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Analysis of the First Treatise on Machine Elements: Codex Madrid I

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Abstract

In this article, the manuscript of Codex Madrid I, Leonardo da Vinci's workshop drawings collection, is reviewed and the main mechanisms that appear in the aforesaid codex are analysed. It begins with a short reference to Leonardo da Vinci works and, subsequently, Leonardo's manuscripts and Codex Madrid I, in particular, are placed in their historical context. After analysing the historical scope of Codex Madrid I, a compilation of the 100 main drawings of the manuscript is made, composing nine Mechanisms Drawings Boxes. Each Mechanisms Drawings Box is a collage made up of 10 to 14 drawings of mechanical elements that appear in Codex Madrid I, with a brief description of each drawing. This compilation illustrates the wide range of mechanical elements and simple mechanisms of Codex Madrid I, forming, as a whole, a complete treatise on mechanisms, understanding mechanisms as basic elements of machines.

Keywords Leonardo da Vinci · Codex Madrid I · Mechanisms · Machines · History

1 Introduction

On February 14th 1967, The New York Times published astonishing news: the American researcher Jules Piccus had found, by chance, two manuscripts with drawings, sketches and annotations by Leonardo da Vinci in the Spanish National Library in Madrid: the now named Codex Madrid I and Codex Madrid II (Da Vinci, 1504).

The Codex Madrid I is a "technical treatise" where, mainly, extensive studies of mechanical engineering were addressed. The title (in Spanish) assigned to Codex Madrid I, when it was registered in the National Library of Spain, was: "Tratado de

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Estatica y Mechanica En Italiano, Escrito en el año 1493…" (Fig. 1a). Its content is so relevant that it is considered the first monographic treatise on mechanical elements known in this scientific field. The texts and drawings of Codex Madrid I have been finished to high quality (many of the studies presented in Codex Madrid I seem to be a refinement of some of the pages of Codex Atlanticus) and apparently compiled as a complete work with the purpose that was published.

The Codex Madrid II is a notebook whose content is more varied than the Codex Madrid I. The title (in Spanish) the Spanish National Library gave to Codex Madrid II when it was registered was: "Tratados varios de Fortificación, Estática y Geometría escritos en italiano, por los años de 1491" (Fig. 1b). The Codex Madrid II contains designs for musical instruments, technical notes on geometry and the specifications of several projects: architectural projects, civil and military engineering projects, the project for the fortification of the city of Piombino, the canalization project for the deviation of the Arno river in the city of Pisa (only project part) and the project for the foundry of the famous horse of the "Sforza Equestrian Monument" (in full detail).

Concerning the physical characteristics of both codices (Madrid I and Madrid II), the two manuscripts are written on paper, with approximate dimensions of 210 × 150 mm. Codex of Madrid I contains 192 folios and Codex of Madrid II has 157 folios.

The current view of Leonardo da Vinci is that of a wise man, the prototype of a humanist man, characteristic of the Renaissance period (Johnston, 2000). Besides, Leonardo is considered a great artist, scientist and inventor, though this last statement is not entirely true (Cerveró-Meliá et al., 2020; Contreras López, 2016) and needs to be nuanced. The discovery of the Codex Madrid I and Codex Madrid II contributed to enlarging the legend of Leonardo (Reti, 1968).

Although Leonardo proposed the design of some original machines (Bucolo et al., 2020; Laurenza et al., 2011), their functionality is questionable and he didn't manufacture and test these machines (Ceccarelli, 2016; Cerveró Meliá et al., 2018; Gille, 1978).

Fig. 1 The title (in Spanish) assigned to Codex Madrid I and Codex Madrid II by the Spanish National Library

However, it should be noted all the written and, especially, all the pictorial material that he bequeathed as a collection of much of the knowledge of his time, the Renaissance (Reti, 1974).

This paper shows the main mechanisms and basic elements of Renaissance machines compiled in the Codex Madrid I, which is Leonardo da Vinci's main contribution to the history of machines and mechanisms (Bautista Paz et al., 2010; Ceccarelli, 2016).

This paper aims to offer an analysis of Codex Madrid I, to relate the technology compiled in this Leonardo da Vinci's work with other authors of his historical environment, and to give a current view of the aforementioned manuscript by selecting 100 drawings from the main basic machine elements or mechanisms, composing a comprehensive treatise about the mechanisms of Renaissance.

The written output about the life and works of Leonardo is immense, but this paper is focused on the Codex Madrid I and we will only cite those documents that are direct references for making this work.

2 Short Reference to Leonardo da Vinci Works

Leonardo di ser Piero da Vinci (Fig. 2) was born on April 15, 1452 in Anchiano, a district of Vinci, a Tuscan town near Florence, and died on May 2, 1519 in Amboise, a city in central France, where Leonardo was, at that time, as a guest at the court of King Francis I.

Leonardo da Vinci was the illegitimate child of Piero Frusolino di Antonio, a Florentine notary. His mother was Caterina di Meo Lippi, a peasant from the Tuscany region. He grew up in the house of his paternal grandfather, Antonio da Vinci, where he learned to read and write (the latter with some difficulty since he was left-handed, speculatively). At that time, Leonardo also learned the rudiments of arithmetic used by merchants and technicians, also later at Verrocchio's workshop, as it was required for an artist, although he was not instructed in the languages and subjects of the scholars of the time (trivium and quadrivium) and did not master Latin or Greek (Isaacson, 2017).

Fig. 2 Portrait of Leonardo da Vinci attributed to Francesco Melzi (c. 1515–1518)

At the age of 17, Leonardo entered as an apprentice in the workshop of the prestigious artist Andrea del Verrocchio (painter, sculptor and goldsmith), where he received multidisciplinary training in various arts. From that moment on, he began his life of study and artistic and scientific production.

Today, many people consider Leonardo da Vinci a man of his time, epitomizing the humanist ideal of the Renaissance. However, many aspects of his life and works do not allow placing him exactly within the Renaissance spirit, both from a theoretical and a practical point of view."

- The current view of Leonardo da Vinci is that of a wise man, a polymath of the Renaissance: painter, draughtsman, engineer, architect, scientist, botanist, anatomist, sculptor, etc.
- His fame rested mainly on the fact that he was an extraordinary painter, one of the best painters of all time, creator of famous paintings such as *The Last Supper* or *Mona Lisa*.

Leonardo is also known for his plentiful production of manuscript works: his notebooks. His manuscripts are gathered in 23 notebooks on a great variety of subjects: anatomy, astronomy, geology, painting, optics, civil engineering, hydrodynamics, tribology (Hutchings, 2016) and mechanical engineering. Most of Leonardo da Vinci's notebooks were rediscovered in the nineteenth century: Codices Atlanticus, Arundel, Leicester, Paris Manuscripts A-M (12 volumes), Forster (3 volumes), On the Flight of Birds, Trivulzianus and Windsor Royal Library Notebooks; except for the Madrid I and II codices that were discovered after the middle of the twentieth century (Cerveró-Melia, 2021).

