

Ritualisation in early number work

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Published online: 5 May 2018
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Abstract In this paper, we propose a way of thinking about ritual that is new to mathematics education research and that challenges the more common approaches to ritual that dichotomise thinking and acting. We argue for a material, monist conceptualisation of ritual, which we refer to as ritualisation. In the context of early number work, we show that ritualisation can be seen as meaningful—and not simply as rote repetition lacking mathematical sophistication—particularly in relation to a symbolically structured environment. We argue that ritualisation practices can allow entry into new fields of activity and discourse, without going through a phase of merely un-thinking performance.

Keywords Ritualisation · Ritual · Early number · Number naming · Symbolically structured environment

1 Introductory theorising

The notion of ritual has played a vital role over the last century in the study of religion and society, by scholars of anthropology, sociology and history of religion. As religious studies scholar Catherine Bell (1991) writes, the recent, more interdisciplinary research on ritual has provided insight into the “cultural dynamics by which people make and remake their worlds” (p. 3). Not surprisingly, various approaches to the concept of ritual have emerged, each differing in their interpretation of its function and nature. Rituals

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might be seen as serving to maintain certain structures or quieting certain fears; they might be seen as non-verbal forms of communication; or, they might even be seen as ways to regulate human environmental interactions (Rappaport, 1999). In her critique of the vast literature on the notion of ritual, Bell points out that its emergence was shaped by an initial “bifurcation of thought and action” (p. 6), in which ritual is seen as standing in opposition to the conceptual aspects of religion and culture (such as belief, reason, symbols and myths). From this standpoint, rituals are thus seen as thoughtless actions, habits or elements of mimicry that are *merely* physical.

In order to move beyond this dichotomising, Bell chooses to develop the notion of ritualisation, which is carefully shaped in order to focus on practice (which involves thinking *and* acting). In this paper, we will draw on Bell’s notion of ritualisation and study its presence in the context of mathematics teaching and learning. This choice of working with ritualisation (instead of ritual) is motivated by our own ontological commitments, in which we seek to blur the boundary between thought and action, situated as we are in the theories of enactivism (Maturana & Varela, 1987) and inclusive materialism (de Freitas & Sinclair, 2014). Though arising from different traditions, our respective ontological assumptions are both monist in orientation and therefore materialist in refusing to see matter (including the body) as passive and inert, and thus choosing to see continuity and fluidity amongst mathematical concepts, human bodies, human discourses and physical objects. This commonality has motivated our interest in Bell’s concept of ritualisation.

Common across both inclusive materialism and enactivism, therefore, is the view that an individual does not end at the boundary of the skin and that the social, cultural and political are enmeshed in the physicality of each of us (Bateson, 1972; de Freitas & Sinclair, 2014). From the perspective of these stances, if considering the learning of mathematics by children in school, we need to expand the typical view of the individual, to include relations that extend outside the skin and, at the same time, inquire of larger systems, such as the social, cultural and political, how these make a difference to the relations and materiality that constitute each individual.

We use ritualisation as a means of troubling the assumption that certain kinds of physical actions that can occur in the mathematics classroom are necessarily devoid of meaning and agency, or are *merely* repetitive, procedural or formalised, rather than involving the kind of thinking usually associated with conceptual or discursive practices. We are aware that some recent work on ritual, arising in the publications of Sfard and colleagues (e.g., Heyd-Metzuyanim, Tabach, & Nachlieli, 2016; Sfard, 2008; Sfard & Lavie, 2005) has interesting similarities to and differences from ritualisation that we will explore later in this paper. We will suggest that it becomes difficult to explain the emergence of new thinking, if a division is maintained between thought and action.

In the next section, we provide a short overview of the ways in which ritual has been taken up and used in mathematics education research. Then, turning to Bell’s notion of ritualisation, and offering a more elaborated sense of the concept, we describe how it avoids dichotomising thought and action and point to the ways in which the idea of ritualisation differs from and can help illuminate current research directions in mathematics education. Using some empirical data that we analyse in terms of ritualisation, we elaborate some potential insights and classroom implications for taking seriously the idea of ritualisation. Drawing on our analysis, we suggest that ritualisation has a powerful role to play in thinking about the learning of number.

