Educational Studies in Mathematics (2019) 101:141–151
https://doi.org/10.1007/s10649-019-09890-x

Rituals and explorations in mathematical teaching and learning: introduction to the special issue

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Published online: 21 March 2019

Abstract
This special issue comprises seven studies and a commentary piece, which relate to various aspects of rituals and the "ritual-exploration" dyad in learning, teaching and learning-to-teach mathematics. The theme of the special issue arose from Sfard and Lavie's (*Cognition and Instruction*, *23*(2), 237–309, 2005) "ritual" vs. "explorative" dyad, on which a Working Session was held during the PME conference in 2016. Three central themes are addressed in the papers of this special issue: (1) the logic of ritual, and why rituals are so persistent in mathematics classrooms; (2) the co-existence of rituals and explorations, including the question of whether rituals and explorations are a binary or a continuum; and (3) alternative theoretical conceptualisations of the ritual-exploration dyad, focusing in particular on what can be learned about rituals from social and critical theory, as well as socio-linguistics. In this introduction, we first give an overview of each of the papers and its unique contribution. We then synthesise the answers given by the papers to the three themes described above.

Keywords Rituals · Explorations · Theoretical dyads · Commognition · Social theory · Non-deficit discourse

The discourse of mathematics education research is replete with conceptual dyads such as "procedural vs. conceptual", "individual vs. social", "extrinsic vs. intrinsic motivation" and "mathematical dis/ability". To these well-known dyads, a relatively new conceptual pair has been introduced: r itual vs. exploration. Originally, this pair has been conceived by Sfard (2008) as types of *routines*. Ritual routines were defined as routines "whose goal (closing condition) is alignment with others and social approval" (p. 301) while exploration routines were defined as routines "whose goal (closing condition) is production of an endorsed narrative" (p. 298). Later works, drawing on Sfard, defined ritual participation as mathematical performance for

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the sake of connecting with others or "people pleasing" (Heyd-Metzuyanim & Graven, 2016). In two PME Working Sessions in 2016, we examined the affordances and limitations of this dyad and began asking questions about its connection with other prevalent dyads in the field. This special issue emerges from the robust engagement of the working group participants.

Sfard and Lavie (2005) initially coined the terms "rituals" and "explorations" based on a study of 4–5-year-old children learning about numbers. Since then, the conceptual dyad has been found useful for description of Israeli middle school learners (Heyd-Metzuyanim, 2013, 2015), South African elementary school learners (Heyd-Metzuyanim & Graven, 2016) and even instruction of pre-service teachers (Heyd-Metzuyanim, Tabach, & Nachlieli, 2016). Moreover, "ritual participation" has been paired with "ritual instruction" connecting learning and teaching practices (Heyd-Metzuyanim & Graven, 2016).

The growing number of studies looking into ritual and explorative participation has led to the emergence of several central questions. The first regards the relationship between the two types of participation as being consecutive or parallel. Originally, Sfard and Lavie (2005) suggested that ritual participation is an antecedent of explorations, often an inevitable practice that supports learning at the peripheral stage when participants do not have sufficient conceptual tools to follow the logic of the discourse and must rely on imitation. Despite the theoretical appeal of this conjecture, the evidence collected so far points to an alternative possibility that students participating predominantly ritually advance in a parallel trajectory (often leading to failure) to those who participate more exploratively (Heyd-Metzuyanim, 2013, 2015). This alternative is a possibility since, until now, little empirical evidence has been published about students who *started* participating ritually and then progressed to explorative participation.

Another question relates to the dichotomy of ritual and exploration. Is this dichotomy justified? Or is a take on ritual and exploration as a continuum more appropriate? And if a continuum is more appropriate, what are the characteristics of the ground that lies between ritual and exploration? Finally, "ritual" has been proposed by Sfard (2008) as a metaphor. She cited the religious studies scholar Huston Smith in the opening of her chapter on explorations, deeds and rituals, quoting: "Rituals help us … to connect deeply with people… The repetition that ritual always involves sets the present moment in a larger context and infuses it with wider meaning" (Smith, n.d., cited in Sfard, 2008, p. 222). Other than that, however, she did not base her notion of "ritual" on any specific study of rituals in domains outside of mathematical learning. Others in the domain of mathematics education (McCloskey, 2014) have used the term "ritual" in a more anthropological sense, relying on social theorists such as Emile Durkheim (1912/1965) and Catherine Bell (1997). In McCloskey's use, ritual is not paired with another term but rather stands on its own, as a social phenomenon worthy of investigation inside and outside the classroom.