Leonardo, in his production, reworks one of the favourite genres of the Renaissance (the treatise), integrating, or better verifying, theory with experiments: "No human investigation can be called true science if it does not pass through mathematical demonstrations, and if you say that sciences, which begin and end in the mind, have truth, this is not granted, but denied, for many reasons, and first, that in such mental discourses experience does not occur, without which nothing gives certainty of itself" (Da Vinci, 1993).

Leonardo da Vinci's manuscripts, written between 1487 and 1519, add up to a total of worksheets of approximately 7000 pages: most worksheets are in notebooks plus approximately fifty loose sheets distributed by various museums and libraries of various cities in Europe and America (Pooler, 2014).

Leonardo da Vinci's notebooks add up to a total of 3495 duplicate folios, with an approximate total of 7000 pages and are gathered in 23 codices:

- Codex Leicester or Hammer. Bill and Melinda Gates collection, Seattle (USA).
- Codices Paris o Manuscripts A, B, C, D, E, F, G, H, I, K, L and M. Bibliothèque de l'Institut de France, Paris (France).
- Codex Turin or Manuscript On the Flight of Birds. Biblioteca Reale in Turin (Italy).
- Codex Trivulzianus. Biblioteca Trivulziana, Castello Sforzesco, Milan (Italy).
- Codex Atlanticus. Biblioteca Ambrosiana in Milan (Italy).
- Windsor collection. Windsor Castle Royal Library, London (U.K.).
- Codex Arundel. British Library, London (U.K.).
- Codex Madrid I and II. Biblioteca Nacional de España, Madrid (Spain).
- Codices Forster: Codex Forster I, II and III. Victoria and Albert Museum, London (U.K.).

Many researchers and science communicators praise Leonardo da Vinci as the greatest engineer of his time because he was more prolific than his contemporaries and predecessors (in number of designs of mechanisms and machines) (Oliveira, 2019). In addition, his technical designs, compared to those of other engineers, introduce advances and improvements. However, these authors do not consider that, probably, Leonardo had more time than the other engineers because while Leonardo drew and wrote about mechanisms and machines or described projects on various topics, those same predecessor engineers and his contemporaries built those machines, putting those projects into practice (in some of which Leonardo himself collaborated).

However, Leonardo was a pioneer in incorporating text notes to his technical drawings (as an explanatory measure to his proposals for designs or devices) and in adding indicator letters on the drawings (to guide practical design and make clarifications in the legends of these) (Mijksenaar & Westendorp, 2004). In addition, his artistic genius is reflected in his drawings, of great precision and beauty.

Leonardo bequeathed an immense number of designs and studies of machines and mechanisms to History (Pedretti, 1999), but his contribution of technical innovations was minimal. Perhaps due to he didn't publish his findings and his writings had scant diffusion and, on the other hand, he didn't leave any pupils to pass on his legacy. Furthermore, his work did not begin to be published until the nineteenth century, when Charles Ravaisson-Mollien (1848–1919), a French art historian and curator of the Louvre museum, published *"Les Manuscrites de Léonard de Vinci"* (Ravaisson-Mollien, 1881). Since then, a large number of scientists, naturalists, art scholars, historians and science popularizers have analyzed Leonardo's manuscripts and have published a large number of them.

We can talk about the importance of Leonardo da Vinci as an 'engineer', it can be said that he was endowed with immense curiosity but was scattered: he started many things and later abandoned them.

He knew the activities of the Engineers and Architects of ancient Rome as well as that of his predecessors in the Italian Renaissance: based on the analysis of Leonardo's manuscripts, it can be stated that Leonardo knew the work of Vitruvius Marcus Pollio (Vitruvio) (Cigola & Ceccarelli, 2014), Filippo Brunelleschi (Prager, 2012), Mariano di Jacopo (il Taccola) (Ceccarelli, 2021), Leon Battista Alberti or Francesco di Giorgio Martini (Ceccarelli, 2018).

In any case, it can be affirmed that Leonardo da Vinci was a great engineer and it should be noted his 'method' and his detailed analysis of the machines, studying the simplest elements, of which the Codex Madrid I was his main work.

3 Codex Madrid I

In 1964, two manuscripts by Leonardo da Vinci of an incalculable historical-scientific-cultural value were found in the Spanish National Library in Madrid. Both manuscripts (Codex Madrid I and Codex Madrid II) embrace almost seven hundred pages of new Leonardo writings about architecture, geometry, mechanics and navigation topics (Koffler, 1974; Ruiz García, 2012).

Codex Madrid I is a compilation of texts and drawings that can be defined as a comprehensive treatise of Renaissance mechanics, with a great graphical, descriptive and engineering ability (Ceccarelli, 2008). These graphic, descriptive, engineering and detail capabilities of Leonardo's drawings are clearly superior to those of his predecessors and contemporaries.

Codex Madrid I was paginated by Leonardo and has come down to us practically intact, except for sixteen pages (eight folios) that were torn out and appear to have been lost. The pagination care of the sheets and the technical-mechanical drawings suggests that Leonardo bound this work thinking of a possible treatise.

Regarding the publication of the treatise, it is clear that finally Leonardo opted for the most conservative theories of medieval "secrecy", far from humanistic "publication". At this point, it is interesting to mention the conversations between Brunelleschi and Mariano di Jacopo (il Taccola), contemporaries and great figures of architecture and engineering (predecessors of Leonardo) and very close in technical matters but divergent in matters of dissemination, in which the Florentine master (Brunelleschi) warned the Sienese master (Taccola) of the dangers of making technical know-how public (Fernández Correas, 2009).

In Codex Madrid I, Leonardo drew and studied several mechanisms to get different kinds of movement: alternating, swinging and, even, predefined movements.

Although Leonardo da Vince wrote other treatises that included machine studies (Codex Atlanticus, Paris Manuscripts, …) Codex Madrid I is unique as it establishes the first case to deconstruct machines into basic machine elements or mechanisms (Leonardo da Vinci called them "elementi macchinali") (Moon, 2007). Leonardo presupposes the existence of a reader for his work and writes the scientific passage thinking about them. Had Leonardo actually published this work, it would have accelerated the development of machine design.

Codex Madrid I contains close to 1000 drawings of machines and machine elements. The treatise is written in high-quality script and in Leonardo's typical mannerist handwriting. The drawings are also of high quality, clear and complete, and they contain studies about on the fundamental concepts of modern Mechanics, i.e., mechanisms or mechanical elements. These mechanisms are not Leonardo's machines, which are more complex and have particular usages. However, Leonardo tried to prove that by using and combining these simple elements it is possible to obtain machines that perform a specific function (Koffler, 1974).

The codex can be dated between 1492 and 1497, in the first period of Leonardo in Milan (1482–1499). This is when Leonardo was in the prime of life, he was living his fourth decade (Ruiz García, 2012).

Codex Madrid I is a one-volume manuscript that consisted of 192 sheets originally (only 184 sheets are preserved today), double side and dimensions 215 × 145 mm. The main body is made up of 12 notebooks, all containing 8 folios. The third and fourth notebooks have lost some folios (Ruiz García, 2012).

From the point of view of its content, it is a compilation of texts mainly from other Leonardo's treatises. It also offers earlier texts about static and geometry (Moon, 2007).

