non-different from a bracelet). In other words, if the crown is golden and so is the bracelet, it does not follow that the crown is a bracelet. One last remark about equality: it is surely licit to use it reflexively, but such a use appears somehow secondary in that it is lacking any informative value. Indeed, whereas it could be of some use to state that ‘gold = *suvarṇa* = *Au* (in the periodic table)’, it is much less interesting to repeat that ‘gold = gold’. The same holds for non-difference: it is safe to assert that ‘$m.\not{\boldsymbol{2}}\llcorner m$’ (A crown non-different from a crown), but such an assertion is utterly uninteresting.

To summarize, it turns out that, even though the crown is in fact gold, it cannot be said to be equal to gold, nor crown-ness to gold-ness. Nonetheless, this crown is still gold, a fact which renders the assertion ‘The crown is not gold’ ($*m \neq h$) also concurrently false. VM openly declares that non-difference is never reducible to a relation of reciprocal absence (*parasparābhāva*; i.e. $\not{I}^{-1}$). If that were the case, there would exist only simple difference and not any kind of non-difference. This eventuality is simply impossible (*asaṃbhava*), however, because it would be directly contradictory (*virodha*) to non-difference: by hypothesis, the two properties do co-exist (*saha-avasthāna*) in the very same *locus*.[49] If simple difference ($*m \neq h$) were the case, then the relation between gold-ness and crown-ness in a golden crown would be assimilable to a relation to whatever other property, say, horse-hood: if $\langle m, h\rangle \in \not{2}$ was read as $m \neq h$, then a crown would also be not-different from a horse. In other terms, if non-duality was conceived as equality or diversity, we would be pushed back to the starting contradiction (cf. P1): $*m = h$ is false, as is $*m \neq h$. Thus, the crown is (i.e., $\underset{\cdot}{N}$, *V*, and $\not{2}$) surely gold, yet not in the sense implied by equality or difference.

Footnote 48 continued
from its incurved sides, because if the sides were taken away there would be no pot left. In parallel, a side of a pot cannot store water, thus revealing that it is not a pot: i.e. a pot is non-different from a side of itself, but not vice versa. Potter (1977, p. 74–75): “In *Nyāya-Vaiśeṣika* a whole is produced from its parts, but is not constituted by them. Favourite examples in the literature are the pot which is produced by its halves, and the cloth which is produced from the threads which compose it. The pot and the cloth are not aggregates of sherds or threads; the pot is an unified substance, of medium dimension, with its own qualities and relations, a *different* entity from the sum or collection of its components” (italics added; because what I am trying to argue, in this paper, is that a pot is *neither* different from *nor* identical to its parts, simply because it is non-different (*abhinna*) from them). Phillips (1997, p. 147 and fn 84): “Logicians from the earliest period defend […] the position that the whole is more than the sum of its part (excluding heaps, collections, and the like)”. See also, NS 4.2.4-17 (2009, pp. 698–706).

[49] VM-B, p. 73: *atrocyate kaḥ punar ayaṃ bhedo nāma, yaḥ sahābhedenaikatra bhavet? parasparābhāva iti cet, kim ayaṃ kāryakāraṇayoḥ kaṭakahāṭakayor asti na vā ? na cet, ekatvam evāsti, na ca bhedaḥ* ǀ *asti cet bheda eva, nābhedaḥ* ǀ *na ca bhāvābhāvayor avirodhaḥ, sahāvasthānāsaṃbhavāt* ǀ *saṃbhave vā kaṭakavardhamānayor api tattvenābhedaprasaṅgaḥ, bhedasyābhedāvirodhāt* ǀ.

Now, could non-difference be interpreted as a relation of identity? Let us try to interpret the assertion *hāṭakaṃ mukuṭam* in terms of identity following the model of [22]–[24]—the crown is (*Ṇ*) gold, in the sense that the crown should be said to be identical (*I*) to gold:

*[29] $*h \rightleftharpoons \boldsymbol{I} \llcorner m \vee$ [30] $h \,.\, \nexists^{-1} \llcorner (\not{\boldsymbol{I}}^{-1} \llcorner m)$

That is, according to the counterfactual definition of identity, the crown should not be the counterpositive of an absolute absence of a mutual absence with respect to something which is that very entity, i.e. the gold. Here, a first important point: [30] is true for $h \notin |\not{\boldsymbol{I}}^{-1} \llcorner m|$, i.e. 'An instance of gold (*h*) is meant to belong to the singleton $|m| = \{m\}$', which is indeed the case ('A crown is not the counterpositive of an absolute absence of a mutual absence with respect to an instance of gold'). What we are talking about is this crown, which is (i.e., *V*, *Ṇ*, $\not{2}$, and I) this gold: what is at stake here is this very singleton. Non-difference fits the counterpositive definition of identity because these two relations ontologically focus on the very same *artha*. So far, non-difference seems to coalesce dangerously into identity.