2 The construct of ritual in mathematics education research

Researchers in mathematics education have employed the concept of ritual at different scales, ranging from the larger cultural scale of mathematics education as an institution to the much smaller scale of individual student or teacher activity. In terms of the former, Lundin (2011) suggests mathematics education itself is a ritual practice, following the neo-functional anthropologist Roy Rappaport. In his *Ritual and Religion in the Making of Humanity*, Rappaport (1999) describes the nature of rituals in terms of the following features: they are largely determined by others (and not by the performers themselves); they take place in spaces separate from other cultural activities, following their own rhythm and schedule; they are “formalized, punctilious, carefully supervised and controlled” (Lundin & Christensen, 2017, p. 28). Links between religious and classroom rituals are present in the work of Vinner (2007), who views rituals as meaningless, in themselves, but as having central functions in human life, such as giving “security when accomplished” (p. 6) and helping individuals develop their identity. Similarly, for Rappaport, the meaning of the ritual is not “told,” it is acted out; the meaning becomes apparent, to the performers themselves as well as potential spectators, via the structure of the activity itself. There is an echo here of Bateson’s (1972) work on ritual in which he suggests that in a ritual the “name” becomes “the thing named” (p. 409), notably in the transubstantiation of the Catholic communion in which the wine *is* the blood of Christ, not just a representation of it. Later in the article, we suggest that, as mathematicians, we often work in just this manner, with symbols becoming the things symbolised, for example, while they are being manipulated or transformed.

McCloskey (2014) draws on Quantz’ (2011) work in her conceptualising of ritual, which she uses as a means to understand the culture of mathematics education—and especially how that culture manages to push back against reform initiatives. Similarly to others, Quantz defines ritual as “that aspect of action that is formalized, symbolic performance” (p. 36). By performance, he emphasises that rituals are not “simply instrumental,” but rather, are meant to be witnessed by others. The performance is symbolic in that it “represents things indirectly through association” (p. 38). Finally, it is formalised in that the ritual has an expected temporal and spatial organisation. McCloskey’s focus is on classroom-level rituals, rather than on rituals that might be identified in particular instances of mathematics learning or mathematics teaching, which is the phenomenon that we are interested in.

This latter type of accounting can be found in Sfard’s (and colleagues) work on ritual, which focuses on discursive practises of both teachers and students. However, Sfard’s concept of ritual aligns with a more colloquial sense of ritual and is distinguished from both another types of discursive routine, namely *explorations*, and from practical routines, called *deeds*. The three kinds of routine differ according to the perceived goal of the performance. Deeds and explorations are geared, respectively, to changing the world and getting to know it. In contrast, rituals are socially oriented: they are acts of solidarity with those with whom they are performed (thus described, Sfard’s rituals have much in common with those of Quantz). Rituals are also seen as nascent forms of young children’s contact with the world. Indeed, if thinking is a special case of communication, and if communication begins as an inter-personal affair, this contact may only develop through other people. Sfard (2008) argues that ritualised discursive performance should not be put in opposition with what is called meaningful activity (again, this is a point of

view shared by most anthropologists working on ritual). Nonetheless, she writes that, “the whole point in the ritual action is that it is strictly defined and followed with accuracy and precision so that different people can perform it in identical ways (possibly together)” (p. 244). Finally, Sford states that rituals are “about performing, not about knowing,” because there is “no room for a substantiating narrative.” This framing of ritual reifies the dichotomy between thought and action identified by Bell, inasmuch as it erects a clear distinction between doing and thinking, even if not a sharp boundary.

A different understanding of ritual, as involving reason, can be found in the writings of mathematics educators interested in the origins of mathematical concepts, such as Dick Tahta (see Tahta, 1998) and David Pimm (see Pimm, 1993), and those drawing on the historian of mathematics Abraham Seidenberg (Sinclair & Pimm, 2015). Seidenberg, in turn, draws on the much earlier work of the more structuralist approach to ritual of Lord Raglan (1936/1975). In this work, rituals are related to myths, where the latter are seen as “a narrative linked with a rite [ritual]” (p. 117). Citing Harrison (1912), Raglan writes: “A *mythos* to the Greek was primarily just a thing spoken, uttered by the *mouth*. Its antithesis or rather correlative is the thing done, enacted” (p. 328, *italics in original*). As Harrison notes, myths are made with the mouth. Etymological links suggest that the Proto-Indo-European roots of the word ritual are “to reason, count”. We see here an interesting suggestion that doing (ritual) is reasoning and talking (myth) is “just” a correlate. This conceptualisation goes some way in troubling the simple dichotomizing of thinking and acting.

