

Alienation in mathematics education: critique and development of neo-Vygotskian perspectives

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Abstract This paper aims to critique and develop neo-Vygotskian work in mathematics education from (i) within the Vygotskian and activity theoretic tradition, and where necessary from (ii) a Bourdieusian perspective. First, I critique Roth and Radford's (2011) version of Cultural-historical Activity Theory, suggesting that a classroom episode presented as developmental might be seen as a process of alienation. I trace this to the institutional structure of schooling, in which curriculum and pedagogy are alienated from the learner's everyday sense. Next, I examine and critique the Vygotskian 'Funds of Knowledge' approach to critical mathematics education, which seeks to overcome alienation by subjecting the curriculum to the needs of the poor communities the school serves. Here, the critical point on alienation in the Vygotskian approach is better argued in Bourdieu's perspective on educational institutions as reproductive of class domination. Finally, the paper discusses the extent to which these critiques pertain to Vygotskian activity theory in general or only to these versions in particular and draws implications for the development of a critical mathematics education perspective in a synthesis of perspectives from Vygotsky and Bourdieu.

Keywords Activity theory · Vygotsky · Cultural-historical activity theory · Funds of knowledge · Bourdieu · Alienation

1 Introduction

The present paper aims to develop the critical, neo-Vygotskian, cultural-historical activity theory (ChAT) approach to mathematics education: it will do so through a process of critique of existing Vygotskian approaches to mathematics education, mainly from within the Vygotskian ChAT perspective but, where necessary, from Bourdieu's sociology of 'schooling'

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as social reproduction. The result will be a more concrete understanding of how Vygotskian and Bourdieusian perspectives can serve critical mathematics education.

In this paper, I will focus on learners' alienation from mathematics in schools and classrooms. One should recognize such alienation: (a) intuitively and subjectively (to paraphrase Marx: the learner does not feel 'at home' when learning mathematics) and (b) scientifically and objectively (the alienated learner is one that is 'forced' into producing mathematics, e.g., to earn rewards rather than willingly to enjoy the expansion of their personality or capabilities).

For Marx, the subjective and the objective are dialectically related: both have a key role in development. Marx (1844/1964, pp.74–76) developed his argument thus: the producer/worker is first of all alienated from the product of their work/labour, that is, what they produce does not belong to them or is taken from them by an 'other' (e.g., they are offered wages instead). Then the worker is alienated from their own work/labour process itself (e.g., it is directed by and for the wage payer/master), and finally, the worker's labour is alienated from 'humanity' and its essence (the collective struggle to meet humanity's needs), and hence, the worker is alienated from other workers. Marx and Engels developed this to explain private property, alienation within the family, alienation between countryside and town and between 'hand and brain'. I argue that this notion forms the basis for (i) Vygotsky's critique of academic scholasticism, when school conceptions are reduced to 'pure verbalism', and (b) Bourdieu's concept of the 'cultural arbitrary' of schooling as a form 'symbolic violence' against the dominated groups that schooling fails (for further details see Williams, 2011, 2012).

Thus, I argue that alienation is implicit in Vygotsky's notion of the dangers of school concepts reducing to pure verbalism and of schooling leading at best to 'pseudoconcepts': for Vygotsky, real or 'true' concepts require the engagement of academic concepts with everyday concepts and practices (Vygotsky, 1986). The alienation of schooling from everyday life and work makes this very challenging (Vygotsky, 1994).

Similarly, the alienation of the 'dominated' in schooling according to Bourdieu's account comes down to the use of schooling to reproduce the dominance of the powerful in society at large, through the imposition of domination locally in education through the 'arbitrary' way selection and failure is exercised disproportionately on groups and classes with less 'cultural or social capital' (Bourdieu & Passeron, 1977/1990).

I previously argued for a synthesis of neo-Vygotskian ChAT with Bourdieusian perspectives, by relating the 'use value' of mathematically competent labour to Vygotsky's true concepts, the 'exchange value' of mathematics to Bourdieu's cultural capital, and its arbitrary use in schooling for selection (see Williams, 2011, 2012). But here I use these concepts specifically to address the problem of *alienation* of learners from mathematics in schools. To do so, I choose some of the best, and also best known, neo-Vygotskian works in mathematics education to critique: I argue that the alienation of schooling is quite often not addressed by Vygotskian scholars, even those who take Vygotsky's roots in Marx seriously, and consequently, they may overlook alienation and domination in their own practical and theoretical work.

