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The effects of culture on mathematics lessons: an international comparative study of a collaboratively designed lesson

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Abstract
This paper reports the results of an international comparative study conducted in Switzerland and Japan of an elementary school mathematics lesson. The principal aim of the study was to advance understanding of the cultural specificities of a mathematics lesson in its totality using concrete examples of lesson design and implementation and of how cultural factors within and beyond the classroom/school shape and produce mathematics lessons in a particular country. We analysed two Grade 4 mathematics lessons designed and implemented in Switzerland and Japan by pre-service teachers in the context of a project-based international exchange programme. The lesson, initially designed collaboratively by the pre-service teachers of the two countries, was ultimately realised in different ways in each country. Specifically, we found differences between the Japanese and Swiss lessons in the structure of the lesson and validation of solutions. To elucidate these differences and identify the cultural factors that yield them, we analysed the resources developed and used during lesson design and implementation (lesson plans, official documents, and textbooks). Furthermore, we discuss three aspects of mathematics lessons that account for the main characteristics of each lesson: collective or individualistic teaching and learning, problem-solving lessons, and distance between theory and practice.

Keywords International comparative study · Cultural context · Collaborative lesson design

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1 Introduction

Many international comparative studies have been conducted on different aspects of mathematics education (Cai, Mok, Reddy, & Stacey, 2016; Dindyal, 2014; Jablonka, Andrews, Clarke, & Xenofontos, 2018; Leung, Graf, & Lopez-Real, 2006). Large-scale studies such as PISA, TIMSS, and TALIS provide extensive information on education. These studies are complemented by smaller ones that enhance understanding of specific aspects such as students, teachers, and the mathematics curriculum (Cai et al., 2016; Jablonka et al., 2018). Among other aspects, the mathematics lesson became an object of comparative study when the TIMSS video study was conducted in the 1990s. This study highlighted differences in mathematics classroom teaching according to country and existence of the cultural scripts or patterns of mathematics lessons (Stigler & Hiebert, 1999, 2004). Furthermore, the Learner's Perspective Study (LPS) followed up on some aspects of the TIMSS video study, investigating the cultural specificities of mathematics teaching in several countries from the perspective of learners (Clarke, 2013; Clarke, Emanuelsson, Jablonka, & Mok, 2006). These studies empirically confirmed the diversity of mathematics lessons as a process of mathematics teaching and learning.

Our study is also interested in the diversity of mathematics lessons, especially the mechanism that produces such diversity. We consider this mechanism critical in understanding the extent to which the results or implications of mathematics education research obtained in a specific context are appropriate in another. While several comparative studies limit the objects of study to the main components of the lesson (e.g., students, teacher, and mathematics), as far as we know, there is a lack of research on the mechanism of a mathematics lesson in its totality, including internal and external cultural factors that the lesson actors (teacher and students) may not be aware of.

To address the issue of the mechanism of a mathematics lesson in its totality, the institutional perspective of the *Anthropological Theory of the Didactic* (ATD hereafter; Chevallard, 2019) guided us in formulating our research question. The mathematics lesson observed in a classroom is a result of the *didactic transposition* (Chevallard, 1991; Chevallard & Bosch, 2014), which is exposed to different cultural factors (called *conditions* and *constraints*) that shape the nature of mathematics teaching and learning. Teacher and students are not the only ones who play a decisive role in creating a lesson. Other cultural factors, such as mathematical contents, teaching materials, textbooks, the classroom environment (computer, number of students, blackboard, etc.), inspectors, official guidelines, and pedagogical ideas, that exist around the lesson, inside and outside the classroom, also have an influence in creating a lesson. Thus, the institutional perspective of ATD implies the need to investigate the mathematics lesson in its totality including external factors, especially when studying a lesson from the international perspective. As such, the question on the diversity of mathematics lessons is formulated as follows: How do different cultural (or institutional) factors shape and produce specific mathematics lessons in a particular country (institution)?

This paper addresses the abovementioned question through an international comparative study of elementary school mathematics lessons carried out in French-speaking Switzerland and Japan. Based on concrete examples of lesson design and implementation, we investigate the differences and commonalities of mathematics lessons between the two countries and the cultural factors that shape them. This study came into being thanks to an international exchange programme of pre-service teachers and their educators. We collected data on a collaboratively designed mathematics lesson on a common topic, which was implemented separately in each country. In comparative studies, it is often difficult to obtain comparable data of lessons with identical topics (Dindyal, 2014). As the lesson in our study was developed collaboratively, the differences we identify in the implemented lessons are deeply rooted in the educational context of each country. We think that such data enables an interesting comparative analysis revealing the specificities of mathematics lessons in the two countries and cultural factors that shape them. In the following, we present the context of our study (Section 2) and then discuss the theoretical frameworks to characterise the nature of mathematics lessons and identify the cultural or institutional factors shaping such lessons (Section 3). After describing the data analysis method (Section 4), we report the results of the analyses of the implemented lessons and resources developed or used during lesson design and implementation (Section 5). We then discuss the cultural factors, focusing on some particular aspects of mathematics teaching that highlight the differences between the two countries (Section 6). Lastly, we conclude this paper in Section 7.

