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"That Wasn't the Math I Wanted to do!"—Students' Beliefs During the Transition from School to University Mathematics

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Abstract
It is a well-known fact that many students struggle during the transition from school to university mathematics. The aim of our quantitative study was to clarify the role of students' beliefs concerning the nature of mathematics for a successful transition. We distinguish between static beliefs (mathematics as a finished system of rules, facts and formula) and dynamic beliefs (mathematics as a dynamic discipline with applications in everyday life). In particular, we examined whether first year students' beliefs are suitable to predict students' exam achievement and their satisfaction (as criteria for a successful transition) and how students' beliefs develop during the transition. Therefore, we used questionnaires at the beginning and in the middle of the first term at university. Our results indicate that dynamic beliefs decrease during transition, whereas static beliefs remain rather stable. This seems problematic since dynamic beliefs turned out to negatively predict students' dropout intention, while static beliefs are a negative predictor of students' achievement in real analysis. Furthermore, the beliefs assessed in the middle of the term had a stronger predictive power than those at the beginning of the term. Based on these results, we discuss implications for the teaching of mathematics during the transition.

Keywords Transition from school to university mathematics · Beliefs · Dropout

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Introduction

The transition from school to university is a challenging process for many students, likewise in mathematics. This is illustrated by high dropout and subject change rates. In Germany—where our study took place—up to 80% of students quit their studies of mathematics without graduation, 35% of them during their first year at university (Dieter & Törner, 2012) while attending courses in linear algebra and real analysis. Chen (2013) reported similar numbers for the USA. Furthermore, many students fail their first exams or do not even attend them (Geisler & Rolka, 2018). These findings underline the necessity to understand students' problems during the transition phase. In this contribution, we focus on the development and influence of students' beliefs concerning the nature of mathematics during the transition. In particular, we are interested in the relation between students' beliefs and different criteria of success (achievement in real analysis and dropout intention) as well as in the development of students' beliefs over time. In order to better understand the role that students' beliefs play in this transition process, we first give an overview about major differences between mathematics at school and university and second discuss theories of person-environment fit (Lubinski & Benbow, 2000; Nagy, 2006; Swanson & Fouad, 1999), which have already been used in the context of transition from school to university mathematics (e.g. Rach & Heinze, 2017).

The Transition from School to University

Differences Between Mathematics at School and University and Related Students' Difficulties

The starting point for most research concerning the transition from school to university mathematics is students' difficulties during this process. Researchers have investigated those difficulties through various lenses (Gueudet, 2008). Most of them refer to the differences between mathematics at school and at university. These differences involve firstly institutional changes within the new learning environment at university and secondly changes of the learning content, meaning changes in the nature of mathematics and the related tasks as well as required skills. The institutional changes involve a new didactical contract with more autonomy and self-regulated learning for the students (Gueudet, 2008). With regard to the nature of mathematics, two major changes can be identified: the way in which new concepts are presented and the kind of tasks that have to be performed.

In this context, Tall (2008) mentioned three mathematical worlds, which build on one another. The *conceptual-embodied* world is based on experiences with real-world objects, whereas the *proceptual-symbolic* world additionally involves thinkable concepts (e.g. numbers). The *axiomatic-formal* world is based on definitions and proofs. Following Tall (2008), school mathematics is usually located within the *conceptual-embodied* and the *proceptual-symbolic* worlds, while university mathematics belongs to the *axiomatic-formal* world. Accordingly, in school, new concepts are based on intuitive understandings and experiences with examples as well as real objects, whereas at university, concepts[1] are introduced through formal and precise definitions.

[1] These concepts may be new ones or concepts that students have already met in school mathematics (like integrals) but that are defined in university mathematics in a more formal and rigorous way.

Properties of mathematical objects are no longer learned via experimentation with examples but deductively derived from definitions (Tall, 1992).

Since formal definitions do not play an important role in school mathematics, many students might not see the importance of the definitions they learn during lectures (Engelbrecht, 2010). According to Hefendehl-Hebeker (2017), formal definitions are usually formulated very technically and therefore obscure their original meaning and phenomenological roots. Students have to deal with a new scientific language characterised by "a high density of information, which requires symbol sense and a specific reading ability" (Hefendehl-Hebeker, 2017, p. 205). In particular, students sometimes struggle with the translations between the mathematical language and everyday language (Engelbrecht, 2010). Moreover, many lecturers do not emphasise the main ideas in their lectures (Lai & Weber, 2014) or do so only verbally, while students' notes contain mainly what lecturers wrote on the board (Weber, Fukawa-Connelly & Meija-Ramos, 2017).

Typical tasks in school mathematics involve the application of mathematics to solve real-world problems with a strong focus on modelling (e.g. Organization for Economic Cooperation and Development [OECD], 2016), which is also reflected in German national standards (KMK, 2012). However, to solve these kinds of tasks, schematic calculations are frequently used, which means that "most students' perception of being successful in mathematics in school does not involve much inquiry but mostly just the application of different methods" (Engelbrecht, 2010, p. 143). In contrast, most tasks at university involve proofs and cannot be performed using those schematic calculations. By analysing German textbooks for upper secondary students and first semester students, Vollstedt, Heinze, Gojdka and Rach (2014) found that "proofs are underemphasized at school" (p. 47), whereas at university, proofs are very important. Both courses—real analysis and linear algebra—that German freshmen usually attend during their first year at university deal mainly with proof and formal definitions (Halverscheid & Pustelnik, 2013). Tasks with an emphasis on proofs are challenging for many students. Some students find it hard to produce deductive arguments and some even prefer empirical ones (Stylianou, Blanton & Rotou, 2015). With regard to real analysis, Liebendörfer and Ostsieker (2014) observed that many students do not check the conditions of theorems before using them in proofs.