However, let us now consider two additional points: on the one hand, the so-called Principle of the *Indiscernibility of Identicals* (sometimes called *Leibniz's Law*, LL): for all *x* and *y*, if $x = y$ (i.e. $\langle x, y \rangle \in I$), then *x* and *y* have the same properties—which is commonly considered quite uncontroversial. On the other hand, what is known as the Principle of Identity of Indiscernibles (PII): for all *x* and *y*, if *x* and *y* have the same properties, then $x = y$ (i.e. $\langle x, y \rangle \in I$)—which, on the contrary, is highly controversial. Whether or not PII functions, this principle does not apply here anyway. In the assertion under examination stating that *hāṭakaṃ mukuṭam*, there is no trace of the commonality of properties, much less of indiscernibility. And yet, the situation regarding LL is even worse: if LL applied here, then crown and gold would display the same properties, which they do not—simply because we are still dealing with two fully distinct properties (cf. Leibniz 1989, p. 42 and 1981, p. 230).

Let us take a step forward. If non-difference were identity tout court and the indiscernibility of property followed for LL, then non-difference would pass the Substitutivity Test (ST). Still, consider the following case: if *[29] $*m \rightleftharpoons \boldsymbol{I} \llcorner h$ (The crown is identical to the gold), then obviously, by substitution: $m \rightleftharpoons \boldsymbol{I} \llcorner m$ and $h \rightleftharpoons \boldsymbol{I} \llcorner h$ (The crown is identical to the crown, the gold to the gold). The same holds true for a golden bracelet (*kaṭaka*, *b*): if $*b \rightleftharpoons \boldsymbol{I} \llcorner h$, then $b \rightleftharpoons \boldsymbol{I} \llcorner b$ and $h \rightleftharpoons \boldsymbol{I} \llcorner h$. In this case, however, it would follow—again by substitution between identical indiscernibles—that: $*b \rightleftharpoons \boldsymbol{I} \llcorner m$ (This bracelet is identical to this crown), which is pure nonsense—simply because a bracelet, perfectly discernible from a crown, is not a crown. Thereby, non-difference clearly fails the ST and, since fallacies are generated, it appears to be non-reducible to identity tout court. Moreover, this last example is a clear case of non-transitivity: non-difference has thus proven to be a non-transitive relation, while identity of course is—if *x* is identical to *y*, *y* is identical to *z*, *z* is identical to *x* (cf. *supra*, Quine 1981, pp. 134–136).[50]

[50] For an initial survey of identity, substitutivity, and Leibniz's law, cf. Hawthorne (2003: § 2.3, pp. 108–131). In this regard, it is worth noting that "the totality of properties in an individual is always different

Interpreted as a case of qualified cognition (V), non-difference does not even appear as a symmetric relation, and this is because V is certainly not one. It has been shown that, for SVN, the property Gold-ness in crowns is a subset of the set Properties of crowns: $|h_t| \subseteq |\boldsymbol{V}^{(N)} \llcorner m_t|$ (cf. P1), for $V^{(N)}$: $\mathrm{M} \mapsto V^{(N)}[\mathrm{M}]$ and $V^{(N)}[\mathrm{M}] \subseteq \mathrm{M}$. Non-difference can analogously be construed as a relation whose domain is M (Crowns) and whose range is $\not{2}[\mathrm{M}]$ (What is non-different from crowns, e.g. gold-ness, heaviness, etc.): i.e. relation $\not{2}$: $\mathrm{M} \mapsto 2[\mathrm{M}]$, for $\not{2}[\mathrm{M}] \subseteq \mathrm{M}$. What is at stake here is the gold-ness occurring in a crown. Inasmuch as the reference domains are distinct, by virtue of V, the relata cannot be simply inverted as in case of symmetry; what is needed instead is a fully fledged inverse relation. The same is clearly true for different kinds of non-difference interpretations as well, such as causation, 'part and whole', 'consisting of', etc.[51]

Cardinality also could help in distinguishing between non-difference and identity. Indeed, it has been shown that the cardinality of identity is strictly equal to one ($card(I) = 1$; cf. [23]). I will argue here that non-difference can bear a cardinality equal to and greater than one ($card(\not{2}) \geq 1$). The assertion *mukuṭa-hāṭakayor abhedaḥ* clearly begs for a cardinality equal to one, since there is but one crown here, a golden one:

Footnote 50 continued
from the totality of properties in any other individual. In this sense, the totality of properties also becomes a differentiating feature of an individual (fn. 99). [...] Is an individual identical with a bundle of properties without a separate substratum for those properties, or is it different from those properties and serves as their substratum, locus, or receptacle? Ultimately, like *Nayāyikas, Mīmāṃsakas* also maintain that an individual (= substance) is *different* from its properties", Deshpande (1992, pp. 30–31), quoting in fn. 99: *Tantra-vārttika* by Kumārila (comm. on *Bhāṣya* by Śabara, in his turn comm. on Jaimini's *Mīmāṃsasūtra*), Banaras 1903, pp. 250–251; italics added, cf. fn. 48. For the reason alluded to by Deshpande *[29] is false but [30] is true. Indeed, I-relation implies the totality of properties and generates inconsistencies, while its counterfactual redefinition regarding a single property leaves open the possibility to claim [30].

