




Young learners' mathematics-related affect: A commentary on concepts, methods, and developmental trends

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Abstract

This article is a commentary for the special issue on affect and mathematics in young children, written from the perspective of research on affect in mathematics education. The studies in this special issue focus on the individual learners' affective traits and use primarily surveys as the method. The most common type of affect is emotions, but some studies do examine student beliefs and motivation. The analysis of concept definitions and operationalizations identified some inconsistencies between the different articles, especially with how they operationalize anxiety either as sadness, worry, or fear. The results of the studies provide evidence that young learners' affect can be reliably measured and that there is a correlation between affect and achievement. This correlation is weaker than for older students and longitudinal data suggests that the causal direction is more likely from achievement to affect.

Keywords Affect · Anxiety · Early learning · Mathematics education

1 Introduction

The image of mathematics is that of cold logic and rationality. Yet, it invokes a lot of emotions in people. How and when does mathematics become a source for anxiety or pride? The current special issue is looking at the early years of mathematics learning, from kindergarten to first years of primary education. The selection of research articles is valuable for bringing into focus an area that has been somewhat marginal both for research on mathematics-related affect and for research on early learning of mathematics.

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I come from the area of research on mathematics-related affect. There, the focus has often been on the negative disposition towards mathematics around adolescence, when affective problems emerge as a critical issue (e.g., McLeod, 1992; Metsämuuronen & Tuohilampi, 2014; Tuohilampi, Hannula, Laine, & Metsämuuronen, 2014). This special issue's focus on mathematics-related affect in early learning provides an interesting opportunity to explore the origins of this problem and also to critically re-evaluate the existing theories of affect that have been primarily informed by studies on adolescence.

I can think of three main reasons to be interested in affect in mathematics education. First, emotions, beliefs, and other affective phenomena are important elements of mathematical thinking. For example, our understanding of mathematical problem solving would be seriously limited, if emotions were not accounted for (for a review, see, e.g., Hannula, 2015). Second, affect can be seen as a learning outcome, an indicator of past learning experiences. This is relevant in studies like PISA that examine the effectiveness of educational systems. Third, affect is relevant as a predictor of future mathematical behavior. Together with mathematical achievement, also mathematics-related affect influences choosing non-compulsory mathematics, dropping out, career choices, and future mathematical achievement.

The current special issue is highly relevant for the first and the third reason to study mathematics-related affect. While we know quite well the role that affective elements play in problem solving for older students, our understanding of these cognitive-affective problem solving processes for the young learners is rather limited. In their articles, Pietro Di Martino and Maria Chiara Passolunghi, Elisa Cargnelutti, and Sandra Pellizzoni give us additional insight into this area. In a similar fashion, research on the predictive aspect of affect has been predominantly conducted among students older than in the current special issue. Therefore, the articles by Janne Lepola and Minna Hannula-Sormunen and by Pietro di Martino shed some light onto issues so far poorly understood. Specifically, it is interesting to look at the development prior to children entering schools.

The remaining two articles have other values that make them important from the perspective of affect in mathematics education. Research on mathematics-related affect has to a large extent been driven by good instruments. As the measurement of affect in younger populations has specific problems, the testing and further development of an attitude survey by Dominic Petronzi, Paul Staples, David Sheffield, Thomas Hunt, and Sandra Fitton-Wilde has potential to make long-lasting impact. Finally, comparative research in mathematics-related affect (e.g., Bofah & Hannula, 2015; Tuohilampi et al., 2015) points out that findings about affect and even the structure of affective variables are often different across countries. Therefore, the comparative study between young learners from the UK and Chinese Hong Kong by Ann Dowker, Olivia Cheriton, Rachel Horton, and Winifred Mark is an important contribution as it tests the universality of the findings regarding early learners' mathematics-related affect.

I will begin this commentary with a critical analysis of how the five studies have defined the concepts they used and how these have been operationalized. Then, I will continue to discuss the findings of the studies from the perspective of how affective variables relate to each other, how affect develops, and how affect is related to the development of achievement. I will then look at what variables are used to explain affect and, as the last point, I will look at what the articles have to tell about affect in the problem solving processes of young mathematics learners.

