

NOTES AND COMMENTS

## A Cautionary Note on the Use of Monin–Obukhov Similarity Theory in Very High-Resolution Large-Eddy Simulations

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**Abstract** In several recent large-eddy simulation studies, the lowest grid level was located well within the roughness sublayer. Monin–Obukhov similarity-based boundary conditions cannot be used under this scenario, and in this note we elaborate on this fundamental problem and suggest potential solutions.

Keywords Inertial sublayer  $\cdot$  Large-eddy simulation  $\cdot$  Monin–Obukhov similarity theory  $\cdot$  Roughness sublayer  $\cdot$  Surface layer

In the era of petascale computing, very high-resolution (the grid size,  $\Delta = O(1)$  m or finer) large-eddy simulation (LES) of atmospheric boundary-layer (ABL) flows is gradually becoming a norm. For example, in a recent study (Sullivan et al. 2016),  $\Delta = 0.39$  m was utilized in the idealized simulation of the stable boundary layer. It is a well-known fact that all the contemporary LES codes utilize the conventional Monin–Obukhov similarity theory (MOST) as lower boundary conditions (e.g., Heus et al. 2010; Maronga et al. 2015). It is also common knowledge (e.g., Lumley and Panofsky 1964; Monin and Yaglom 1971; Wyngaard 2010) that MOST is only valid for heights  $z \gg z_0$ , where  $z_0$  is the aerodynamic roughness length. MOST is not applicable for  $z < \alpha h$ , where h denotes the height of the roughness elements. Typically,  $\alpha$  is assumed to be between 2 and 5 based on laboratory studies (see Raupach et al. 1991, and references therein). Unfortunately, a number of recent LES studies (e.g., Beare et al. 2006; Basu et al. 2011; Maronga 2014; Sullivan et al. 2016; Udina et al.

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Fig. 1 The surface layer and its sublayers (taken from Raupach and Legg 1984)

2016) prescribed the lowest grid levels  $(z_1)$  within the range  $0 < z_1 < \alpha h$  and inappropriately invoked MOST-based boundary conditions. The purpose here is to highlight this undesirable oversight.

The schematic in Fig. 1 depicts the canonical structure of the atmospheric surface layer, with the roughness elements located on the substrate surface z = 0.<sup>1</sup> The so-called displacement height is denoted by *d* (Jackson 1981; Raupach and Legg 1984). MOST (including the logarithmic law of the wall) is valid within the inertial sublayer (ISL), where the lower limit of the ISL is  $z_*$  (=  $\alpha h$ ) and the upper limit is usually taken as the 10% of the boundary-layer height. In the roughness sublayer (RSL; also known as the transition layer), surface-wake generation and interactions dominate and the flow is quite heterogeneous in comparison with that in the ISL. Furthermore, due to enhanced mixing, wind shear in the RSL is significantly less than that in the ISL. In his classic book (pp. 142–143), Townsend (1976) wrote:

For fully rough flow at large values of Reynolds number,  $z_{\circ}$  is commonly about onetenth of the average height of the roughness elements and the logarithmic distribution can be valid only at heights considerably larger than that, say for  $z/z_{\circ}$  greater than fifty.

In other words, according to Townsend (1976):  $h/z_o \approx 10$  and  $z_*/z_o \approx 50$ . Other estimates and formulations of  $h/z_o$  from experimental studies have been summarized in, e.g., Raupach et al. (1991) and Jiménez (2004). One of the first studies documenting values of  $z_*/z_o$  in the ABL was due to Tennekes (1973), who put an estimate of 100 along with a candid note "of course, that is relatively arbitrary, because firm estimates of the accuracy cannot yet be made." In the following years, significant contributions were made by Garratt (1978, 1980, 1983, 1992) who, based on observations from the Koorin experiment, reported  $z_*/z_o$  to vary from 35 to 150 for the wind profile under unstable conditions. For temperature, he found  $z_*/z_o \approx 100$  (Garratt 1980). For stably stratified conditions,  $z_*/z_o$  for the wind profile was found to be significantly lower, approximately in the range of 11–55 (Garratt 1983). Based on past field and laboratory studies (e.g., Garratt 1980, 1983; Raupach et al. 1991), it is quite evident that, in addition to atmospheric stability, the geometric nature of roughness elements also contributes significantly to the variability in this ratio. Recently, Huang et al.

<sup>&</sup>lt;sup>1</sup> Some studies (e.g., Garratt 1980, 1983) use a different convention and assume the origin of z at the zero-plane displacement.