The aforementioned differences between school and university mathematics and the related mathematical worlds can also be found with regard to the subdomain of calculus and real analysis. Whereas calculus at school is still often about schematic calculating, working with graphs and manipulating formulae, at university, real analysis is much more abstract and involves mainly tasks related to proving (Tall, 2008). These differences are also reflected in the contents and their presentation in calculus/real analysis textbooks for school and university students (Witzke, 2015).

Person-Environment Fit During the Transition

Following theories of person-environment fit (Lubinski & Benbow, 2000; Nagy, 2006; Rach & Heinze, 2017; Swanson & Fouad, 1999), in the process of transition, students leave an environment (school) whose characteristics fit their own characteristics (the fit was sufficient in the sense that students successfully graduated from school). According to these theories, students' success at university is influenced by the fit between their characteristics (e.g. cognitive prerequisites, learning behaviour, interests and attitudes) and this new environment, which is very different from school—as illustrated above. In particular, the fit between students' cognitive prerequisites, learning behaviour and the learning environment (especially the requirements of university) determines the students' achievement. The achievement, in turn, determines whether the students meet the university's demands and therefore affects the so-called *satisfactoriness*. Students' *satisfaction* is the result of sufficient fit between their interests as well as their attitudes and the learning environment (Lubinski & Benbow, 2000; Swanson & Fouad, 1999). Whereas *satisfactoriness* reflects the viewpoint of the university (e.g. is he or she a satisfying student, does he or she meet the university's demands), *satisfaction* reflects the experiences of the student (e.g. is he or she satisfied with his or her studies). Both dimensions, *satisfactoriness* and *satisfaction*, are important for a successful transition. In the case of low *satisfactoriness*, usually indicated by low achievement, the students may have to quit their studies due to university restrictions. This is the case, for example, if a student has finally failed a necessary exam. Low *satisfaction* can lead to a voluntary decision to drop out. We are aware of the fact that *satisfaction* and *satisfactoriness* are not independent in the sense that low *satisfaction* might influence students' learning and therefore their success in exams as a criterion of *satisfactoriness*. Furthermore, low achievement can lead to less *satisfaction* and might lead students to voluntarily drop out.

According to Haak (2017), incongruences between personal characteristics and the environment lead to a personal crisis. For Clark and Lovric (2008), this crisis is even an integral part of the transition from school to university mathematics. Swanson and Fouad (1999) pointed out that this crisis leads to adjustment behaviour, but they did not describe this behaviour in detail. Haak (2017) proposed two possibilities to overcome the crisis: students can either adapt the environment's characteristics or their personal characteristics (e.g. attitudes, learning behaviour). The first means to leave the learning environment by changing the subject or dropping out (Haak, 2017), since students have only little influence on the characteristics of mathematics courses at university. Regarding a possible adaption of personal characteristics, the development of these characteristics *during the transition* can be even more important for students' success than their characteristics at the beginning of their studies. Indeed, with regard to engineering students, Bengmark, Thunberg and Winberg (2017) found that beliefs measured close to exams are better predictors of success than those measured at the beginning of one's studies. Moreover, Schiefele, Streblow and Brinkmann (2007) reported that students who drop out differ from those who continue their studies, more concerning interest and learning behaviour at the time of the dropout than at the beginning of their studies.

Beliefs

The notion of mathematical beliefs has been of growing interest in mathematics education during the last decades. However, beliefs are not easy to conceptualise which leads to what Pajares (1992) called "a messy construct". We refer to beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). Like many authors, we locate beliefs within the affective domain (Hannula, 2012; McLeod, 1992; Philipp, 2007). This is reasonable since beliefs are crucial for the development of attitudes and emotions towards mathematics (McLeod, 1992). However, beliefs are more cognitive in nature than attitudes and emotions (McLeod, 1992) and can be seen as subjective knowledge (Pehkonen & Pietilä, 2003). The relevance of studying beliefs lies not only in their affective effects but also in their influence on learning behaviour. Schoenfeld (1985) stated that one's beliefs determine how one works on mathematical tasks and how much effort is put into them. Furthermore, beliefs influence which information are learned since they function as a regulation system (Pehkonen & Törner, 1996). Thus, students' beliefs seem to be related to their achievement. Stylianou et al. (2015) found that high-performing students hold more elaborated beliefs about proofs (e.g. "Proofs serve the purpose of not only validating mathematical conjectures, but also of communicating and explaining new ideas" (Stylianou et al., 2015, p. 110)) than low-performing students.

In this contribution, we will focus on students' beliefs concerning the nature of mathematics. Following Grigutsch and Törner (1998), we distinguish between rather static beliefs and dynamic beliefs. The static beliefs are characterised by the formal and exact structure of mathematics. Mathematics is seen as a coherent system of rules, facts and formulas. Sometimes, these rules and formulas are even experienced as unconnected and mathematics is reduced to a toolbox, while doing mathematics primarily means calculating and using such rules and formulas (e.g. Grigutsch & Törner, 1998; Liljedahl, Rolka & Rösken, 2007). The dynamic beliefs focus more on the development of new mathematical knowledge. This development involves processes which include experimental and inductive activities. Here, mathematics is seen as a creative field of research with applications in other sciences and everyday life. These dynamic beliefs also stress relations between different mathematical objects and statements (Grigutsch & Törner, 1998; Liljedahl et al., 2007).