In this special issue, the first paper, by Irit Lavie, Aya Steiner and Anna Sfard (2018), presents the latest developments in Sfard and her colleagues' conceptualisation of ritual routines and the process of "de-ritualisation" (the process by which ritual routines turn into exploration routines). In their paper, Lavie et al. set out to extend their earlier work by refining and operationalising more clearly the concept of "routine". To define routine, the authors introduce the concepts of *task situation*, the situation in which a person feels the need to act, and *precedent*, any past event judged as similar enough to this current task situation to justify doing now what was done then. Routine evoked in a given task situation is now defined as a "task-procedure" pair, where task comprises all elements of precedent events a person considers need to be repeated, and procedure is a prescription for actions performed then that have to be applied in the present situation as well. In this way, Lavie et al. (2018) refine former

definitions of routines proposed by Sfard (e.g., 2008), where routine was defined according to the "how" and the "when" of a procedure. In addition, the refined definitions in this paper move away from the earlier Sfard and Lavie (2005) definition, where rituals and explorations were defined according to their goal (connecting with others vs. producing mathematical narratives). In doing so, Lavie et al. attempt to circumvent the problems associated with finding "goals" of routines, which were not operational enough, and more specifically, did not help in accounting for the actual moves of routine performers. The concepts of "task situation", "precedent" and "de-ritualisation" form significant extensions of Sfard and Lavie's (2005) earlier work, providing a spectrum of intermediate possibilities between ritual and exploration. In addition, the refinement of definitions provides a clear set of analytical and operational tools for analysing routines in learning-teaching discourse. The paper also provides a reflection on the 12-year journey from Sfard and Lavie's (2005) earlier work towards these current understandings.

Departing from Sfard and Lavie's (2005) conceptualisation of rituals and explorations, Alf Coles and Nathalie Sinclair (2018) offer an alternative view of rituals. Building mostly on the work of Bell (1991) and Manning (2016), they concentrate on the term "ritualisation". Coles and Sinclair claim their approach "challenges the more common approaches to ritual that dichotomise thinking and acting", counteracting what they read as previous accounts in mathematics education literature of ritual as meaningless performance. According to their definition, ritualisation

> are those practices in the mathematics classroom that: (a) set themselves apart as distinct and privileged compared to other activities (e.g., by following a fixed agenda or occurring at periodic intervals); (b) are embedded in a symbolically structured environment; and, (c) do not bring what is being done across the threshold of discourse or systematic thinking.

Moreover, ritualisation is a useful concept for blurring the divide between thinking and doing, mind and body. These theoretical claims are supported by data from Canada and the UK, illustrating young children's learning of numbers via a Gattegno chart.

In the same vein, viewing rituals (or ritualisation) as a given, perhaps even "natural", social phenomenon, Andrea McCloskey, Gwendolyn Lloyd and Courtney Lynch (2017) enter a US elementary classroom with a view of rituals as a lens for examining "the cultural nature of teaching, learning and learning to teach mathematics". This lens, according to McCloskey et al., is suggested as a means to assist us in understanding the persistence of ritual practices in US classrooms (and beyond) despite decades of promoting "reform" practices. Taking on an ethnographic methodology, McCloskey and her colleagues (2017) show that the same characteristics of rituals identified in cultural studies (formalised, traditionalised, symbolic and performative) could also be found in mathematics classroom practices.

Sally-Ann Robertson and Mellony Graven (2018) examine a South African classroom where students' limited mastery of the language of instruction (English) provides a constant source of struggle for the teacher aiming for more exploratory talk in her classroom. Their study builds on socio-linguistic theory to stress the limiting of opportunities for exploration when students (as well as the teacher) must teach and learn mathematics in a language other than their mother tongue. Robertson and Graven do not use the term "ritual", but instead draw on Barnes's (2010) dyadic notion of "right answerism" vs. "exploratory talk" to reveal that the former, while providing a short-term survival mechanism for mathematics classroom talk, undermines meaning making opportunities.