Codex Madrid I has a great correspondence with the Codex Atlanticus: A series of devices, such as gears, springs, hinges, mechanical elements of clocks, scales, weights, screws, etc., are outlined in the Codex Atlanticus and developed in the Codex Madrid I, with drawings with great detail and quality. There is a connection with other codices: the Codices Forster or Manuscripts A and H, from Codices Paris (Cerveró-Meliá et al., 2020).

4 Historical Scope

In this section, we analyse the Codex Madrid I (Leonardo da Vinci, 1974), placing the mechanical knowledge that it contains into the historical context (the Renaissance) (Lefèvre, 2004) and justifying the claim that it is the first known treatise where the main mechanisms and simple machines are compiled and studied.

The culture of the Renaissance is marked by the exaltation of the dignity of man, thought inherited from classical thinking: in Sophocles's Antigone (Sophocles, 1994), it was said: "Wonders are many, and none is more wonderful than man". The culture of the Renaissance took into account other topics: the exaltation of nature, the critical spirit, using reason as the fundamental instrument of any mental process, adding mathematics as a complement to literary thinking, leaving aside the principle of authority, appreciating the importance of experimental practice, etc.

Renaissance ideas were shared by the Italian intellectual elite of the fifteenth and sixteenth centuries. The Byzantine intellectuals that emigrated to Italy after the fall of Constantinople (1453) contributed to this school of thought (Ruiz García, 2012).

The life and work of Leonardo da Vinci allow placing him within his historical dimension: the Renaissance spirit, both from a theoretical and practical point of view (Ceccarelli, 2008, 2016).

Leonardo lived in one of the most cultured environments in the European Renaissance. He didn't develop original comprehensive theories in the scientific field, but he was a persistent observer and a singular exponent of a time that changed the world. His contribution to the effective progress of the science of his time was not great, perhaps because of the limited dissemination of his writings. However, he left testimony of his defence of experimentation as a method of intellectual (and scientific) work, his claim to technique and of machines as instruments of progress and, above all, their general conception of reality governed by the laws of necessity, pragmatism, reason and proportion. Leonardo tried to demonstrate the superiority of visual creation over literary text in his work (Innocenzi, 2019; Ruiz García, 2012).

One of the most important innovations of the Renaissance was resorting to drawing as an instrument of demonstration and communication. Following this idea, the graphic conventions necessary to visualize machines and suggest their features were defined. This art of describing machines became an instrument of technical-scientific study. Leonardo da Vinci was not original at this point but maybe he is the greatest exponent of this art.

Leonardo, as humanist, intended to understand nature. He tried to decipher its secrets and formulate its laws, in an attempt to solve the infinite problems posed by nature.

As we have commented, the 'Renaissance' is characterized by the emergence of the so-called "engineers" that surpasses the category of "master of war machines", as the craftsmen-builders of the Middle Ages used to be called (Ceccarelli, 2008; Moon, 2007).

Leonardo can be classified as an 'engineer'. He knew the activities of the Engineers and Architects of ancient Rome as well as that of his predecessors in the Italian Renaissance. Analysing Leonardo's manuscripts, it can be stated that Leonardo knew the work of Vitruvius (Cigola & Ceccarelli, 2013, 2014), Filippo Brunelleschi (Prager, 2012), Mariano di Jacopo (il Taccola) (Ceccarelli, 2021), Leon Battista Alberti or Francesco di Giorgio Martini (Ceccarelli, 2008, 2018; Merrill, 2015).

As predecessors of these Renaissance engineers, it is worth mentioning the Muslim engineers and, before them, the Greek and Roman engineers. It is clear that "De architectura" by Vitruvius was the reference treatise for Renaissance architects and engineers, but Archimedes (c. 287–212 B.C.) had already made several contributions to the study of machines (such as the principle of the lever, the screw of Archimedes or certain siege engines) and Hipparchus of Nicaea (c. 190–120 B.C.) had established the principles of trigonometry. Especially, the contributions that various members of the Alexandrian School made to the evolution of mechanics and machines were fundamental: Ctesibius (c. 285–222 B.C.), Philo of Byzantium (c. 280–220 B.C.), Heron of Alexandria (c. 10–70 A.D.) (Papadopoulos, 2007) or Pappus of Alexandria (c. 290–350 A.D.).

The contributions of several Muslim authors are also very important (Al-Hassani & Reference Collection, 2015) and should be recognized as heirs to the knowledge of the School of Alexandria, continuing the advance of mechanical engineering (before the Renaissance) and as precursors of machine treatises: the most important figures are the Banu Musa brothers (ninth century), Ibn Khalaf al-Muradi (eleventh century) and Ibn al-Razzaz al-Jazari (1136–1206) (Hassaan, 2014a, 2014b). After Leonardo da Vinci, the machine manuscripts of Taqi-al-Din (1521–1585) deserve special mention (Al-Hasan, 1978; Gökdoğan & Uymaz, 2020).

On the other hand, the contribution in technology and machines of the engineers of ancient China would deserve a separate chapter, it is enough to mention two highly complex mechanical devices for the time in which they were created: The Compass Cart or south-pointing Chariot (Han Dynasty, 202 B.C.–220 A.D.) and the Water-Powered Astronomical Clock Tower. The first device was a cart with a heading mechanical control system that, when the cart moved or turned, always pointed south (Bautista Paz et al., 2010; Lu, 2015; Needham et al., 1986). The Clock Tower was a famous water-powered automatic device whose construction was completed by Su Song in 1087 (Bautista Paz et al., 2010; Lu, 2015).

The most important Chinese treatise on machines, before Codex Madrid I, is the "Nong Shu" (Agricultural Treatise, 1313) by Wang Zhen, which contains a large number of illustrations of tools and machines used in agriculture (Bautista Paz et al., 2010; Lu, 2015; Needham et al., 1986). No other treatise of this importance will appear until the "Tian Gong Kai Wu" (The Exploitation of the Works of Nature, 1637) composed by Song Yingxing (Bautista Paz et al., 2010; Lu, 2015; Needham et al., 1986). “Yuanxi Qiqi Tushuo Luzui” (Collected diagrams and explanations of wonderful machines from the far west), published in 1627, was the fruit of cooperation between the Jesuit missionary Johann Schreck (J. Terentius) and Wang Cheng (Zhang & Renn, 2006).

If we talk about the first works of the "engineers" of the Renaissance in Europe, these appear in the Trecento. These "engineers" compiled the designs of machines rescued from antiquity or devised by themselves and made treatises for intellectual purposes, to give greater prominence to technical culture (Fernández Correas, 2009). We can start with the military treatise by Guido di Vigevano (Texaurus regis Francie, 1335) as the predecessor of the notebooks of the Italian engineers of the Renaissance, as well as the military-technical writings of the German school, including Konrad Kyeser (Bellifortis, c. 1405) (Lefèvre, 2004; Moon, 2007).

Among the Italian engineers of the Renaissance, we can speak of up to two generations of Italian engineers. Among the authors of the first generation, we find Filippo Brunelleschi, Taccola, Leon Battista Alberti or Giacomo Fontana (Ceccarelli, 2008; Moon, 2007). In an intermediate time, preceding the second generation, we can cite Roberto Valturio or Francesco di Giorgio Martini, marking the definitive transition to the second generation, of which Da Vinci was a notable representative. These two authors preceded and coincided in time with Leonardo, even Leonardo collaborated in some of his Works (Lefèvre, 2004; Moon, 2007).