3 From ritual to ritualisation

As mentioned above, Bell seeks to develop a way of thinking about ritual that does not take the a priori assumption that thought and action are disjointed. As with Sford, she also focuses on performances, but connects them with a particular strategy:

Ritualisation is a way of acting that is designed and orchestrated to distinguish and privilege what is being done in comparison to other, usually more quotidian, activities. As such, ritualisation is a matter of various culturally specific strategies for setting some activities off from others, for creating and privileging a qualitative distinction between the ‘sacred’ and the ‘profane,’ and for ascribing such distinctions to realities thought to transcend the powers of human actors. (1991, p. 74)

Ritualisation, on this reading, does not distinguish action from thought but rather marks what is sacred from what is profane. While some ritualisations might be formal and punctilious, Bell does not take this as some kind of defining characteristic:

That is to say, formalizing a gathering, following a fixed agenda and repeating that activity at periodic intervals, and so on, reveal potential strategies of ritualisation because these ways of acting are the means by which one group of activities is set off as distinct and privileged vis-à-vis other activities. (p. 92)

In other words, it is the privileging of certain activities over others that is the function and purpose of ritualisation. This might be done through formality and punctiliousness but need not be. Indeed, in her reflections on the rituality of art-making, and in line with Bell’s view of

ritualisation, Manning (2016) emphasises the variation within repetition that arises in ritual, writing that they are “ever-changing, altered by conditions of futurities in the making” (p. 68). Like Heraclitus’ adage that one can never step in the same river twice, Manning evokes the way in which a repetitive practice initiates change, both in the practitioner and the event. In contrast to characterising rituals as unthinking and formalised, therefore, on Manning’s reading, they are “capable of shifting the field of experience” (p. 67).

Far from being set in opposition to thought, both Bell and Manning—in different ways—see ritualisation as a particular form of thinking. For Bell, it is non-discursive thinking:

Ritualisation is embedded within the dynamics of the body defined within a symbolically structured environment. An important corollary to this is the fact that ritualisation is a particularly ‘mute’ form of activity. It is designed to do what it does without bringing what it is doing across the threshold of discourse or systematic thinking. (p. 93)

With Bell, we resist making the jump from acknowledging something does not cross “the threshold of discourse or systematic thinking,” to assuming it is therefore meaningless or unthinking. For Manning, rituals activate; they “invent forms of value emergent from the ritual itself” (p. 71). That these forms of value are not explanatory, like Sfard’s substantiating narrative, does not mean they are to be excluded from experiences of knowing and thinking. In fact, we will be arguing that ritualisations can be part of sophisticated mathematical activity.

4 Ritualisation in the mathematics classroom

Like Sfard, we are interested in the activities of students and teachers in the classroom rather than in broader, institutional phenomena. But we are interested in investigating ritualisation as a means of disrupting the boundary between acting/performing and knowing in mathematics education research. We define ritualisation as those *practices* in the mathematics classroom that: (a) set themselves apart as distinct and privileged compared to other activities (e.g., by following a fixed agenda or occurring at periodic intervals); (b) are embedded in a symbolically structured environment; and, (c) do not bring what is being done across the threshold of discourse or systematic thinking.

In this section, we report on empirical data arising from two settings, drawing out elements of ritualisation in order to ground some of the suggestions above and, through reflection on the data, to develop further our theorisation of the potential for ritualisation in the mathematics classroom. In particular, we are interested in how the notion of ritualisation might help us understand significant aspects of learning mathematics—and in particular, learning number—that may be overlooked by more narrow definitions of ritual.

The data we draw on comes from joint research we have been conducting, in the UK and Canada, around the early learning of number (e.g., see Coles & Sinclair, 2017). Through the use of particular tools, to support learning, our aim was to explore ordinal approaches to developing number sense, starting with work to develop automatic and fluent number naming of any part of the count sequence. One tool we have been using, and that features in both sets of data on which we report, is the “Gattegno tens chart” (see Fig. 1). This chart can be used initially to work on number naming (the teacher

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000

Fig. 1 An example of Gattegno’s whole number tens chart

might tap on “300”, “50” and then “7”, the children choral speak back in unison “three hundred (and) fifty-seven”). The teacher might also tap on a number and get children to choral speak back the number that is one greater, or ten greater, or one hundred greater, or ten times bigger, or ten times smaller, etc. What the chart makes available is the structure of our written number system, not via a cardinal linking of numerals and objects, but via a more ordinal or relational sense of making links amongst numerals themselves, for example, how the spoken number names form patterns, and how you can get from one number to the next (see Coles, 2014, for further possibilities for working with children on the chart). The Gattegno chart is one example of a “symbolically structured environment” in the mathematics classroom.

4.1 Participants and settings

The UK portion of the research occurred in a primary school context in which Alf worked with a grade 1 student who had been identified, by the classroom teacher, as the lowest attaining child in the class. There were 13 sessions in total, which each lasted approximately 25 min, involving the use of the Gattegno chart and an iPad app *TouchCounts* (not reported on here). Before each session, Alf planned a small number of tasks to offer the student, tasks that were usually developed in collaboration with Nathalie, based on her viewing of the video recordings taken at the previous session. These tasks were designed to take advantage of the particular affordances of the chart.