Taking us to the domain of university mathematics education, Olov Viirman and Elena Nardi (2018) present biology students' ritualised and exploratory participation in mathematical modelling activities in a Norwegian university. Their study examines the back and forth movement in the students' activity between exploratory and ritual practices and connects these practices to movement between discourses: the biology discourse with which students in the study are relatively more comfortable, and the discourse of difference equations which is relatively unfamiliar to the students. The paper focuses particularly on a quintessential routine in mathematical modelling, assumption building, and traces students' starting from ritualised engagement in the shape of "guesswork" and gradually turning towards exploratory formulations of the assumptions underpinning their models. This study exemplifies the continuous nature of the ritual-explorative dyad, and as well as shines light on the potential of linking mathematical activities to problems from other domains (such as biology) as a means of encouraging explorative participation.

Continuing in this vein, Talli Nachlieli and Michal Tabach (2018) examine the possible reasons for the prevalence of instructional practices that afford opportunities for ritual participation. Starting from a definition of "ritual-enabling" vs. "exploration-requiring" instructional routines, they map nine videotaped lessons from different English-speaking countries in the TIMSS video study. First, they observe the prevalence of ritual-enabling routines in all (but one) lessons. They then foreground the nested nature of these routines and show that there is often a back-and-forth movement between exploration-requiring and ritual-enabling routines. Finally, drawing on commognitive theory (Sfard, 2008) and their current observations, they propose several reasons for the prevalence of ritual-enabling routines. These include preparation for meta-level learning, assisting students in making their first steps in a new discourse and establishing procedures as a basis for explorations.

Einat Heyd-Metzuyanim, Margaret Smith, Victoria Bill and Lauren Resnick (2018) take us from the domain of students' learning of mathematics to teachers' learning to teach mathematics in professional development settings. Adopting Sfard's (2008) perspective on ritual as a form of legitimate peripheral participation (Lave & Wenger, 1991), Heyd-Metzuyanim and her colleagues (2018) report on the actions of two middle school US teachers who are making their best efforts to adopt dialogic, cognitively demanding teaching practices. These authors show how characteristics originally developed to describe ritual routines in mathematics—namely rigid rule following, imitation of more knowledgeable others and lack of coherence between procedures and goals—can be used to describe the two teachers' initial attempts to implement Smith and Stein's (2011) "5 practices for orchestrating productive mathematics discussions". These practices include choosing cognitively demanding tasks, anticipating students' responses, monitoring their work, sequencing and linking students' presentations and using Accountable Talk™ (Michaels, O'Connor, & Resnick, 2008) in classroom discussions. Gradually, as they make more and more attempts, these teachers' instructional practices become more explorative, that is, more flexible, coherent and connected. The authors conclude that the interpretation of teachers' learning as movement from ritual to explorative participation can shed light on not only why "reform" or "dialogic" teaching practices are so rare and difficult for teachers to adopt, but also how professional development efforts are probably too short-term to achieve the desired change.

The final paper in this special issue is a commentary, authored by Jill Adler. Adler (2019) reflects and ponders about the different issues and questions raised by the seven papers, focusing in particular on the notion of routines. We will pick up on her reflections in the concluding section of this introduction to the special issue.

In the call for papers to be contributed to this special issue, we encouraged authors to relate their work to one or more of the following themes:

1. *The logic of ritual* as a way of moving from deficit models that concentrate on what is *not being done* or learnt in the mathematics classroom towards understanding possible drivers for the persistence of what is being done.
2. *The coexistence of rituals* and explorations rather than the polarisation of these two modes of participation.
3. *Other views of ritual*, different from Sfard's (2008) commognitive framework, that have been used in the literature and that may shed light on, or add to, our understanding of rituals and explorations.

In what follows, we synthesise the insights developed from the seven papers in this special issue regarding these themes.

1 The logic of ritualsIn what follows, we synthesise the insights developed from the seven papers in this special issue regarding these themes

In one way or another, all the papers in this issue relate to the logic of ritual, rituals or ritualisation. The most prominent statements about this theme, however, come from four papers by Lavie et al., Nachlieli and Tabach, McCloskey et al. and Coles and Sinclair. Each of these papers offers a somewhat different explanation of why rituals in mathematics learning and teaching are so prevalent.