First, in Roth and Radford (2011)—abbreviated here to R&R—one finds a detailed account of a Vygotskian-inspired developmental classroom understood by R&R as affording a zone of proximal development (ZPD) for mathematics: I choose this study to critique not only due to its Vygotskian inspiration but also because it provides an admirable degree of detail of the classroom process that makes the praxis visible. This transparency allows one to interpret the

interaction for oneself, and I see there a process of alienation from mathematics, rather than a zone of development. I see both subjective and objective alienation in this classroom: subjective alienation in the withdrawal of some of the children from engagement in their learning and objective alienation in the curriculum task and pedagogy that obliges actions without sense. I trace my objections to R&R's theorization through their erroneous account of Vygotsky's theory of what others call 'internalisation', and then to their version of the 'learning paradox'. I will argue that for Vygotsky, the learner has to be an active participant in making sense of everyday activities with mathematical concepts, and that, without this, classroom activity descends through 'pure verbalism' into alienation. I will also suggest their theoretical adoption of the learning paradox (i.e., Plato's argument that the learner cannot know what it is they are to learn before they have learnt it) as a part of ChAT implicates alienation and is a quite unnecessary and unwelcome addition to the theory. It is important to note that the theoretical critique here does not need to draw on Bourdieu's framework, but that appealing to Bourdieu allows one to see the alienation as latent in the 'arbitrariness' of a school curriculum being inflexibly imposed with no 'everyday sense' for some learners. (Bourdieuists argue that it is precisely the lack of everyday sense that is discriminatory in favour of the middle classes).

Second, I move to the 'Funds of Knowledge' curriculum development work in critical mathematics education of Moll, Gonzales and others, based explicitly on Vygotskian theory and an explicitly critical perspective on the 'everyday practices' of the community outside school. I choose to focus on this project because, unlike in Roth and Radford's example, they overtly seek 'sense' in the knowledge of the poor and working class 'dominated' communities. Because this 'Funds of Knowledge' work aims to expose the curriculum to the stress of 'everyday practices' in the lives of their poor and disadvantaged communities, one can expect that this approach might deal with the concerns raised in my R&R critique. In practice, however, one can also critique some of this work, especially that in mathematics education, because (i) the funds of knowledge often seem to be recognized as characteristics of middle-class cultural capital that schools already privilege (e.g., they find workers who were business people or teachers 'back home') and (ii) they generally fail to find the critical mathematics in these local practices' 'funds'. I will argue that Bourdieu's perspective proves to be much sharper, insofar as it explains why schools and teachers may identify the 'capital' in these funds more readily than its critical 'use' in supporting the poor and underprivileged.

2 Zone of proximal development or a moment in the process of alienation?

First then, I critique R&R and their version of Vygotskian activity theory and the learning paradox. When I refer to 'R&R', I indicate the account by these two authors in one particular book, and in particular, their claims about development and the learning paradox therein. There is, of course, much I value in the book and in these authors' various accounts of activity theory in mathematics education: this critique would not be worth doing otherwise. R&R provide an account of cultural-historical activity theory (in what they call 'the Vygotsky-Leontiev-Holzkamp lineage') in the context of an early-algebra project and one classroom learning-and-teaching episode in particular. It will be helpful to give the reader a summary of this episode and how it is understood from R&R's ChAT perspective: it is only because of the detailed presentation of their analysis of this episode that this critique is possible.

The episode took place in a grade 4 Canadian classroom, as a group of three (9 to 10 years old) children tackled the task involving saving money in a piggy-bank: 6 dollars in the beginning and 3 dollars in each subsequent week. The curriculum focus was on the transition from repeated addition to multiplication, formulated as a key moment in learning algebraic generalization. A worksheet instructed the children to (i) model the task using counters and cups, one cup to hold the counters collected by the end of the first week ($3+6$), one cup for those collected by the end of the second week ($3+3+6$) and so on, up to the sixth week (it seems Teresa, Aurelie and Mario have all done this before the transcript of the focal episode begins), and then (ii) to fill in the gaps in a table on a worksheet showing what has been done. This worksheet task starts their troubles which are carefully analysed and re-analysed in R&R.

A partially completed table requires them to fill in values for each week, from week 1 to week 6, and in the first row, expressions are expected like this: ' $3+6$; $3+3+6$; $3+3+3+6$; ...; $3+3+3+3+3+6$ '. Then in the next row the worksheet requires the children to write expressions using multiplication signs (these being forced by the worksheet template): $3+6$; $2\times 3+6$; $3\times 3+6$; ...; $6\times 3+6$.

This structuring of the model building is designed, one understands, to draw the children's attention to the multiplicative structure implicit in the repeated addition process. As the transcript opens, Teresa has completed the worksheet as required and more or less disengaged from the group. Aurelie is frustrated by the worksheet and bangs on the desk crying 'plaintively': 'I don't understand and I'll never understand!' which arguably actualizes a 'subjective' alienation. Teresa responds: 'just copy mine'—which Aurelie does, inaccurately, perhaps actualizing an 'objective' alienation from her work, that is, from the essence of mathematics and from her teacher and maybe her classmates as well.