The case of Francesco di Giorgio Martini is significant because the importance of his career has been dwarfed by the shadow of Leonardo today, who was inspired by Martini's work and copied his drawings (Ceccarelli, 2018; Merrill, 2015; Moon, 2005, 2007). However, Francesco di Giorgio was a well-known engineer in Italy in the late fifteenth century, whose works were known and highly regarded. Leonardo did not have as much recognition as Francesco di Giorgio in his lifetime, moreover, let us remember that Leonardo's manuscripts were not published until a long time later. In Leonardo's defence, it should be noted that the work of Francesco di Giorgio had been inspired by Taccola's work (Ceccarelli, 2021; Moon, 2007).

Leonardo da Vinci developed his works between 1480 and 1515, the Codex Madrid I was produced between 1492 and 1497. Leonardo da Vinci's manuscripts set up the beginning of a period between the end of the fifteenth century and the end of the seventeenth century where a large number of treatises on machines appeared (Ravier-Mazzocco, 2013) The next tables (Table 1 and Table 2) list the main authors and works about machines that came before and after Leonardo and his works.

Table 1 shows the indications on the early machine treaties suggested by (Moon, 2007), revised and expanded. Table 2 develops a revised summary of the compendium of the first collections of machine drawings proposed by (Lefèvre, 2004).

To appreciate how knowledge evolved between Antiquity and the Renaissance, the Vitruvian man can be taken as an example (Riavis, 2020). In the images of Fig. 3 this evolution can be observed: Fig. 3a is an interpretation of the original Vitruvian man ("De Architectura" by Vitruvius, illustrated edition by Cesare Cesariano, 1521) (Martínez Díaz, 2020); Fig. 3b is a drawing by Taccola (De Ingeneis, c. 1450); Fig. 3c is a drawing by Francesco di Giorgio Martini (1470–1480); and Fig. 3d is the famous drawing of the Vitruvian man by Leonardo da Vinci (c. 1490).

To be more specific and understand how technical knowledge emerged and evolved between antiquity and the Renaissance, the evolution of piston pumps can be taken as an example. This evolutionary process can be seen in the drawings of Fig. 4: Fig. 4a is a drawing made by Al-Jazari (1204–1206) (Hassaan, 2014a); Fig. 4b is a drawing by Taccola (c. 1450, first European representation of a piston pump) (Ceccarelli, 2021); Fig. 4c is a drawing by Francesco di Giorgio Martini (1475–1480); and Fig. 4d is a drawing by Leonardo da Vinci (*Codex Madrid I*, 1492–1497) (Rubio et al., 2022).

5 The Mechanisms of Codex Madrid I

Shortly after the Codex Madrid was rediscovered in 1965, Ladislao Reti translated the text into English and showed that da Vinci had attempted to compile a basic compendium of machine elements (Leonardo da Vinci, 1974).

Prestigious researchers checked that the main mechanical constructive elements were presented in the drawings in Codex Madrid I: (Reti, 1974) compared the drawings of machine elements and mechanisms in Leonardo's Codex Madrid I with the basic list of 22 constructive elements proposed by Franz Reuleaux in his 19th-century book on machine design "The Constructor". More recently, Moon confirmed this list (Moon, 2007) and made an advanced study of the machine elements and mechanisms that Leonardo described in the Codex Madrid I, including a comparison with the works of Timoshenko (Moon, 2009).

What mechanical elements can be seen in the Codex Madrid I? Ball and rolling bearings; several types of gears and gear sets; cams and followers; flywheels; belt drives

Table 1 A summary of 'theatre of machines' books up to the seventeenth century

Time	Author	Short title
c. 27 BC	Vitruvius Pollio	*De Architectura*
850	The Banu Musa brothers	*Book of Ingenious Devices (literally: "The Book of Tricks")*
eleventh century	Ibn Khalaf al-Muradi	*The Book of Secrets in the Results of Ideas (copy of 1246)*
1204–1206	Ibn al-Razzaz al-Jazari	*The Book of Knowledge of Ingenious Mechanical Devices*
c. 1225–1250	Villard de Honnecourt	*The Sketchbook of Villard de Honnecourt*
1331	Wang Zhen	*Nong Shu (Agricultural Treatise)*
1335	Guido di Vigevano	*Texaurus regis Francie*
1366–c. 1405	Konrad Kyeser	*Bellifortis*
1393–1455	Giacomo Fontana	*Bellicorum instrumentorum liber*
c. 1445–1516	Giuliano da Sangallo	*Opusculum*
c. 1450	Marianus Jacobus/Taccola	*De Ingeneis*
1451–1516	Bonaccorso Ghiberti	*Zibaldone*
1470–1480	Francesco di Giorgio	*Trattato di Architettura*
1472	Roberto Valturio	*De re militari*
1480–1515	Leonardo da Vinci	*Notebooks [unpublished]*
1540	Vannuccio Biringuccio	*De la Pirotechnia*
1551	Taqi al-Din	*The Sublime Methods of Spiritual Machines*
1556	Georgius Agricola	*De Re Metallica*
1578	Jacques Besson	*Theatre des instruments mathematiques et mechaniques*
1584	Jean Errard	*Le Premier Livre des Instruments Mathematiques Mechaniques*
1588	Agostino Ramelli	*Livre des diverses et artificieuses machines*
1607	Vittorio Zonca	*Novo teatro di machine et edifici*
1607–1614	Heinrich Zeising	*Theatri Machinarium*
1615	Salomon de Caus	*Les Raisons des Forces Mouvantes avec diverses Machines*
1617–1618	Jacob de Strada	*Künstlicher Abriss allerand Wasser, Wind, Ross und Handt Mühlen*
1627	Johann Schreck (J. Terentius) and Wang Cheng	*Yuanxi Qiqi Tushuo Luzui (Collected diagrams and explanations of wonderful machines from the far west)*
1629	Giovanni Branca	*Le machine*
1637	Song Yingxing	*Tian Gong Kai Wu (The Exploitation of the Works of Nature)*
1661	Georg Böckler	*Theatrum machinarum novum*
1672	Otto von Guericke	*Experimenta nova, ut vocantur Magdeburgica*
1695	Phillipe De Le Hire	*Traite de mecanique*

(open and crossed); chain drives (several models of chain); ratchet and pawl mechanism, joints; couplings; and many more drawings of mechanical elements and functional studies that, in this work, we will try to summarize all of them in 100 drawings.

The following pages include a sequence of nine drawings collages with Leonardo's simple machines, displaying 100 drawings of 100 mechanisms-machine elements and functional studies from Codex Madrid I, which serve as a basis for correctly interpreting machines as a set of mechanisms and machine elements.