Beliefs During the Transition from School to University Mathematics

Since the nature of mathematics changes during the transition from school to university, it is not surprising that many students experience a big gap between school and university mathematics (Di Martino & Gregorio, 2019; Hernandez-Martinez, 2016). The experience of this gap can be the result of unfulfilled expectations (Hirst, Meacock & Ralha, 2004) and incongruences between students' established beliefs and the reality they face at university (Andrà, Magnano & Morselli, 2011; Witzke, 2015). Daskalogianni and Simpson (2001) introduced the notion of beliefs overhang, describing students' beliefs that have evolved for a long time during school but are not fitting to university mathematics. In this case, the beliefs are "inappropriate for students' adjustment to advanced mathematics, and result in a problematic move from school to university" (Daskalogianni & Simpson, 2001, p. 99). This leads to the question, which beliefs actually fit to university mathematics and therefore are beneficial during the transition.

From a theoretical point of view, both static and dynamic beliefs can fit to the mathematics students' encounter at German universities. On the one hand, students usually have to work on weekly so-called "task sheets" to gain extra points or the admission for the exams. These sheets mainly include tasks with a focus on proving. Many of these proofs cannot be performed using well-known schemes and algorithms. New ideas (at least from the viewpoint of the students) and creative thinking are necessary. One can argue that students have to undergo a process of proving, which involves failure and dead ends, to solve their tasks and that dynamic beliefs should be beneficial in this case. On the other hand, during lectures, mathematics is presented in a rather static way and appears as a coherent system. Proofs are usually presented in their final version, while the process that lies behind often remains invisible. This is exactly what Hersh (1991) referred to in his distinction between the *front* and the *back* of mathematics where "the 'front' of mathematics is mathematics in 'finished' form, as it is presented to the public in classrooms, textbooks, and journals. The 'back' would be mathematics as it appears among working mathematicians, in informal settings, told to one another in an office behind closed doors" (p. 128).

In line with these theoretical considerations, university teachers hold static beliefs as well as dynamic beliefs (Grigutsch & Törner, 1998)—which could be another indicator that both kinds of beliefs fit to university mathematics. Nevertheless, some university teachers clearly articulate that they want their students to hold dynamic rather than static beliefs (Kuklinski et al., 2018), indicating that they consider dynamic beliefs to be advantageous. Furthermore, many supporting programmes and bridging courses aim to increase students' dynamic beliefs (Biehler et al., 2018).

Indeed, dynamic beliefs seem to have a positive influence during the transition, since they positively correlate with interest in mathematics during the first year at university (Kolter, Liebendörfer & Schukajlow, 2016; Liebendörfer & Schukajlow, 2017). Qualitative approaches have raised the hypothesis that static beliefs seem to result in less interest in university mathematics (e.g. Daskalogianni & Simpson, 2002; Liebendörfer & Hochmuth, 2013) and can lead to typical mistakes like not checking the conditions of a theorem before using it for a proof (Liebendörfer & Ostsieker, 2014).

Dynamic beliefs concerning the nature of mathematics often go hand in hand with constructivist beliefs concerning the learning of mathematics, whereas static beliefs correlate with transmissive beliefs of learning (Manderfeld & Siller, 2018), which are considered to be less useful for university mathematics. In addition, students with a rather fragmented (and therefore static) view on mathematics tend to take a surface approach to mathematics learning, which mainly involves memorising and reproduction (Liston & O'Dnoghue, 2009). These students are less successful in exams at university than students with a deep-learning approach (Crawford, Gordon, Nicholas & Prosser, 1994). Geisler and Rolka (2018) found significant differences concerning dynamic beliefs between students who succeeded in their first university exam, those who failed and students who did not even attend their exam. The latter showed the lowest acceptance of dynamic beliefs, while the successful students indicated the highest consent to dynamic beliefs. A similar relation could not be found for static beliefs. According to Baars and Arnold (2014), not attending exams is a useful indicator for dropout.

On the basis of these findings, dynamic beliefs seem to be more beneficial for a successful transition than static beliefs. This seems problematic because most upper secondary students hold static rather than dynamic beliefs concerning the nature of mathematics (Baumert et al., 2000). Geisler and Rolka (2018) found that even mathematics university students as a whole agree more with static beliefs than with dynamic ones. In contrast, Törner and Grigutsch (1994) identified predominantly dynamic beliefs among mathematics freshmen.

Changing Beliefs

Since beliefs are considered to evolve over a long time and remain rather stable (McLeod, 1992; Philipp, 2007), there is a debate between researchers whether and how beliefs can be changed. Assuming that beliefs which do not fit to university mathematics (in the sense of person-environment fit) lead to a personal crisis as well as adjustment behaviour and that this crisis can be overcome by either dropping out or adjusting personal characteristics (Haak, 2017), the question whether beliefs can be changed becomes crucial. Recent studies showed that beliefs can be changed and that the learning environment can play a substantial role for this change. Liljedahl et al. (2007) have used a specially designed university course to shift pre-service elementary school teachers' beliefs from static towards more dynamic ones. Furthermore, changes in attitudes and beliefs of secondary school students have been observed, when entering specially designed learning environments (Liljedahl, 2018). We argue that the changes in the nature of mathematics and the learning environment during the transition may induce change in students' beliefs as well.