Lavie et al., continuing previous works in the commognitive lineage (Lavie & Sfard, 2016; Sfard, 2008; Sfard & Lavie, 2005), explain rituals simply as being the initial, necessary step that novices make into a new discourse community. Through their lens, ritual is the entry ticket, the sine qua non of any learning process. The necessity of ritual stems from the lack of sufficient precedents in the precedent-search-space (PSS) of the child—those responses that are associated with a certain task situation. This limited PSS leaves the child only with the possibility of imitating an adult (or expert) in interpreting the task situation and performing a procedure. Yet, the necessity of ritual beginnings in no way means that rituals need not give way to explorations. In fact, the process of learning is precisely the de-ritualisation of routines, namely expanding the PSS of the learning for certain task situations. The persistence of rituals, Lavie et al. claim, stems from the situated nature of school learning. In school, one's searches for precedents are unlikely to exceed the boundaries of school mathematical discourse. The "walls" separating discourses are too high, they claim, for school mathematics to be truly connected to most people's lives. Moreover,

> Ritualization is as if inscribed in the mission of school: The mathematics teacher aims at equipping students with procedures that are generally considered as potentially useful in a wide variety of situations. It is thus the way students perform numerical calculation, solve equations, prove theorems, or explore functions that the teacher is trying to monitor most closely.

Since the focus on the "how" (procedure), rather than the "why" or "when", represents the epitome of ritual routines, traditionally, therefore, school learning gravitates towards rituals almost by definition and against the teacher's better judgement.

Nachlieli and Tabach continue this line of thought; yet, they seem to offer a more optimistic vision of rituals or "ritual-promoting" instruction. First, they claim, similarly to Sfard (2008) and Lavie et al., that ritual-enabling routines are necessary for certain learning outcomes. In particular, they focus on the movement from "object-level" learning, which consists of "expanding one's discourse about a familiar mathematical object" to meta-level learning, which "involves a change in the meta-rules of the discourse, a transition to a discourse in which words are used differently". Whereas object-level learning can proceed seemingly uninterrupted via exploration-requiring routines, meta-level learning, by definition, seems to require an expert to step in and lead the learner. Nachlieli and Tabach show this nicely in the case of a US teacher attempting to teach students the rules of exponents. Whereas learning to derive the rules of $a^m a^n = a^{m+n}$ or $a^m{:}a^n = a^{m-n}$ (object-level) proceeds smoothly in this 8th–grade classroom along explorative routines, the rule that $a^0 = 1$ encounters insurmountable difficulties. Nachlieli and Tabach explain that this rule, unlike the preceding ones, in fact requires a meta-discursive shift. The students need to learn that unlike in previous rules, which were justified similarly to the rules of the arithmetic discourse by relying on tangible properties of the natural numbers, here, the reasons for justifying a new rule are based only on consistency with previous discourses. Nachlieli and Tabach claim that there is no way for the students to pass through this meta-level shift without the guidance of the teacher, who indeed turns to ritual instructional routines in the face of the students' difficulties.

Taking a wider view of rituals as a form of cultural activity, McCloskey and her colleagues (2017) seek "to understand participants in mathematics classrooms as both creators and products of culture: inheritors, perpetuators, constructors, and subverters of the practices that have occurred before and are occurring around the participants' own practices." Their analysis of a 5th-grade classroom in the USA reveals that classrooms share characteristics of other cultural activities that have been characterised as ritual. These include the activities being formalised, traditionalised, symbolic and consisting of performance. McCloskey et al. show that even those practices that have been introduced by "reform" ideas become subjected to ritualisation:

> Practices that are typically associated with "reformed", "ambitious", or "student-centered" teaching practices, such as requiring students to show their mathematical work, incorporating number lines as representations, and drawing on multiple resources when planning mathematics instruction, can sometimes undermine reasoning and sense-making.

For these authors, ritual is thus not exactly a phenomenon that should be explained, but rather a lens through which cultural patterns can be made visible in the classroom.