Table 2 Prominent sources of early machine drawings (1250–1650)

Time	Author	Document type
c. 1225–1250	Villard de Honnecourt	A, B
1335	Guido di Vigevano	C2
1366–c. 1405	Konrad Kyeser	C2
c. 1410–1430	German Master Gun-makers	C1
c. 1410–1450	Marianus Jacobus/Taccola	B, C2
1472	Roberto Valturio	D1
c. 1475–1490	Anonymous of the Hussite Wars	C1 (Hussite Wars 1420–1436)
1470–1480	Francesco di Giorgio Martini	D1
1480–1515	Leonardo da Vinci	B
1486	Vitruvius	D3 (Original work c. 27 BCE)
c. 1510–1546	Antonio da Sangallo	B
1537	Niccolo Fontana/Tartaglia	D2
1551	Taqi al-Din	B, C2
1556	Georgius Agricola	D1
1558	Berthold Holzschuher	A, B
1567	Giuseppe Ceredi	D1
1569	Jacques Besson	C3
1577	Guidobaldo del Monte	D2
1584	Jean Errard	C3
1588	Agostino Ramelli	C3
1588	Pappus of Alexandria	D3 (Original work c. 340)
1589	Heron of Alexandria	D3 (Original work c. 35-c. 75)
c. 1590–1635	Heinrich Schickhardt	A, B
1597	Buonaiuto Lorini	D1
1607	Vittorio Zonca	C3
1607–1614	Heinrich Zeising	C3
1615	Salomon de Caus	C3
1617	Jacob de Strada	C3
1629	Giovanni Branca	C3

A: Workshop drawings of design and construction drawings

B: Workshop drawings of sketch-books and notebooks

C: Presentational manuscripts or books of collections of drawings with or without explanatory text

C1: Practitioner booklets

C2: Representational manuscripts

C3: Theatres of machines

D: Presentational manuscripts or books of drawings in treatises and editions

D1: In technological treatises

D2: In treatises on mechanics

D3: In editions of classical sources

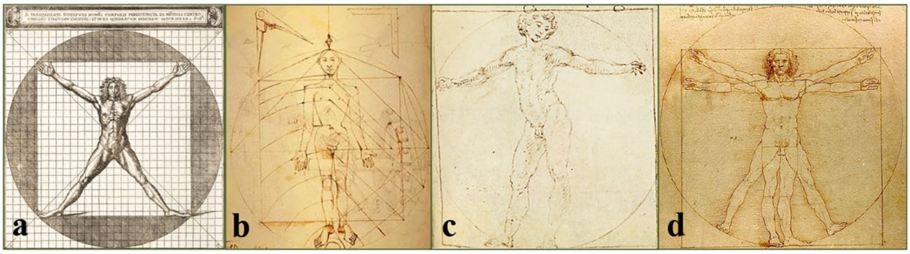


Fig. 3 Drawings of the Vitruvian man by various authors

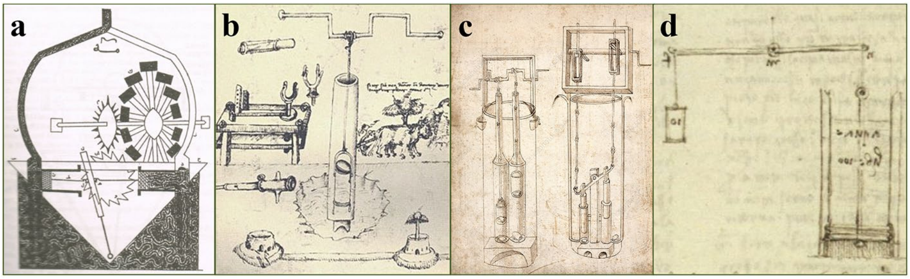


Fig. 4 Drawings of piston pumps by various authors

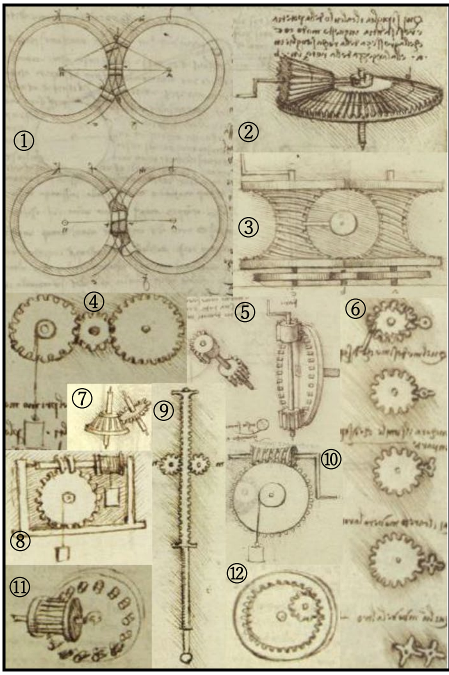
These nine drawings collages or Mechanisms Drawings Boxes, together with the 100 drawings, serve as a basis for correctly interpreting machines as a set of mechanisms and machine elements, and the great contribution of Leonardo's studies to machine engineering as a science can be appreciated.

In these 100 drawings (Figs. 5, 6, 7, 8, 9, 10, 11, 12 and 13), we can see a compilation of 100 main mechanisms and machine elements that Leonardo illustrated in the Codex Madrid I, several of which were aimed to be integrated into Leonardo's watch design Project (Fig. 14a) (Robey, 2012). In the next Mechanisms Drawings Boxes (Figs. 5, 6, 7, 8, 9, 10, 11, 12 and 13) the drawings are very briefly described.

In the Mechanisms Drawings Box 1 (MDB-1) several drawings of gears are shown. Among others, various devices can be seen: a study of gears with teeth or pins (MDB-1.1); spur gears (MDB-1.4); bevel gears (with teeth, MDB-1.2, or with pins MDB-1.7); worm gear systems (MDB-1.3, MDB-1.8 and MDB-1.10); crown wheel gears (MDB-1.5 and MDB-1.11); internal gear (MDB-1.12); rack and gears (MDB-1.9); or a study of transmission ratio in gears (MDB-1.6).

In the Mechanisms Drawings Box 2 (MDB-2) several drawings of screw spindles are shown (MDB-2.4, MDB-2.5, MDB-2.6 and MDB-2.8). Drawings of a double helix reversing mechanism (MDB-2.9), a gear regulator de-vice (MDB-2.1), a rack and gears drive (MDB-2.10), a planetary-epicyclic gear (MDB-2.7) or train gear reduction systems (MDB-2.2 and MDB-2.3) can also be seen in MDB-2.

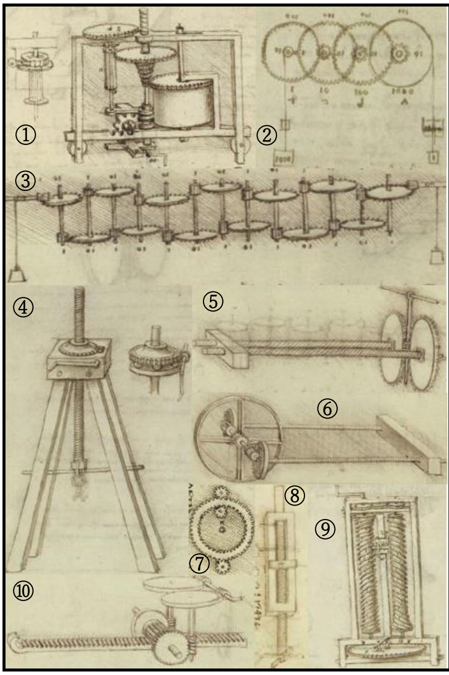
The Mechanisms Drawings Box 3 (MDB-3) shows different drawings of rolling bearings and rolling studies (MDB-3.1, MDB-3.2, MDB-3.3, MDB-3.4, MDB-3.5, MDB-3.6 and MDB-3.9). Leonardo da Vinci's drawing of the radial anti-friction ball bearing (MDB-3.3) is a representative icon of the rolling bearing in the history of technology.