4.2 Methodological approach

In terms of analysing and reporting on our research, we lean towards reporting on “telling” rather than “typical” examples (Rampton, 2006, p. 408) with the aim of sensitising the reader to new possibilities around the nature and significance of ritualisation rather than asserting causal connections. Our approach is one of “particularization” (Krainer, 2011, p. 52), i.e., looking in detail at particular cases in order to draw out more general principles, as we seek “paradigmatic examples” (Freudenthal, 1981, p. 135), two of which we offer below. Also, since we are interested in the way the concept of number is created in/through/with the Gattegno tens chart—in which body actions are central, both gestures on the chart and movements between the children—we pay particular attention to the rhythms of voice, gesture and body position. Indeed, we will use the poetic transcription method from linguistic anthropology, which aims to document repetition and rhythm in discourse (Staats, 2008).

4.3 Working one-to-one (UK)

The transcript¹ that follows is from the first session Alf worked with Aidan (a pseudonym), and is at the start of the video recording.

So, what I'd like to do,
can I point to some numbers
and you tell me what they are?

Okay

So if I said that one

[*Alf points to 2*] **Two**

[*Alf points to 3*] **Three**

[*Alf points to 6*] **Six**

[*Alf points to 8*] **Eight**

[*Alf points to 9*] **Nine**

[*Alf points to 7*] **Seven**

[*Alf points to 5*] **Five**

[*Alf points to 1*] **One**

You know all of them!

[*Alf points to 3*] **Three**

Lovely, so, what about that one

[*Alf points to 300*] **Three
hundred**

[*Alf points to 400*] **Four hundred**

[*Alf points to 500*] **Five hundred**

[*Alf points to 600*] **Six hundred**

[*Alf points to 700*] **Seven hundred**

[*Alf points to 800*] **Eight hundred**

[*Alf points to 900*] **Nine hundred**

[*Alf points to 700*] **Seven hundred**

[*Alf points to 200*]

Two hundred

[*Alf points to 100*]

One hundred

¹ In this transcription and those that follow, student voices are in bold font, actions are in brackets and italicised, the teacher voice is in plain text. Spacing and indentation aim to give a sense of rhythm.

Wow, fantastic!
So, any other number on here
you want to do?

I want to do that one

[Aidan points to 20,000]

Right!
What did you tell me that one was?
[Alf points to 2,000] **Two
thousand**

So have you got any guess for that one?
[Alf points to 20,000] **I don't know**

Well, do you know that number?
[Alf points to 20]
because can you see it's
[Alf points to 20,000]
got that number
[Alf points to 20]
at the beginning
[Alf points to 20,000]

Do you know that number?
[Alf points to 20]

[Aidan's eyes narrow and nose scrunches (a grimace?),
he glances at the camera and shakes his head.]

Similar to the hesitation in his first answer on the hundreds row, with the thousands row, Aidan does not know how to say 3000 initially, but once Alf has supplied the word “thousand”, he appears confident saying all the others, thus showing awareness of the structure. Even in this short time, it seems as though Aidan has caught onto something about the symbolic structure of the chart and how symbols in each row are to be verbalised. And it is the symbolic structuring of the environment that allows Aidan the opportunity to develop this awareness of structure, in amongst a repetitious and perhaps seemingly “unthinking” activity. We note he is able to make these extensions despite being unsure about how to say “20” (both in the context of saying “20,000” and when asked to say “20” in the tens row).

Moreover, in relation to Bell's insistence that ritualisations are meant to privilege and distinguish, we see in this transcript the setting off of one set of actions from the other—in this case, the privileging of big numbers that are related in a particular way. Alf does not point to the numbers between 11 and 20, nor to numbers outside of the rows of 10s, 100s and 1000s; instead, he marks a particular class of numbers as being worth saying, worth repeating, worth attending to.

Something hard to convey in transcript form is that the rhythm of Alf's asking a question followed the pattern of Aidan's responses. In other words, when Aidan answered quickly, another question followed quickly. When Aidan took longer, Alf's pace slowed. Agency was shared, in other words, with Aidan performing a ritualisation that was being led by Alf, but exerting influence over events.

The final element of ritualisation that we observe, relates to it being “designed to do what it does without bringing what it is doing across the threshold of discourse or systematic thinking” (Bell, 1991, p. 93). Alf does not, for example, try to link the number names with “place value” columns,