Finally, Coles and Sinclair, similarly to Lavie et al., argue that ritual (or ritualisation) is a necessary phenomenon that should not be avoided and rather should be acknowledged for its beneficial role in learning. Within their conceptualisation, ritualisations involve "repetition; a symbolically structured environment; and little or no attempt to bring what is being done across the threshold of discourse". Coles and Sinclair argue that most of the conceptualisations of ritual in mathematics education contrast ritual activity with thinking, understanding or "mathematical ways of knowing". They suggest we should see ritualisation as "a way of privileging certain activities over others", rather than as a rote action or submission to authority. This alternative view, they claim, avoids the dichotomisation of thinking-doing which underplays the role of the body and embodiment in learning.

2 The coexistence of rituals and explorations

As explained above, ritual was originally proposed by Sfard and Lavie (2005) as an antonym of explorations. Yet, presentations in the PME 2016 Working Session suggested that the two forms of participation (or routines) may sometimes co-exist or that the division between ritual and explorations may not be so clear-cut. Three papers in our special issue relate most prominently to this point: those by Viirman and Nardi (2018), Nachlieli and Tabach (2018) and Heyd-Metzuyanim et al. (2018).

Viirman and Nardi declare at the outset that the dichotomisation of rituals and explorations may be unhelpful:

> Students' reasoning is often described using binaries such as *instrumental* and *relational understanding* (Skemp, 1976) or *creative* and *imitative reasoning* (Lithner, 2008). Although perhaps not initially intended as such, binaries like these are often used as dichotomies, presupposing a strict divide between the two (Nardi, 2017). Indeed, this is sometimes the case also concerning the treatment of the commognitive constructs of ritual and exploration.

To explore the "dichotomisation" of rituals and explorations, Viirman and Nardi analyse the discourse of first year university biology students working on modelling problems, focusing in particular on routines of assumption building. They show in these routines the back-and-forth movement seen in students' participation between ritual engagement with certain mathematical procedures (e.g., related to the discourse of difference equations that is relatively unfamiliar to the students) and explorative engagement within the relatively familiar biology discourse. They conclude that:

> … a more fluid view of the apparent dichotomy of the ritual-exploration dyad – where rituals morph into explorations through recognizing flexibility and broadened applicability – might help tap into the great interpretive potential that we, and others (e.g., Presmeg, 2016) see in the commognitive framework.

Nachlieli and Tabach (2018) take another approach to the question of the ritual-explorative binary. They endorse the binary and even substantiate it by offering clear operationalisations of teaching routines that afford explorative vs. ritual participation. However, they also show that rituals and explorations (or, in their words, "ritual-enabling" and "exploration-requiring" opportunities to learn, OTL) almost always co-exist in a lesson and are even nested within each other. After analysing nine lessons from around the world and finding that all but one included a combination of ritual and explorative OTLs, they declare:

> … to lead to explorative mathematical discourse, teaching should include both ritual-enabling and exploration-requiring opportunities to learn. The proper mixture of those can only be finetuned by the teacher, over time, to meet the needs of her teaching goals.

Finally, Heyd-Metzuyanim and her colleagues (2018) adopt the ritual-explorative binary but show how the progression from ritual to explorative engagement in the teaching practice of orchestrating classroom discussions is slow and gradual. The two teachers examined in their paper are shown to start from relatively rigid imitation of the discussion-orchestration moves that they observe being enacted by the leaders of the professional development program. For example, they choose tasks that are cognitively demanding but not appropriate for their students' ability, and use discussion-eliciting talk moves without really inquiring into students' thinking. Gradually, their actions in the classroom

become more explorative, connected to the specific goals of the lesson and to the students' thinking processes. Yet, this movement is so gradual that it cannot be described in dichotomous terms. Heyd-Metzuyanim et al. (2018) state that often "the process of change from ritual to explorative participation tends to be slow and may never be completed".

This claim echoes the theorising of "de-ritualisation" of Lavie et al. (2018) as the process by which ritual routines become explorative routines:

> In between ritual and exploration, there is a whole spectrum of intermediate possibilities, which in the context of learning may be seen as stages in the development of a routine.

3 Alternative views of ritual

Three papers in this special issue relate directly to our call for alternative perspectives to Sfard and Lavie's (2005) "commognitive" perspective of rituals and explorations: those by McCloskey et al., Coles and Sinclair and Robertson and Graven. Each of these papers offers alternatives in different ways. Whereas McCloskey et al. and Coles and Sinclair focus on alternative views of ritual anchored in social theory, Robertson and Graven offer an alternative socio-linguistic view of the ritual-explorative dyad.