Mechanism description of drawings:

MDB-1.1	Pins and teeth gears
MDB-1.2	Bevel gear
MDB-1.3	Double globoid worm system
MDB-1.4	Spur gears
MDB-1.5	Pin-teeth rotary reversing mechanism
MDB-1.6	Transmission ratio study in gears
MDB-1.7	Crown bevel gear and lantern pinion
MDB-1.8	Endless screw or worm drive
MDB-1.9	Rack and pinion
MDB-1.10	Globoid worm drive
MDB-1.11	Crown wheel gear and lantern pinion
MDB-1.12	Internal gear

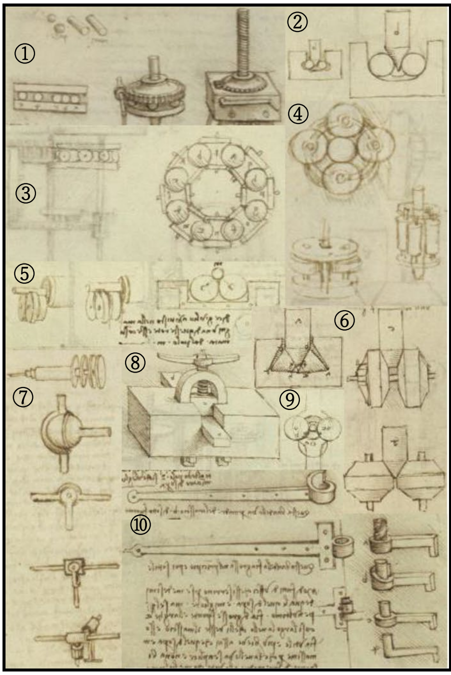
Fig. 5 Mechanisms Drawings Box 1 (MDB-1)



Mechanism description of drawings:

MDB-2.1	Gear regulator device
MDB-2.2	Gear reduction system
MDB-2.3	Lantern gear reduction system
MDB-2.4	Screw jack with roller bearing
MDB-2.5	Double screw spindle
MDB-2.6	Double screw spindle drive
MDB-2.7	Planetary-epicyclic gear train
MDB-2.8	Screw spindle
MDB-2.9	Double helix reversing mechanism
MDB-2.10	Rack and pinion drive

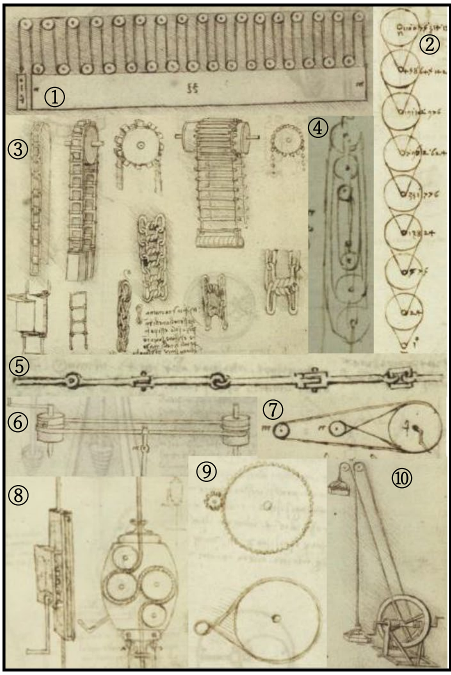
Fig. 6 Mechanisms Drawings Box 2 (MDB-2)



Mechanism description of drawings:

MDB-3.1	Axial anti-friction ball bearing
MDB-3.2	Ball bearings for a conical pivot
MDB-3.3	Axial ball bearing
MDB-3.4	Rollers bearings for shaft
MDB-3.5	Disks bearings
MDB-3.6	Roller bearings for a conical pivot
MDB-3.7	Universal joints
MDB-3.8	Spring-making machine
MDB-3.9	Rollers bearing for shaft
MDB-3.10	Standard and self-closing hinges

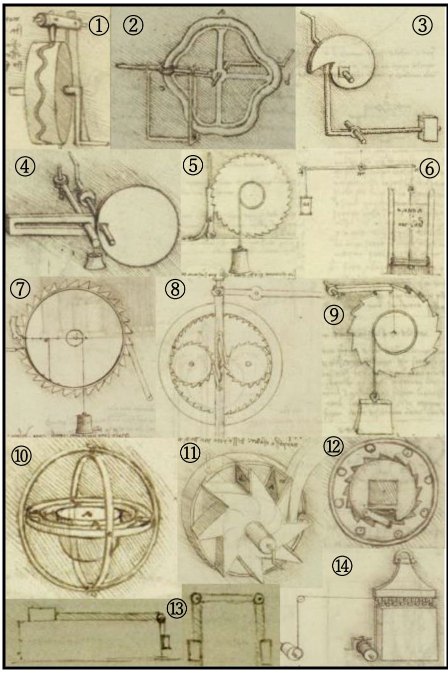
Fig. 7 Mechanisms Drawings Box 3 (MDB-3)



Mechanism description of drawings:

MDB-4.1	Multiple pulley mechanism
MDB-4.2	Belts & pulleys reduction system
MDB-4.3	Models of chains
MDB-4.4	Block and tackle system
MDB-4.5	Models of chains
MDB-4.6	Belt and pulley mechanism
MDB-4.7	Open and cross belt drives
MDB-4.8	Manual lift mechanism
MDB-4.9	Analogy between gear and belt transmission
MDB-4.10	Crane with drum hoist winch

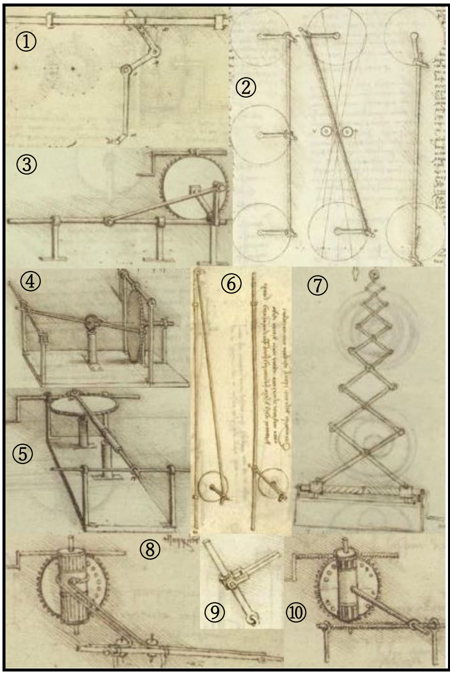
Fig. 8 Mechanisms Drawings Box 4 (MDB-4)



Mechanism description of drawings:

MDB-5.1	Cylindrical cam mechanism
MDB-5.2	Cam mechanism
MDB-5.3	Cam actuated lever
MDB-5.4	Eccentrical cam mechanism
MDB-5.5	Ratchet and pawl mechanism
MDB-5.6	Piston & cylinder
MDB-5.7	Ratchet and pawl mechanism
MDB-5.8	Ratchet and pawl mechanism
MDB-5.9	Ratchet and pawl mechanism
MDB-5.10	Gimbals mechanism
MDB-5.11	Water wheel
MDB-5.12	Ratchet and pawl mechanism
MDB-5.13	friction measurement system weight-driven
MDB-5.14	Pulley system for weight-driven clock gear

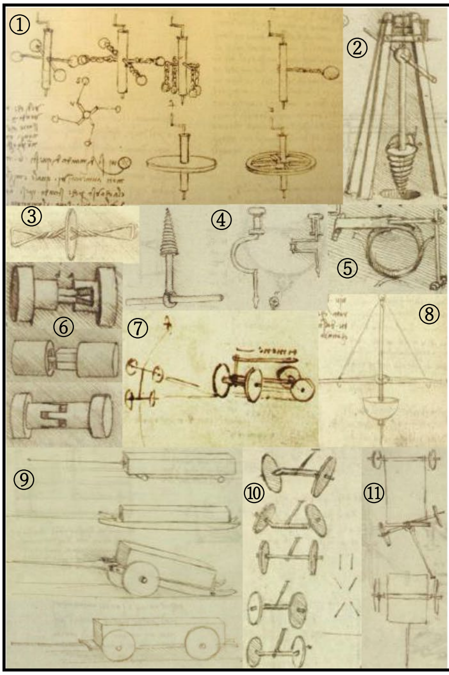
Fig. 9 Mechanisms Drawings Box 5 (MDB-5)



Mechanism description of drawings:

MDB-6.1	Linear motion generation mechanism
MDB-6.2	Four-bar linkage
MDB-6.3	Linear motion generation mechanism
MDB-6.4	Linear motion generation mechanism
MDB-6.5	Linear motion generation mechanism
MDB-6.6	Slider crank mechanism
MDB-6.7	Lazy tongs or Nürnberg shears
MDB-6.8	Linear motion generation mechanism
MDB-6.9	Three degrees of freedom joint
MDB-6.10	Linear motion generation mechanism

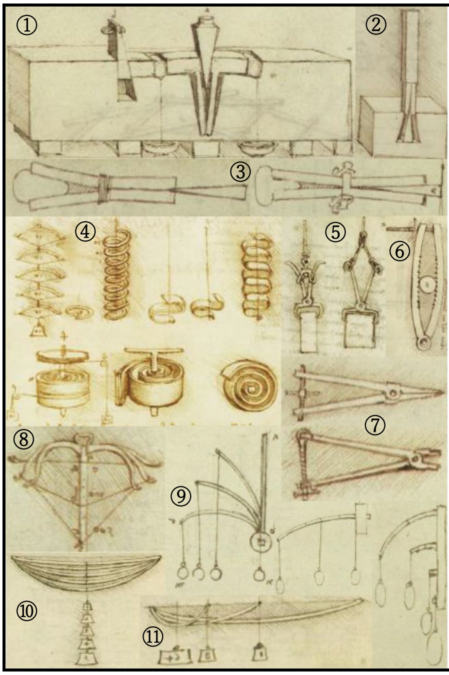
Fig. 10 Mechanisms Drawings Box 6 (MDB-6)



Mechanism description of drawings:

MDB-7.1	Flywheel designs
MDB-7.2	Drilling tool
MDB-7.3	Rope flywheel
MDB-7.4	Bit brace tools
MDB-7.5	Brake mechanism
MDB-7.6	Couplings
MDB-7.7	Study of the curve tracing of a cart
MDB-7.8	Manual drilling tool with rope
MDB-7.9	Study of weight drag
MDB-7.10	Study of wheels tilt
MDB-7.11	One and two-axle carriage design

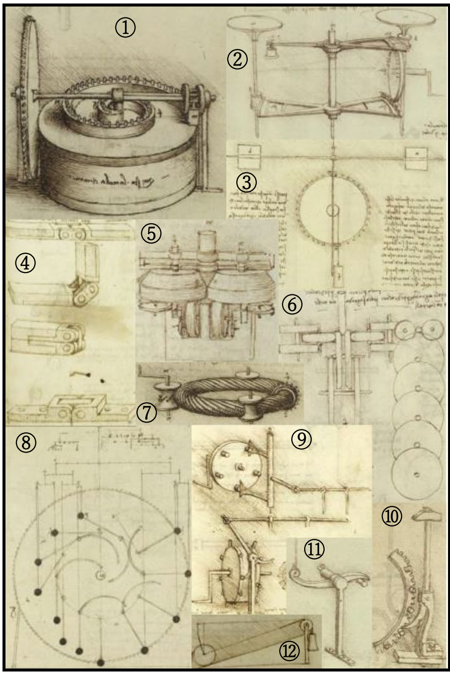
Fig. 11 Mechanisms Drawings Box 7 (MDB-7)



Mechanism description of drawings:

MDB-8.1	Wedge mechanisms
MDB-8.2	Wedge mechanism
MDB-8.3	Wedge mechanism
MDB-8.4	Springs
MDB-8.5	Grippers
MDB-8.6	Clamping system
MDB-8.7	Pliers
MDB-8.8	Crossbow deformation study
MDB-8.9	Deformation studies of a cantilever beam
MDB-8.10	Deformation study of a bi-supported beam
MDB-8.11	Deformation study of bi-supported beams of different lengths

Fig. 12 Mechanisms Drawings Box 8 (MDB-8)



Mechanism description of drawings:

MDB-9.1	Central cylindrical drum for a clock
MDB-9.2	Reciprocating transformation mechanism
MDB-9.3	Verge clock escapement
MDB-9.4	Positional study for a joint
MDB-9.5	Friction wheels
MDB-9.6	Friction wheels
MDB-9.7	Circular endless screw
MDB-9.8	Study of a mechanism for perpetual motion
MDB-9.9	Two intermittent mechanisms
MDB-9.10	Alternative mechanism actuated lever for a clock
MDB-9.11	Mechanism drive for a clock
MDB-9.12	Inclined plane mechanism

Fig. 13 Mechanisms Drawings Box 9 (MDB-9)

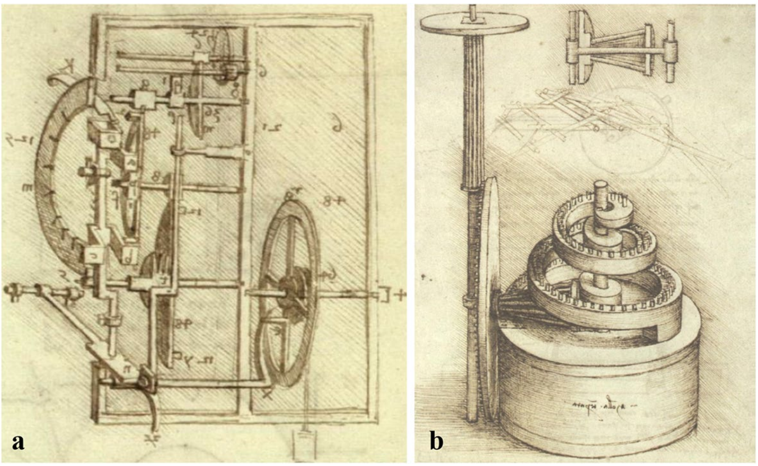


Fig. 14 Leonardo mechanical clock project drawings

Drawings of a spring-making machine (MDB-3.8), hinges (MDB-3.10) or universal joints (MDB-3.7) can also be seen in MDB-3.