Indeed, Kolter et al. (2016) showed that dynamic beliefs held by elementary pre-service teachers decreased during the first term at university while their static beliefs increased slightly. In a mathematics course with secondary pre-service teachers, Kuklinski et al. (2018) found no changes in students' dynamic beliefs during the first term at university and only a weak decrease of their consent to static beliefs, but here, increasing students' dynamic beliefs was a clearly stated goal of the university teacher. However, it remains uncertain whether these results can be transferred to a traditional mathematics course with pre-service upper secondary teachers and mathematics majors.

Research Questions and Hypothesis

Our guiding research interest lies in the question which beliefs "fit" to the mathematics and learning environment that students encounter when entering university and which beliefs are therefore beneficial during the transition. In particular, we are looking at:

i. the connection between students' beliefs and different criteria of success and
ii. the change of students' beliefs during the transition.

This leads to the following sets of research questions:

Connection Between Beliefs and Success

> In which way can beliefs concerning the nature of mathematics predict students' dropout-intention and their achievement in the exam at the end of the semester? Are students' beliefs in the middle of the term better predictors of the dropout-intention and the achievement than the beliefs at the beginning of the term?

In line with theories of person-environment fit, we expect students' beliefs to predict both, dropout intention (as a criterion for low *satisfaction*) and the achievement in the exam (as a criterion for *satisfactoriness*). Since dynamic beliefs appear to be more beneficial during the transition (e.g. Crawford et al., 1994; Geisler & Rolka, 2018), we believe that static beliefs will be a negative predictor of the achievement in the exam (H1) while dynamic beliefs will positively predict the exam outcome (H2). Regarding dropout intention, we assume the opposite relations: static beliefs will be a positive predictor of dropout intention (H3) while dynamic beliefs will be negatively related to dropout intention (H4).

As described before, an insufficient fit between students' personal characteristics—in our case, their beliefs—and the learning environment will lead to adjustment behaviour (Swanson & Fouad, 1999). Students can react by changing the environment (which means to change subject or drop out from university) or they can adapt their own characteristics (in our case, their beliefs) (Haak, 2017). We argue that the outcome of this adjustment process is more relevant for students' achievement and dropout intention than their beliefs at the beginning of their studies. This leads to the hypothesis that students' beliefs in the middle of the term will be stronger predictors of the achievement in the exam and the dropout intention than their beliefs at the beginning of the term (H5). This hypothesis is supported by recent studies (e.g. Bengmark et al., 2017; Schiefele et al., 2007).

Change of Beliefs During the Transition

Recent studies have shown that students' beliefs can change within the first year at university (Kolter et al., 2016). However, we argue that in order to better understand students' problems during the transition, it is of great interest to find out whether there are differences in this development between rather successful and unsuccessful students. This leads to the questions:

> How do students' beliefs concerning the nature of mathematics change during the first weeks at university? Are there differences in this change between students who succeed in their exam at the end of the semester and those who fail or between students with and without dropout-intention?

Due to the fact that recent studies provided inconsistent results (e.g. Kuklinski et al., 2018; Kolter et al., 2016), we have no special hypothesis concerning the overall development of students' beliefs during the first term, but since beliefs are rather stable, we expect only small or medium effects. To the best of our knowledge, there is currently no quantitative study examining differences in the development of beliefs between different groups of university students during the transition from school to university. Therefore, the research questions posed under ii) are rather explorative. However, we assume that beliefs of successful students will undergo changes in a more beneficial way (e.g. increasing dynamic beliefs, decreasing static beliefs) than beliefs of unsuccessful students.

Methods

Design and Instruments

We used paper and pencil questionnaires during the real analysis lecture in the first week of the winter term (T1) and 8 weeks later (T2) to access students' beliefs concerning the nature of mathematics. Demographic data (gender, age, school leaving marks) was also included at T1. Since students can rarely be aware of their intention to drop out during the first week at university, their dropout intention was only captured by the second questionnaire (T2). The timing was carefully chosen: on the one hand, we did not want to measure dropout intention too early since we wanted the students to give a realistic assessment. On the other hand, our experiences over the past years show that many students decide to drop out during the winter break around Christmas. That is why we decided to hand out the second questionnaire in mid-December, which falls within the middle of the term. The instruments used in the questionnaires can be found in Table 1.

Sample

Our sample consists of freshmen at a large public university in Germany. The students are enrolled in the bachelor (pure mathematics) or teacher education programme for upper secondary school and attended the course real analysis. A total of 226 students participated in the first questionnaire in the first week of the term. One hundred thirty-one students filled out the second questionnaire in the middle of the term. We could match $N = 104$ students (54.7% male, M(age) = 19.5, SD = 2.2) who participated in both questionnaires. We only used data of those for our analysis. Sixty-two of these students (60%) also attended the real analysis exam at the end of the term.