2 Context of the study

The principal methodological tool of this comparative study was the collaborative development of a mathematics lesson by Swiss and Japanese pre-service teachers. This happened within the more general context of a pre-service teachers and professors exchange programme, namely the Projet d'Étudiants et d'Enseignants-chercheurs en Réseaux Sociaux (Student and Researcher Social Networks Project) (PEERS) carried out by the Lausanne University of Teacher Education (HEP Vaud). The project comprised pre-service teachers' exchanges around a jointly defined research project by a team of pre-service teachers from the HEP Vaud in association with a team from the partner university. PEERS is supervised by a professor from each institution and combines face-to-face and distance collaborative work phases. The two authors of this paper supervised PEERS with the Joetsu University of Education.

The group,[1] constituted by five Japanese and four Swiss pre-service primary school teachers and the two professors, first met through Skype meetings organised three times in fall 2017. The selected general theme of PEERS and mathematical theme was the collaborative development of a problem-solving geometry lesson for Grade 4 students. This development adopted the lesson study process (Hart, Alston, & Murata, 2011; Huang, Takahashi, & Pedro da Ponte, 2019) as follows. The group spent 1 week in Joetsu in October 2017 to design the task, study the topic, and plan a lesson together. At the end of the week, the first draft of the lesson plan was ready. During the winter, the two teams developed their lesson separately and taught it several times. The Japanese team taught the lesson twice as a mock lesson and twice to Grade 4 classes including about 35 students in the school attached to the university. In the Swiss team, each pre-service teacher taught the lesson in her/his practicum classroom of about 20 students. The class was observed by the rest of the team and followed by a post-lesson discussion. This discussion led to changes in the lesson plan for the next lesson. After three Skype meetings, the Japanese team spent 1 week in Lausanne in February 2018. During this week, the whole group observed the last Swiss lesson, watched the video of the last Japanese lesson, and discussed any differences and commonalities.

[1] The word *group* refers to the group of seven pre-service teachers and two professors. The word *team* refers to the Japanese team and the Swiss team respectively.

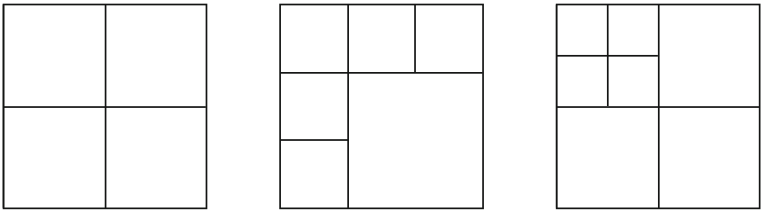

Fig. 1 Possible solutions for 4, 6, and 7

The group selected a problem from a Swiss textbook (Danalet, Dumas, Studer, & Villars-Kneubühler, 1999). The instruction is as follows: ‘‘Divide a square into several squares, but not more than 20. Find as many solutions as possible’’ (see Fig. 1 for some expected solutions). During the week in Joetsu, the members of the group analysed the problem together. They discussed the possible strategies and answers, as well as the difficulties students might encounter. For example, the group anticipated the strategy of counting the number of all squares including the overlapping ones (e.g., 5 squares for the left case in Fig. 1), while the problem requires ‘dividing’ a square; the nesting strategy of creating a square within a square; the perceptive strategy of finding squares without paying attention to the height and width of each small ‘square’ (e.g., 12 ‘squares’ by dividing one side into 3 and another side into 4); and the difficulty to come up with solutions involving different-sized squares, while it is easy to find ‘regular solutions’ for 4, 9, and 16 and so forth. Based on such analysis of the problem, the group developed a lesson plan. The lesson plan by the Swiss team is available on the Lausanne Laboratory Lesson Study website (www.hepl.ch/3LS).

3 Theoretical frameworks

This study addressed two issues. First, it identified the characteristics of the lessons observed in each country and showed the differences between them. Second, it identified the cultural elements that shape such characteristics and differences. Next, we describe the analytical tools formulated to investigate these issues.

3.1 Analytical tools to characterise the lesson

Concerns regarding international comparative studies of mathematics classrooms include establishing a common aspect to compare and developing a tool to characterise classroom activities. In our study, we focused on two aspects, namely, the structure of lessons and the roles of the teacher and students associated with it. Like previous studies that compared lesson structures (Clarke et al., 2007; Stigler & Hiebert, 1999), we also considered this a critical aspect, because it provides the overall features of students' activities and the teacher's instruction. However, the structure itself is not enough. Our interest extended to the nature of mathematics learning in the classroom. Therefore, after identifying the structure, we analysed the roles of the teacher and students in the development of the mathematical knowledge during classroom activities.