nor focus particularly on the relative size of each row (beyond saying to Aidan that three thousand is less than three million). The aim of the activity, that we have a snapshot of in this transcript, seems to be a fluent naming of number symbols. There is no attention paid to “why” questions. There is an arbitrary (Hewitt, 1999) element to our number naming, i.e., the initial labels in English “one” to “nine” and (ignoring the exceptions of the numbers 11–19) the words for subsequent power of ten, “hundred”, “thousand”. Alf provides these labels as and when they are needed and, in a sense, because they are arbitrary, there would be nothing to bring across the threshold of discourse or systematic thinking beyond the labels themselves. More significantly, therefore, Alf also does little, verbally, to point to the patterns within the chart *across* the rows. The one place he does this is at the end of the transcript reported, when he attempts to draw Aidan’s attention explicitly to the “20” in “20,000” (we note this was not successful). It would have been possible to draw explicit attention to the repeated features of counting in hundreds and thousands but this was not done. As adults, confident in counting, we draw on number names, certainly below a certain size, with no deliberation at all. We do this repeatedly and without bringing our performance (of number naming) across the threshold of discourse or systematic thinking, hence, we argue, through ritualisation. Of course, the performance itself is discursive but, the point is that, in general, there is no discourse *about* our naming. One possibility open to Aidan in this activity appears to be that he could, from the outset, begin operating with number naming in the same process of ritualisation as the expert counter—without the labour of having to go through a phase of deliberate and systematic thinking before the activity becomes automatic.

We recognise there is a danger that what we are saying here could be read as advocating starting learning mathematics through a rote approach in which students develop a behaviourist response to a stimulus with no thought about what they are doing. However, we are arguing that it is the potential for symbolically structured environments that allows a possible entry into ritualisation, in the context of learning mathematics, which can be meaningful, fast and energising for the learner. There are no manipulatives or other concrete metaphors for number, which need to be translated into number symbols to be used effectively. Operating directly on the numbers, doing and knowing come together: the doing of the talking, as well as the visual choreography involved in reading the table, is precisely what it means to be effective at number naming.

4.4 Working with a whole class (Canada)

Concurrent to Alf’s work with Aidan, research conducted in Canada was led by Nathalie and took place in a grade 1 French immersion classroom (English speakers being taught mathematics, as well as other school subjects, in French). The classroom was chosen because of the teacher’s interest in trying both the chart and *TouchCounts* app, which she had learned about during her Master’s programme. Nathalie worked closely with the classroom teacher to develop lessons using both tools. Sometimes the classroom teacher would lead the lesson and sometimes Nathalie would do so. The lessons involved whole classroom interactions, sometimes at the whiteboard and sometimes using a projector connected to an iPad. Many of the tasks used in the classroom were the same as the ones used in the UK setting. Nathalie visited the classroom once per week for twelve weeks in total and each lesson lasted approximately an hour. The lesson we report on here was the first visit.

After having asked children to name particular numbers on the Gattegno tens chart, Nathalie changed the task and asked them to say which number comes after a given number. They had correctly responded to the prompts of 25 and 27, when a student asked, “Can we do one on the bottom row?”, indicating the 100s row. The transcript has been translated into English, from the French. If the original was in English, this has been noted through the use of brackets.

What is this number here?

touching 600

Whoa!

Six hundred

And what is the number after six hundred?

touching 600

Seven hundred

Noooo! It is not seven hundred!

Noooo. That is seven hundred.

touching 700

But the number after six hundred?

touching 600

Six hundred...

touching 600

ONE!

touching 1

Six hundred...

touching 600

ONE!

touching 1

As French immersion grade 1 children, knowing the name for 600 is quite an achievement. And it is not surprising that the children said that the number after 600 would be 700. Although incorrect, it generalises from a ritualisation practice that worked well for stating the number after 6 and even the number after 26 (change the 6 to a 7). Nathalie offered the correct response, saying the number six hundred and one while touching 600 (which is towards the bottom right of the chart) then 1 (which is at the top left), in a large gesture. The word “one” was said slowly and in a very low tone. The children did not speak or move. Nathalie repeated the words and gestures again, this time faster. As in the episode we analysed above, the meaning of the number 601, in the context of this activity, emerged from the symbolic structure of the grid and the gestural and verbal associations, both of which broke dramatically from the children’s response of “seven hundred” as the number following six hundred. The double touching (of 600 and then 1) across rows² and the change of emphasis in the voice (from *six cent* to *um*) initiated off one set of actions as distinct and privileged from the sequence of either one, two, three, etc. or one hundred, two hundred, three hundred, etc. Indeed, rather than saying explicitly “you have to skip a row,” the ritualisation initiated by Nathalie was designed to offer a way of thinking “without bringing [it] across the threshold of discourse or systematic thinking” (Bell, p. 93).

Nathalie then continued with 300.

² The previous activity also involved double touching, responding to which number follows 25, where Nathalie would touch 20 then 5 and then 26. In the case of 601, the double touching skips an entire row (the tens row), which is apparent in the large gesture required to go from 600 all the way up to 1.

What is this number?

touching 300

Three

Three hun...

Three hundred

And what is the number after three hundred?

Four

Four hundred

Three hundred and one

Three hundred one!