(2016) analyzed an extensive database comprising several direct numerical simulations and wind-tunnel experiments. They reported the parameter  $\alpha (=z_*/h)$  to be within the range of 1.2–2.2 for neutrally stratified flows and, in agreement with earlier studies, also found  $\alpha$  to be strongly dependent on the geometric details (e.g., roughness element shape and interelement spacing).

Based on the above discussion, we strongly advocate that for future LES studies, the lowest grid level ( $z_1$ ) should not be prescribed at heights  $z < z_*$ . In lieu of any universal criterion, we recommend that the modelling community follows a tentative guideline of  $z_1 > 50z_0$ . Thus, for a value of  $z_0 = 0.1$  m (e.g., in the GABLS-1 LES intercomparison study<sup>2</sup>), one should use  $z_1 > 5$  m.

Please note that similar guidelines are already followed in the dispersion modelling and wind engineering communities. In the context of regulatory modelling applications, the US Environmental Protection Agency (Bailey 2000) recommended that wind speed and direction data should always be collected from  $z \ge \max(20z_0, 1 \text{ m})$ . Blocken et al. (2007) noted the following condition as one of the four criteria for computational fluid dynamics simulations:

A distance  $y_P$  from the centre point P of the wall-adjacent cell to the wall (ground surface) that is larger than the physical (or geometrical) roughness height  $K_S$  ( $y_P > K_S$ ).

For fully rough flows, the sand-grain roughness ( $K_S$ ) is related to the aerodynamic roughness length (Blocken et al. 2007):  $z_o \approx K_S/30$ . In other words,  $y_P$  (i.e.,  $z_1$  in our notation) should be >30 $z_o$ .

We point out that our proposed guideline may be insufficient for transitional rough surfaces. Recently, Marusic et al. (2013) analyzed surface-layer data from both laboratory experiments and field measurements from the SLTEST site over Utah's western desert. They proposed a conservative limit of  $z_*$  as a function of the friction Reynolds number ( $Re_\tau$ ),

$$\frac{z_*u_*}{v} = 3\sqrt{Re_\tau}.$$
 (1)

For the SLTEST data, Marusic et al. (2013) reported:  $u_* = 0.188 \text{ m s}^{-1}$ ,  $v = 1.8 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ , and  $Re_{\tau} = 6.3 \times 10^5$ . Using Eq. 1, we obtain  $z_* = 0.23 \text{ m}$ , while the sand-grain roughness ( $K_S$ ) for this case is 2 mm. Since this dataset represents a mildly transitional rough surface (Marusic et al. 2013), we utilize Eqs. 4.4 and 4.5 of Kunkel and Marusic (2006) to estimate  $z_o = 3.5 \times 10^{-5} \text{ m}$ . Clearly, the ratio  $z_*/z_o$  is several order larger than the corresponding fully rough values. If one were to perform a large-eddy simulation for this specific SLTEST case, the lowest grid point would need to be >0.23 m.

We speculate that for most idealized LES problems the guideline of  $z_1 > 50z_0$  suffices. As a viable alternative, one could also use modified MOST relationships (with an RSL correction) as boundary conditions in conjunction with  $z_1 < z_*$ . For example, Physick and Garratt (1995) proposed such a formulation for mesoscale models that could be utilized in LES codes (with minor modifications). Other empirical formulations also exist in the forest and urban canopy turbulence literature (e.g., de Ridder 2010; Arnqvist and Bergström 2015). For neutrally stratified flows, the generalized law-of-the-wall proposed by Huang et al. (2016) is another potential candidate. Incorporation of such a formulation in contemporary LES codes should not be a challenging task.

For idealized simulations over homogeneous surfaces, several LES codes utilize the MOST-based boundary conditions locally (e.g., Stoll and Porté-Agel 2008) and others use a

<sup>&</sup>lt;sup>2</sup> The acronym GABLS stands for GEWEX (The Global Energy and Water Cycle Exchanges Project) Atmospheric Boundary Layer Study.

planar-averaged option (e.g., Basu and Porté-Agel 2006). Even though the concept of the RSL is formally applicable for ensemble averages, we recommend employing the RSL corrections for both types of boundary conditions.

For LES of the stable boundary layer, several studies have shown that with increasing resolution the surface fluxes decrease monotonically, and in turn, the boundary-layer height decreases. As an illustrative example, refer to Fig. 1 and Table 1 in Sullivan et al. (2016). We hypothesize that, due to the lack of RSL corrections in the very high-resolution simulations of Sullivan et al. (2016), mixing near the surface has been reduced spuriously, and (partially) contributed to an artificial resolution sensitivity.

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