Between these two apparent extremes, in what R&R name a zone of proximal development, Mario also says he does not get it. He draws the teacher's attention, who then engages with Mario, leading him through the task (e.g., asking him to read the sheet, prompting 'how many 3 s?', pointing to spaces he must fill in on the worksheet etc.) and apparently hinting in various ways. Communicating with pointing gestures to required actions, and vocal intonation feeding back if Mario's action is leading in the right direction, she leads Mario to what the task demands. Mario is initially frustrated, but makes some false 'tries' in response to the teacher, who in turn expresses frustration. Eventually, however, the teacher and Mario 'tune' in until it seems Mario has figured out what is required and has started to fill in the correct numbers in the spaces on the sheet. The teacher, Jeanne is satisfied and says she thinks he has got it now and leaves. Mario completes the worksheet table as required, checks against Teresa's sheet and reflects after some time 'Me, I understand now'. R&R show how—from being tense and frustrated at the beginning of the interaction—Mario has become relaxed and calm, emotionally reflecting a change in his evaluation of his progress in the activity, the task and his perception of his learning. The main contribution of R&R's whole book is to place emotions with motives in the centre of the ChAT methodology, and the key idea here is that Mario's emotions involve his assessment of his state of progress towards achieving a goal. In his case, his initial frustration realizes his need to make sense, while his quiet calm invokes his satisfaction that he has finally understood. I admit that this does not feel like a subjective experience of alienation, but will suggest that he may be objectively alienated even so.

R&R see this as an important societal interaction for Mario and the teacher: they have both engaged in a joint learning-teaching activity of what Vygotsky called 'obuchenie'. Both in their different ways learn from this dialogue. By completing the sheet, Mario has objectified the multiplicative structure inherent in the pattern, while Jeanne has 'learnt to teach' algebraic

reasoning to Mario. R&R argue this constituted a zone of proximal development (ZPD), at least for Mario, completing the worksheet mathematics of ‘algebraic generalization’ and maybe even the ‘object-motive’ of the activity which R&R suggest is ‘to reflect algebraically’ (p. 31).

In this summary, of course, I have missed out and glossed over much. But I miss out a lot partly because R&R miss out a lot: the research team has been in this classroom for at least weeks (and in the school over years). But the reader gets little of this context in the account or in the analysis. R&R hold it important to analyse and interpret the event as it occurs and without evidence or insights from this wider context—while sometimes calling on micro-, second-by-second details of the episode itself (e.g., plotting a graph to show the discontinuity in pitch of a spoken syllable that represents an emotional ‘tone’ interpreted as ‘excitement’, on p. 81). As such, their transcript and analysis include the children’s reference to the research team, when Aurelie is copying Teresa’s worksheet, Teresa tells Aurelie she is on camera, and Aurelie responds ‘I’m not copying’. Thus, she suggests she knows she is not supposed to copy and copies anyway.

Essentially, one is not told how or why the task was designed. R&R acknowledge that without the wider context there is much one cannot know and might guess at: importantly, we do not discover much about what Teresa and Aurelie, or even Mario, said they thought they learnt from the lesson, which could have constituted evidence of reflection.

Interestingly also, the text does not include an account of the mathematics education research context. In the transition from arithmetic to algebra, learners can have difficulties with questions that require them to accept ‘lack of closure’, and this may be related to a shift of attention when processes and relations become new objects of mathematical attention. Mario’s initial frustration might be related to his difficulty in accepting the lack of closure apparently demanded in the worksheet, writing $3+3+6$ instead of just 12! Perhaps, in the context of the task, his expectation of closure of the arithmetical operations makes more sense (one might like to know that one has 12 dollars by the end of the second week, perhaps) than writing the unclosed expressions, a peculiarly ‘mathematical’, scholastic thing to do. Thus, the gap between the task and the learner’s everyday, spontaneous conceptions could explain Mario or Aurelie’s alienation from the mathematics the worksheet forces on him (i.e. the scientific conception, in Vygotskian terms): there is no evidence here that a ‘true concept’ has formed.

In conclusion, the task and worksheet activity does not invite the children to mathematise algebraically, it arbitrarily imposes it; when this does not make sense to a learner, the pedagogy obliges the student to conform behaviourally to the demands of the worksheet, without any ‘sense’ of a need for algebraic formulation arising. This is what Vygotsky means by ‘pure verbalism’, and this type of senseless, arbitrary ‘imposition’ and ‘obligation’ is what Marx means by alienation. Insofar, as this senselessness is imposed as an ‘arbitrary’ of the schooling ‘culture’, a ‘cultural arbitrary’ tending to lead some students (more than others) to disengage, it is what Bourdieu calls ‘symbolic violence’.

Is Mario’s mathematical development from what he has ‘understood’ really so different from that of Aurelie who says she ‘will never understand’. Mario’s subjective perception of what he has done (and thus the emotional tone of his reflection) is based on his evaluation of what he has achieved, given the teacher’s feedback, and is different from Aurelie and is expressed positively. I agree with R&R’s interpretation that Mario’s self-evaluation is that he has made an appropriate response, and so he considers he has learnt. But I argue this learning might not be mathematically developmental and in fact might be seen as compliance with the teacher’s evaluation that he has understood when in fact, objectively, he may have understood

little or nothing about algebraic reasoning or reflection as such. Thus, one might read both children's efforts in this activity as alienated, but perhaps in two different ways: while Aurelie is aware that she does not understand, Mario may not understand but thinks that he does.

In the next section, I turn to R&R's theorisations of the Vygotskian ZPD and possible theoretical blind spots in R&R that hide the alienation of learning in their account of this activity.