3.1.1 Structure of the lesson

Our first question was how to characterise lesson structure. Answering this enables us to identify significant phases in mathematics teaching and learning and cultural specificities. In this study, we adopted two criteria to characterise lesson structure. The first was the mode of work in terms of interaction with others in the classroom, which shows how students engage in activities. We suggested three modes: *individual*, where the student works alone; *group*, where the student works with a few peers; and *collective*, where students work in a whole-class setting guided by the teacher. These modes of work imply the social interaction of the students and the teacher in the classroom (Clarke et al., 2006; Funahashi & Hino, 2014). The second criterion was the mode of work in terms of the development of mathematical knowledge and teaching progression, which shows the different phases in which students access the mathematical knowledge being constructed in the classroom. We used four phases: *introduction*, where a problem is introduced; *research*, where students try to solve the problem; *sharing*, where students' ideas are shared or developed; and *synthesis*, where the mathematical knowledge to be taught is summarised. These four phases roughly correspond to the phases of Japanese mathematics lessons called *mondai kaiketsugata jugyō* (structured problem-solving lesson in English; see Stigler & Hiebert, 1999) and to the different situations in the *Theory of Didactical Situations* (TDS hereafter; Brousseau, 1997) of French origin. This rough correspondence was the rationale behind our choice to identify the specificities of lessons in the two countries. The former type of lesson usually consists of four or five phases: *hatsumon* (introduction of a problem), *jirikikaiketsu* (individual and/or group work), *neriage* (whole-class collective work), and *matome* (synthesis) (see also Shimizu, 1999). The latter (TDS) characterises the process of mathematics teaching and learning in terms of the states of mathematical knowledge—*situation of action*, *situation of formulation*, and *situation of validation*, corresponding roughly to research and sharing—and the process involving the teacher, namely, *devolution* and *institutionalisation*, which roughly corresponds to introduction and synthesis. Several authors have highlighted the parallel between the phases in the two traditions (Clivaz, 2015; Miyakawa & Winsløw, 2009; Shimizu, 2006). Furthermore, our four phases (introduction, research, sharing, and synthesis) can be considered as boundary objects by which the two lessons may be compared. These boundary objects play a key role in the international comparative studies by heightening both validity and comparability (Clarke, Mesiti, Cao, & Novotná, 2017).

3.1.2 The roles of the teacher and students

To closely analyse the differences and commonalities between the two lessons, we focused on the roles of the teacher and students in the problem-solving process. Many theoretical perspectives consider this focus an important aspect of mathematics learning that shows how the teacher and students are concerned with the development of mathematical knowledge in the lesson. It also enabled us to clarify the teacher's management of classroom activities and nature of the mathematical knowledge developed in the lesson. For instance, from a TDS perspective, the notion of *didactical contract*, which is implicitly established between the teacher and learners, deals with this focus. The didactical contract determines ''what each partner, the teacher and the student, will have the responsibility for managing and, in some way or other, be responsible to the other person for'' (Brousseau, 1997, p. 31). From the earlier mentioned ATD perspective, this focus is considered in terms of *the topogenesis* that accounts for the organisation of different places (or roles) occupied (or played) by different kinds of actors (students or teacher) with respect to the knowledge being constructed (Chevallard, 1991).

3.2 Perspective for discussing cultural factors

While these frameworks provide us with the aspects of classroom activities to be compared, they do not allow us to characterise the exterior factors shaping the lesson. To address this issue, we adopted a theoretical construct within ATD to investigate factors beyond the classroom (Bosch & Gascón, 2006; Chevallard, 2002). In the theory, the lesson implemented in the classroom is considered a result of *didactic transposition*, which is influenced by the *conditions* that support the realisation of such a lesson and *constraints* that hinder it. ATD implies that these conditions and constraints may have a different origin beyond that identifiable in the classroom and proposes a classification called *the levels of didactic codetermination*: *civilisation–society–school–pedagogy–discipline–domain–sector–theme–subject* (Bosch & Gascón, 2006; Chevallard, 2002). Artigue and Winsløw (2010) confirmed that this perspective enables capturing, in the context of an international comparative study, the extensive factors affecting mathematics education. We also believe that this perspective helped in identifying and discussing the various cultural elements that shape the lessons in our project.

4 Data analysis method

The data were collected from the previously mentioned exchange project. We videotaped most activities related to the collaborative development of the mathematics lesson and its implementation, including Skype meetings, discussions in the face-to-face meetings, preparatory lessons, implementation of lessons, and the post-lesson discussion. In this paper, the principal data for the comparative study were the video data of the last lessons from the two countries, and materials or resources including the lesson plans the pre-service teachers developed and used in lesson design and implementation. Regarding the lesson plans, the first draft was collaboratively written in English. Then, detailed lesson plans were written separately in both countries in the pre-service teachers' language (Japanese and French) and revised several times after the lessons. They were later translated into English for sharing in the project. The video data of the Japanese lesson were transcribed first and then translated into English, while data from the Swiss lesson were transcribed in French, and then only the parts needed to write this article were translated into English, because both authors understand French.