McCloskey et al. (2017) build on the first author's previous work regarding ritual as a lens to view culture in the classroom (McCloskey, 2014). This theorising offers the following definition of ritual, which is taken from the critical theorist Richard Quantz (2011): "Ritual is that aspect of action that is formalised, traditionalised, symbolic, and performance".

By "formalised", McCloskey et al. (2017) mean, for example, the rigid use in a 5th-grade classroom of a number line for representing fraction multiplication, even though other representations may be more meaningful for students. This "formalisation" is connected, according to the authors, with the number line being the single form presented by the textbook for solving fraction multiplication and division problems.

By "traditionalised", McCloskey et al. (2017) refer to certain classroom practices being impacted by tradition and older generations. For example, the KFC (keep, flip, change) procedure for solving fraction division is brought into the classroom through the older sibling of one of the students. This occurred despite the teachers trying to avoid the procedure in order to promote "understanding".

The "symbolised" aspect of ritual prompted the authors to attend to a certain practice that has meaning for the participants "over and beyond the specific situational meanings". For example, in their study, teachers freeing themselves from the textbook was regarded by the teacher leader and the young teacher advised by her to be a symbol of professional expertise.

Finally, the "performance" aspect of ritual is exemplified in McCloskey et al.'s (2017) study by the practice, observed in the 5th-grade classroom, of asking students to "show your work". "Showing your work" was treated as an indication of conceptual understanding. Yet, sometimes, students who did show understanding (at least according to the researchers' interpretations) were forced to "show their work" in ways that were limited to only that which was considered an acceptable method (or an expected method) by the teacher.

We thus see the four elements of ritual providing McCloskey and her colleagues with a means of sifting through and highlighting practices in the classroom that may go unnoticed, being so transparent to the local participants in this classroom culture. This is a very different approach to ritual from that of studies in the commognitive lineage, since it does not attempt to

characterise certain routines (or practices) as ritual vs. others that are not. Nevertheless, McCloskey et al. conclude by relating to issues that are also captured by the commognitive definition of ritual. For example, the "formalised" and "traditionalised" aspects, which anchor action in external authority (textbook, teacher leader), are very similar to Sfard and Lavie's (2005) original characterisation of rituals as relying on external authority. Also, the "performance" aspect, at least as it is exemplified in McCloskey et al., is related to a focus on procedures rather than end results, precisely the aspect of routines that earns the "ritual" label according to Lavie et al.

Coles and Sinclair (2018) offer a different view of ritual, focusing on the term "ritualisation". They build on the work of the religious studies scholar, Catherine Bell, whose book "Ritual theory, ritual practice" (1991) theorises rituals as providing insight into "the cultural dynamics by which people make and remake their worlds" (p.3, cited in Coles & Sinclair, 2018). For Coles and Sinclair, the important aspect of Bell's theory lies in that she "seeks to develop a way of thinking about ritual that does not take the a priori assumption that thought and action are disjointed". On this reading, "Ritualisation … does not distinguish action from thought but rather marks what is sacred from what is profane". Coles and Sinclair critique former conceptualisations of ritual, including that of Sfard and Lavie (2005).

> Sfard states that rituals are "about performing, not about knowing," because there is "no room for a substantiating narrative." This framing of ritual reifies the dichotomy between thought and action identified by Bell, inasmuch as it erects a clear distinction between doing and thinking, even if not a sharp boundary.

Having defined ritualisation as "the privileging of certain activities over others", Coles and Sinclair (2018) turn to examine their own repetitive actions (serving as the teachers of 1st-graders in two different settings) when working with young learners on developing automatic and fluent number naming, with the goal of developing the learners' number sense. Coles and Sinclair make unique usage of discourse representation (in the form of special spacing and indentation to give a sense of the discourse's rhythm). In doing so, they show how sense-making and repetition are intricately interwoven in the pointing of the teacher to numbers on the Gategno chart, coupled by the students' rhythmic chanting of the number names. They conclude that:

> In both cases [analysed in the paper], we see ritualisation at work: the setting apart as distinct and privileged activity; the environment is symbolically structured; there is little or no attempt to bring what is being done across the threshold of discourse, in the sense of discussing why particular patterns exist or what they are.

