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ERRATUM

Erratum to: A regularized Newton method without line search for unconstrained optimization

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The original version of this article unfortunately contained an error in Lemma 1. However, the main theorems of the paper are still true. The corrections of Lemma 1 and the corresponding statements are given below:

The proof of Lemma 1 in [1] is not correct since it uses a wrong formula of Taylor's theorem. Thus, we first modify it. Let $h : R \to R$ be defined by

$$h(t) = f(x^k + td^k(\nu)),$$

where f is the objective function and $d^k(\nu)$ is a search direction. Then Taylor's theorem on h is written as

$$h(1) = h(0) + h'(0) + \int_0^1 h''(\tau)(1-\tau)\mathrm{d}\tau.$$

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Using the formula, the equations on page 326, lines 4–5 from below become

$$\begin{aligned}
&f_k - f(x^k + d^k(\nu)) \\
&\quad = -{g^k}^T d^k(\nu) - \int_0^1 d^k(\nu)^T \nabla^2 f(x^k + \tau d^k(\nu)) d^k(\nu)(1-\tau)\mathrm{d}\tau \\
&\quad = d^k(\nu)^T (H_k + E_k(\nu)) d^k(\nu) - \int_0^1 d^k(\nu)^T \nabla^2 f(x^k + \tau d^k(\nu)) d^k(\nu)(1-\tau)\mathrm{d}\tau.
\end{aligned}$$

Moreover, the last equation in the proof (page 327, line 2) becomes

$$\begin{aligned}
&f_k - f(x^k + d^k(\nu)) - \eta_1(f_k - \phi_k(d^k(\nu), \nu)) \\
&\quad = \frac{2-\eta_1}{2} d^k(\nu)^T (H_k + E_k(\nu)) d^k(\nu) \\
&\qquad - \int_0^1 d^k(\nu)^T \nabla^2 f(x^k + \tau d^k(\nu)) d^k(\nu)(1-\tau)\mathrm{d}\tau.
\end{aligned}$$

Then Lemma 1 on page 326 is modified as follows.

Lemma 1 *It holds that*

$$\begin{aligned}
&f_k - f(x^k + d^k(\nu)) - \eta_1(f_k - \phi_k(d^k(\nu), \nu)) \\
&\quad = \frac{2-\eta_1}{2} d^k(\nu)^T (H_k + E_k(\nu)) d^k(\nu) \\
&\qquad - \int_0^1 d^k(\nu)^T \nabla^2 f(x^k + \tau d^k(\nu)) d^k(\nu)(1-\tau)\mathrm{d}\tau.
\end{aligned}$$

Next we modify proofs of Lemmas 2, 3, 5, 10 and 19 that directly follow from Lemma 1. These modifications are summarized as follows.

Statement and proof of Lemma 2

The fist inequality in the proof of Lemma 2 (page 327, lines 9–11) is based on Lemma 1. Thus it should be modified as

$$\begin{aligned}
&f_k - f(x^k + d^k(\nu)) - \eta_1(f_k - \phi_k(d^k(\nu), \nu)) \\
&\quad \geq \frac{1-\eta_1}{2} d^k(\nu)^T (H_k + E_k(\nu)) d^k(\nu) \\
&\qquad - \int_0^1 d^k(\nu)^T (\nabla^2 f(x^k + \tau d^k(\nu)) - H_k) d^k(\nu)(1-\tau)\mathrm{d}\tau.
\end{aligned}$$

Modifying the corresponding terms in the subsequent inequalities, we have the following correct statement of Lemma 2.

Lemma 2 *Suppose that Assumption 1 holds. Then,*

$$\begin{aligned} & f_k - f(x^k + d^k(\nu)) - \eta_1(f_k - \phi_k(d^k(\nu), \nu)) \\ & \quad \geq \frac{1}{2}\left((1-\eta_1)c_2\nu\|g^k\|^\delta - 2\int_0^1 \|\nabla^2 f(x^k + \tau d^k(\nu)) - H_k\|(1-\tau)\mathrm{d}\tau\right)\|d^k(\nu)\|^2. \end{aligned}$$

Proof of Lemma 3

The proof of Lemma 3 is based on Lemma 2, which is modified as above. Thus the equation on page 328, line 3 is now

$$\lim_{\nu\to\infty}\int_0^1 \|\nabla^2 f(x^k + \tau d^k(\nu)) - H_k\|(1-\tau)\mathrm{d}\tau = 0.$$

Moreover, the next inequality on page 328, line 5 should be

$$2\int_0^1 \|\nabla^2 f(x^k + \tau d^k(\nu)) - H_k\|(1-\tau)\mathrm{d}\tau \leq (1-\eta_1)c_2\nu\|g^k\|^\delta.$$

We do not change the statement of Lemma 3.

Proof of Lemma 5

The first inequality in the proof of Lemma 5 (page 329, line 1 from below) is based on Lemma 1. Thus it should be

$$\begin{aligned} & f_k - f(x^k + d^k(\nu)) - \eta_1(f_k - \phi_k(d^k(\nu), \nu)) \\ & \quad \geq \frac{1}{2}\left((2-\eta_1)\lambda_{\min}(H_k + E_k(\nu)) - 2\int_0^1 \|\nabla^2 f(x^k + \tau d^k(\nu))\|(1-\tau)\mathrm{d}\tau\right)\|d^k(\nu)\|^2. \end{aligned}$$

We do not have to change the subsequent inequalities and the statement of Lemma 5 because we have $-\int_0^1 L_g \mathrm{d}\tau = -2\int_0^1 L_g(1-\tau)\mathrm{d}\tau$.

Proofs of Lemmas 10 and 19

The first inequality and equality in the proof of Lemma 10 (page 333) is based on Lemma 2, and hence they should be

$$
\begin{aligned}
&f_k - f(x^k + d^k(\nu)) - \eta_1(f_k - \phi_k(d^k(\nu), \nu)) \\
&\quad \geq \frac{1}{2}\left((1-\eta_1)c_2\nu\|g^k\|^\delta - 2L_H\|d^k(\nu)\|\int_0^1 (1-\tau)\tau \mathrm{d}\tau\right)\|d^k(\nu)\|^2 \\
&\quad = \frac{1}{2}\left((1-\eta_1)c_2\nu\|g^k\|^\delta - \frac{L_H}{3}\|d^k(\nu)\|\right)\|d^k(\nu)\|^2. \\
&\quad \geq \frac{1}{2}\left((1-\eta_1)c_2\nu\|g^k\|^\delta - \frac{L_H}{2}\|d^k(\nu)\|\right)\|d^k(\nu)\|^2.
\end{aligned}
$$

Note that we may get a slightly better result if we do not use the last inequality. The last inequality is derived just because we do not want to change the statement of Lemma 10. The same modifications are necessary for the proof of Lemma 19 on page 345.

Note that since the statements of Lemmas 3, 5, 10 and 19 are not changed, the above modifications do not affect the main results.

Reference

1. Ueda, K., Yamashita, N.: A regularized Newton method without line search for unconstrained optimization. Comput. Optim. Appl. **59**, 321–351 (2014)