2 Concepts and their operationalizations

The field of affect is conceptually complex as discussed over the years (e.g., Furinghetti & Pehkonen, 2002; Hannula, 2012; Hannula et al., 2016; McLeod, 1992; Zan, Brown, Evans, & Hannula, 2006). I have suggested that the theories and terminology for affect can be structured using three dimensions: (1) The type of affect: cognitive (beliefs), motivational (desires), or emotional (feelings); (2) The temporal grain size of analysis: rapidly changing affective states or the relatively stable affective traits; and (3) The scope of theorizing: physiological (embodied), psychological, or social (Hannula, 2012). I will use these dimensions to analyze the articles in this special issue.

The articles are similar to each other with respect to two of these dimensions. Each of the studies looks at affect as a relatively stable trait and theorizes affect as an individual rather than a social phenomenon. Only Passolunghi and colleagues extend their theoretical frame slightly to the area of neurophysiological theories as they include working memory and processing speed as factors in their analysis.

The main distinction between the papers is the type of affect they focus on. In the current special issue, the focus is clearly on emotions, but also beliefs (Dowker et al.; Di Martino) and motivation (Lepola and Hannula-Sormunen) are addressed.

There are three studies that included anxiety as a concept. Closer examination of these articles identifies some interesting differences in how they define and operationalize anxiety. In their article about the development of a mathematics anxiety instrument, Petronzi and colleagues describe mathematics anxiety as “worrisome thoughts” and measure it using a scale with items consisting of three simple emoticon response choices: happy, uncertain, and sad. The items are formulated as event descriptions with a statement ending with the words “I feel.” Also, Dowker and colleagues claim to focus on the “predominantly cognitive ‘worry’ component,” and the items measuring anxiety include the question “how worried or relaxed you would feel” and the response options vary from “Mr Worry” to “Mr Happy.” The third article using the term anxiety by Passolunghi and colleagues defines anxiety “as a feeling of tension or fear” and measures it “by asking participants how anxious they feel” on a Likert scale from “no fear” to “very much fear.”

These differences illustrate the more general “problematique” of defining and operationalizing the affect concepts. On the level of definition, Passolunghi and colleagues are interested in anxiety as an emotion, while the other two articles define it as cognitive “worry.” However, the survey items and the response options indicate other differences between the three papers. Petronzi and colleagues ask about emotions: feeling happy, uncertain, or sad; Dowker and colleagues ask also about emotions, but somewhat different emotions: feeling worried or happy; and Passolunghi and colleagues ask yet a different emotion: fear. The differences highlight that the distinction between emotional and cognitive dimension of affect is blurred. Although worry is defined as a cognitive component in anxiety research, it essentially taps onto the dimension of subjective experience (e.g., Buck, 1999) of emotions. Moreover, these three papers operationalize anxiety along three different pairs of negative and positive emotions: happy–sad, worry–happy, and fear–no fear. However, when you look at it, these are not the same thing.

In addition to anxiety, Dowker and colleagues measure also “Self-rating,” “Liking,” and “Unhappiness.” In my own framework (Hannula, 2012), self-beliefs are in the cognitive dimension of affect while the other two belong to the dimension of emotions.

Moving to research methodology, the three papers mentioned above use Likert-type self-report scales. This has been a typical approach for studying mathematics-related affect for a long time (see, e.g., Leder & Forgasz, 2006; Liljedahl & Hannula, 2016; McLeod, 1992). Likert-type surveys are designed for adult respondents, and the method does not necessarily suit well studying younger populations. To better accommodate young respondents, Petronzi and colleagues tested their participants in small groups and read the items aloud. A similar solution was made by Dowker and colleagues, who tested each participant individually, read the items aloud, and, moreover, used pictorial rating scales.

However, Likert-type surveys were not the only methods used. Di Martino and Lepola and Hannula-Sormunen used somewhat different approaches to measuring affect.

Di Martino used self-reports as well, but with a narrative approach. He analyzed the stories children had written to identify the three components of their attitude towards mathematics: emotional disposition, the vision of mathematics, and the perceived competence in mathematics. The important distinction here is that it is the researcher who determines the affect of the child, not the child him- or herself.

Lepola and Hannula-Sormunen did not rely on self-reports. Instead, they asked teachers to rate the child's task-related behavior, which was then used to determine child's motivational orientation on three dimensions. However, the dimensions ended up having high correlations with each other and it seems that at this age, the teacher observations differentiated these three orientations only minimally. This is not a specific problem to this study, but a more general issue in research on mathematics-related affect. The affective components are typically highly correlated and it is often difficult to disentangle one from the other.