Three hundred...

touching 300

one

touches 1

And this number here?

touching 800

Eight hundred

Eight hundred

touching 800 twice

And what is the number after eight hundred?

Eight hundred and one

Eight hundred...

touching 800

one

touching 1

And what is the number after eight hundred one?

touching 800

one

touching 1

Eight hundred two.

Eight hundred...

touching 800

two

touching 2

Eight hundred three.

Eight hundred four

Eight hundred five,

Eight hundred six,

Eight hundred seven

Eight hundred eight

Eight hundred nine.

After eight hundred nine?

After eight hundred nine?

Eight hundred ten!

Yes! Eight hundred...

touching 800

ten

touching 10

After eight hundred ten?

Eight hundred twenty.

No! Not eight hundred twenty.

Eight hundred...

touching 800

eleven

touching 10

touching 1

Eight hundred eleven

After eight hundred eleven?

Eight hundred twelve

Eight hundred thirteen

Eight hundred fourteen

Eight hundred fifteen

[It's going to take forever]

There are several ways in which this passage could seem as merely ritualistic: there is a lot of repetition; there is chanting in the 801 to 809 span and in the 812–815 span; the children seem to be saying number names without knowing what they are doing or why; Nathalie seems to be controlling the actions. We would like to argue, however, that the excerpt involves ritualisation.

After Nathalie offered 601 as the number that follows 600, she invited the children to consider what came after 300. Several children repeated the strategy they had used with 600 and suggested 400. Many voices can be heard in the video, and the word “quatre” is most prevalent, but eventually a voice can also be heard saying “three hundred and one”.³ Nathalie re-voiced the 301 in the same way she had done for 601 (with the same shift in emphasis of her voice), while also simultaneously gesturing to the 300 and the 1 on the Gattegno chart, using the large gesture that moves from the row of hundreds all the way to the row of ones. This was repeated for the case of the number after 800. And following this, the children spontaneously began to choral speak the numbers from 802 to 809. This choral speaking is now the children’s ritualisation, one that sets off the saying of eight hundred and two, eight hundred and three, etc., as a distinct span of numbers that is privileged in that it follows a rhythm that the children know well. They are essentially communicating the fact that once you know the structure of eight hundred and one, you can keep going by just changing the last word. The chanting represents the joint dynamics of their voices in which their activity does not cross the “threshold of discourse” but nonetheless acts as a particular form of thinking. As in the UK example, the performance itself is discursive, but it is in the sense that there is no talk *about* what is going on (the teacher does not “tell” the children what to do, for example) that we say the activity does cross the threshold of discourse.

The hesitation after 809 is not surprising in that the sequence of touching for 809 is very different from that of 810, because of the structure of the chart, but once that has been cued, the children again spontaneously continue their chanting. We stopped the transcript at the point where one child said “this is going to take forever” because it signals a shift—at least for this child—from a confident desire to exercise a pattern of number naming, to a realisation that counting by ones is a slow, plodding way to proceed. We speculate that the child could have been thinking that it would take forever to get to 900, having shifted from seeing repetitions in blocks of 10 to seeing them in blocks of 100. Here, the child does “cross the threshold” to talk about the activity and, for us, this suggests it makes no sense to posit the earlier chanting was meaningless or without thought, since it would then seem impossible to explain the arrival, *ex nihilo*, of a powerful mathematical awareness.

5 Discussion

In both contexts, where we have offered empirical data, the teacher is using the same configuration of our number system, the Gattegno chart, and asking students to say back either the number she or he pointed to or the number one more than the one pointed to. In both cases, we see ritualisation at work: the setting apart as distinct and privileged activity; the environment is symbolically structured; there is little or no attempt to bring what is being done across the threshold of discourse, in the sense of discussing why particular patterns exist or

³ There is an incorrect generalisation at work here because one says “vingt-et-un”, “trente-et-un,” etc., but not “cent et un.”

what they are. We explore each of these in turn, suggesting in each case that its presence is consistent with mathematical knowing and thinking.

The setting apart of a distinct and privileged activity is necessary for learning in the sense that learning is developing a certain kind of awareness of relations, patterns, structure. In both cases, the particular sequence of numbers was at the heart of the activity. While in Alf's case, it involved the naming of the hundreds, in Nathalie's case, it worked to produce ranges of numbers *in* the hundreds. The privileged activity was working on the symbols and on the patterned naming of them.