Preliminary Analysis

Only the data from students who participated in both questionnaires were used to assess the reliabilities and factorial structures of the used instruments. All instruments have at least sufficient reliabilities, except the static beliefs scale. We eliminated the item "Fundamental to

Table 1 Instruments with number of items, reliability (cronbach's α) and sample items

Construct	Source	# items	α(T1)/α(T2)	Sample item
Beliefs dynamic	Laschke & Blömeke, 2013	6	0.68/0.77	Mathematics involves creativity and new ideas.
Beliefs static	Laschke & Blömeke, 2013	6 (5)	0.55 (0.66)/0.69 (0.72)	Mathematics means learning, remembering and applying.
Dropout intention	Own formulation, following Blömeke, 2009	1	-	Recently, I really thought about quitting my studies of mathematics.

All items had to be answered on a 5-point Likert scale, where 1 indicated *totally disagree* and 5 *totally agree* mathematics is its logical rigor and preciseness" which increased reliability from 0.55 to 0.66 (T1) or, respectively from 0.69 to 0.72 (T2). An explorative factor analysis (varimax rotation) with the number of factors set to two confirmed the factorial structure of the beliefs scale. All items have factor loadings > 0.5 with cross-loadings < 0.3 except the previously mentioned item "Fundamental to mathematics is its logical rigor and preciseness", which has no high loadings on either factor (0.36 and − 0.08).

Since we only used the data from those students who participated in both questionnaires (T1 and T2), we wanted to know whether those students differ from the group which only participated in the first questionnaire. We therefore used a multivariate analysis of variance (MANOVA) to compare the students' school leaving marks (German Abiturnote) and their beliefs at the beginning of the term (T1). The results can be found in Table 2.

School leaving marks between 1 (*very good*) and 4 (*sufficient*); answers to beliefs scales between 1 (*totally disagree*) and 5 (*totally agree*)

The MANOVA revealed significant differences between students who participated only in the first questionnaire and those who filled out both questionnaires ($F(3, 209) = 4.78$, $p < 0.01$, $\eta^2 = 0.06$). In particular, students who participated in both questionnaires have better school leaving marks ($F(1, 211) = 3.61$, $p < 0.01$, $\eta^2 = 0.05$) and show a higher acceptance of dynamic beliefs ($F(1, 211) = 1.98$, $p < 0.01$, $\eta^2 = 0.03$) than those who participated only in the first one. We therefore assume that students who were included in our analysis have to be seen as positively selected. However, according to Cohen (1988), these effects are rather small.

Furthermore, we checked whether students enrolled in a teacher education programme and bachelor students differ concerning their beliefs and school examination marks. We conducted a MANOVA including all students who participated in both questionnaires. The results can be found in Table 3. Students enrolled in a teacher education programme reported slightly higher acceptance of static beliefs and lower acceptance of dynamic beliefs than bachelor students at the beginning of their studies. However, only the differences concerning static beliefs are significant ($F(1, 96) = 6.02$, $p < 0.05$, $\eta^2 = 0.06$). In the middle of the term, no relevant differences between both groups of students can be identified.

School leaving marks between 1 (*very good*) and 4 (*sufficient*); answers to beliefs scales between 1 (*totally disagree*) and 5 (*totally agree*)

Table 2 Means, standard deviations and results of the MANOVA (comparison of students who participated only in T1 and those who participated in both questionnaires)

	Only T1 ($N = 113$)		T1 and T2 ($N = 100$)		η^2
	M	*SD*	*M*	*SD*	
School leaving marks	2.5	0.59	2.24	0.62	0.05**
Beliefs static T1	3.63	0.56	3.54	0.6	0.01
Beliefs dynamic T1	3.63	0.65	3.82	0.54	0.03*

*$p < 0.05$, **$p < 0.01$

Table 3 Means, standard deviations and results of the MANOVA (comparison of students enrolled in a teacher education programme and bachelor students)

	Teacher education ($N = 73$)		Bachelor ($N = 27$)		η^2
	M	*SD*	*M*	*SD*	
School leaving marks	2.23	0.61	2.22	0.65	0.00
Beliefs static T1	3.61	0.61	3.28	0.53	0.06*
Beliefs dynamic T1	3.77	0.57	4.01	0.38	0.04
Beliefs static T2	3.65	0.59	3.58	0.62	0.00
Beliefs dynamic T2	3.38	0.67	3.56	0.55	0.02

*$p < 0.05$

Results

Connection Between Beliefs and Success

We used hierarchical linear regressions (method inclusion) to assess the connection between students' beliefs and their achievement in the exam as well as their dropout intention. Model 1 only consists of students' school leaving marks. Model 2 consists of the beliefs at T1, while the school leaving marks are still controlled. In model 3, the beliefs at T2 were added. Since we expected significant correlations between students' beliefs at the beginning and the middle of the term, we screened for multicollinearity of the independent variables included in the models. For all performed regressions, multicollinearity was no problem (VIF > 0.4; tolerance < 2.2).