There is one more concept to discuss, the Spontaneous Focusing on Numerosity (SFON; Lepola and Hannula-Sormunen), even if it is not clear whether SFON belongs to the affective domain. Lepola and Hannula-Sormunen define SFON as a "separate attentional process." Although attentional processes are usually considered as cognitive processes, I argue that SFON includes also affective elements. To some extent, SFON is like a belief in that it influences how the child sees the world—whether and where they see mathematics around them. A child with high SFON behaves as if having an implicit belief that quantities are relevant. SFON has also a motivational element in that it influences how the children interact with a potentially mathematical situation, whether they decide to focus on the mathematics in the situation. In a way, they have a motivation to focus on quantities. Yet, SFON seems rather unlike beliefs or motivation.

To summarize, the articles in this special issue focus primarily on the emotional dimension of affect, with less focus on motivation and beliefs. All articles conceptualize affect on the level of the individual and look at the relatively stable affective traits. The articles differ in their definitions and conceptualizations, and the reader should be aware that the use of terminology is—as typical for the area of mathematics-related affect—somewhat inconsistent across the different articles.

3 Structure of affect

Research on mathematics-related affect often highlights the need to study affect as a structure (see, e.g., Hannula, 2011 or Liljedahl & Hannula, 2016). It is not surprising that the positive emotions, beliefs, and motivation tend to correlate positively with each other. However, when one of our own studies allowed us to compare the structures of different age groups, we observed more coherence (i.e., scale reliabilities and correlations between variables were

higher) in grade eight than in grade four (Hannula & Laakso, 2011). This would suggest that the reliabilities and correlations might be even lower for early learners. Yet, in the current study, all quantitative affect measures had good reliabilities.

While the scale reliabilities were good in the studies reported here, two studies raise some concerns regarding the problems of measuring early learners' affect. First, Petronzi and colleagues' study was purposefully aiming to develop an instrument to study early learners' mathematics anxiety. The confirmatory factor analysis of an earlier established two-factor structure failed to reach good fit indexes. While the new exploratory analysis produced an acceptable single-factor solution, some elaboration is warranted.

In educational settings, data are almost always clustered in schools and classes and, consequently, a learner's affect often has a teacher effect. It is likely that this effect is stronger in the beginning of education, when the learners have little experience with school mathematics. The pupils in Petronzi and colleagues' study came from two schools and probably just a handful of teachers. Hence, one possible explanation for the different factor structures is that the teachers in the latter study did not represent the full variation of teachers in the earlier (and larger) study. Whatever the reason, the fact that factor analyses suggested different factor structures in two different samples underlines the need to use large samples and to use multilevel analyses when studying early learner affect.

Dowker and colleagues' comparative study raises similar concerns regarding the universality of how learner affect is structured. We see that the correlations between variables are quite different in the UK sample and the Chinese sample, leading to quite different regression models. This may indicate a difference between the two countries in general, but in the light of what was just discussed above on the Petronzi and colleagues study, it is possible that the observed differences are between teachers and schools selected in the samples rather than between countries.

4 Development of affect and achievement

The developmental trend of affect has been well confirmed to be declining over the school years (e.g., McLeod, 1992), especially the transition from primary to secondary school being detrimental to mathematics-related affect (e.g., Michael, Panaoura, Gagatsis, & Kalogirou, 2010). Yet, for example, Metsämuuronen and Tuohilampi's (2014) data show a decline in affect already from grade three to grade six. In this special issue, Di Martino provides evidence for the negative development taking place already over the first years of school, between grade one and grade three in all the three components of attitude they examine: vision, emotional disposition, and perceived competence. The other longitudinal study, by Lepola and Hannula-Sormunen, does not show a decline in motivational orientations from kindergarten to grade one, but their methodology that used teacher evaluations might not provide comparable measures for this transition from one school level to another.

One big question in research about mathematics-related affect is the relationship between affect and achievement. The positive correlation between affective variables and achievement has been well documented in the mathematics education literature (e.g., Dowker, Sarkar, & Looi, 2016; Leder & Forgasz, 2006; McLeod, 1992). However, determining causal relationships has been more difficult.