[51] In the example of crown and gold, for SVN, 'A crown is not-different from the gold [it is qualified by]' ($\langle m, h\rangle \in \not{2}$; for $\not{2}: \mathrm{M} \mapsto \not{2}[\mathrm{M}]$ and $\not{2}[\mathrm{M}] \subseteq \mathrm{M}$), but not vice versa: $*\langle h, m\rangle \in \not{2}$, for $*\not{2}: \not{2}[\mathrm{M}] \mapsto \mathrm{M}$ and $*\mathrm{M} \subseteq \not{2}[\mathrm{M}]$, which is clearly illicit. A well-formed inverse relation would instead be: $\langle h, m\rangle \in \not{2}^{-1}$, for $\not{2}^{-1}: \not{2}[\mathrm{M}] \mapsto \mathrm{M}$, for $\not{2}[\mathrm{M}] \subseteq \mathrm{M}$; let us say in active form, 'A specimen of gold does not differ from the crown [it qualifies]'; QED. For the same reason, in the case of a blue pot, 'A pot (g) is non-different from blue-ness (n_t) [by which it is qualified ($V^{(N)}$)]', but not viceversa; for $\langle g, n_t\rangle \in V^{(N)}$; $V^{(N)}: \mathrm{G} \mapsto V^{(N)}[\mathrm{G}]$, $V^{(\mathrm{N})}[\mathrm{G}] \subseteq \mathrm{G}$; $\langle g, n_t\rangle \in \not{2}$, $\not{2}: \mathrm{G} \mapsto \not{2}[\mathrm{G}]$, $\not{2}[\mathrm{G}] \subseteq \mathrm{G}$; and $\langle g, n_t\rangle \in \not{2}^{-1}$, $\not{2}^{-1}: \not{2}[\mathrm{G}] \mapsto \mathrm{G}$, $\not{2}[\mathrm{G}] \subseteq \mathrm{G}$. This last case concerning *guṇa* is revelatory. In general, it has been shown that in $V^{(N)}$ the *viśeṣya* is the *avacchedaka* of the attributed *viśeṣa* (cf. P1, [4]-[7]). This feature occurs in what is qualified, indeed: *viśeṣyāvacchinna-viśeṣaḥ*. A pot (*dravya*) is non-different from blueness (*guṇa*) because blueness occurs in the pot, and not pot in blue-ness. Therefore, in naming blueness we are talking about a qualification of the pot; in other words, there is no blueness but in the pot and, for this reason, the pot is non-different from one of its qualifications. SVN displays its heuristic power here. Hanging onto domain-range truth conditions, one must not yield to the temptation to be pulled back to the start and reinterpret non-difference as a vague notion of 'being'. It is true that 'The pot is blue' because $V^{(N)}: \mathrm{G} \mapsto V^{(N)}[\mathrm{G}]$; here, only pots exist (that is why: $\not{2}: \mathrm{G} \mapsto \not{2}[\mathrm{G}]$). But, 'Blueness is non-different from the pot' is false because it relies on: $\not{2}: \mathrm{N} \mapsto \not{2}[\mathrm{N}]$, an interpretation which, in turn depends on: $V^{(N)}: \mathrm{N} \mapsto V^{(N)}[\mathrm{N}]$, a relation having Blue as its domain and connecting this quality with that by which it is qualified, here a pot (i.e. 'A blueness qualified by a pot'; which is quite a piece of nonsense). So, the second temptation to resist, here made evident, is that of reifying *guṇas*. Indeed, there is nothing but a pot, here. (cf. fn. 47). On Nyāya-Vaiśeṣika ontology, cf. Potter (1977, pp. 38–146) and Phillips (1997, pp. 44–51).

[31] $(\text{h}\ .\ \text{ƻ} \llcorner m)\neg\mathbb{N}\llcorner(=1_t)$
yad abhinnatā-avacchedakāvacchinna-paryāptitvaṃ tad ekatva-nirūpitam, saiva abhinnatā hāṭaka-niṣṭhā mukuṭa-nirūpitā; 'Non-difference, conditioned by a crown, is limited by a specimen of gold, for card(ƻ) =1(since they are but the same object)'.