We qualitatively analysed these data. First, we identified the modes of work of students and teachers in the lesson based on the two aspects described earlier: individual, group, and collective work on one hand and introduction, research, sharing, and synthesis on the other. Based on this analysis, we captured the structure of the lesson showing the description of the overall classroom activities and differences between the two lessons. To visualise the comparison of the lesson structure, we used a graphical representation like in the LPS study (Clarke et al., 2006).

Next, using classroom data, we investigated the roles of the teacher and students in the problem-solving process and identified the differences and commonalities between the two lessons. Specifically, we focused on the teacher and student activities to validate answers. In addition to the importance of validation in mathematics (Balacheff, 1987; Lakatos, 1976), we believe that the differences between the two lessons are more explicit in these validation activities than in others, since the lesson was developed collaboratively and the same problem was used in both countries.

After identifying the differences between the Swiss and Japanese lessons, we further investigated the nature of the two lessons and found the cultural factors shaping them through the analysis of the resources referred to during lesson design, namely, the last lesson plans written separately by the two teams of pre-service teachers, official documents, and textbooks. Finally, we examined the cultural factors in terms of the level of didactic codetermination, focusing on the differences identified in the comparative analysis of the lessons and resources.

5 Results

In this section, we present the results of the comparative analysis of the implemented lessons, focusing on the differences in lesson structure and validation in the classroom, and the resources developed and used during lesson design and implementation.

5.1 Lesson structure: what is at the centre of a mathematics lesson?

Figure 2 shows the results of the data analysis of the lesson structure. The two lessons were of similar duration (65 min for the Swiss lesson and 60 min for the Japanese lesson), followed the lesson plan, and had the same succession of phases. In the Swiss classroom with 20 students and Japanese classroom with 35 students, most solutions were found. While the task was designed collaboratively in the face-to-face workshops organised in Japan, its implementation in Switzerland and Japan differed. Differences are evident in various parts including between the lesson structures (see Fig. 2) and in the same phase. We focused on the research and sharing phases, which comprise the main phases of the two lessons.

In the Swiss lesson, the most important phase was the research phase based on time spent (45% of the lesson) and preoccupation during lesson preparation. This phase alternated between group work and the teacher’s interventions to explain or give hints to the students or to highlight correct or incorrect solutions (see validation below). The Swiss pre-service teachers regarded the sharing phase as a conclusion of the research

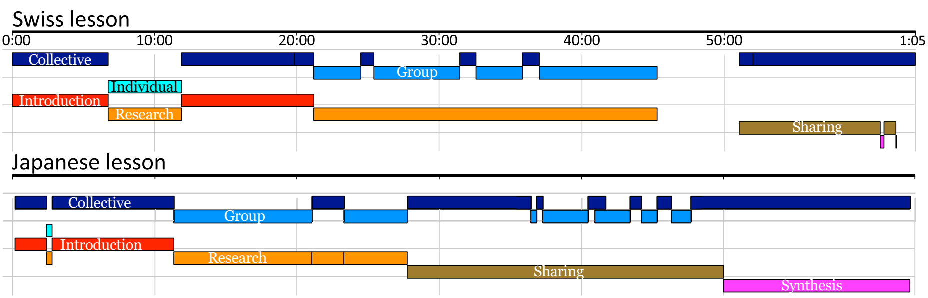

Fig. 2 Structure of the two lessons

phase, which was led by the teacher in front of the class and involved commenting on and evaluating students' solution. In contrast, the sharing phase comprised the main part of the Japanese lesson, began before reaching the halfway point of the lesson, and took the longest time (37% of the lesson). The research phase, with a few interruptions, was considered as preparation for the sharing phase. During the sharing phase, the teacher asked the class about the validity of the solutions, gave several short periods for the groups to work on specific cases, and conducted a short debate with the whole class to reach a conclusion. This phase, called *neriage*, is considered a central phase in Japanese problem-solving lessons (Takahashi, 2008; Inoue, 2011).

5.2 Validation

In both the Swiss and Japanese lessons, the solutions were validated during the research and sharing phases, which occupied more than half the class time. This constituted the main and recurring difficulties for both the Swiss and Japanese pre-service teachers in designing, teaching, and discussing the lesson. For example, the Swiss lesson plan says:

> Show on the board correct and [...] incorrect solutions (diagonal, cut in half). Define the criteria for a correct solution with the students, and write them on the blackboard to make a checklist [of incorrect solutions] to which the students will have to refer before showing a solution to the teacher.

This list reflects the main preoccupation of the Swiss team to deal with many students coming to the teacher. In fact, during the group research phase of the Swiss lesson (orange part in Fig. 2), the students went to the teacher and asked whether their solutions were correct. The Swiss teacher dealt with the students individually in front of the board as follows:

> Student: Teacher, is this okay?
> Teacher: Ah, a box inside the square. […] unfortunately, [student's name], I cannot accept your solution, because the squares in the square, in the square ...