Coles and Sinclair conclude that their conceptualisation of ritualisation as privileging certain actions over others not only offers a way to avoid "dichotomies of performing/knowing and memorising/knowing" but also helps to avoid teaching dilemmas such as "whether to work on 'understanding' or 'fluency' first", a question, which they add "arises from the separation of knowing and acting".

Unlike Coles and Sinclair, and McCloskey et al. (2017), who focus on offering an alternative to the conceptualisation of "ritual", Robertson and Graven (2018) propose an alternative dyad—namely "right answerism" vs. "exploratory talk". This dyad is anchored in the socio-linguistic field and brings into focus the importance of students having a level of linguistic fluency in the language of learning and teaching (LoLT) in order for classroom talk to progress from imitative talk to exploratory talk. They draw on Barnes's (2010) notion of "right answerism" vs. "exploratory talk" and the distinction made by Cummins (1979) between basic interpersonal communication skills (BICS) and cognitive academic language

proficiency (CALP) to analyse the classroom talk practices of a grade 4 teacher. This teacher attempts to elicit exploratory mathematics talk, but her efforts are frustrated by her learners' limited linguistic fluency in English (the LoLT). Her learners have limited BICS (let alone CALP) in English and yet they are expected to engage with the specialised language of mathematics in the classroom in English. Since her learners do not speak English at home and have only limited exposure to English in their out-of-classroom environment, classroom mathematics talk is almost doomed to remain at the level of imitative repetition of classroom-established mathematics narratives. Robertson and Graven argue, as indeed do others, that learners require access to their home language as a resource for "talk(ing) their way into understanding" (Barnes, 2010, p. 7). Thus, drawing on socio-linguistic theorists and constructs, they reveal the additional challenges faced by teachers when attempting to shift classroom talk towards exploratory talk in a language that is unfamiliar to their learners. This said, the notions of right answerism and exploratory talk correspond in several respects to Sfard and Lavie's (2005) ritual explorative conceptualisation. Right answerism can be seen in the paper as a primarily imitative discourse serving "teacher pleasing" goals while exploratory talk (though largely absent from the transcript) was considered as "a ready tool for trying out different ways of thinking and understanding" (Barnes, 2010, pp. 7–9) en route to producing mathematical narratives for their own sake (i.e., explorative participation in Sfard and Lavie's, 2005 terms).

4 Conclusions

The seven papers in this special issue offer a wide and innovative view on the role of rituals and explorations in mathematical teaching and learning. The studies vary in their geographical settings (USA, Israel, Norway, South Africa, Canada, UK, Australia and Hong-Kong), participants' ages (1st-graders to university students and teachers), content matter (learning mathematics and learning to teach mathematics) and theoretical frameworks (commognition, social and critical theory, socio-linguistics). These papers exemplify the potency of the "ritual-exploration" dyad (or of ritual concept as a stand-alone concept). Yet, they also open up new questions. These are picked up and reflected upon thoughtfully in Adler's (2019) commentary. One of the questions Adler raises is whether it is possible to avoid deficit discourses and the ways by which the "ritual-exploration" dyad feeds into these discourses. Adler examines the papers closely and concludes that even those claiming to avoid deficit discourses inevitably lapse into deficit descriptions when describing the emergence of new actions or practices. She concludes that "it is impossible in our work to point to new presences … if these are not juxtaposed with descriptions of their prior absence."

An associated question concerns the connection between studies of the ritual-exploration dyad to literature in other domains. Adler mentions, in particular, the literature on educational development focusing on "reform" efforts in developing countries "that drew attention to the 'widespread failure' of educational aid programs promoting learner-centred pedagogy". She thus asks, "could research inspired by the various binaries in educational discourse on the one hand, and routines as unit of analysis on the other, also benefit from engaging in conversation?"

We join Adler in her hope that the seven papers in this *Educational Studies in Mathematics* special issue spur further research on the roles of rituals, ritualisation, de-ritualisation and explorations in the teaching and learning of mathematics. We wish the reader a pleasant explorative tour!

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