However, let us try to interprete ƻ as an *avayavāvayavin* relation ('Part and whole') in which it turns out that multiplicity is structurally embedded: *aśvo svāṅgābhinnaḥ* ('Non-difference between a horse (a) and its own limbs ($\dot{n}$)'; for $\langle a, \dot{n}\rangle \in$ ƻ) or *ghaṭaḥ kapāladvayābhinnaḥ*, ('Non-difference between a pot (g) and its own halves (k)'; for $\langle g, k\rangle \in$ ƻ). That, just because *avayavī-avayavābhedaḥ* ('Non-difference between the whole ($\bar{\imath}$) and its constituents (v)'; for $\langle \bar{\imath}, v\rangle \in$ ƻ). Thus:

[32] $(a\ .\ \text{ƻ}^{-1} \llcorner \dot{n}_t)\neg\mathbb{N}\llcorner(\geq 1_t) \vee (g\ .\ \text{ƻ}^{-1} \llcorner k_t)\neg\mathbb{N}\llcorner(\geq 1_t) \vee (\bar{\imath}\ .\ \text{ƻ}^{-1} \llcorner v_t)\neg\mathbb{N}\llcorner(\geq 1_t)$
yad viparītābhinnatā-avacchedakāvacchinna-paryāptitvaṃ tad ekatvādi-nirūpitā, saiva viparītābhinnatā (aśva-; or ghaṭa-; or avayavī-)niṣṭhā (aṅgatva-; or kapālatva-; or avayavatva-)nirūpitā; $^{(t)}$'The inverse relational abstract of non-difference relation, conditioned by the constituents (such as limbs or halves), is limited by the whole (such as a horse or a pot), for card(ƻ) ≥1'. Iff, in s.n., $(\exists x, \forall y \mid \bar{\mathrm{I}}x, \mathrm{V}y)$ ($\langle x, y\rangle \in$ ƻ).[52]

Looking closer, even interpretations based on *upādānakāraṇa* (^{u}K) or *viśeṣaṇa-viśeṣya-bhāva* (V) might display the same feature. Moreover, all of the above cases are 'one-to-many' relations. Multiplicity might be introduced into the domain as well, however, thereby obtaining 'many to one' and 'many to many' relations of non-difference. For instance, *vahnyabhinne prakāśanadāhakārye*, 'The effects of making light and heat are non-distinct from fire', or *bāṣpābhinnā meghāḥ*, 'Clouds are non-different from water vapour'. Let us take now a step forward by considering, e.g., the 88 notes corresponding to the standard 88 piano keys (K=$\{k_1, \ldots, k_{88}\}$, for card(K)=88). Now, a non-difference relation can be construed having as its domain every possible piano piece, written or not-yet-written, potentially counting infinite notes (P=$\{p_1, \ldots, p_n\}$ for card(P) = $\aleph_0$; i.e. aleph-zero, the cardinality of the set of all natural numbers): thus, having dom(ƻ) = P and ran(ƻ) = K, i.e. ƻ: P$\mapsto$K. Although it is pointless to say that every possible piano piece is equivalent, equal or identical to the 88 notes corresponding to the 88 piano keys, it

[52] The relation between the whole and the totality of its components appears a particularly complex case; i.e. $(v_t\neg\ \text{ƻ} \llcorner \bar{\imath})\neg\mathbb{N}\llcorner(\geq 1_t)$, in s.n. $(\exists x, \forall y \mid \bar{\mathrm{I}}x, \mathrm{V}y)$ ($\langle x, y\rangle \in$ ƻ). For instance, could a horse non-different from all its limbs be also said identical to them? Would this case pass ST? In order to avoid these difficulties I have chosen a more nuanced solution: 'A horse non-different from one or some of its limbs'; the aforesaid quantification begs for the introduction of the bizarre non-difference inverse: *viparītābheda* (ƻ$^{-1}$). Cf. also fn. 51. Further investigations are required.

could be perfectly sound to state that the former are non-different from the latter. In standard notation: $(\forall x, \forall y \mid Px, Ky)\ (\langle x, y\rangle \in \cancel{2})$. Once more, a symmetric inversion of the *relata* is not possible. It is simply false that the 88 notes corresponding to the 88 piano keys are non-different from every possible piano piece: i.e., $^{*}\langle y, x\rangle \in \cancel{2}$ is false, since only $\langle y, x\rangle \in \cancel{2}^{-1}$ is true. Obviously, the same could be said about writing systems and literature or about the five DNA-RNA nitrogenous bases and living beings.[53] A novel is non-different from, say, the Latin alphabet, but not vice-versa (i.e., ⟨novel, alphabet⟩$\in \cancel{2}$ and ⟨alphabet, novel⟩$\in \cancel{2}^{-1}$ are true, but *⟨alphabet, novel⟩ $\in \cancel{2}$ is false). In the same way, organisms are non-different from their nitrogenous basis, whereas these latter cannot be said to be simply non-different from the former (i.e., ⟨organisms, n-basis⟩$\in \cancel{2}$ and ⟨n-basis, organisms⟩$\in \cancel{2}^{-1}$ are true, but *⟨n-basis, organisms⟩$\in \cancel{2}$ is false).