There are two main theoretical perspectives to the relationship between these two overall constructs. One line of theorizing emphasizes how learners' affective relationship with

mathematics is strongly influenced by experiences of success and failure. This line of thinking highlights the relevance of mathematical attainment to the development of mathematics-related achievement. However, there is another strong line of research looking at how negative affect is detrimental to learning. Mathematics anxiety, low self-efficacy, and lack of motivation influence learning negatively and consequently future mathematics attainment declines. Both of these lines of research have accumulated lots of empirical evidence, and there is strong evidence that the relationship between self-beliefs and achievement is reciprocal (Hannula, Bofah, Tuohilampi, & Metsämuuronen, 2014). Also, the relationship between mathematics-related emotions and achievement seems to be reciprocal, with the effect from achievement to emotions being more dominant (Hannula et al., 2014). Moreover, there seems to be a developmental trend where the direction of effect seems stronger from achievement to affect in young learners and it gradually shifts so that for young adults, the main direction of effect is from affect to achievement (Hannula et al., 2014; Hannula, Majjala, & Pehkonen, 2004; Metsämuuronen, 2017).

In this special issue, three papers address the relationship between affective variables and mathematics performance, shedding more light to the early development of the reciprocal relationship between affective variables and mathematics achievement. Lepola and Hannula-Sormunen examined the longitudinal development of mathematical competencies and motivational orientation from kindergarten to grade two. Their results show that at the entry to school, pupils' mathematical competencies have an effect on their motivational orientation, and that this soon becomes a factor influencing further learning. Passolunghi and colleagues' study provides further evidence that learner affect (mathematics anxiety) influences their mathematics performance in grade four. However, Dowker and colleagues study calls for caution. While their study of first graders found both self-beliefs and unhappiness to be significant predictors for performance in the English sample, the affective variables proved to not be significant predictors in the Chinese sample. Hence, it seems that the direction of the effect in the early ages might differ across countries. Taking together all these three studies, we can see that the correlations between performance and affect are statistically significant, but lower than typically among older learners. Moreover, the dominant direction in this age group seems to be from achievement to affect rather than vice versa.

5 Factors influencing affect

Another approach to research on mathematics-related affect has been to examine the effects of different individual and contextual background variables. In this special issue, such analyses on the effects of socio-economic background or school type are mostly missing.

One of the well-established findings in mathematics-related affect research is the gender difference (e.g., Leder & Forgasz, 2006; McLeod, 1992). While the gender differences in mathematics attainment seem to be influenced by the test type and the educational context, the gender differences in mathematics-related affect during adolescence have been found consistently. Except for self-efficacy, these differences seem to be much smaller or even non-existent in early learning (Dowker et al., 2016). In this special issue, only Dowker and colleagues analyzed the effect of gender and even they had the information about gender only for their UK sample. Their results show that already in first grade male learners have more positive affect than female learners.

6 Affect and problem solving

Two of the studies in this special issue look specifically at problem solving. However, they define problem solving in quite different ways. Passolunghi and colleagues define arithmetic problem solving as a primarily linguistic problem of interpreting the information and the question, and thereafter choosing and completing the right procedure. Remarkably, this is what Di Martino in his article criticizes as stereotypical problem solving leading to mechanical solution approaches. These types of problems are essentially different from non-routine problems that are the primary interest for research in mathematical problem solving.

Passolunghi and colleagues' findings show that anxiety correlates negatively with performance in arithmetic problem solving in grade four, and that anxiety explains more of the variation in performance than processing speed or working memory. These results provide additional evidence for the importance of anxiety also for young learners' problem solving performance, where evidence has been conflicting.

Di Martino's study, on the other hand, shows that the children's understanding of what a problem is and how they approach the problems is largely influenced by school mathematics. In kindergarten, children perceive problems in a broader sense, and they feel competent to solve problems. By grade three, their approach to problems has narrowed and their self-confidence and enjoyment as problem solvers has decreased. These results suggest that many of the difficulties for teaching problem solving competencies might be related to the early stages of introducing problem solving in schools.

7 Conclusions

Research on mathematics-related affect has gradually extended to new theoretical avenues, to new methodological approaches, and to new populations. The studies in this special issue give some snapshots of the growing literature of affect in early mathematics learning.

The studies in this special issue reflect the tradition of using survey instruments to understand the development of affective traits. They show that affect can be measured reliably in such young populations. The results indicate that the correlation between affect and achievement exists already very early, even if this correlation is weaker than for older learners. The dominant direction of effect seems to be from achievement to affect.

As these studies come almost exclusively from European countries, results may be not universally generalizable. Comparative studies of early learners may well show larger differences in affect than what is so far observed in older populations, because the young learners have shorter histories in a universally rather homogenous school mathematics education.

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