



Some Other “No Hole” Spacetimes Properties Are Unstable Too

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Abstract

Two interesting “no hole” spacetime properties (being epistemically hole free (g), not being future nakedly singular) are unstable in the fine topology.

Keywords General relativity · No hole conditions · Stability · Naked singularities · Epistemic holes · Determinism · Time machines · Global spacetime structure

1 Introduction

Recently [1] has shown that a certain “no hole” spacetime property, effective completeness, is unstable in the fine topology (so a “hole free” spacetime can be arbitrarily close to spacetimes which have “holes”). In this note I point out that the very same example can be used to show that (a) another “no hole” property, being epistemically hole free in the geodesic sense, is \mathcal{F} -unstable, and that (b) the spacetime property of not being future nakedly singular is \mathcal{F} -unstable as well.

Why do these observations matter? First, “no hole” conditions are an inherently interesting issue in the study of global structure of spacetime. Second, one may hold a view that stability is a necessary condition for being a physically significant spacetime property (see e.g. [2] for an expression of this position). “No hole” properties are posited in many contexts of foundational relevance—examples include a distinction between spurious and physically relevant instances of indeterminism (a “dirty open secret” of Sect. 3.8 of [3]), definitions of time machines [4], or connections between metaphysical principles such as principle of sufficient reason and principles of model selection in physics (see Sect. 2.3 of [3]). However, since many of the “no hole” properties one could try to employ in these context turn out to be unstable, a dilemma arises: abandon these “no hole” properties and turn to other, more promising spacetime properties which could do the similar work, or deny that stability is a necessary condition for physical significance.

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2 Background Information

Recall¹ [6] that spacetime (M, g_{ab}) has an epistemic hole (g) iff (def) there are two future inextendible timelike geodesics γ and γ' with the same past endpoint such that $I^-[\gamma]$ is a proper subset of $I^-[\gamma']$.² Spacetime (M, g_{ab}) is said to be future nakedly singular iff (def) there is a point $p \in M$ and an incomplete future directed timelike or null geodesic $\gamma: I \rightarrow M$ such that γ is fully contained in $I^-(p)$; otherwise, spacetime is not future nakedly singular.

Certain qualitative features of Misner spacetime will be of relevance for what follows (see [7, pp. 170–174] and [8] for more detailed “Discussion” section). The maximally extended Misner spacetime consists of two regions: the maximal globally hyperbolic Taub region and a chronology violating NUT region, separated from the Taub region by a Cauchy horizon at $t = 0$. In what follows I assume that the temporal orientation has been chosen in such a way that the NUT region lies to the future of the Taub region. The Taub region is timelike and null geodesically incomplete. There are two classes of incomplete causal geodesics: winding to the right along the compact spacelike section, and winding to the left. Assuming that the maximally extended spacetime is Hausdorff, exactly one of these classes can be completed by extending them through the surface $t = 0$.³ Thus: every inextendible extension of the Taub region is timelike and null geodesically incomplete. (All spacetime extensions are taken to be smooth; these two extensions are actually analytic, and there is a orientation-reversing isometry between them; see [10].

Recently, [1] constructed an example of a spacetime (M, g_{ab}) (isometric to a two-dimensional Minkowski spacetime compactified along one null direction) which is geodesically complete (therefore also effectively complete), and a sequence of spacetimes $(M, g_{ab}(n))$ such that in every neighborhood of (M, g_{ab}) in the \mathcal{F} topology there is a spacetime $(M, g_{ab}(n))$ which is isometric to a portion of maximally extended Misner spacetime (which, in turn, is not effectively complete). The full details of that construction (essential in what follows) are to be found in [1]. Since the null compactified Minkowski spacetime (M, g_{ab}) of [1] is geodesically complete, it is not future nakedly singular. It is also easy to see that since Minkowski spacetime is epistemically hole free (g), so is Minkowski spacetime compactified along one null direction.

A spacetime property P is called τ -stable if for any spacetime (M, g_{ab}) with P there is a τ -neighborhood O of (M, g_{ab}) , such that every spacetime in O also has P (in some topology τ). Otherwise P is unstable. Recall also that an open

¹ Definitions of standard notions and notational conventions will follow [1, 5].

² A stronger condition is being epistemically hole free (f), where γ and γ' have finite total acceleration.

³ Actually, as [7] show, there is a third class of geodesics which remains incomplete in any maximal extension which remains a non-Hausdorff topological manifold. Thus, observations concerning epistemic holes and naked singularities will continue to hold even if one allows for violations of the Hausdorff condition at the Cauchy horizon—although it seems that the observation of [1] concerning effective completeness relies on the fact that the extensions are Hausdorff (in a non-Hausdorff extension of the Taub region both left- and rightward geodesics are completed, so the “unwinding” map Θ will map complete geodesics into complete ones; compare Fig. 1 of [1]. See also [9] for a discussion of related issues.

neighborhood of some spacetime (M, g_{ab}) in the \mathcal{F} topology over the set of Lorentzian metrics on a given manifold M is defined by the set of all spacetimes (M, g'_{ab}) such that $\text{Max}_M[h^{am}h^{bn}(g_{ab} - g'_{ab})(g_{mn} - g'_{mn})] < \epsilon$, where h^{ab} is a positive definite metric on M and ϵ is a positive real number. The fine topology is called so because it has plenty of open sets; for example, a one parameter family of spacetimes $(M, \lambda g_{ab})$ where $\lambda \in (0, \infty)$ is not a continuous curve in the \mathcal{F} -topology. However, for this very reason it is convenient to use the \mathcal{F} -topology for proving instability results. Moreover, if a spacetime property fails to be stable in the \mathcal{F} -topology, it also fails to be stable relative to any coarser topology. See [11, 12] for an in-depth discussion of these issues.