This last remark might cast new light—from an *advaita* perspective—on a classical issue concerning identity. The case of Rāma and his description as 'prince of Ayodhyā' has already been discussed above. The case is analogous to the famous 'Scott = author of Waverley'. It is well known that this case and its potentially paradoxical consequences have been analysed in detail, firstly through the distinction between names, descriptions, and denotations.[54] Yet, it might still be usefully rephrased in terms of non-difference: 'Rāma is non-different from the prince of Ayodhyā'—just as 'Scott is non-different from the author of Waverley'. It will come as little surprise that these assertions cannot pass the ST, since they involve properties which are distinct and highly informative ('being called *Rāma*' and 'being the prince of a city called Ayodhyā') even if referring to the very same referent (the man called Rāma). Nonetheless, pushing the argument even further and assuming that Dāśarathi Rāma was a real living human being—just as sir W. Scott was—it could be said that Rāma is non-different from his DNA-RNA nitrogenous bases or his biochemical bases in general—and the same for Scott.

[33] $(r\ .\ \cancel{2}^{-1} \llcorner b_t)\urcorner\ \mathbb{N} \llcorner(\geq 1_t)$
yad viparītābhinnatā-avacchedakāvacchinna-paryāptitvaṃ tad ekatvādi-nirūpitam, saiva viparītābhinnatā rāma-niṣṭhā mahābhūtatva-nirūpitā;
$^{(t)}$'Rāma (r) is non-different from his biochemical bases (b)'.[55]

[53] Regarding the three pyrimidines (cytosine, thymine, uracil) and the two purines (adenine, guanine) and their role in composing nucleic acids (DNA and RNA), cf. Carey (2008, pp. 1164–1166).

[54] Cf. end of § 2 and fn. 27. Russell (1905, p. 483): "If we say 'Scott is the author of Waverley', we assert an identity of denotation with a difference of meaning"; Russell (1919, pp. 173–175): "[…] a consideration of the difference between a name and a definite description. Take the proposition, 'Scott is the author of Waverley'. […] A name is a simple symbol […]. On the other hand, 'the author of Waverley' is not a simple symbol [… but] a description, which consists of several words, whose meaning are already fixed, and from which results whatever meaning is to be taken as the 'meaning' of the description".

[55] The inverse ($\cancel{2}^{-1}$) is required in reason of quantification, cf. fn. 51–52. Gross elements (*mahābhūta*) —*ākāśa* (ether or space), *vāyu* (air), *tejas* (fire), *āpas* (water), *pṛthivī* (earth)—are almost universally accepted in Indian cosmologies, starting with the Sāṃkhya system; in this passage, I make free use of it. On Sāṃkhya cosmology, cf. Larson (1987, pp. 65–72); regarding "material substances" in Nyāya-Vaiśeṣika system, see Potter (1977, p. 73).

The same holds for Scott as well, because every human could be said to be non-different from his/her biology.

> [33_{a}] $(p_t \urcorner\ \boldsymbol{\not{2}}^{-1} \llcorner b_t) \urcorner\ \mathbb{N}\ \llcorner (\geq 1_t)$
> *yad viparītābhinnatā-avacchedakāvacchinna-paryāptitvaṃ tad ekatvādi-nirūpitam, saiva viparītābhinnatā puruṣatva-avacchedakāvacchinnā mahā-bhūtatva-nirūpitā*; 'Humanhood (*puruṣatva*, p_t) is non-different from its biochemical-base-hood (b_t)';
> iff $|p_t| \subseteq |\ \boldsymbol{\not{2}}^{-1} \llcorner b_t|$ (card($\not{2}$) ≥ 1); in s.n. $(\forall x, \forall y \mid \mathrm{P}x, \mathrm{B}y)\ (\langle x, y\rangle \in \not{2}\ )$.

Clearly [33_{a}] has nothing to do with the identity we evoked when talking about Rāma or Scott, since it involves general properties and no longer deals with a singularity, much less defined descriptions. Being distinctly relational in nature, [33_{a}] could not be straightforwardly reduced to a predicative schema either, nor does it claim that 'Humankind is its biology'—only that the former is non-distinct from the latter.

Conclusions

The assertion 'A golden crown' displays an evident case of *sāmānādhikaraṇya* ($\underset{\cdot}{N}$), syntactical homogeneity and coreferentiality. The notion of $\underset{\cdot}{N}$-relation is nevertheless extremely vague and requires further interpretation. It has been shown that: $\underset{\cdot}{N} \neq E$; $\underset{\cdot}{N} \subseteq V$; $Q \subseteq \underset{\cdot}{N}$; $I \subseteq \underset{\cdot}{N}$; $\not{2} \subseteq \underset{\cdot}{N}$. Thus, $\underset{\cdot}{N}$ might or might not be said to be reflexive, symmetric, or transitive, depending on the chosen interpretation. For instance, if $\underset{\cdot}{N}$ is supposed to be a particular case of V—as the assertion 'A golden crown' suggests—it will be non-symmetrical, non-transitive and reflexive only in a secondary, uninformative, sense.