Table 4 summarises the results of the linear regression to predict students' achievement in the analysis exam (method inclusion). All four models significantly explain the achievement ($p < 0.001$). Model 1 shows that students' school leaving marks can explain 26% of the variance in the exam achievement ($R^2 = 0.26; \beta = -0.51; p < 0.001$)—they remain a significant predictor in all four models. The inclusion of the beliefs at T1 (model

Table 4 Results (standardised beta-coefficients) of linear regressions with dependent variable exam achievement

Independent variable	Model 1***	Model 2***	Model 3***
School leaving marks	− 0.51***	− 0.46***	− 0.42***
Beliefs static T1		− 0.24*	0.11
Beliefs dynamic T1		0.15	0.2
Beliefs static T2			− 0.51**
Beliefs dynamic T2			− 0.05
R^2	0.26	0.34	0.47
ΔR^2	-	0.08*	0.13**

$N = 62$, method inclusion, *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$

2) increases the explained variance significantly ($\Delta R^2 = 0.08; p < 0.05$), indicating that the static beliefs are a negative predictor of the exam outcome ($\beta = -0.24; p < 0.05$). In model 3, the beliefs at T2 are added ($\Delta R^2 = 0.13; p < 0.01$). The static beliefs at T2 turn out to be a negative predictor ($\beta = -0.51; p < 0.01$), while the static beliefs at T1 have no predictive power anymore. Model 3 can explain 47% of the variance in the exam outcome. However, dynamic beliefs do not significantly predict the exam achievement. Thus, (H1) can be confirmed and (H2) has to be refuted.

The results of the hierarchical linear regression (method: inclusion) to predict students' dropout intention can be found in Table 5. The first two models, consisting of the school leaving marks (model 1) and additionally the beliefs assessed at T1 (model 2), cannot significantly explain students' dropout intention ($R^2 = 0.02$ respectively 0.06). In line with this global analysis of the two models, neither the school leaving marks nor the beliefs at T1 are significant predictors. Adding students' beliefs in the middle of the term increases the explained variance significantly to 24% ($\Delta R^2 = 0.18$; $p < 0.001$) (H5) and reveals that the dynamic beliefs are a negative predictor of dropout intention ($\beta = -0.53$; $p < 0.001$), confirming H4. We find no significant effect of the static beliefs, which refutes H3.

Change of Beliefs During the Transition

Table 6 shows means and standard deviations of students' beliefs at both measurement points T1 and T2:

We used independent t tests to compare all students' beliefs at the beginning of the winter term and in the middle of the term. The t tests reveal a significant decrease of dynamic beliefs ($t = 7.8$, $p < 0.001$, $d = 0.77$), while the static beliefs remain nearly constant ($t = -1.83$, $p = 0.07$). To enable more differentiated insights into the development of the beliefs, we distinguished between students who succeeded in their real analysis exam (more than 50 out of 100 points) and those who failed. Furthermore, we distinguished between students without dropout intention ($M < 3$) and with dropout intention ($M \geq 3$).

Table 5 Results (standardised beta-coefficients) of linear regressions with dependent variable dropout intention

Independent variable	Model 1	Model 2	Model 3***
School leaving marks	0.15	0.12	0.04
Beliefs static T1		0.11	− 0.01
Beliefs dynamic T1		− 0.14	0.15
Beliefs static T2			0.09
Beliefs dynamic T2			− 0.53***
R^2	0.02	0.06	0.24
ΔR^2	-	0.04	0.18***

$N = 104$, method inclusion, ***$p < 0.001$

Table 6 Means and standard deviations of students' beliefs assessed at T1 and T2

Beliefs		All students participating in the study		Students without dropout intention		Students with dropout intention		Students who succeeded in the exam		Students who failed in the exam	
		$N = 104$		$N = 74$		$N = 30$		$N = 28$		$N = 34$	
		M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
T1	Static	3.53	0.59	3.45	0.6	3.71	0.54	3.23	0.6	3.57	0.6
	Dynamic	3.84	0.54	3.9	0.53	3.68	0.54	3.97	0.42	3.8	0.55
T2	Static	3.63	0.6	3.54	0.6	3.83	0.56	3.28	0.58	3.85	0.54
	Dynamic	3.43	0.65	3.63	0.58	2.93	0.54	3.63	0.52	3.39	0.43

Answers between 1 (*totally disagree*) and 5 (*totally agree*)

ANOVAs with repeated measures were used to check whether the development of beliefs differs between students who succeeded in the real analysis exam and those who failed. Concerning the static beliefs (Diagram 1(left side)), the ANOVA reveals a significant mean effect of the time ($F(1, 60) = 5.87$; $p < 0.05$; $\eta^2 = 0.09$) and the exam achievement ($F(1, 60) = 10.08$; $p < 0.01$; $\eta^2 = 0.14$) as well as a significant interaction *time*exam achievement* ($F(1, 60) = 5.39$; $p < 0.05$; $\eta^2 = 0.08$). The acceptance of static beliefs increases for students who failed in their exam, while students who succeeded show no change.

With regard to dynamic beliefs, we find no differences in the development between students who were successful in the exam and those who failed (Diagram 1(right side)). Both groups of students report decreasing consent to dynamic beliefs. Consequently, the ANOVA only shows a significant main effect of the time ($F(1, 60) = 42.55$; $p < 0.001$; $\eta^2 = 0.42$) but neither a significant mean effect of the exam achievement ($F(1, 60) = 2.7$; $p > 0.1$; $\eta^2 = 0.04$) nor a significant interaction *time*exam achievement* ($F(1, 60) = 0.4$; $p > 0.5$; $\eta^2 = 0.01$).