The symbolic structuring of the environment is not only "in" the chart and so cannot be assumed simply by the presence of the chart. Rather, structure emerges from the interactions of teacher and student(s) and chart. In both examples, the pointing of the teacher follows particular (but different) patterns which highlight elements of the symbolic structuring offered by the chart. For Alf, the pattern he focused on was in the common naming of rows and the repetition of that naming within any row. For Nathalie, the pattern she focused on involved the repetition of pointing to 1, 2, 3 ..., 9 (at the end of the number) when counting up in 1s from any starting point. In both examples after some initial hesitancy the students are able to follow the patterns with speed and confidence (in that moment). We would conjecture that the students are able to anticipate some of the next moves in ritualisation of both examples; and, furthermore, that this capacity would equate to a (mathematical) knowing (in that moment).

To cross the threshold of discourse or systematic thinking, as discussed in our analysis of the individual cases, would have involved the teacher, for example, stopping the class or student and drawing their attention to the similarity across what they are doing, or asking or offering an explanation for why what they are doing follows the pattern it does. None of this takes place (with the possible exception of Alf asking Aidan if he can "see" the 20 in 20,000). And, in later videos, the pattern remains of the teacher focusing on doing, without introducing discourse *about* that doing. We see here a link to Bateson's (1972) discussion of what we might learn from Balinese culture:

We can either have the habit of automatically looking before we cross the street, or the habit of carefully remembering to look. Of the two I prefer the automatic. (p. 182)

Bateson contrasts these alternatives as: anxious taking-care (consciously thought about each time) and automatic, rote caution (performed). In an analogous manner, as we were teaching in the contexts described above, we were wanting to develop a habit of automatic number naming or counting, rather than something to be consciously thought about. We imagine that part of what made these actions energetic and engaging for children was just how *little* had to be memorised (because of how our number names are structured), for them to be able to say any number and count up to any number automatically. Of course, in the extracts, we have shown the students were just at the start of this journey, but even at the end of the first few minutes of the French immersion class that we transcribed, it seems plausible that the children could have said the next number after *any* natural number that ended in a zero, one, two, ..., up to eight. They are not making use of any sense of the size of the number (cardinality) in order to do the counting we observe, nor do they need to, and we acknowledge this may trouble some readers. But, we would question whether, as mathematicians, we would be making use of cardinality at all if asked, for example, to give the next number after eight hundred and two (see Coles & Sinclair, 2017).

Bateson's (1972) view was that, in ritual, "the name is the thing named. The bread is the Body, and the wine in the Blood" (p. 409). In both of the excerpts presented, students are being asked to work with number in just this manner. In the practices in which we are involving students, the number is the name of the number. No other meaning is required or asked for, there is no metaphor; just metonymy.⁴ Over time, of course, students must and will develop richer and more complex associations with particular numbers (such as the multiples of 1 to 10). But we do not see the development as one from unthinking behaviour to mathematical meaning. As mathematicians, if we are asked to perform an algorithm (e.g., in calculating 23×75), we call on the number names to perform the calculation in just the automatic manner in which children are being asked to in our two examples. The name or symbol stands for the number during the calculation process, perhaps only at the end to be considered for its reasonableness or meaning in a context.

Finally, in our examples, there is also repetition, but this is not to say that the same question is being asked of students over and over again. Rather, it is the same *kind* of question that is being asked repeatedly (to say the number pointed to, with Alf, and to say the number one more than the one pointed to, with Nathalie). The variation within the repetition allows for students to catch onto the rhythm and ritualisation. Having discovered that what Nathalie wants is "eight hundred one", by "eight hundred three", the whole class has joined in. The comment "It's going to take forever" is evidence for us that there is no sharp division between the "doing" of this activity and thinking. This student, albeit colloquially (and not in the target language of French), introduces the concept of forever to express, we assume, a sudden awareness of how many numbers there are when counting in 1s. We argue that conceiving the "ritualising" part as unthinking offers no story for how such a sudden awareness arose; rather, we see the expression of an insight, here, as evidence that the previous work, chanting, had been meaningful and given rise to agency.

Through ritualisation, the focus is not on rote and meaningless activity that later has to become mathematical. Rather, students are offered access to mathematical structure in a manner that allows them to surprise themselves in terms of what they can do and know. We recognise some readers may have a concern about *what* students show they can do, but we suggest that a worry that they are not "really" showing any understanding of number betrays the very assumption we are questioning, that ritualisation practices such as communal chanting carry no meaning.

6 Conclusion

The notion of ritual is one that has powerful explanatory potential in relation to mathematics education, both looking at schooling as a whole and at specific classroom-based practices. In the ways in which ritual is currently conceptualised in mathematics education, we see an unnecessary distinction between thought and action. While rituals are not seen as meaningless, they are characterised in contrast to practices of understanding mathematics and, in Sfard and Lavie (2005), rituals are seen as an initiation into more mathematical ways of knowing, with agency for the learning only found in the latter.

⁴ In a metonym, a "part" stands for the "whole" (in our case, the number name is standing for the entire number concept, for the purposes of the tasks described); in a metaphor, a substitution is required, where one set of relations is made sense of by analogy to another set of relations (e.g., numbers and Dienes blocks).