It has also been shown that equivalence (*tulyatva*; E) first and foremost entails one shared property (*taddharmavattva*, td_t) among many. It has also proven to be a symmetric ($\rightleftharpoons$) and transitive relation whose cardinality is strictly greater than one. According to [10], the equivalence between generic elements a and b can be expressed in NL as: $((b.td_t) \rightleftharpoons \boldsymbol{E} \llcorner (a.td_t)) \urcorner \mathbb{N} \llcorner (\geq 2_t)$; iff $(a \neq b) \wedge ((a, b) \in |td_t|) \wedge$ (card $(E) \geq 2$). In keeping with these truth conditions, interpretations of equivalence show that: $E \neq \underset{\cdot}{N}$; $E \neq Q$; $E \neq I$; $E \neq \not{2}$. That is, an equivalence relation, stricto sensu, is to be considered distinct from coreferentiality, equality, identity, and non-difference.[56]

[56] Equivalence can be said to be reflexive only in a secondary, uninformative, and highly context-sensitive sense. Only in this secondary reflexive case could it be said that $Q \subseteq E$ and $I \subseteq E$ (cf. reservations expressed in §5 and examples about *dik* and *vaidya*). In this case, a radical change in truth conditions (i.e. card(E) ≥ 1) occurs; this could be considered equivalence *lato sensu*. Nonetheless, $E \subseteq V$. Let us try to interpret equivalence [8] (i.e., *so gaur etasya gos tulyaḥ*) under qualification: $(g'.g_t) \urcorner \boldsymbol{V} \llcorner (g.g_t)$, 'The qualifier-ness, conditioned by cow-ness in this cow, is limited by cow-ness in that cow', iff $(g, g') \in \mathrm{G}$ and $g' \neq g$; which is a trivial, but true, case of reflexive qualification ('Cow-ness qualified by that very cow-ness') in two distinct instances.

Equality (*samaniyatatva*; Q) has proven to mean invariable concomitance, or mutual pervasion, with regards to names or expressions. It appears to be a reflexive (albeit in a secondary sense, because in this case it lacks any informative value), symmetric, and transitive relation, whose cardinality is greater than or equal to one. According to [17] the equality between two generic expressions '*a*' and '*b*'—in relation to their primary meaningfulness (*śakyatā*, $\acute{\boldsymbol{S}}$) and individual manifestations with respect to a given generic property (*taddharmavattva*, td_t)—can be expressed in NL as: $(\text{‘}a\text{’} \neg\acute{\boldsymbol{S}} \llcorner td_t) \rightleftharpoons \boldsymbol{Q} \llcorner (\text{‘}b\text{’} \neg\acute{\boldsymbol{S}} \llcorner td_t)) \neg\mathbb{N} \llcorner (\geq 1_t)$; iff $(|\text{‘}a\text{’}| = b \in (|td_t| = \mathrm{TD}) \wedge (|\text{‘}b\text{’}| = a \in (|td_t| = \mathrm{TD}) \wedge (card(\mathrm{TD}) \geq 1)$. The above relation of equality between terms can be promptly interpreted as a case of identity (I) and non-difference ($\not{2}$) of their extension. If $\langle \text{‘}a\text{’}, \text{‘}b\text{’} \rangle \in Q$, then for AE also $\langle \text{‘}a\text{’}, \text{‘}b\text{’} \rangle \in I$ and $\langle \text{‘}a\text{’}, \text{‘}b\text{’} \rangle \in \not{2}$. If linguistically $\langle$'gold', '*Au*'$\rangle \in Q$, then 'gold' is extensionally non-different from '*Au*' ($\langle$'gold', '*Au*'$\rangle \in \not{2}$): every case of equality is, *arthataḥ*, also non-duality, but not the other way around ($\langle$gold, crown$\rangle \in \not{2}$, yet 'gold'$\neq$'crown' $\wedge$ |'gold'| $\neq$ |'crown'|). In summary: $Q \neq E$ (cf. §1 and fn. 56); $Q \subseteq \not{2}$, and consequently $Q \subseteq (\underset{.}{N} \subseteq V)$; lastly, I is Q *arthataḥ*, while Q is I *śabdataḥ* (cf. § 2–3 and formulas [22]–[26]).

Identity (*tādātmya*; I) appears as a reflexive (although somehow paradoxically, cf. *supra* fn. 28), symmetric, and transitive relation whose cardinality is strictly equal to one. Identity, as 'not the counterpositive of a difference resident in something of the same kind', can be expressed, for the generic primitive *a*, as (cf. [22]-[25]): $(a \rightleftharpoons \nexists^{-1} \llcorner (\not{I}^{-1} \llcorner a')) \neg\mathbb{N} \llcorner (1_t)$; iff $(a \notin (|\not{I}^{-1} \llcorner a'| = \overline{\mathrm{A}'})) \wedge (\mathrm{A}' = \{a'\})$. In brief: $I \neq E$ (cf. §1 and fn. 56); $I \subseteq \not{2}$, and thus $I \subseteq (\underset{.}{N} \subseteq V)$; again, I and Q are the *artha* and *śabda* sides of the same coin (cf. previous point).