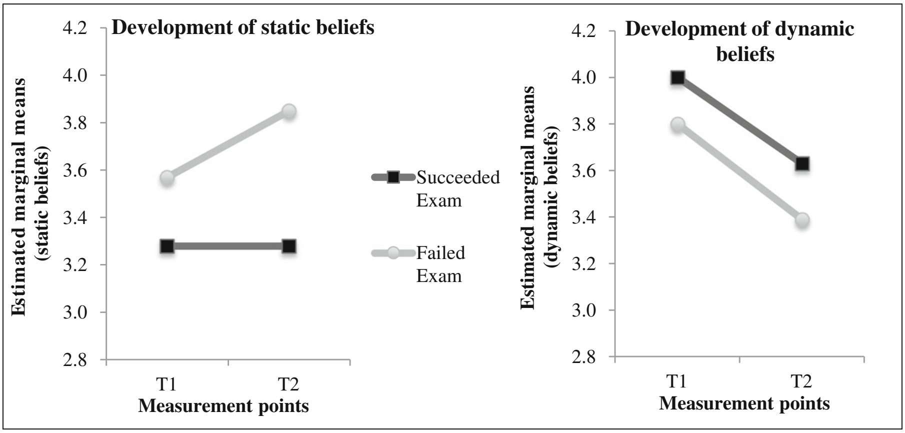

Diagram 1 Comparison of students who succeed in the exam and those who fail. Left, development of static beliefs. Right, development of dynamic beliefs. Answers between 1 (*totally disagree*) and 5 (*totally agree*)

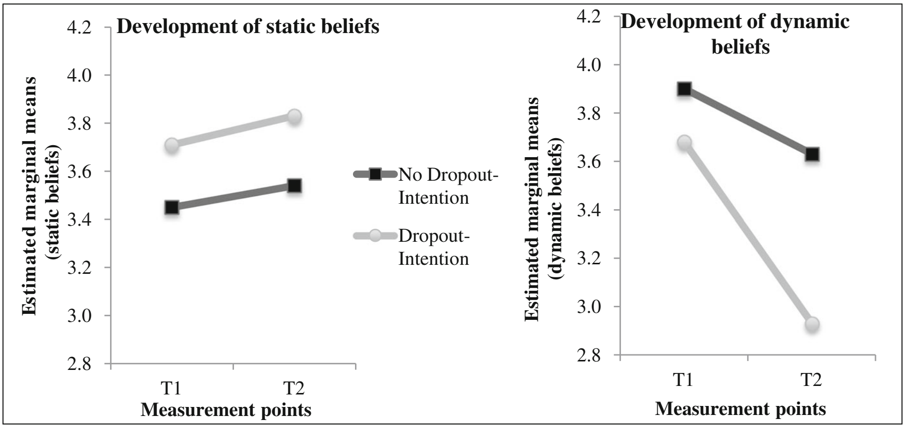

Diagram 2 Comparison of students with dropout intention and those without dropout intention. Left, development of static beliefs. Right, development of dynamic beliefs. Answers between 1 (*totally disagree*) and 5 (*totally agree*)

We compared the development of beliefs between students with and without dropout intention by using ANOVAs. For static beliefs (Diagram 2(left side)), we see a significant main effect of the dropout intention ($F(1, 102) = 5.77$; $p < 0.05$; $\eta^2 = 0.05$). The main effect of the time ($F(1, 102) = 3.01$; $p > 0.1$; $\eta^2 = 0.03$) and the interaction *time*dropout intention* are not significant ($F(1, 102) = 0.04$; $p > 0.5$; $\eta^2 = 0.00$). Thus, there is no significant difference in the development of static beliefs in both groups.

The development of the dynamic beliefs differs between students with dropout intention and those who did not think about quitting their studies of mathematics (Diagram 2(right side)) with a significant main effect of the time ($F(1, 101) = 92.38$; $p < 0.001$; $\eta^2 = 0.48$) and dropout intention ($F(1, 101) = 18.23$; $p < 0.001$; $\eta^2 = 0.15$) as well as a significant interaction *time*dropout intention* ($F(1, 101) = 20.15$; $p < 0.001$; $\eta^2 = 0.17$). Both groups of students report decreasing consent to dynamic beliefs, but this decrease is considerably stronger for students with dropout intention.

Discussion

According to theories of person-environment fit, the fit of students' personal characteristics and the characteristics of the university influences *satisfaction* and *satisfactoriness*. In this study, we focused on students' beliefs concerning the nature of mathematics. As expected, students' beliefs are related to their exam achievement (as a criterion for *satisfactoriness*) and their dropout intention (as a criterion for low *satisfaction*).

Static beliefs go along with lower exam achievement, which is in line with the results of Crawford et al. (1994). Liebendörfer and Ostsieker (2014) have hypothesised that static beliefs can lead to typical mistakes in exams (e.g. using theorems without checking the necessary conditions). Moreover, static beliefs are often connected with a surface approach to mathematics learning (Crawford et al., 1994). Even if—theoretically—static beliefs fit to the way mathematics is presented during lectures, it seems that static beliefs are not beneficial during the transition. Therefore, keeping ones' static beliefs can be viewed as a form of beliefs overhang (in the sense of Daskalogianni & Simpson, 2001).

While many previous quantitative studies focused on students' achievement (e.g. Crawford et al., 1994; Rach & Heinze, 2017), we took into account students' dropout intention as well. We found a strong negative relation between students' dynamic beliefs and dropout intention. This is in line with our previous findings that students who do not attend their exams—according to Baars and Arnold (2014) an indicator for dropout—show lower acceptance of dynamic beliefs (Geisler & Rolka, 2018).