3 What is wrong with 'internalisation' in R&R's interpretations of Vygotsky?

R&R repeatedly visit Vygotsky's account of what most authors call internalisation, drawing on his analysis of the way an infant learns to point. But in my reading, R&R misunderstand Vygotsky in an apparently small but vital way that is germane to this issue. I suggest this leads them to misunderstand learning in activity. R&R begin by quoting Vygotsky (1989 p.56):

[A]ll cultural development has three stages: development itself, for others and for oneself (e.g. a demonstrative gesture—at first it is simply a failed grasping movement aimed at an object and designating an action; then the mother understands it as an instruction, and finally, the child begins to point.) (p. 69).

R&R then go on to say that when an infant makes a 'random gesture' that seems to the carer or parent as though the infant might be pointing, the carer or parent interprets it as pointing, and consequently, reaches out and gives the infant the object that the grasping gesture suggests, and thereby begin to 'teach' the infant to point to desired objects. So R&R say:

A movement receives the sense of an action of a particular kind first by the culturally competent individual before this sense comes to be actualised by the child. In the example Vygotsky provides, there first is a random movement. The child does not know its cultural signification; it does not (yet) know to point. Rather the parent who sees the child move understands it as a pointing gesture.(p. 69)

But this is significantly different from Vygotsky's own account as I read it and as discussed in John-Steiner and Mann (1996); and as it is also to be found in Vygotsky (1978, p.56). In my reading, the infant began this interaction with a goal in reaching out to grasp some desired object, one imagines an object such as a dummy or shiny toy—this is not a 'random movement'. Indeed, it is the desire to grasp an object which is interpreted by a culturally competent helper, one who empathises with the infant's frustration when it fails to grasp it. Only because the carer is motivated to help the infant relieve their frustration, they progress the infant's action towards fulfilling its goal. No doubt emotion plays a crucial role as a moment in this interpretation—one need only imagine the effect of an emotionless robot waiting for an infant's random arm motion under instructions to shape it into a 'pointing' function.

The developing infant then has to notice the carer's action, recognise its association with their own grasping movement, and probably practise this on a number of occasions; one imagines the infant perhaps dropping the toy and reaching/pointing to see the effect of these on the adult. Eventually, one imagines the infant looks at a desired object, reaches out, consciously 'pointing' and looks to the carer (if necessary perhaps maybe even drawing their attention with a cry). The evolution of 'reaching out to grasp' into 'pointing to indicate', as a means to

act on an object and achieve a desired outcome, is then a completed development of the pointing function as a communicative tool.

The joint nature of this activity is now quite clear: there was no random gesture or action on the child's part that was interpreted as pointing. Rather an intention and an attempt to act was interpreted and understood, and so assisted by the carer in joint activity. The emotion, motion and motive originated with, and were initiated by, the infant in this sequence at least as much as by the adult. Of course, I agree with R&R's interpretation of Vygotsky that the infant's initial gesture did not constitute 'pointing' and that the child could not have consciously desired to 'learn to point', and of course this pointing 'function' exists in the culture, and on the social, interpsychological plane before it comes into existence in the child's independent functioning, that is, before it exists as an intrapsychological function. But that is not the point; learning emerged from the activity in part structured by the child's goal to 'grasp' an object, and it was the joint achievement of this that allowed 'emergence' of new cultural, communicative knowledge in a way that made sense for both infant and carer. In all this story, of course, I invite the substitution of 'pointing' by 'mathematics': there has to be a need, and a goal for the learner, one that involves learning together to use a new tool in order to satisfy the need.

Let me clarify the difference between what R&R say and how I claim Leontiev views 'joint activity' here: the agency, goal and initiative of the child is an *essential* moment, and the intervention of the adult/teacher in joint activity only makes sense to the learner on that basis. The child makes an intentional move (maybe seduced by the attractive object carefully placed by the adult); learning to point is completely dependent on this. And the same is essentially valid for all normal, 'situated learning' in everyday practice, as explained by Lave and others, but also by Holzkamp (2013), for whom learning should expand the possibilities of the learner to act, and as far as possible consciously be the subject of this expansion.

A caveat here: R&R rightly say that the object-motive for collective activity does not in general originate within the individual subject. You could say that the 'desire for shiny objects' itself provides a goal and that joint play provides an activity motive that has been fashioned historically in previous millennia of cultural activity. In a sense then, there is no 'beginning' in any one individual or act. Nevertheless, I argue that an activity of the learner-with-others, one that is simply 'for another', is by Marx's definition an alienated activity, and that learning 'for the other' is alienated learning (Lave & McDermott, 2002). Thus, in their promotion of a 'learning paradox', R&R may be offering a theoretical rationale for alienated learning activity.