Non-difference (*abheda*; $\not{2}$) has been shown to be a reflexive (although in a secondary sense), non-symmetric, and non-transitive relation whose cardinality is greater than or equal to one. Every instance of non-difference appears to be a case of co-reference and qualified cognition (*viśiṣṭa-jñāna*), but not the other way around. Indeed, the assertion *daṇḍī puruṣaḥ* qualifies a man by means of a staff, though it does not follow this man is non-different from his staff (cf. fn. 45). Thereby: $\not{2} \subseteq (\underset{.}{N} \subseteq V)$. It follows that SVN rules $\not{2}$, for $\not{2}[\mathrm{A}] \subseteq \mathrm{A}$. That is, non-difference is an instance of closure as well, because the set 'Non-different from what belongs to the generic set A' is A-closed under the relation $\not{2}$ (i.e. $\not{2}$: $\mathrm{A} \mapsto \not{2}[\mathrm{A}]$). On one hand, in cognitions connecting a pot and its colour (*nīlo ghaṭaḥ*, a case of $V^{(\underset{.}{N})}$) or a crown and its material (*hāṭakaṃ mukuṭam*, a case of ^{u}K; cf. end of §4), the relata in both cases are to be understood as non-different, yet in a different sense. The same clearly holds true for 'part and whole' relations (A, *avayavāvayavi-bhāva*). On the other hand, non-difference cannot even be reduced to identity; although they could appear as highly resembling each other, they do not collapse into other. In fact, their cardinality prevents such coalescence (card(I)=1 *vs.* card($\not{2}$)≥1), together with the fact that I is always symmetrical and transitive while $\not{2}$ never is (cf. interpretations under V, ^{u}K, or A), but also with the structural involvement of distinct properties (e.

g., being gold and being a crown) in $\cancel{2}$. To sum up, it could be stated that: $((A \cong {}^{u}K) \subseteq \cancel{2}) \subseteq (\underset{\cdot}{N} \subseteq V)$; $\cancel{2} \neq E$ (cf. *supra* and fn. 56); $Q \subseteq \cancel{2}$, $I \subseteq \cancel{2}$.[57]

In light of the above, let us take now a step forward. Non-difference between two generic properties a_t and b_t was expressed in §4 (cf. [31]–[33$_a$]) as: $(b\boldsymbol{.}\ \boldsymbol{\cancel{2}} \llcorner a)\neg\mathbb{N}\llcorner (\geq 1_t)$. Nevertheless, this definition can be further developed through [22]–[25] (i.e. the Gadādhara's counterfactual definition of identity), the application of SVN, and the plain reading of the literal meaning of *a-bheda* (i.e., 'non-difference'). Difference, as shown, is expressed as *paṭo ghaṭo na*: $g\boldsymbol{.}\ \boldsymbol{\cancel{I}}^{-1} \llcorner p$ ('A cloth is not a pot'; cf. [19]–[21]). However, non-difference clearly negates difference. Since 'The crown is gold', the assertion 'The crown is not gold' will be false: *mukuṭaṃ hāṭakaṃ nety na*, or $\nexists(h\boldsymbol{.}\ \boldsymbol{\cancel{I}}^{-1} \llcorner m)$ (recall here steps [19]-[23]). *Abheda* thus proves to be a peculiar relation which negates difference. Yet, it involves more than one property (e.g. the generic a_t and b_t) referring to the same potentially more than one locus (card ≥1). As has been said, *abheda* cannot collapse into mere identity, which involves, as we have seen, 'the same kind' (*svasajātīya*) and a cardinality equal to one. A counterpositive definition of non-difference might be more of the same: *samānādhikaraṇa-dharmāntara-avacchinna-bheda-apratiyogitvam abhedaḥ*, 'Not being the counterpositive of a difference occurring in another co-occurring property'.[58]

[34] $(((b \downharpoonleft\!\upharpoonright \boldsymbol{\underset{\cdot}{N}} \llcorner a_t) \rightleftharpoons \nexists^{-1} \llcorner (\boldsymbol{\cancel{I}}^{-1} \llcorner a_t))\neg\ \mathbb{N} \llcorner (\geq 1_t)$
yad atyantābhāvīyapratiyogitā-avacchedakāvacchinna-paryāptitvaṃ tad ekatvādi-nirūpitam; tatra yaiva atyantābhāvīyapratiyogitā sāmānādhikaraṇyatā-avacchedakāvacchinnā sā anyonyābhāvīyapratiyogitā-nirūpitā, eṣaiva anyonyābhāvīyapratiyogitā etaddharmavattva-nirūpitā, tadviparyayeṇa ca; yathā yaiva taddharma-niṣṭha-sāmānādhikaraṇyatā sā etaddharmavattva-nirūpitā, tathā yaitaddharma-niṣṭha-sāmānādhikaraṇyatā sā taddharmavattva-nirūpitā; 'The relational abstract absolute absentee-hood ($\nexists^{-1}$), conditioned by the