The beliefs measured in the middle of the term turn out to be stronger predictors than those measured at the beginning of the term, which supports our hypothesis that the outcome of the development during the transition is more important than students' personal characteristics at the beginning of their studies.

In line with previous studies (Burton & Ramist, 2001; Rach & Heinze, 2017), the school leaving marks are a strong predictor of the exam achievement—in this case the most significant one in all models we tested. In contrast, the school leaving marks have no significant effect on students' dropout intention. This is clearly in line with theories of person-environment fit, which suggest that cognitive prerequisites mainly affect *satisfactoriness* but not *satisfaction*.

With regard to the development of students' beliefs, previous studies have reported inconsistent results (e.g. Kolter et al., 2016; Kuklinski et al., 2018). Our results show that dynamic beliefs decrease during the first term at university, which supports the findings of Kolter et al. (2016). Overall, the static beliefs remain rather stable in the whole sample. One might expect that students enrolled in the teacher education programme and those enrolled in the bachelor programme differ concerning their beliefs. We found only small differences with regard to static beliefs. Bachelor students start their studies with slightly less consent to static beliefs and report a moderate growth of their consent. However, decreasing dynamic beliefs are prevalent in both groups of students.

In contrast to previous studies that only examined the development of beliefs in a whole sample, our results enable a more differentiated perspective by comparing the development of successful and rather unsuccessful students: The decrease of dynamic beliefs is significantly smaller for students without dropout intention than for those who thought about quitting their studies. Students who succeeded in their exam report nearly constant static beliefs, whereas unsuccessful students indicated even increasing consent to static beliefs. It seems that successful students start their studies of mathematics with rather beneficial beliefs and do not undergo substantial changes concerning their beliefs, whereas less successful students already begin their studies with rather disadvantageous beliefs and undergo changes to even less beneficial ones. This conclusion holds for both criteria of a successful transition: the exam achievement and the dropout intention. In both cases, unsuccessful students change their beliefs into a direction that turned out to be disadvantageous. With respect to Haak (2017)—who supposed that students can overcome the crisis which emerges from incongruences between characteristics of themselves and the learning environment by adjusting their characteristics—we find no hints of functional adjustments. Instead, the changes in students' beliefs have to be seen as dysfunctional. The question remains in which way a functional adjustment is possible.

Limitations and Strengths

As stated above, students whose data we analysed seem to be positively selected. We therefore argue that those students, who were excluded from the analysis because they only participated in the first questionnaire, might have undergone an even less beneficial development of beliefs during the transition than those who failed their exam and/or indicated that they thought about dropping out. However, further investigations are necessary to confirm this hypothesis. Besides, our results rely on self-reports and the questionnaires had to be filled in during lectures. Students who do not regularly attend the lectures are not captured by our study.

Another limitation of the study lies in the small sample that only consists of students from one university. Our findings might therefore reflect local peculiarities: all students in our sample attended real analysis and linear algebra during their first term. At some other German universities, students enrolled in a teacher education programme attend only linear algebra during the first term. Moreover, in some other countries (e.g. the USA), linear algebra and real analysis are advanced courses, while freshmen usually attend calculus courses at the beginning of their university studies.

A strength of our study lies in the distinction between successful and unsuccessful students while investigating the changes in their beliefs during the transition from school to university. We found clear evidence that the changes differ between these groups of students and that unsuccessful students undergo a less beneficial change of beliefs. In line with theories of person-environment fit, we focused on objective criteria (exam achievement) and subjective criteria (dropout intention) of success in the transition to gain a more comprehensive picture of the role that students' beliefs play in this phase.

Conclusion and Outlook

All in all, we found evidence that dynamic beliefs are beneficial for a successful transition from school to university mathematics while static beliefs seem to be disadvantageous. Furthermore, our results indicate that dynamic beliefs decrease during the first term at university whereas static beliefs remain rather stable.

These results have some implications for teaching at university. When planning undergraduate courses, especially for freshmen, lecturers should try to foster students' dynamic beliefs. Since traditional lectures tend to draw a rather static picture of mathematics, the way in which mathematics is presented in the lectures could be reconsidered. Instead of showing the *front* of mathematics (in the sense of Hersh, 1991), students should be invited to the *back*: For example, some shorter proofs could be developed together with the students during lectures in order to highlight the dynamic character of mathematics and to strengthen students' dynamic beliefs. In order to structure this collaborative proving process, lecturers might build on Pólya's (1957) four steps to mathematical problem solving: *understanding the problem*, *devising a plan*, *carrying out the plan* and *looking back*. Especially *understanding the problem* and *devising a plan*—those steps that carry students to the *fron*t of mathematics—but also *looking back* are usually absent in traditional German mathematics lectures (Rach, Siebert & Heinze, 2016). However, some universities have introduced lectures with collaborative proving and problem solving activities as well as bridging courses that try to foster students' dynamic beliefs (Biehler et al., 2018; Kuklinski et al., 2018).

Furthermore, tasks should be created in a way that allows insightful experiences of success. Such moments of "illumination," how Liljedahl (2005) calls it, can shift students' beliefs towards more dynamic ones.

Finally, further investigations are required to clarify why even dynamic beliefs of successful students decrease during the transition. Using a qualitative approach may gain deeper insights.

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