At another moment, when the teacher got a correct solution from a student, she reacted as follows:

> Teacher: How many did you get? 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13. Yes. It's not very precise, but a solution of 13 is fine for me.
> Student: Okay! Phew.

The roles of the teacher and students are clear: The student is to solve the problem, and the teacher is to evaluate his/her solution. The teacher knows the correct solution ('fine for me').

In comparison, in the research phase of the Japanese lesson, the students did not show the teacher their solution. Instead, the teacher moved from one group to another and asked questions. For instance, the teacher intervenes when a student gave the answer shown in Fig. 3 ($4 \times 5 = 20$):

> Teacher: This one, are they really all squares? Can you think about it?
> Student: Okay. [Teacher leaves]

The teacher tried not to say whether the solution was correct or not (even though students may know that the teacher's question implies its inappropriateness). The teacher's interventions

Fig. 3 A Japanese student's solution during the research phase

during the research phase were only for incorrect solutions, never for the correct ones. Thus, regarding the roles of the teacher and students, the students had to reflect on and solve the problem in their group. It was not the time for the teacher to validate a student's answer and provide the correct answers. Rather, students had to deeply reflect on the problem. In addition, the teacher gathered data on students' answers during this time to manage the problem-solving process. In fact, the teacher intervened once to the whole class during the research phase (the collective part [21:03–23:17]):

> Teacher: Now, many of you could make 4, 9, and 16, but has any group succeeded for the other cases?
> […]
> Teacher: Please look at the blackboard for a while. How did you make it? Someone here has an interesting idea. Can you all see this? What about this?
> Student: First, there are 9 squares in this one block, and it is the same here, 9 squares for this block. 9 times 2 equals 18, and adding these makes 19, 20, so I could make 20 in this way.

The teacher shared a student's solution for 20 squares (after dividing a square into 4, 2 squares are divided again into 9). This intervention was not for the validation but to convey an important idea that the squares could be of different sizes to accelerate student's research. As such, the teacher controlled students' problem-solving time. After this whole-class moment, the class returned to the group work of the research phase.

Comparing the validation in the Japanese and Swiss lessons, the Japanese team's way of validating solutions is close to the definition of validation by Margolinas (2004) within TDS: 'The student decides by himself about the validity of his work [...] thanks to the interactions with the *milieu*' (p. 24; our translation). In comparison, the Swiss team focused on evaluation: 'The validity of the student's work is evaluated by the teacher in the form of an irrevocable judgement' (p. 24; our translation).

This characteristic is also evident in the sharing phase (bronze part in Fig. 2), usually called *mise en commun* (putting in common) in French and *neriage* in Japanese. In the sharing phases in the Swiss lesson, the teacher mainly explains the two basic strategies (dividing a square and regrouping squares) and provides the missing solutions with students' inputs. The teacher guarantees the validity of the solutions. For example, the teacher explains the strategy of dividing a square (from 4 squares to 7), as shown in the following transcripts:

> Teacher: We had 4 squares before, and then suddenly we have how many do you think?
> Student: 7.
> Teacher: 7, exactly, that's great, after a second, you're right, we get 7. In the 7 squares, if we cross again another square, how many could we get?

In the sharing phase of the Japanese lesson, by contrast, the teacher asked students to present their solution in front of the classroom individually with the video projector and to explain how they reached it. She tried to not directly validate students' answers but to transfer the responsibility of validation to the students and the class, as in the following transcripts:

> Teacher: He said he could make 15! […]
> Student: It looks like a rectangle. […]
> Teacher: He said this is 15. What do you think? Does it look okay?

At another moment:

> Teacher: 17? How about 17? [...] 17. Okay, come up here.
> Teacher: Can you tell us how you found it?
> Student: I don't know.
> Teacher: You don't know!? [student goes back] Can you all see this?

As a novice teacher, the Japanese pre-service teacher did not always succeed in letting the students or class validate their solutions. However, the differences between validation in the two lessons are clear. The pre-service teachers in both countries were also aware of these differences. After observing the Swiss lesson and watching the video of the Japanese lesson together, they reported their collective reflections during the workshop in written form in English:

> Validation of answers during "neriage": JP = other students / CH = the teacher mostly. In Japan [it] is very important to exercise the students/children to think about HOW they find a solution. By explaining from peers to peers (and not the adult explaining), other students will understand it more because it comes from another student like them. It also helps the students to confirm that he understood well the problem and it induces a discussion and deep-thinking on the topic. (Notes of workshop, PEERS week in Lausanne)

Based on the pre-service teachers' sharp and concise description of the difference between who is validating the solutions, we summarise the differences between the two lessons in two mutually related aspects of mathematics lessons. The first aspect is the overall form of mathematics teaching and learning: *collective* or *individualistic*. In the Japanese lesson, *neriage* is a moment for the whole class including students and the teacher to validate learners' answers and further develop their ideas. Even in other phases, the teacher often tries to manage the whole class (see the duration of collective work in the Japanese lesson in Fig. 2). In the Swiss lesson, the teacher individually validates students' answers in both the research and sharing phases. The second aspect is the *didactical contract* mentioned earlier. The teacher directly validated students' answers in the Swiss lesson, but not in the Japanese lesson.