[57] Relations *A* and ${}^{u}K$ share some occurrences (A ≅ ${}^{u}K$), although not all; because, e.g., if *mahābhūtas* are simultaneously part (*avayava*) and material cause (*upādāna-kāraṇa*) of a living being, yet for Nayāyikas halves are not the material cause of a pot—which instead, in *satkāryavāda* sense, is the clay—but the *samavāyi-kāraṇa* ('causal substrate' or 'substantial cause'; Matilal 1975, p. 44). Moreover, although they are expressible in oblique cases also (so not every instance of *A* and ${}^{u}K$ can be said, syntactically homogeneous from a linguistic point of view), they, nevertheless, appear as radical extensional instances of coreference; e.g. *aśvasya aṅgāni* (the limbs of a horse), or *tilāt tailam* (oil *from* sesame [seeds]). Cf. fn. 45 for the opposite case of *daṇḍī puruṣaḥ*.

[58] This formulation might sound somehow paradoxical at first; nonetheless, regarding this peculiar notion of a kind of non-reflexive identity (*abheda*), which is at once not reducible to reflexive identity *stricto sensu*, as well as 'compatible with difference' (*bhedasahiṣṇu*)—or rather with the coreference of different properties—see among the others Mohanty (2000, pp. 55–56): "[for an *advaitin*], the 'blue' and the 'lotus'—in 'the blue lotus'—are fundamentally identical. The quality is of the nature of the substance. […] It would seem, then, that for Śaṃkara there is only one category, namely substance, and one relation, namely *tādātmya* (being its essence), which is a form of identity that 'tolerates' differences (*bhedasahiṣṇu*)"; and Chakrabarti (2001, p. 219): "In the Advaita (and the Bhāṭṭa Mīmāṃsā) view […] the relation between a substance [*guṇin*] and its qualia [*guṇa*] is that of identity in and through difference (*bheda-sahiṣṇu-abheda*). That is, a substance and its qualia are neither utterly identical nor utterly different".

mutual absentee-hood ($\underset{\cdot}{\not{I}}^{-1}$), in turn conditioned by this [generic] property (a_t), is limited by coreferenceness ($\underset{\cdot}{N}$), and vice versa; moreover, just like the coreferenceness, conditioned by this [generic] property (a_t), is limited by at least one specimen of that [generic] property ($b \in |b_t|$), so the inverse coreferenceness ($\underset{\cdot}{N}^{-1}$), conditioned by that [generic] property (b_t), is limited by at least one *specimen* of this [generic] property ($a \in |a_t|$); for a cardinality greater than or equal to one'; iff $(|a_t| \neq |b_t|) \wedge (|a_t| \cap |b_t| \neq \varnothing)$ (i.e., a_t and b_t are not the same property but the intersection of their domains is not empty); in s. n. $(\forall x, \forall y \mid \mathrm{A}x, \mathrm{B}y)\ (\langle x, y\rangle \in \not{2}\ ) \leftrightarrow ((\mathrm{A} \neq \mathrm{B}) \wedge (\langle x, y\rangle \in \underset{\cdot}{N}))$.[59]

In conclusion, non-difference seems to peculiarly reverse the claims of both Leibniz's law (LL) and the Principle of Identity of Indiscernibles (PII). Apparently, *abheda* does not claim (as LL does) that the same referent must have the same properties it already has, which would coalesce into mere identity—which, although true, might even sound like a linguistic short circuit, as Wittgenstein has pointed out. Nor does it claim (as PII does) that what possesses the same properties is the very same referent, since different properties are at stake here. What *abheda* appears to claim—at first glance generating another linguistic short circuit just as identity might—is that distinct properties referring to the same *locus* cannot be said to be fully different. This is a crown, surely; but this crown is nothing but gold. What cognition has—etymologically—abstracted from the referent must indeed be *located* there again. The application of this analysis—prompted in the first instance by VM—to the issues of language, knowledge and knowledgeability, causation, and first and foremost to the relation between manifestation (*jagat*) and *brahman*, requires further investigation. Such investigation will be attempted in the following part of this article.

Compliance with ethical standards

Conflict of interest The author states that there is no conflict of interest.

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[59] Regarding the yathā-tathā operator ('just like-so'; ⇃↾), in order to express in NL the non-empty intersection between two sets, cf. Anrò (forthcoming: § 4.3).

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