The Mechanisms Drawings Box 4 (MDB-4) shows different drawings of pulley & belts systems (MDB-4.1, MDB-4.2, MDB-4.4, MDB-4.6 and MDB-4.7) and several chains drawings (MDB-4.3 and MDB-4.5). In addition, other devices and study drawings can be seen: a crane (MDB-4.10), a manual lift mechanism (MDB-4.8) or an analogy between gear and belt transmission (MDB-4.9).

In the Mechanisms Drawings Box 5 (MDB-5) several drawings of cam mechanisms (MDB-5.1, MDB-5.2, MDB-5.3 and MDB-5.4) and ratchet and pawl mechanisms (MDB-5.5, MDB-5.7, MDB-5.8, MDB-5.9 and MDB-5.12) are shown. Drawings of a water wheel (MDB-5.11), a friction measurement system (MDB-5.13) or a multi-pulley system (MDB-5.14) can also be seen in MDB-2. Especially interesting are the drawings of a three-axis gimbal mechanism (MDB-5.10) and a piston pump (MDB-5.6).

The Mechanisms Drawings Box 6 (MDB-6) shows an interesting selection of linkage mechanism drawings: linear motion mechanisms (MDB-6.1, MDB-6.3, MDB-6.4, MDB-6.5, MDB-6.8 and MDB-6.10), a four-bar mechanism (MDB-6.2), a slider crank mechanism (MDB-6.6), or Nuremberg scissors mechanism (MDB-6.7). Also shown is a drawing of a three-degree-of-freedom joint (MDB-6.9).

The Mechanisms Drawings Box 7 (MDB-7) shows several drawings of carriage studies (MDB-7.7, MDB-7.10 and MDB-7.11), a study of weight drag (MDB-7.9), drilling tools (MDB-7.2, MDB-7.4 and MDB-7.8), flywheel designs (MDB-7.1 and MDB-7.3), couplings (MDB-7.6) and a brake mechanism (MDB-7.5).

In the Mechanisms Drawings Box 8 (MDB-8) different drawings of deformation studies (MDB-8.8, MDB-8.9, MDB-8.10 and MDB-8.11) and wedge mechanisms (MDB-8.1, MDB-8.2 and MDB-8.3) are shown. Drawings of springs (MDB-8.4), grippers (MDB-8.5), a clamping system (MDB-8.6) and clamp pliers (MDB-8.7) can also be seen in MDB-8.

Finally, in the Mechanisms Drawings Box 9 (MDB-9) several drawings of mechanisms for a clock are shown (MDB-9.1, MDB-9.2, MDB-9.3, MDB-9.9, MDB-9.10 and MDB-9.11). In addition, other device and studies drawings can be seen: a circular endless screw (MDB-9.7), friction wheels (MDB-9.5 and MDB-9.6), a positional study for a joint (MDB-9.4), a study of a mechanism for perpetual motion (MDB-9.8) and study of an inclined plane (MDB-9.12).

The authors consider that the classification of the 100 drawings of *"elementi macchinali"* proposed in this work is a representative summary of the nearly 1000 drawings that Codex Madrid I contains. Despite the extent of the proposed mechanism classification, they are left out of this classification such interesting studies as the study of rotary movements generated by a flywheel and crankshaft system (Fig. 15a); or the study of a locking

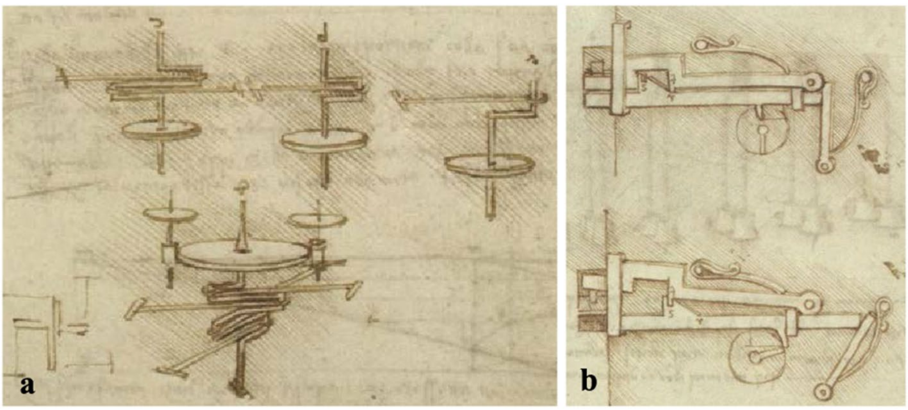


Fig. 15 Leonardo's drawings of mechanical systems: **a** flywheel-crankshaft **b** locking

system (Fig. 15b): with the key in the locked position, system locked by spring (upper part) and with the key in the open position (lower part).

The drawing of a device as interesting as the one in Fig. 14-b was also left out of the proposed classification of "elementi macchinali". The drawing shows a rotation speed regulating mechanism for a clock, but it belongs to the same study as the drawing MDB-9.1 (Central cylindrical drum for a clock), shown in Fig. 13.

6 Conclusions

Leonardo da Vinci was, above all, a master of painting, who also delved himself into many other subjects: physics, anatomy, building and machine design, and city planning, and he also left a huge written work. His pictorial genius is also reflected in his splendid drawings, of great precision and, on many occasions, of sublime beauty. However, Leonardo da Vinci protected or hid his findings and his manuscripts, which led to the opinion that much of Leonardo's scientific-technical work was speculative or not original.

In this paper, some of the main contributions of Leonardo da Vinci in the field of mechanism and mechanical elements have been shown and analysed.

After reviewing Leonardo's Codex Madrid I, this paper focuses on studying the principles of generating solutions most used by Leonardo: inspiration by nature; the principles of mechanisms or his organic vision; in-depth analysis of machines to decompose them into *"elementi macchinali";* synthesis which enables composing machines for the integration of mechanical elements and mechanisms; the analogies between different machine elements; and the economy in the design by suppressing the superfluous. All these advances allowed his mechanical designs to be better than those of his contemporaries.

The works of Leonardo and, in particular, the Codex Madrid I have been placed in their historical scope and we have noted that representative genial personalities like Leonardo da Vinci and several others, could reach the heights in the Mechanism and Machine Science thanks to the contributions of many others predecessors.

In this article, after reviewing the manuscript of Codex Madrid I, in our opinion, the most important drawings of machine elements and mechanisms are shown and described: A compilation of the 100 main mechanism drawings of the manuscript is made.

This drawings compilation illustrates the variety of mechanical elements and simple mechanisms that exist in Codex Madrid I, forming, as a whole, a comprehensive treatise on mechanisms (under-standing mechanisms as basic elements of machines), representation of the knowledge of this matter in the late fifteenth century.

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Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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