It must be recognised here that this 'pointing' example is ideally situated and not paradigmatic of much mathematics encapsulated in schooling (Engeström, 1991). Therefore, pedagogy must try to find ways to make such concepts ascend to the concrete, which Vygotsky argued can only happen when they are employed in practical problem solving tasks, where the scientific ideas can 'make sense' by offering deeper understandings of the everyday, spontaneous concepts, thereby becoming 'true concepts'. This is a key issue for mathematics education, as it explains why it is so difficult to design and implement developmental activity within the confines of school: the initiative is all with the teacher who has to engage the student in a task that requires a relevant mathematical development. The danger is that the teacher successfully seduces the student to feel engaged in the arbitrary of schooling per se, while disengaging from anything that looks like critical mathematics.

Now, let us empathise with Mario's frustration: he appears to be in the hands of an algebra task of schooling, but he does not know why—he cannot know 'the point', according to R&R, until after he has completed the task owing to the ChAT version of the 'learning paradox'. Mario does not know what the object is that he is expected to try to grasp (I use the grasp-

object-point here metaphorically as well as theoretically: i.e. I equate ‘grasping’ with acting on, trying to transform the ‘object’ and so meet some ‘need’).

By way of an alternative, if there had been a meaningful goal that Mario had been seduced into trying to ‘grasp’, one that could engage Mario’s attention to a desired object (to solve a problem) maybe he would have reached out and tried to grasp or engage with a goal (before he had completed it), and one that leads to the emergence in reflection after the event of a new mathematical point—‘to multiplicatively model patterns’, say. For sure, this would require the assistance of others in joint activity. Maybe then on completion of the task Mario could reflect on the use of the new mathematics (e.g., its multiplicative structure) in this and other task contexts: he might reflect that it is good to see a pattern that can be useful in some problems and tasks. Incidentally, then, Mario might also reflect on mathematics and its relevance to his life. Perhaps then we would all be persuaded that Mario was clearly the subject of his own development, and the joint work could be described as a ZPD. This is a very demanding and perhaps often impossible expectation for schooling, but I read this as what Vygotsky had in mind when he spoke of a dialectic of academic and everyday concepts.

Such an approach is sometimes visible in mathematics education that reaches out beyond the school to everyday or workplace tasks in which mathematics can prove its value in use or as ‘use value’ (e.g., Engeström, 1991; Moll, Amanti, Neff, & Gonzalez, 1992; and Roth himself, in Lee & Roth, 2001). It is also sometimes the case that a mathematical context itself can have practical sense for new mathematical activity, where such a context has become experientially real for learners. But the motive for ‘critical’ mathematics often arises from some social or cultural interest from outside academia that is stimulated in the classroom by the teacher with a research team, such as the work done to investigate pollution in the local creek that Roth engaged with.

4 Joint activity and the division of labour in the classroom: putting alienation back in the middle of the Vygotskian account

Leontiev (1981) describes an ideal of joint activity in the case of the hunting party engaging together in a ‘primal hunt’: apparently, there is a division of labour between those involved, but it is reasonably transparent and sometimes may even be benign (at least they all ‘see’ who does what and who gets what, even if the shares are not equitable). I see the joint activity of infant and carer in similar vein: the infant seeks to grasp a desired object, the carer helps the infant satisfy its perceived needs by grasping an object and the outcome is bliss for all.

But in many systems where activity is involved, the division of labour, distribution and exchange is not like this. As Marx points out, the labour process in class societies is typically alienating due to the power structures involved. One cannot neglect this possibility in any activity process and certainly not in schooling (see Lave & McDermott, 2002; Williams, 2011, 2012).

In the typical factory, where the division of labour is extreme, the worker may not even be aware of the nature of the product of their labour: they are micro-managed to produce, and they are paid wages. The mediation of wages between the worker and the use of their product alienates them from their own labour, whose use value is always for ‘others’ and from the owners who take their product. Analogically, Lave and McDermott (2002) develop Marx’s 1844 text: substitute grades for wages, learning for labour, schools for factories, teachers and head teachers for managers, the state for the owners of factories and you have alienation of learners from learning, and from teachers, in schooling (see also Engeström, 1987, 1991).

In Williams (2011), I critiqued and developed this notion of alienation, pointing out that the learning is not ‘labour’ as such, but that its product is (suitably qualified, differentiated and certificated) the unusual commodity ‘labour power’, and this involves contradictions between its use and exchange value. For instance, its exchange value is associated with its relative value (insofar as it is more productive for capital than that of others) while its use value might be in its consumption in critical work. Schooling is riddled with such contradictions.

Thus, the teacher and the children in classroom activity may not be engaged in fact in the same activity, as defined by a common object-motive. As Leontiev suggests, many children in early schooling-activity may have the object-motive of ‘pleasing the teacher’, or later ‘getting grades’, while the teachers and their supervisors manage and direct the learning, grading and selection processes of schooling—they ‘add value’ or ‘improve results’. Thus, one has two different and often contradictory activities that alienate teacher from learner and both from mathematics.

Thus, I conclude that a viable interpretation might be that Mario has learnt what was intended, but that the imposed curriculum by virtue of his commitment to engage with and please his teacher—if extended and habitual—may proceed to an alienation from mathematics as it could be and should be, and as it needs to be, to be critical. In this process of alienation, the root cause is in the dual operation of (i) a non-negotiable curriculum imposed on problem solving activity and (ii) a pedagogic relationship primarily in the hands of the teacher and the curriculum.