Students may ask the teacher to validate their answers in the Swiss lesson, but not in the Japanese lesson. In the actual lesson, the Swiss students showed that they were used to a didactical contract where they 'need the teacher to tell me if I am right or wrong' as expressed by Sierpinska (2007). In contrast, although the Japanese teacher did not always play her role well, she did try to leave the responsibility of validation to the students in the research phase and to the whole class in the *neriage* phase.

5.3 A comparative analysis of resources

To clarify the differences between these two lessons and identify the cultural factors that yield such differences, we conducted a comparative analysis of the resources used. In the analysis of different resources, we inductively looked for the factors shaping the lesson in each country.

5.3.1 Lesson plans

The lesson plan was a principal resource developed by the Japanese and Swiss pre-service teachers when designing and implementing the lesson. In the first phase of collaborative design, they drafted the lesson plan together, then finished it in two teams in each country, and revised it according to the implemented lessons. Here, we discuss the final versions of the lesson plan.

The lesson plan produced by the Japanese team included the title, overall explanation of the experimental lesson, goal and idea of the lesson with a table showing the chronological progression of classroom teaching, plan for writing on the board, and worksheet for students. This lesson plan follows the ordinary format of a Japanese lesson plan (e.g., Fernandez & Yoshida, 2004; Miyakawa & Winsløw, 2013).

The lesson plan produced by the Swiss team included a list of necessary materials, lesson goals (also given as a list), instruction for observations, a table chronologically showing the actions of the teacher and students, and plan for writing on the board. This was provided following the Japanese lesson plan format.

An interesting difference in the lesson plans is the importance of the *collectivity* in classroom activities based on the collective teaching and learning mentioned earlier. The Japanese lesson plan includes several remarks—more than in the Swiss one—to promote the collective development of problem solving in the whole-class setting, for example, in the 'teacher's support' column:

- When an incorrect answer is given, (the teacher) takes it to the whole class and checks why it is wrong.
- To share successful cases in the whole class, students stick origami on the blackboard.
- To share a student's idea with the whole class, project the origami using the video projector.

The importance of the collectivity in the classroom should be a principal factor that shapes the nature of the Japanese lesson and the teacher's didactical choices. More time is taken for the *neriage* and sharing (see Fig. 2), which were implemented in the whole-class setting. Furthermore, a student's prominent idea was shared with the class so that the other learners could use it to further their investigation. In addition, the teacher's role during the research phase was not to individually validate students' answers, because the class collectively validates these. Even during the sharing (*neriage*) phase, the teacher occasionally asked the students to work again on specific cases, because there was a collective process of developing mathematical ideas in the flow of the lesson. Finally, a video projector was used to facilitate the sharing and collective development of mathematical ideas.

5.3.2 National or regional curricula

When writing the lesson plan, both teams referred to the national or regional curricula. The goals of the lesson in the Swiss lesson plan are cited from the geometry section of the regional curriculum (Table 1): 'Pose and solve problems to structure plan and space'.

The Japanese team, on the other hand, did not situate this lesson in the domain of geometry but in that of 'relation of numerical quantities', in which students are required to identify the pattern of changes in quantity. The goal given in the Japanese lesson plan was to 'be able to discover different ways to divide a square, identify the pattern, and apply it'.

This choice of goal was based on the national curriculum. The goal of Grade 4 geometry in the Japanese national curriculum is specific to geometric objects and does not conform to the problem chosen in our project. The Japanese national curriculum (MEXT, 2008) states the following:

> ''Through the analysis of geometric figures focusing on their components and positions, be able to understand plane figures such as the parallelogram and rhombus, as well as solid figures such as the rectangular parallelepiped''. (our translation)

This difference in teaching goal is likely another reason the Japanese lesson included the synthesis phase (*matome*), while the Swiss lesson did not. The Japanese pre-service teachers could not achieve their goal without including the synthesis phase, in which the pattern of increase of the number of squares is summarised and discussed, whereas the Swiss team could achieve their goal without this phase, as their objective was to realise the activities in Table 1.

Table 1 Goals of the lesson in the Swiss lesson plan

- Elements for problem solving (in connection with *modelling*): solving geometric problems
  - ... by imagining and using visual representations (codes, diagrams, graphics, tables, ...)
  - ... by sorting and organising data
  - ... by communicating its results and interpretations
  - ... by asking questions and defining a framework of study
  - ... by mobilising ... mathematical tools
- Pose and solve problems to structure the plan
  - ...by representing plane figures (...) using a sketch (...)
- Decomposition of a plan surface into elementary surfaces, and recomposition

5.3.3 Textbooks

Textbooks were another resource pre-service teachers used for their collaborative design. The division of squares activity was taken from a Swiss textbook. One characteristic of the Swiss mathematics textbook (Danalet et al., 1999) is that it includes a collection of problems that often do not provide explicit indication about the mathematical knowledge to learn (at least for students) and without a suggested order in which to build a teaching sequence. Problem solving for the Swiss team was built on the notion of focusing on the process of resolution, not on the acquisition of specific mathematical knowledge. This is one reason the Swiss lesson did not allocate time for synthesis.