3 Two Observations

I am now in position to state two observations concerning spacetimes $(M, g_{ab}(n))$ of [1].

Fact 1 *Every spacetime $(M, g_{ab}(n))$ has epistemic hole.*

First, note that every $(M, g_{ab}(n))$ is isometric to a portion of Misner spacetime (again, see [1] for the justification of this claim). So it is sufficient to argue that (this portion of) Misner spacetime has epistemic hole.⁴ Take any timelike geodesic γ' entering the NUT region. γ' intersects the surface $t = 0$ at some point q . Consider now some future inextendible, incomplete timelike geodesic γ which does not cross $t = 0$ (but circles around, never reaching $t = 0$). Since both γ' and γ wind around the spacelike section in the Taub region, one can easily find some point p for the common endpoint. $I^-(\gamma')$ will consist of a portion of the Taub region and a portion of the NUT region. Since γ winds around, it is fully contained in $I^-(q)$. So every point r in $I^-(\gamma)$ is also in $I^-(\gamma')$. But γ is incomplete; in particular, it does not enter the NUT region. So $I^-(\gamma)$ is a proper subset of $I^-(\gamma')$.

Fact 2 *Every spacetime $(M, g_{ab}(n))$ is future nakedly singular.*

Consider any point p in the NUT region; such p lies to the future of the Taub region (to see that it is indeed the case, consider any timelike curve leaving the Taub region and passing through some r at $t = 0$ to p). Take an inextendible, incomplete geodesic γ confined to the Taub region. Naturally, γ is contained to the past of $t = 0$ surface. Hence any extension of the Taub region is future nakedly singular.

⁴ It can be shown that any proper extension of the maximal globally hyperbolic region of Misner spacetime has epistemic hole [13]; this property carries over to Misner-like spacetimes.

4 Discussion

These two observations (in conjunction with the crucial observation of [1] concerning neighborhoods of null compactified Minkowski spacetime in the \mathcal{F} -topology) imply that a geodesically complete, effectively complete, epistemically hole free (g) and not future nakedly singular spacetime can be arbitrarily close in the fine topology to a spacetime which is not effectively complete, fails to be epistemically hole free (g), and which is future nakedly singular.

This is bad news. As [6] (see also [14, 15] for related remarks) had recently pointed out, most of the “no hole” conditions which have been proposed in the literature (see e.g. [16–19]) rely on mappings between a given spacetime and other spacetimes. In certain contexts (for instance, when trying to make a distinction between physically significant and physically insignificant spacetimes) this is a rather undesirable property, because in order to see whether a condition holds one needs to look at a large set of spacetimes, which in turn seems to presuppose a distinction one wishes to make precise with the use of a “no hole” property. In contrast, being epistemically hole free (g) and not being future nakedly singular are internal to the spacetime; indeed, these are the only two known “no hole” properties of this nature, so their instability is particularly unwelcome.

One could debate the significance of the construction of [1] and the above two facts by noticing that the result seems to depend on the permissiveness of the \mathcal{F} -topology and causally pathological properties of the Misner spacetime. One could then demand that, for the purpose of discussions concerning stability of “no hole” conditions, either:

1. another, more reasonable, choice of topology is made (for instance, recently constructed global topologies of [12] may be promising in that regard), or that
2. attention is restricted to a particular subset of spacetimes (and continue with either the fine topology or some other topology on the restricted set; see [20, pp. 250–257] for a discussion of related issues in the context of proving \mathcal{F} -stability of geodesic incompleteness when restricted to certain FLRW spacetimes). This could be a set of spacetimes which satisfy some causality condition (after all, Misner spacetime is well-known for its bad causal behavior; note though that many other spacetimes of relevance, including maximally extended Kerr spacetime, also have bad causal behaviour) or some other spacetime property.

Denaro and Dotti [21] were recently able to show a form of the Strong Cosmic Censorship hypothesis holds in Misner spacetime, namely that the Cauchy horizon in Misner spacetime is unstable against certain type of perturbations: in a one-parameter family of solutions through the Misner spacetime (in a theory with either (a) minimally coupled massless scalar field, (b) Maxwell field, or (c) in pure (Ricci-flat) gravity), all spacetimes apart from Misner develop a curvature singularity at $t = 0$. This could be taken as an indication that these “no hole” instability results are a result of considering too large set of spacetimes, *per* 2. above.

In a subtle sense the “no hole” properties considered here would be, then, simultaneously stable and unstable (relatively to different choices of the universe for a topology one is considering). There is a family F_{Misner} of spacetimes $(M, g_{ab}(n))$ going through the null compactified Minkowski spacetime (M, g_{ab}) . (M, g_{ab}) satisfies all three “no hole” properties considered by [1] and in this note, but all other elements of this family violate all three of these properties. However, through any of spacetimes $(M, g_{ab}(n))$ goes another family of spacetimes F_{SCC} , those considered by [21], such that only $(M, g_{ab}(n))$ violates these “no hole” properties, but other members of F_{SCC} satisfy them.

Finally, let me point out that all these “no hole” properties have been shown to be unstable using one and the same family of spacetimes (in this sense Misner spacetime not only looks like, *pace* [22], a counterexample to almost everything, but also as an example for almost anything). In order to better understand under which conditions “no hole” properties hold or fail, and when do they do so in a stable or unstable manner, it would be useful to have examples of spacetimes separating these “no hole” properties, so that nearby spacetimes would be, for example, epistemically hole free and not nakedly singular, but would fail to be effectively complete (and so on for other properties of interest). Hopefully this could also provide examples of instabilities of “no hole” properties in spacetimes which are less causally pathological than Misner spacetime.

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