Instead, a critical mathematics education curriculum and pedagogy must reverse the priorities between the everyday and the academic, seeking activities that will engage the learner’s interests and develop criticality, but that will also reveal functional mathematics. In this work, the learner and teacher—if they are to consciously realize themselves as agents of critical mathematics—must construct solutions to meaningful problems (and these might indeed even be problems arising within mathematics). This does not imply they do this from first principles or by themselves, because the collective and its culture, including the teacher and the curriculum, might offer mathematical tools that help solve problems that engage their interests.

Now the question arises, is this critique necessarily a critique of Vygotsky and Activity Theory, or only of R&R’s version of it? And if indeed alienation in schooling is not a part of Activity Theory as it is commonly understood, how might we develop a more critical Activity Theory? In my reading, the version of Vygotsky R&R present was valid except only in one small, but very significant respect: the initial goal of the child-as-Subject is essential in allowing the carer to ‘help’ the child achieve their goal. This ‘small’ point is one that goes to the heart of the difference between academic scholasticism and everyday, practical work; and it goes to the heart of the difference between academic/scientific and spontaneous everyday concepts. These differences can only be overcome by overcoming the encapsulation of schooling, the pseudo-concepts and the alienation of learning from useful work.

In this discussion, R&R introduce the learning paradox from Plato (via Bereiter), that is, that the learner cannot know what it is they are to learn before they have learnt it. They reformulate the original paradox as a paradox within Activity Theory: that the learner cannot know the motive of collective activity before they have engaged in it (of course I agree the individual subject is generally not fully conscious of the motive of collective activity). This is close to Leontiev’s discussion of the emergent nature of motives in activity (I have discussed this elsewhere in Black, Williams, Hernandez-Martinez, Davis, Pampaka, & Wake, (2010) and Black & Williams (2012). But in fact what Leontiev (1981) argues is that the individual subject

(e.g., the school learner) engages in activity with a motive but—if the activity is developmental—may emerge with a new motive: indeed, the emergence of new motives is what chiefly characterizes Leontiev’s notion of development. Thus, in his example of a student studying a history text as the ‘object’ of an activity of schooling, that is, preparing for an exam, the student may emerge from this work with the new motive of wanting to understand history! This is rather different from the learning paradox, wherein it is claimed the learner cannot know in advance (recall the ‘random movement’) the motive (that only the carer or curriculum/the teacher knows). Rather, we should say that in a ZPD the learner begins activity with one set of motives, goals and consciousness and develops through joint work in transforming some common object, emerging with new consciousness, goals and motives.

This problem then in practice arises in a narrowness and inflexibility of the curriculum and pedagogy of school mathematics, which pays its dues to ‘algebra’, but not enough to the problems that the learner might encounter and be motivated to engage with. How to break out of this encapsulation of scholastic mathematics? (Cf. Engeström, 1991). Many have tried (including work by Roth and colleagues, see, e.g., Lee & Roth, 2001), and it is to one such project that I turn next. In the ‘Funds of Knowledge’ approach, there is a deliberate engagement of teachers and the curriculum with serious community issues and problems, and these particularly deprived communities provide critical mathematics educators with real objects which motivate community activity of the kind avoided above.

5 A funds of knowledge approach to mathematics education

For Moll in the USA, who develops a Vygotskian activity theoretic approach to research and development aimed at empowering poor immigrant families and children in schools, funds of knowledge are ‘the essential cultural practices and bodies of knowledge and information that households use to survive, get ahead or to thrive [...] acquired primarily, but not exclusively, through work and participation in diverse labour markets’ (Moll et al., 1992 p. 21).

This notion is used to conceptualise Moll’s research team’s work with teachers and researchers who reach out to the families and communities to identify these ‘funds’ and seek to use them in schooling activity that their children will recognize. Clearly, this provides potentially the kind of ‘everyday’ practices and concepts that might help concretise the academic-scientific concepts for their children. The teachers involved are trained by the research team (including educational researchers and anthropologists) to visit homes and explore the family’s knowledge, and then transform this knowledge to help them teach the children of these poor communities more effectively.

Strikingly, the teachers-researchers find all kinds of expertise and dispositions that can be thought of as constituting cultural resources for teaching. The family is very often found to be highly supportive of schooling, encouraging their children to conform and supporting their children in doing their homework and so on. The immigrants often have a wealth of knowledge from their work experience, and their often extended families collectively have a range of expertise in trades and the professions.

Most of this work was conducted with a focus on literacy teaching. However, mathematics (and to some extent science) funds of knowledge were less obvious in the local community. This hidden nature of mathematics and science makes the strategy more challenging, but not impossible, for the visiting teacher-researchers, for reasons that are unclear (but which I will

return to below). In brief, these families and communities have ‘funds of knowledge’ and dispositions that can be useful to mathematics in schooling.