In contrast, a chapter of the Japanese textbook consists of an amalgam of various elements such as problem situations, summaries of specific mathematical knowledge to learn, and exercises (see, e.g., Hitotsumatsu & Okada, 2015). The Japanese pre-service teachers tried to make explicit in the lesson plan specific mathematical knowledge as an objective, although they adopted a problem situation from the Swiss textbook. This is also because of the structure of Japanese problem-solving lessons (Stigler & Hiebert, 1999), which includes a summary of mathematical content.

Also noticeable is that the teacher's part of the Swiss textbook mentions that the sharing phase is a privileged moment for validation and provides the following instruction regarding how the teacher should validate students' answers:

> 'It is the students who provide the proof of what they say. The teacher is careful not to decide on good and bad proposals himself. However, the teacher demands solid validation by pushing the students to go to the end of their point of view.' (Danalet et al., 1999, p. 21)

Clear here is the distance between theory and practice in the Swiss lesson. While the curricular document prescribes the way of validation, it was not really implemented in the Swiss classroom. In comparison, this distance seemed smaller in the Japanese lesson, as the Japanese pre-service teacher tried to promote students' autonomous and independent learning, as prescribed in the Japanese national curriculum (MEXT, 2008).

5.4 Summary of results

The analysis of the data collected in the context of a project-based international exchange programme allowed us to reveal several differences between the two lessons designed first collaboratively and then implemented separately in Japan and Switzerland by pre-service teachers. Specifically, we identified the differences in the structure of the lesson and the teachers' ways of validating the solutions. In addition, the analysis of the resources developed and used during lesson design and implementation implied some cultural factors that yielded such differences. These cultural factors will be elaborated further in the next section.

6 Discussion

In this section, we discuss the cultural elements according to the levels of didactic codetermination, focusing on the three aspects of lessons referred to in the previous analyses: collective versus individualistic teaching and learning, characteristics of the problem-solving lesson, and distance between theory and practice. Additionally, we discuss the specificities of the lessons implemented by the pre-service teachers in the two countries, to what extent these lessons are representatives of each culture.

6.1 Collective or individualistic teaching and learning

A question related to the difference in the collective dimension of the lesson is what cultural factors support or hinder collective or individualistic teaching and learning in each country. One obvious factor that supports—or is even required to carry out—collective teaching and learning in the Japanese class is the number of students. In a classroom of 35 students (about 20 students in the Swiss case), it is difficult for the teacher to take care of each individually. The whole-class validation in the *neriage* phase is a solution for this constraint. In addition, the Japanese classroom is equipped so that the teacher can control a whole-class discussion and share individual work: the big blackboard in front and large display on the side (Fig. 4). Compared with this setting, the Swiss classroom's blackboard size is about half of the Japanese one, and no display is available to project students' solutions. These conditions or constraints afford or hinder collective teaching and learning at the *school level* in terms of codetermination.

Another factor that supports collective teaching is homogeneity or the idea of equality at the *society* or *civilisation level*. In Japan in general, the teacher tries to manage the whole class so that every student learns equally in the same way. Instructions are not for a particular learner in the classroom. This is why the teacher shared students' solutions in the classroom, even during the research phase. The importance of the *neriage* and *matome* phases as a dialogue with the whole class is presumably the effect of this factor. The teacher's highly developed skill of *bansho* (board writing, see Yoshida, 2005; Tan, Fukaya, & Nozaki, 2018) is also a result of this factor. On the other hand, in Switzerland and Europe, the idea of individualism and 'differentiation' is considered a way of promoting equity. What is necessary for each learner is different, and therefore, a teacher's individualised intervention for each student or each group is necessary during the research phase and the sharing and synthesis phases, since only the teacher can know individual needs.

Fig. 4 Blackboard and large display in the Japanese classroom

6.2 Problem-solving lesson

At the levels of *pedagogy* and *discipline*, we consider the shared notion of teaching in the teachers' community of each country as a crucial factor shaping teachers' role in the classroom. In Japan, the national curriculum emphasises students' autonomous and independent learning (MEXT, 2008). In general, Japanese teachers share the idea that in the problem-solving lesson, they should not directly validate students' answers, as this is the role of other students. The students in our lesson knew this *didactical contract*, and few asked the teacher to validate their answers.

In contrast, the Swiss teachers' community shares the *open problem* (Arsac & Mante, 2007) *tradition*. Here, the problem-solving process is important, not the product thereof. This effect is obvious in the Swiss mathematics textbook (Danalet et al., 1999), which includes only the problem situations for many problems and no explicit concepts or ideas for students to learn. The Swiss lesson in our project included almost no synthesis phase. Here, the teacher did not take much time to introduce the problem and did not often intervene when solving the problem, because in this tradition, 'the research must be student-owned' (Danalet et al., 1999, p. 19), except when evaluating solutions. As such, problem solving is a shared idea in both countries. However, the interpretation of this idea and its further development differ.