In this paper, we have argued for a re-framing of the notion of ritual in a mathematics classroom, towards the idea of ritualisation. We suggested that ritualisations involve: repetition; a symbolically structured environment; and, little or no attempt to bring what is being done across the threshold of discourse. Ritualisation is a way of privileging certain activities over others and is not characterised by un-thinking-ness, rote action or submission to authority. We drew on empirical data to illustrate some kinds of mathematics classroom practice that, we argued, could be analysed in terms of ritualisation and in which important mathematical activity was taking place, without crossing the threshold of discourse about that activity.

We want to suggest that there are several benefits from the re-framing described above, which we now discuss in turn. Firstly, the notion of ritualisation provides a non-dichotomised way of treating phenomena that does not demote the body and hence might be of use to scholars working in an embodiment tradition. Through acknowledging a role for non-discursive thinking, ritualisation might also prove productive for more socio-cultural colleagues wanting to take account of the role of the body in mathematics teaching and learning.

In addition to avoiding dichotomies of performing/knowing and memorising/knowing, we also see ritualisation as a concept that helps us navigate dyads such as procedural/conceptual or instrumental/relational, particularly in relation to the notion of “practice” in a mathematics classroom (we note that work to find the common relationship between poles of a supposed dichotomy has a long history in mathematics education, for example, Hiebert and Lefevre (1986); Reid and Brown (1999)). The classroom excerpts described in this article, exemplifying ritualisation, point to ways of working that avoid dilemmas, such as whether to work on “understanding” or “fluency” first (a question that arises from the separation of knowing and acting). We see a critical role for the structured symbolic system that is being offered to students, in allowing immediate access both to fluent activity and to the potential for mathematical structuring and exploration. In contrast to Sfard’s notion of ritual discourse, which is seen as preventing agency, both of our examples showed how the ritualisation was exactly what gave rise to agency.

A further feature of our focus on ritualisation that we see as potentially productive is how our analysis draws attention to marking a distinction between what is privileged and what is not, or what is sacred and what is profane, which we extend to what is aesthetic and what is not. Rather than the view of ritual as social and performative, we are highlighting the way in which ritualisation can draw attention to what is distinct or important, mathematically. Part of attending to mathematics is knowing what to attend to, in other words, knowing what is important or significant, which we think of as being marked by what is aesthetically relevant. Through ritualisation practices, such as those exemplified in this paper, there is an aligning of attention of teacher and student and, in that process, a sharing of those significant mathematical features of the symbolically structured environment. We do not pay attention to the font or size or colour of the number symbols, for example, but we do pay attention to their vertical and horizontal alignment. Through ritualisation, we observe students coming to pay attention to just those features of the environment in a short space of time, with seemingly little effort and with enthusiasm.

The focus on ritualisation sidesteps, for us, the question implied by Sfard and Lavie’s (2005) work of whether ritual has to come before exploration. Ritualisation can involve exploration from the very beginning. Alf’s question to Aidan “any other number on here you want to do?”, asked within a few minutes of starting work, indicates how there can be both repetition and space for exploration, even when entering a new field of activity and discourse. The student in the class Nathalie was teaching who exclaimed “It’s going to take forever”

demonstrates an awareness of mathematical structure arising in the course of repeated activity (adding one to a number) that is not made a subject of discourse in the sense of being talked about.

Although both our examples concern the early learning of number and the same symbolically structured artefact (the Gattegno tens chart), we contend that ritualisation is possible as an entry into many other parts of the curriculum. We suspect that the offer of a structured environment might well necessitate working, from the start, on larger “wholes” than might be typical, in the way that the Gattegno chart gives equal access to much bigger numbers than would be typically dealt with in the early years of schooling and has the potential to deal with the “whole” of the number naming system. Hewitt’s (2016) “Grid algebra” offers one example of a symbolically structured environment in that curriculum area. Like early number, algebra can often be associated with memorisation and meaningless rote learning. However, like the Gattegno chart, working in Grid algebra involves a choreography of spoken words, symbols and gestural actions on a lattice. These actions seem particularly significant in that they embody mathematical relations that can be expressed in movement rather than discursively. In a different way, we can see opportunity for ritualisation in geometry as well and Tahta’s (1981) way of working with Nicolet films is a compelling example of ritualisation in a symbolically structured environment of moving shapes. We are not wanting to argue in favour of practices other scholars have studied and found to be formulaic and devoid of meaning. Rather, we are wanting to suggest that there are, perhaps little recognised, ways of working in mathematics that allow novices opportunities to act in sophisticated ways with new symbols, from the very beginning, and which can promote agency and energetic learning.

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