This concept of knowledge in the community is conceptualised following Vygotsky as ‘everyday concepts’—it will be helpful to think of these concepts concretely as *concepts-in-activity*, which one makes sense of in ‘everyday practices’. These then might be regarded as an important knowledge base for the school curriculum, which is formulated as ‘scientific concepts’ in Vygotsky’s sense. The work of the school is to introduce scientific concepts and metacognition (or better, self-regulation), but these are made sensible by the everyday concrete experiences which the science/mathematics illuminates. In this view, the work of schooling is to make a form of hybrid, one which Vygotsky (following Marx) expresses as ‘making the abstract concrete’.

However, a striking feature of the ‘funds’ that teachers are able to recognize is that they may do little to challenge the normative nature of schooling: this is hinted at in the quote from Moll above, in which it is indicated that schooling should help the children ‘to get ahead’ (presumably of their fellows). What do the teachers find when they visit the families in their poor communities? According to Gonzales et al. (1995):

They all found parents who were engineers, teachers, and small business owners in Mexico, who pulled up stakes and now work in jobs far below their capabilities in order to obtain a ‘better life and education for their children’. (p. 458).

And:

... some schools in Mexico were academically ahead of the United States, and discipline was stricter (Gonzales et al., p.461).

In other words, the teachers recognized the funds the immigrants had, and stereotypes about immigrants that presume that their poverty implies cultural impoverishment were challenged. But their predominantly middle-class school values might go otherwise untested: the parents who were teachers and engineers were acknowledged as an asset, while experienced cleaners and carers perhaps might not be. Thus, I am left with some doubt about the transformative effects that they claim. I suspect the normal rules of schooling and classroom competition may apply in these teachers’ schools—the rules by which some poor immigrants, those well ‘funded’, more academically prepared, (like Teresa in R&R perhaps?) progress and others, those with fewer ‘funds’ of whatever currency, (like Aurelie, perhaps?) may still tend to drop by the wayside.

This suggests one must consider ‘funds of knowledge’ as cultural capital in Bourdieu’s sense (Bourdieu & Passeron, 1977/1990). Bourdieu understands the institution of schooling as a means to reproduce society and social structures, including class structures, and so preserve the dominance of the privileged, dominant classes. Thus, the particular nature of cultural capital that counts in schooling is that already generally possessed by the dominant classes. Schooling from the beginning therefore privileges these classes. Bourdieu sees this cultural capital as essentially arbitrary, but claims that schools misrecognize this arbitrary nature: and insofar as it is arbitrary, the school system imposes a ‘symbolic violence’ on the dominated classes.

What is this ‘arbitrary’ stuff that confers arbitrary power on those who possess this cultural capital? For Bourdieu, it varies across context: each ‘cultural field’ has its own somewhat autonomous power structure in each moment of its history: thus being a member of the golf

club may be important in some fields but a generation later the power has moved on. Mathematics purely as a qualification confers some power in the academic field now, but some years ago it was Latin: to the extent that its power is independent of its use in practice it is 'arbitrary'. One might argue for instance that passing exams—precisely the means to access scarce resources, elite status and even financial opportunities—involves largely arbitrary know-how, of little functional use value (see Williams, 2012).

To the extent then that the teacher-researchers from school recognize the cultural capital in the funds of their poor communities (e.g., that they were formerly small business owners rather than their workers) as of benefit to the school enterprise, then they are reproducing cultural capital. The logic, then, might be that the transformed 'funds of knowledge' school simply reinforces their mission of class reproduction, inflicting symbolic violence on the poor in cultural capital, whether immigrant poor or not. I say here 'might be', because Bourdieu's account does leave a small door open to another interpretation, which I see in his key term 'insofar as', when he says 'insofar as it is arbitrary'. For instance, Bourdieu & Passeron (1997/1990):

All pedagogic action (PA) is, objectively, symbolic violence insofar as it is the imposition of a cultural arbitrary by an arbitrary power. (p.5)

Then, contra Bourdieu, I suggest that: 'Insofar as pedagogic action supports the critical mathematical competence of the learner, it objectively undermines the cultural arbitrary and its arbitrary power.'

I therefore question to what extent it is possible that the school simply identifies cultural capital in the community funds that further inflicts violence on the dominated classes, and to what extent it might identify with 'critical mathematical competence'? And, is it possible for the school and its agents to recognize the critical cultural resources of the poor apart from those that the school already values, and so to recognize their use value, and to hybridise schooling with these resources in ways that serve the poor communities themselves, as such?

For regular schools, teachers and academics to engage in a critical 'funds of knowledge' enterprise, that is, one that is critical of schooling, may seem in some ways contradictory. I am often told that 'schools don't want critical pedagogy, they want improved exam results'. But that does not mean it is impossible. The orthodox rhetoric of schooling claims that education is 'for all', for the elevation of the poorest in society, contradicting the statistical 'facts' that poorer students get less education out of schools (going to less effective schools, taught by less qualified teachers and dropping out of education earlier than more upper class students). It is in such contradictions that projects like Moll's can sometimes find the space to adopt a critical perspective.