This difference, revealed at the *discipline* level, might be rooted in the *pedagogy* level. In Japan, the *neriage* phase is the most important part of the lesson, 'the proving ground of teachers' knowledge and skills' (Takahashi, 2008, p. 156) citing (Fujii, 2008). Here, 'solving of the problem by each student at the beginning of the lesson is a preparation for Neriage' (Takahashi, 2008, p. 150), whereas for French-speaking teachers, the research phase is the main part of the lesson and the sharing phase is considered the conclusion thereof.

6.3 Distance between theory and practice

An examination of the prescriptions of the teacher training texts in the French-speaking part of Switzerland or the French mathematics education research texts shows that they are surprisingly close to the format of the Japanese lesson. As mentioned, the structure of mathematical activities characterised in the TDS is similar to that of the Japanese problem-solving lesson. Japanese scholars such as Shimizu refer to the TDS to relate *matome* "with the function of the situation of 'institutionalisation' (Brousseau, 1997)" (Shimizu, 2006, p. 142). Furthermore, the expected mathematics learning is also similar: Students develop mathematical ideas on their own through interaction with *a didactical milieu* in the TDS and through the mathematical activities in an autonomous and independent manner in the Japanese problem-solving lesson (see, e.g., Miyakawa & Winsløw, 2009).

However, in comparison with the Japanese lesson, we found a substantial distance between theory and practice in the Swiss teachers' instructions, especially regarding the validation by students. The theory here should be understood, in a broader sense, as the ideas that affect teaching practices. In terms of the ATD, it denotes the theoretical block (or *logos*) of *didactic praxeology* that explains, chooses, and justifies the teacher's instructional choices (Chevallard, 2019). The prescriptions in the French-speaking part of Switzerland were therefore not working as logos of Swiss pre-service teachers' practices. How this is different from the Japanese lesson implies the existence or absence of a crucial condition to fulfil the gap between theory and practice. The implementation of a prescription in teaching practice is not an immediate process. Rather, it requires a means to allow teachers to practically investigate how to implement prescriptions in official documents into classroom teaching.

The *discipline*, *pedagogy*, and *school* levels of didactic co-determination seem to indicate a gap that does not exist in Japan between the prescriptions (from school authorities, textbooks, universities) and actual school classroom practices. In Japan, the lesson study or open lesson constitutes an important condition that supports teachers' activities to delve into the theory behind the practice and the practice implied by the theory (Miyakawa & Winsløw, 2013; Takahashi, 2014). Thus, the distance between theory and practice brings us to discuss a system of fundamental conditions, namely, the *paradidactic infrastructure* that supports teachers' practices in classroom teaching (Miyakawa & Winsløw, 2013, 2019) and its outcomes.

6.4 Specificities of the lessons by the pre-service teachers

To what extent are the two lessons analysed in this paper representatives of the two countries respectively? Is it possible to generalise the differences identified in this comparative study to ordinary mathematics lessons of the two countries? We consider that the lessons implemented by the pre-service teachers in our project reflect, to a large extent, the characteristics of ordinary mathematics lessons in each country. The first reason is that the pre-service teachers who participated in our project were accustomed to the mathematics lesson of their respective country through their personal experiences at schools and their teaching practicums. The Japanese pre-service teachers, who were in the fourth year of the teacher training programme, have observed the standard Japanese problem-solving lesson several times during their teacher training. In contrast, the Swiss pre-service teachers rarely experienced a problem-solving lesson as school students. Even though during their pre-service training, problem-solving lessons that included student validation were recommended in university, it was seldom observed during their practicum. The second reason is that the cultural factors we have discussed above promote or hinder a specific way of mathematics teaching, beyond the teacher's personal knowledge and skills. That is to say, they are usually not within the control of the individual teacher. However, lessons by pre-service teachers and in-service teachers are predictably different. Therefore, further studies are necessary to generalise our conclusions.

7 Conclusion

Our research showed the important differences between Japanese and Swiss mathematics lessons despite their collaborative design and identified the cultural factors that produce them. We used a project-based international exchange programme as a methodological tool for uncovering cultural factors, which are often taken for granted and rarely discussed within the country. Aligned with Chan et al. (2018), this international comparison allowed us 'to reveal a breadth of possibilities for attention and learning not currently recognised in the separate local teaching communities' (p. 94), such as the three aspects discussed above. Further, in addition to the results of past studies that have been focusing on the main components of the classroom practice (teacher, students, interactions among them, or mathematics) (Cai et al., 2016; Clarke et al., 2006), our study of a mathematics lesson in its totality moved forward our understanding on the classroom practice, in particular how the different cultural elements inside and outside the classroom affect the teaching practice in the cases of Japanese and Swiss pre-service teachers.

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