However, in Gonzales, Andrade, Civil, and Moll (2001), one finds a familiar experience in the accounts of the funds of knowledge researchers and teachers:

Marta Civil, the mathematics educator in our group, admitted that seeing Señora María develop this drawing and listening to her say things such as, 'Waist is 70 cm, let's add 6 because we need 3 for each pleat, and then we divide by 4' was as 'mesmerizing and mysterious to her as mathematics lectures are to so many people seeing bits and pieces but not getting the whole picture.' Yet, it did not seem as mysterious to the other women in the group because they were much more familiar with the practice. (p. 124)

Just so: a great deal of work may need to be done to enlighten an everyday practice with school mathematics. Such an approach can make demands well beyond those schools currently expect, and what they traditionally value. This is a huge challenge to the institution of

schooling and to its function in reproducing cultural capital. Can schools realistically justify the investment required to make such everyday practices give sense to academic mathematics? Perhaps, in fact, the value has to be seen as moving in the other direction: can academic/scientific mathematics add value to the community by making sense of their everyday struggles?

Rather than expecting the required practical knowledge in *existing* cultural practices in the community, I suggest rather that schooling should offer scientific concepts that might help the community's everyday practices with mathematical-scientific concepts. Such 'true concepts' may not confer much relative advantage of some over their peers in 'getting ahead', but may be of use to the students and their communities, indeed, these concepts may be made useful by the communities themselves, in their own activity outside schools.

6 Conclusions: is there a problem with alienation in activity theory?

Now to the final question: in the above critiques, I made some progress in criticizing from within the Vygotskian and Marxist cultural psychology perspective itself. I argued that such an immanent critique was sufficient to develop a Vygotskian theory that engages learners in activity that makes sense, but in which new mathematically-enhanced activity is emergent. A key problem was identified in the encapsulation of schooling and the separation of learning from useful work.

But the consideration of a neo-Vygotskian project that breaks the encapsulation of schooling, when considered from the critical standpoint of Bourdieu, was also found wanting. In this case, one sees some limitations of Vygotskian notions, and the need for a more developed class analysis of culture, schooling and capital, one that extends beyond activity theoretic traditions.

Has activity theory ignored the alienation of learning and learners in schooling activity? Previously in Williams (2012), I have argued that alienation was played down in the Vygotskian activity theoretic literature and argued that a synthesis with Bourdieusian perspectives on class reproduction is needed to understand schooling. In this paper, I have gone beyond this previous argument by applying this approach to two cases of praxis in mathematics education and that I now seek to generalize. The key question is: what can be done within the ChAT tradition and what requires an additional sociological class perspective?

At first, it may seem a small step to add to the Vygotskian perspective a Bourdieusian notion that intellectual labour has been historically appropriated by schooling institutions in arbitrary ways that serve to maintain the dominance of the dominant classes, including their privileged access to education. I argue then that Vygotskian or ChAT perspectives should be consistent with Bourdieu's view of schooling, educational cultural capital and symbolic violence. These concepts are then helpful in explaining why it might be difficult—if not impossible—for schools to stimulate critical funds of knowledge in their communities. One can predict all kinds of political reactions to any such activity.

However, anticipating political reaction to critical mathematics education does not necessarily obviate the possibility of such action: the contradictions in the education system allow one to argue that schooling should sometimes have 'use value', for the poor and dominated communities some schools serve. All kinds of alternative curricula and pedagogy have found the space to operate in poor communities, especially in schools where it is recognized that the examined and certificated curriculum has little to offer the majority of its students. In practice, this requires that mathematics curricula and pedagogy are required to address problems of

significance to the students and their community, whether that be related to employment, health, work, the law etc., or simply critically demystifying the social and political dominance of those in power.

In conclusion, I claim to have shown how Activity Theorists and practitioners sometimes interpret their neo-Vygotskian work as ‘creating a zone of proximal development’ where a credible alternative view is that there is actually alienation of learners in progress. I have accounted for this in some erroneous or partial interpretations of Vygotsky’s theory of learning and development, and to an unhelpful notion of the learning paradox. But I also pointed to Vygotsky’s and Vygotskians’ relative lack of attention to the alienating role of schooling in a classed society. I argue then that this lack of attention to class and oppression needs explicitation, such as that offered to sociocultural theory by Marx, Bernstein or Bourdieu (in the work of Lave and others, including my own), but also by Freire, for example, (the list of others who make class explicit being long). Without this, apparently faithful, Vygotskian work will be prone to the criticism of conservative praxis.

I showed that to a degree such criticisms of Vygotsky can be challenged in that his theory does attend to relevant notions in the social division of labour, especially between schooling and work. However, I also note that Bourdieu’s sociological concepts can reveal the class effects in schooling much more sharply. Finally, I argued that Vygotskian and Bourdieusian perspectives could be applied together consistently in critiquing practice and that each might shed light on mathematics education in a complementary way, such as in the two cases tackled here. In short, this is the paper’s contribution to the ongoing project of synthesizing the cultural psychology of Activity Theory and Bourdieusian sociology in pursuit of a critical theory of mathematics education.

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